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The rotation of eigenspaces of perturbed matrix pairs II

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Abstract. This paper studies the perturbation theory for spectral projections of Hermitian matrix pairs $(H, M)$, where $H$ is non-singular Hermitian matrix which can be factorized as $H = G J G^*$, $J = \text{diag}(\pm 1)$, and $M$ is positive definite. The class of allowed perturbations is so restricted that the corresponding perturbed pair $(\wtd H, \wtd M) = (H + \delta H, M + \delta M)$ must have the form $\wtd H = \wtd G J \wtd G^*$, $J = \text{diag}(\pm 1)$ and $\wtd M$ is positive definite. The main contribution of the paper is a $\sin \Theta$ theorem which generalizes the main result from the first part of the paper to this more general setting. Our estimate, in its most general form, depends on a uniform norm bound on a set of all $J$-unitary matrices which diagonalize $G^*G$. The second main contribution is a new sharp uniform estimate of a norm of a all $J$-unitary matrix which diagonalize $G^*G$ such that $H = G^*JG$ is a quasi-definite matrix. The case of a quasi-definite pair is therefore the case where our bounds are most competitive. We present numerical experiments to corroborate the theory.