The relation of Connected Set Cover and Group Steiner Tree

Khaled Elbassioni*, Slobodan Jelić†, Domagoj Matijević‡

Abstract

We report that the Connected Set Cover (CSC) problem is just a special case of the Group Steiner Tree (GST) problem. Based on that we obtain the first algorithm for CSC with polylogarithmic approximation guarantee as well as the first approximation algorithms for the weighted version of the problem and the version with requirements. Moreover, we argue that the inapproximability result of GST will carry on to the weighted version of the CSC problem.

Keywords: set cover, connected set cover, weighted connected set cover, group Steiner Tree, node weighted group Steiner Tree, covering Steiner Tree problem

1. Introduction

Let U be the universe of elements, $\mathcal{S}$ family of subsets of U such that $\bigcup_{S\in\mathcal{S}} S = U$ and $G = (\mathcal{S}, E)$ connected graph on vertex set $\mathcal{S}$. We say that subfamily $\mathcal{R} \subseteq \mathcal{S}$ is *set cover* with respect to the instance $(U, \mathcal{S})$ if every $u \in U$ is covered by at least one set from $\mathcal{R}$. The *set cover problem* introduced by Chvátal [5] is to find the subfamily $\mathcal{R}$ of minimal size. The more general version of the problem is typically called the *weighted set cover problem* where each set from family $\mathcal{S}$ has a nonnegative weight associated with it. The task then is to find the minimum weight subfamily of sets which covers entire universe U.

A *connected set cover* with respect to the instance $(U, \mathcal{S}, G)$ is a set cover $\mathcal{R}$ with respect to $(U, \mathcal{S})$ such that the subgraph $G[\mathcal{R}]$ induced by $\mathcal{R}$ is connected. The *connected set cover problem* (CSC) on $(U, \mathcal{S}, G)$ is a problem of finding a connected set cover with respect to $(U, \mathcal{S}, G)$ with minimum number of sets (vertices). Analogously to the weighted version of the set cover problem, we define the weighted connected set cover problem (WCSC) with the task of computing the connected set cover with minumum weight subfamily of sets (vertices).

In this paper we will study the relation of CSC and WCSC and well-studied Group Steiner Tree (GST) problem introduced by Reich and Widmayer [14] motivated by the problem of wire routing with multiport terminals in physical VLSI design. Let $G = (V, E)$ denote a graph with edge weight function $w : E \to \mathbb{R}^+$ and family of subsets of vertices $\mathcal{G} = \{g_1, g_2, \ldots, g_k\}, \quad g_i \subset V$ which will be called groups. The task is to find a subtree T that minimizes the cost function $\sum_{e\in E} w(e)$ such that $V(T) \cap g_i \neq \emptyset$ for all $i \in \{1, \ldots, k\}$. This problem is called the Group Steiner Tree problem with respect to the instance $(G, \mathcal{G}, w)$. We fix the following notations: $k = |\mathcal{G}|$, $N = \max_{1\leq i\leq k} |g_i|$ and $n = |V|$.

It's well known that GST is at least as hard as the set cover problem. Namely, it can be shown that the set cover can be reduced to GST by building a star graph with leaves that corresponds to the sets from $\mathcal{S}$ and each group corresponds to exactly one element from U. All sets (leaves) covering one (fixed) element belong to the same group. Weights of edges can be defined as weights of set that are connecting particular sets to the root of the star graph (Figure 1.1).

*Max-Planck-Institut für Informatik, Department 1: Algorithms and Complexity, Campus E1 4, Room 307, 66123 Saarbrücken, Germany; e-mail: elbassio@mpi-inf.mpg.de, phone: +49 681 9325 107

†Josip Juraj Strossmayer University of Osijek, Department of Mathematics, Trg Ljudevita Gaja 6, Room 19, 31000 Osijek, Croatia; email: sjelic@mathos.hr, phone: +385 31 224 822

‡Josip Juraj Strossmayer University of Osijek, Department of Mathematics, Trg Ljudevita Gaja 6, Room 41, 31000 Osijek, Croatia; email: domagoj@mathos.hr, phone: +385 31 224 825

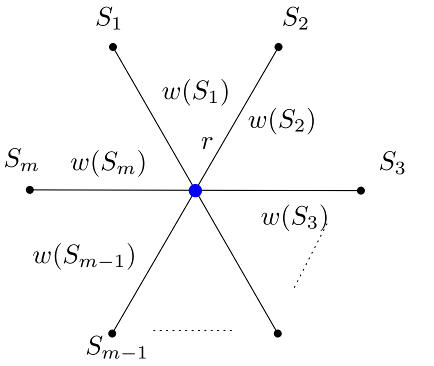

Figure 1.1: Reduction of set cover to GST problem

1.1. Previous work

CSC has been independently investigated from GST by several papers before. In fact, the main motivation for introducing the connectivity requirement in classical set cover came from biology, namely from the problem of reserve selection for conservation of species (see [3]). Cerdeira and Pinto [3] formally introduced the CSC problem and studied some valid inequalities for the convex hull of the set of incidence vectors of connected covers.

Shuai and Hu [15] gave two polynomial algorithms for CSC problem on graph where each vertex has degree less than or equal to 2 and proved that for any $0 < \rho < 1$ there is no approximation algorithms with approximation ratio $\rho \ln n$ for CSC problem on graphs where at most one vertex has degree greater than 2, unless $\mathsf{NP} \subset \mathsf{DTIME}(n^{\text{poly} \log n})$.

In the paper by Zhang, Gao and Wu [16] first two approximation algorithms for CSC are given. First algorithm is a combination of approximation algorithms for set cover and Steiner tree with minimum number of Steiner points with approximation ratios of α and β, respectively. In the first phase of algorithm the set cover is computed with the approximation ratio α. In the second phase, Steiner tree with minimum number of Steiner points is computed on the set cover from the first phase in order to resolve eventual disconnectedness. It was proved that the described algorithm has approximation ratio of $\alpha + \beta + \alpha\beta(D_c - 1)$ where D_c is the length of the longest path in graph G between two non-disjoint sets. Second algorithm Zhang et al. [16] describe uses the greedy strategy that generalizes the greedy algorithm of set cover with the approximation ratio of $1 + D_c H(\gamma - 1)$ where $H(\cdot)$ is the harmonic function and $\gamma = \max\{|S| : S \in \mathcal{S}\}$.

Note that theoretically both algorithms do not provide a very good bounds since D_c can grow as large as $O(n)$, as we demonstrate with the following example.

Example 1..1. *Suppose that $n \in \mathbb{N}$ is even and $n \geq 6$. Universe U is given by $U = \{1, 2, \ldots, n\}$ and $\mathcal{S} = \{S_1, \ldots, S_n\}$ where $S_1 = \{1, 2, \ldots, n/2 + 1\}$, $S_{n/2} = \{1, 2, \ldots, n/2\}$, $S_{n/2+1} = \{n/2 + 1, \ldots, n\}$, $S_n = \{n/2 - 1, n/2, \ldots, n\}$ and $S_i = \{i - 1, i\}$ for $1 < i < n/2$ and $n/2 + 1 < i < n$. Graph $G = (V(G), E(G))$ is given by $V(G) = \mathcal{S}$ and $E(G) = \{\{S_i, S_{i+1}\} : 1 \leq i \leq n - 1\}$ (see Figure 1.2). Optimal solution is $\mathcal{R}^* = \{S_{n/2}, S_{n/2+1}\}$. However, the approximation algorithms of Zhang et al. will be forced to pick either S_1 or S_n and the mechanisms they use to overcome possible disconnectedness will incure $O(n)$ additional sets (vertices).*

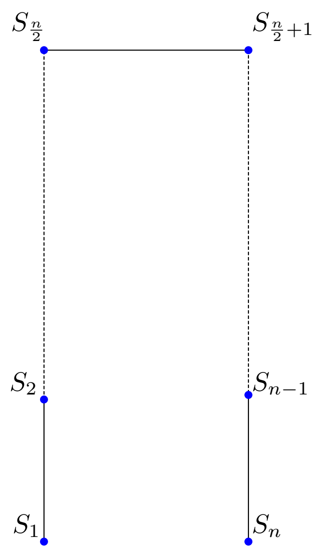

Figure 1.2: Bad example for the approximation algorithms proposed by Zhang et al.

On the other hand, GST is an older and more studied problem. Grag, Konjevod and Ravi [8] gave first polylogarithmic approximation algorithm which with high probability finds a group Steiner tree of cost within $O(\log N \log n \log\log n \log k)$ of the cost of the best group Steiner tree. Using a randomized approach, they solved the problem on trees with the approximation ratio of $O(\log k \log N)$ which they extend to general graphs by probabilistic approximation of metric spaces due to Bartal [2]. Some generalizations of GST has also been studied. Khandekar et al. [11] gave approximation algorithms for Fault-Tolerant Group-Steiner Problems. In their work they address the *node-weighted* GST problem with both node and edge weights on graph G and provide the $O(\sqrt{m}\log m)$ approximation algorithm for that problem where m is a number of vertices in graph G. Konjevod, Ravi and Srinivasan [12] considered the GST were each group in $\mathcal{G}$ has a nonnegative integer requirement associated with it. They provide the polylogarithmic approximation algorithm for the problem of determining a minimum-weight tree spanning at least the required number of vertices of every group.

Very recently Naor et al. [13] presented a very interesting quasi-polynomial-time randomized online algorithm for the node-weighted GST problem with a polylogarithmic competitive ratio.

1.2. Our results

In this paper we show that CSC is just a special case of GST. Namely, we show that CSC and GST where all edge weights are set to 1 are equivalent problems. Although the reduction works in both direction, we will mainly exploit the fact that we can transform CSC instance to equivalent GST instance, find the solution of GST by some of the known algorithms, and transform the solution back to CSC. Doing so will immediately imply the better approximation algorithms for the CSC, WCSC and the generalization of CSC with requirements.

More precisely, results of Garg, Konjevod and Ravi [8] imply polylogarithmic approximation algorithm for connected set cover with approximation ratio $O(\log^2 m \log\log m \log n)$ where is $n = |U|$ and $m = |\mathcal{S}|$. The algorithms of Khandekar et al. [11] will be used to approximate the weighted CSC no more than $O(\sqrt{m}\log m)$ times optimal. Note that this is the first algorithm with approximation guarantee for the WCSC problem[1]. Note that the recent results of Naor et al. [13] provide the first algorithm with polylogarithmic approximation guarantee. Their algorithm is online (i.e. does not need any knowledge of family $\mathcal{G} = \{g_1, g_2, \ldots, g_k\}, \quad g_i \subset V$ in advance). However, it runs only in quasi-polynomial running time.

Finally, the results of Konjevod, Ravi and Srinivasan [12] will be used to solve CSC with requirements $r_e \in \mathbb{N}$, where each element $e \in U$ has to be covered at by at least r_e sets.

2. CSC is just a special case of GST

Here we prove our main result, i.e. CSC problem is equivalent to GST problem with all edge weights set to 1. We do that by showing that WCSC is equivalent to the GST problem with all edge weights set to 1 and nonnegative weights associated to graph nodes. Precise definitions follows.

Definition 2.1 (Weighted Connected Set Cover (WCSC)). *Given a set U of elements, $\mathcal{S}$ a family of subsets of U, graph G such that $V(G) = \mathcal{S}$ and $w_N : \mathcal{S} \to \mathbb{R}^+$, find a subfamily $\mathcal{R}$ of $\mathcal{S}$ such that every element of U is covered by at least one set of $\mathcal{R}$, subgraph $G[\mathcal{R}]$ of G induced by $\mathcal{R}$ is connected and $\sum_{S\in\mathcal{R}} w_N(S)$ is minimized.*

Definition 2.2 (Node weighted Group Steiner Tree). *Suppose that we are given a graph G with node-weight function $w_N : V(G) \to \mathbb{R}^+$ and family of subsets of vertices $\mathcal{G} = \{g_1, g_2, \ldots, g_k\}, \quad g_i \subset V$ which will be called groups. We have to find subtree T that minimizes cost function $\sum_{v\in V(T)} w_N(v)$ such that $V(T) \cap g_i \neq \emptyset$ for all $i \in \{1, \ldots, k\}$.*

In the following we prove that above two problems are equivalent.

Theorem 2.1. *Suppose that $(U, \mathcal{S}, G, w_N)$ and $(G, \mathcal{G}, w_N)$ are instances of WCSC and node-weighted GST, respectively. We can reduce WCSC to node-weighted GST and conversely, i.e these problems are equivalent.*

Proof. We are given an instance $(U, \mathcal{S}, G, w_N)$ of WCSC. Let define $\mathcal{G} = \{g_u\}_{u\in U}$ such that

[1] Zhang et al. [16] left the weighted variant of CSC as an open problem.

$$g_u = \{S \in \mathcal{S} : u \in S\}. \quad (2.1)$$

The instance $(G, \mathcal{G}, w_N)$ of node-weighted GST is obtained. If T is a solution subtree, we can check that $\mathcal{R} = \{S_v : v \in V(T)\}$ is solution of our WCSC problem. Indeed, $G[\mathcal{R}]$ is connected minimum weighted subgraph of G that covers all elements of universe U. Since T is tree, it follows that $G[\mathcal{R}]$ is connected. First, we check covering constraint. Suppose that there is at least one element u' such that it is not covered by no one element of $\mathcal{R}$. It means that for every $S \in \mathcal{R}$ we have that $u' \notin S$. By (2.1) it follows that there is $g_{u'}$ such that there are no vertices in T that are in group $g_{u'}$. It is contradiction to feasibility of T in node-weighted GST. Second, suppose that there is a better solution $\mathcal{R}'$ such that $w_N(\mathcal{R}') < w_N(\mathcal{R})$. Now, we can construct tree T' by cycle deletion such that $w_N(T') < w_N(T)$ and $V(T') \cap g \neq \emptyset$ which contradicts optimality of T.

Now, we will prove that node-weighted GST is reducible to WCSC. The instance $(G, \mathcal{G}, w_N)$ of node-weighted GST is given. Let $U = \mathcal{G}$ and $\mathcal{S} = \{S_v\}_{v \in V(G)}$ such that

$$S_v = \{g \in U : v \in g\}, \quad v \in V(G). \quad (2.2)$$

The instance $(U, \mathcal{S}, G, w_N)$ is obtained. If $\mathcal{R}$ is optimal node weighted connected cover, we can check that tree $T = (\mathcal{R}, E(T))$ is optimal solution to the node weighted GST problem, where $E(T)$ is obtained from $E(G[\mathcal{R}])$ by cycle deletion. In other words, we claim that T is minimum weight subtree that includes at least one vertex from each group. If there is one uncovered group there is one uncovered element in solution of WCSC. If we suppose that there is another tree T' such that $w_N(T') < w_N(T)$ and $V(T') \cap g \neq \emptyset$ for all $g \in \mathcal{G}$, that it follows that $\mathcal{R}' = \{S_v\}_{v \in V(T')}$ is connected cover such that $w_N(\mathcal{R}') < w_N(\mathcal{R})$. It contradicts optimality of $\mathcal{R}$. □

Clearly, CSC is a special case of WCSC where $w_N(v) = 1$ for all $v \in V(G)$. Similarly (as in Theorem 2.1), it can be proved that CSC problem is equivalent to the GST problem where all edges have equal weights (i.e. $w(e) = 1$ for all $e \in E(G)$).

Theorem 2.2. *CSC and GST with all edge weights equal to* 1*, are equivalent. In other words, any CSC instance can be reduced to GST instance and vice versa.*

2.1. Algorithms for (W)CSC

Garg et al. in [8] proved the following theorem.

Claim 2.1. *For any $\epsilon > 0$, there is a polynomial-time algorithm that with probability $1-\epsilon$ finds group Steiner tree whose cost is*

1. *$O(\log N \log k)$ times the cost of the optimal tree, if the input graph is a tree;*
2. *$O(\log N \log n \log \log n \log k)$ times the cost of the optimal tree on general graphs.*

By Theorem 2.2, the same algorithm can be used to solve the CSC problem by reducing the CSC to equivalent GST problem first. Hence, the polylogarithmic approximation algorithm for CSC is obtained.

Khandekar et al. in [11] studied fault-tolerant versions of group Steiner tree problem. Given (directed or undirected) a graph G with edge or node weights, a root vertex $r \in V(G)$ and a collection of groups $\mathcal{G} = \{g_1, \dots, g_k\}$ that are subsets of $V(G) \setminus \{r\}$, the task is to find a minimum weight subgraph H of G that contains two edge or vertex-disjoint paths from each group $g \in \mathcal{G}$ to the root r.

They proved the following theorem.

Claim 2.2. *There is a polynomial-time algorithm that approximates within $O(\sqrt{n} \log n)$ the fault-tolerant version of group Steiner tree problem.*

We can use algorithms provided in [11] to solve the WCSC problem, since that is, by our knowledge, the only algorithm with approximation guarantee known that can be used to approximate the node-weighted GST problem. The algorithm is more precisely stated below.

Since reduction in the proof of Proposition 2.1 defines group g_u for each element $u \in U$ in (2.1), it follows that number groups in node-weighted GST will be equal to number of elements in U, more precisely $k = |U| = n$. Maximal group size in node-weighted GST can be viewed as the maximal number of sets in $\mathcal{S}$ which cover some fixed element $u \in U$ in WCSC instance. Hence, $N \leq m$ where $m = |\mathcal{S}|$.

Algorithm 2.1. returns WCSC whose weight is less $O(\sqrt{m} \log m)$ times the optimal.

Algorithm 2.1.: Algorithm for WCSC

Input: universe U, family of sets S, graph G, node weight function w_N

Output: connected cover $\mathcal{R}$

transform WCSC instance to equivalent node-weighted GST instance $(G, \mathcal{G}, w_N)$ as in the proof of Proposition 2.1;

for $r \in V(G)$ **do**

  Find subgraph H using algorithm of Khandekar et al. which contains two vertex-disjoint paths from root r to each group $g \in \mathcal{G}$;

take the subgraph H which has a minimal weight;

return family $\{S_v \in \mathcal{S} : v \in V(H)\}$;

2.2. Algorithms for (W)CSC on trees

Note that the first part of Claim 2.1 together with Theorem 2.2 imply the $O(\log m \log n)$ algorithm for CSC if the input graph is a tree.

When the input graph is a node weighted tree, the node-weighted GST can as well be substantially better approximated. Namely, it can be shown that any node weighted input tree can be transformed into an equivalent edge weighted tree. These trees have the same solution subtree as we shown in the following theorem.

Theorem 2.3. *Suppose that T is node weighted tree with node weight function $w_N : V(T) \to \mathbb{R}^+$ and root r. The instance of node-weighted GST problem is denoted by $(T, \mathcal{G}, w_N, r)$. There is a tree T' with edge weight function $w_E : E(T') \to \mathbb{R}^+$ such that the optimal solution of an instance $(T, \mathcal{G}, w_N, r)$ can be reconstructed from optimal solution of an instance $(T', \mathcal{G}, w_E, r)$ in polynomial time.*

Proof. We can take $V(T') = V(T) \cup \{r'\}$ where r' is the copy of root r, $w_N(r') = w_N(r)$ and $E(T') = E(T) \cup \{\{r, r'\}\}$. We define function $w_E : E(T') \to \mathbb{R}^+$ such that

$$w_E(\mathrm{pe}(v)) = w(v), \quad v \in V(T') \setminus \{r\},$$

where $\mathrm{pe}(v)$ denotes parental edge of $v \in V(T') \setminus \{r\}$. T^* is optimal solution subtree of $(T', \mathcal{G}, w_E, r)$. If we take vertices of T^* and induce subtree of T on these vertices (if $r' \in V(T^*)$ we will take root r), we will obtain subtree whose node weight is equal to the edge weight of T^*. Obviously, it is optimal solution of $(T, \mathcal{G}, w_N, r)$ since each another subtree T'' such that $w_N(T'') < w_N(T^*)$ contradicts optimality of T^* in $(T, \mathcal{G}, w_E, r)$ instance. □

By the first part of Claim 2.1 and Theorem 2.2 it follows that WCSC can be solved within the approximation ratio of $O(\log m \log n)$ when the input graph is a tree.

2.3. Connected set cover problem with requirements

Zhang et al. in [16] introduced the fault-tolerant connected set cover problem with the uniform requirement m. Solution $\mathcal{R}$ is (k, m)-CSC if it induces k-connected subgraph of G and each element $u \in U$ is covered by at least m subset. They gave approximation algorithm for $(2, m)$-CSC that has performance $(PD(G) - 1)(1 + H(\gamma - 1))$ where $PD(G)$ is the maximum length of a path in graph G with internal vertices of degree two. Note that this guarantee can again be as bad as $O(n)$. In this section we generalize the $(1, m)$-CSC problem where each element $u \in U$ has arbitrary nonnegative integer requirement r_u and show that such problem can be solved in polylogarithmic approximation ratio by using result of Konjevod et al. [12] for Covering Steiner Tree problem (CST).

Definition 2.3 (Covering Steiner Tree problem - (CST))**.** *Suppose that we are given a graph G with edge-weight function $w_E : V(G) \to \mathbb{R}^+$ and family of subsets of vertices $\mathcal{G} = \{g_1, g_2, \ldots, g_k\}$, $g_i \subset V$ which will be called groups. We have to find the subtree T that minimizes a cost function $\sum_{e \in E(T)} w_E(e)$ such that $V(T) \cap g_i \neq \emptyset$ for all $i \in \{1, \ldots, k\}$ and $V(T)$ contains at least k_g vertices from each group $g \in \mathcal{G}$. Number k_g is called the requirement of group $g \in \mathcal{G}$ and $\mathcal{K} = \{k_g \in \mathbb{N} : g \in \mathcal{G}, k_g \leq |g|\}$ is set of all requirements.*

Konjevod et al. in [12] solved CST when the input graph is a tree using the technique of LP relaxation and randomized rounding described in [8] and obtained a randomized polynomial-time approximation algorithm for the covering Steiner problem on trees, which with constant probability produces a solution of value at most $O(\log N \log(K \cdot k))$

times optimal, where K is the largest requirement. Using Bartal approximation of metric spaces they extended their algorithm to general graphs with the cost of introducing an additional stretch of $O(\log n \log\log n)$ where is $n = |V(G)|$. Hence, they compute with high probability CST of cost not more then $O(\log n \log\log n \log N \log(K \cdot k))$ times the cost of the optimal tree.

In the following, we provide the formal definition of CSC problem with requirements.

Definition 2.4 (Connected set cover problem with requirements - (CSC-R))**.** *Given a set U of elements, $\mathcal{S}$ a family of subsets of U and graph G such that $V(G) = \mathcal{S}$, find a minimum size subfamily $\mathcal{R}$ of $\mathcal{S}$ such that every element of U is covered by at least r_u sets in $\mathcal{R}$ and subgraph $G[\mathcal{R}]$ of G induced by $\mathcal{R}$ is connected. Number $r_u \in \mathbb{N}$ is called requirement for element $u \in U$. Set of all requirements is denoted by $\mathcal{P}$.*

Reduction from CSC-R problem to CST problem, where all edges have equal weights, can be performed in same way as in Proposition 2.1. Requirement r_u of elements $u \in U$ will be requirement of corresponding groups g_u that consist of all sets that cover element u. On the other hand, requirement k_g of group $g \in \mathcal{G}$ will be requirement of the element g in CSC-R problem.

Theorem 2.4. *CSC-R is equivalent to the CST problem where all edges in the graph G have equal weights.*

As a result the algorithm of Konjevod et al. applies for CSC-R problem in order to compute the solution of CSC-R whose value is at most $O(\log^2 m \log\log m \log(R \cdot n))$ where R is the largest requirement. Size of the largest group in CST problem corresponds to the largest number of sets from $\mathcal{S}$ that cover some fixed element $u \in U$.

3. Inapproximability of CSC

Halperin and Krauthgamer [9] gave polylogarithmic inapproximability result for the GST problem. More precisely, they proved that for every fixed $\epsilon > 0$ the GST admits no efficient $\log^{2-\epsilon} n$ approximation, where n denotes the input size, unless $\mathsf{NP} \subseteq \mathsf{ZTIME}(n^{\text{polylog}(n)})$. That holds even for Hierarchically Well-Separated Trees (HST), and hence for general trees as well. Since node weighted GST on trees is reducible to GST on trees (see Theorem 2.3) , it follows that same inaproximability result holds for node weighted GST when input graph is a tree. By Theorem 2.1, it follows that weighted CSC is also $\Omega(\log^{2-\epsilon} n)$- hard, for all $\epsilon > 0$, even when the input graph is a tree.

Theorem 3.1. *Weighted Connected Set Cover problem is $\Omega(\log^{2-\epsilon} n)$-hard, for all $\epsilon > 0$.*

4. Conclusion

In this paper we found relation between two combinatorial problems, Connected Set Cover and Group Steiner Tree. Doing so, we are the first one to argue that the CSC problem can be approximated within the polylogarithmic approximation ratio. By similar arguments we gave the first algorithm for weighted version of CSC that has been raised as an open and interesting problem in [16]. Very recent results by Naor et al. [13] raise an interesting question whether there exist a polynomial algorithm for an on-line variant of the (weighted) CSC problem with a polylogarithmic approximation guarantee.

Inaproximability results showed that the weighted CSC problem is $\Omega(\log^{2-\epsilon} n)$-hard. However, it is still not clear whether the same inapproximability results holds for CSC. Note that obtaining a better bounds for CSC will immediately imply a bounds for GST with uniform edge weights (e.g. all edge weights are one) and vice versa. It is worth mentioning that the construction of Halperin and Krauthgamer [9] uses information of edge weights and the same approach cannot be used for GST with uniform edge weights.

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