19TH CENTURY MATHEMATICS

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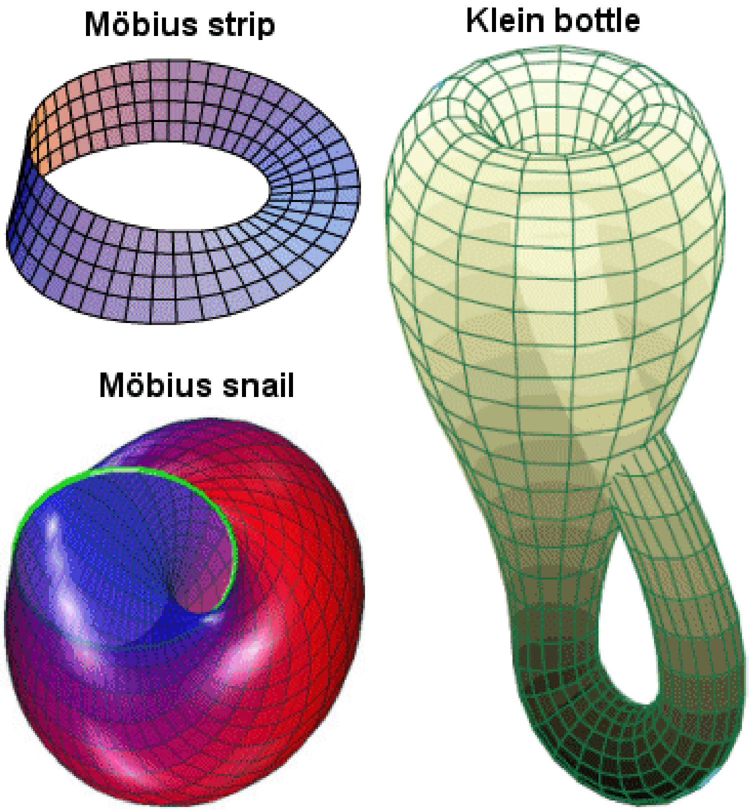
- age of revolution → France and Germany
- "the three L's": Lagrange, Laplace and Legendre
- advance in mathematical analysis and periodic functions → Joseph Fourier's study
- Argand Diagrams
- Gauss → the "Prince of Mathematics"
- elliptic geometry → Riemann
- Babbage → "difference engine,,
- Boolean algebra

Complexity and abstraction

- Weierstrass – Bolzano
- Riemann – Weierstrass – Cauchy
- discovery of the Möbius strip



GALOIS (1811-1832)

- A romantic figure in Franch mathematical history
- fundamental discoveries in the theory of polynomial equations
- group

AN EXAMPLE OF GALOIS' RATHER UNDISCIPLINED NOTES

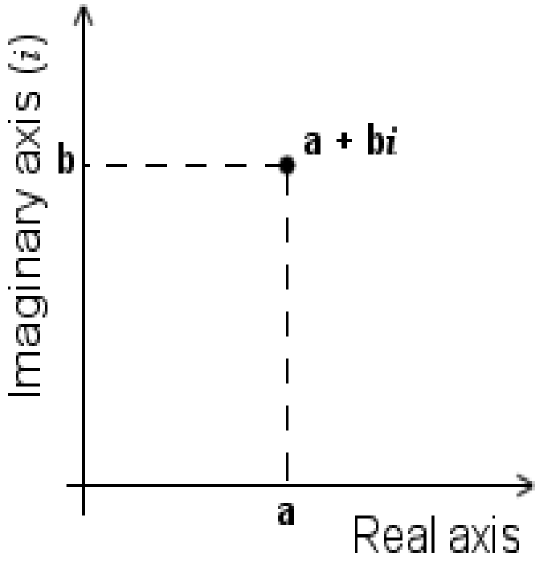
Gauss (1777-1855)

- "Prince of Mathematicians"
- prime numbers
- “mathematics is the queen of the sciences, and the theory of numbers is the queen of mathematics”

Gauss

- exposition of complex numbers and of the investigation of functions of complex variables
- Fundamental Theorem of Algebra

A complex number such as a + b*i* (where *i* is the square root of -1) can be represented graphically using a real axis and an imaginary axis (using the imaginary unit *i*).



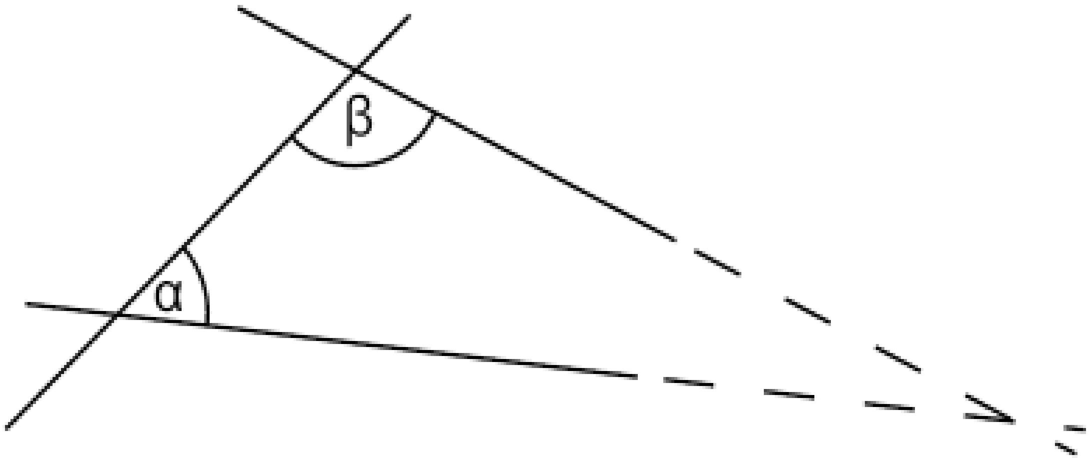
The imaginary and real numbers together form a plane, known as the complex plane.

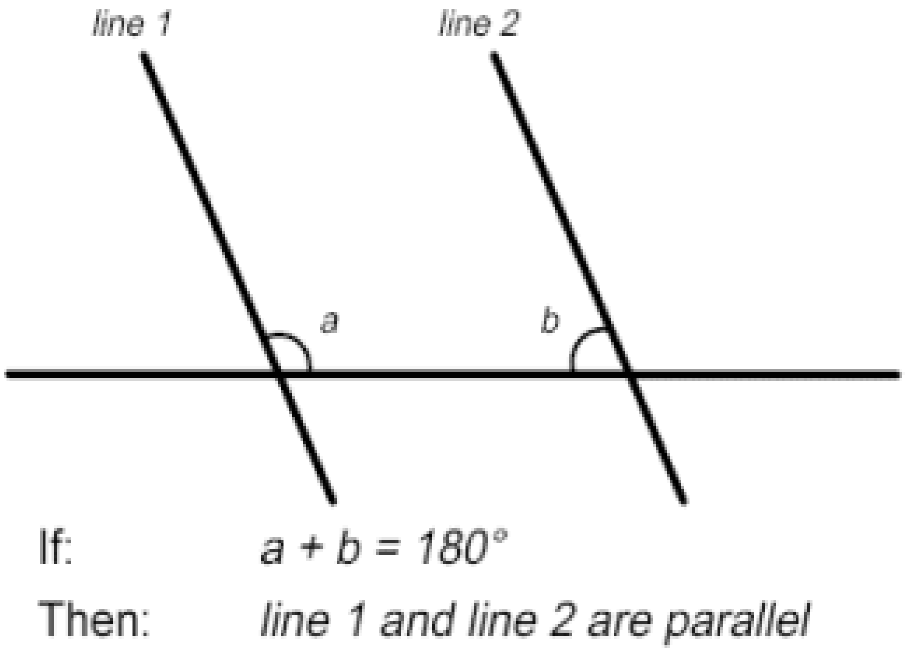
BOLYAI (1802-1860)

- a Hungarian mathematician
- obsessed with Euclid's fifth postulate

Euclid's parallel postulate states that:

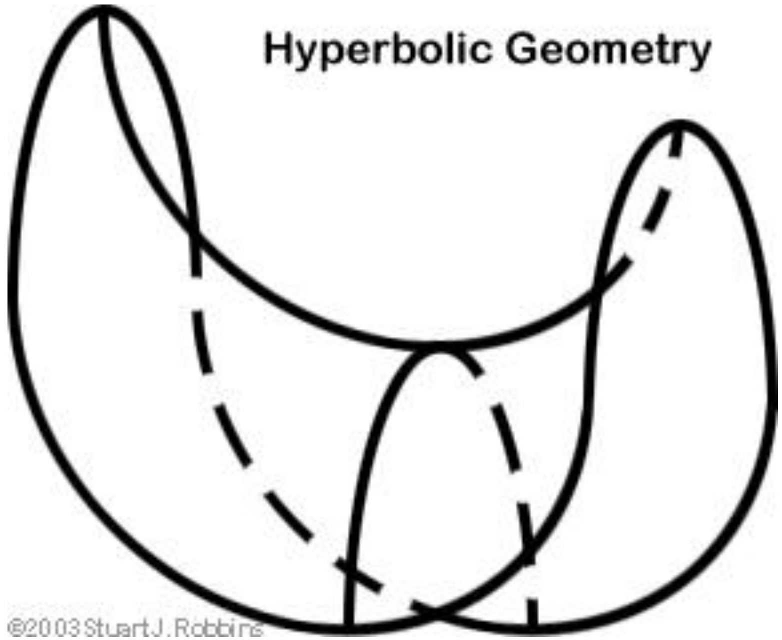
If a line segment intersects two straight lines forming two interior angles on the same side that sum to less than two right angles, then the two lines, if extended indefinitely, meet on that side on which the angles sum to less than two right angles.





Bolyai

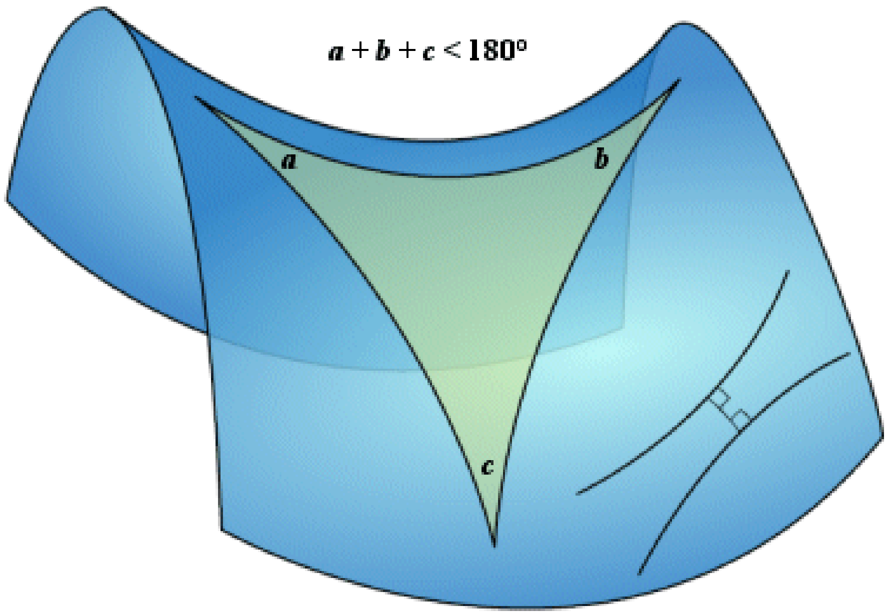
- -"imaginary geometry" (now known as hyperbolic geometry)
- a radical departure from Euclidean geometry
- the first step to Einstein's Theory of Relativity



LOBACHEVSKY (1792-1856)

- Russian mathematician
- hyperbolic geometry
  (published in 1830)
- Lobachevskian geometry
  or Bolyai-Lobachevskian
  geometry
- mathematical achievements - Dandelin-Gräffe method, and the definition of a function

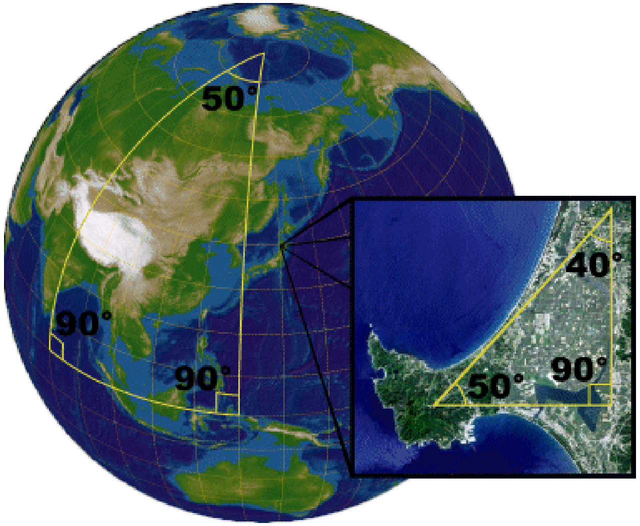
HYPERBOLIC GEOMETRY = BOLYAI-LOBACHEVSKIAN GEOMETRY



RIEMANN (1826-1866)

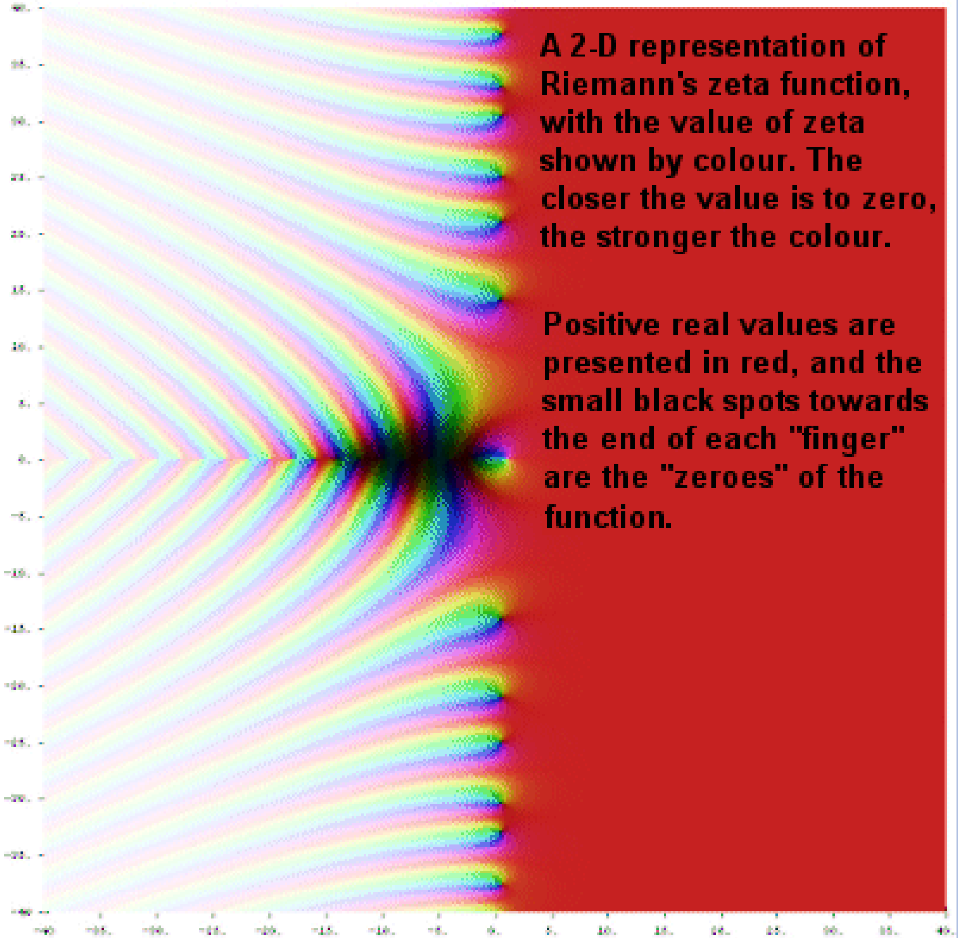
- from northern Germany
- tried to prove mathematically the correctness of the Book of Genesis
- elliptic geometry
- Riemann surfaces

The easiest way to think of elliptic geometry is to consider a triangle drawn on a sphere. The interior angles sum to more than 180°, although, on a smaller scale, Euclidean geometry is a good approximation.



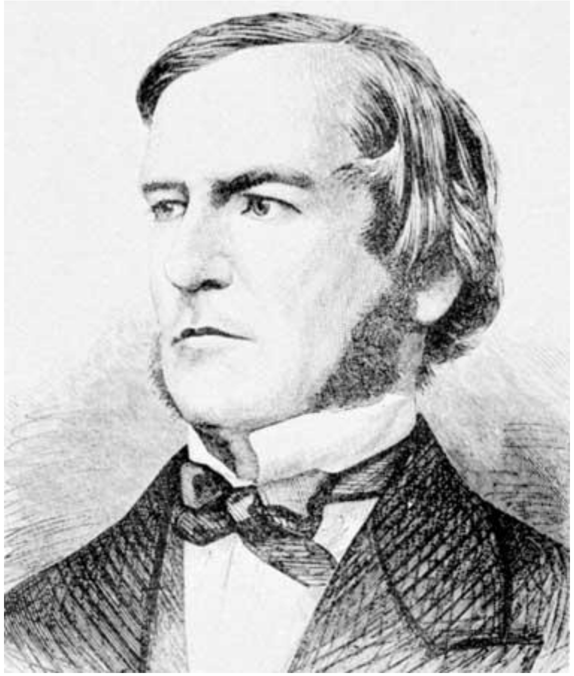
RIEMANN

- broke away from all the limitations of 2 and 3 dimensional geometry
- zeta function
- the Riemann Hypothesis



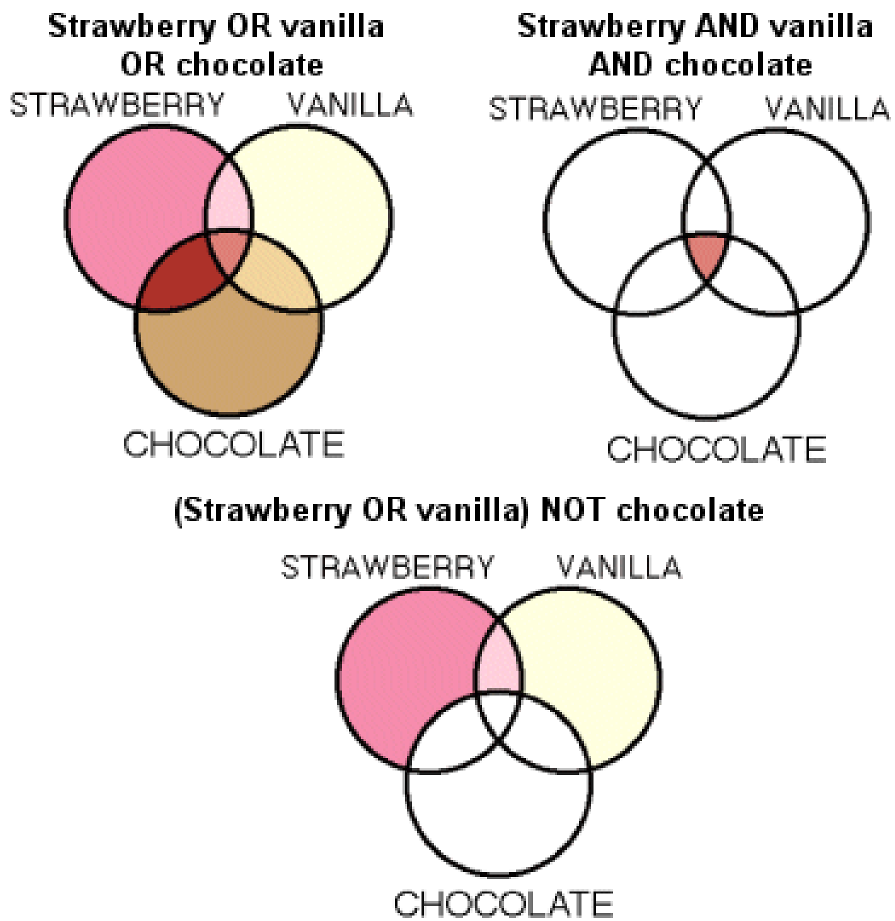
BOOLE (1815-1864)

- The British mathematician and philosopher
- “calculus of reason”
- Boolean algebra
- AND – OR – NOT
- a founder of the field of computer science



Boolean algebra in logic

Boolean logic can be used to compare and manipulate sets using just three operators: AND, OR and NOT. For example:



- The operations are usually taken to be: conjunction(AND,*) $\wedge$ disjunction(OR,+) $\vee$ negation(NOT) $\neg$
- Truth tables

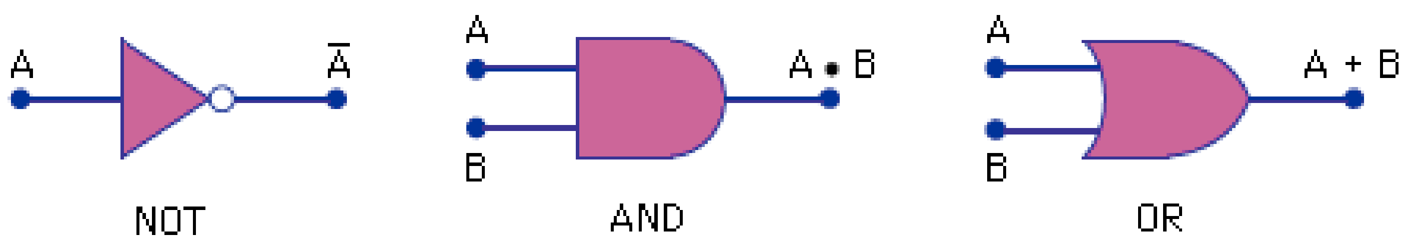
Boolean logic in computer science

- Claude Shannon recognisedthat Boole's work could form the basis of mechanisms and processes in the real world and that it was therefore highly relevant

A	0	1
$\bar{A}$	1	0

A	0	0	1	1
B	0	1	0	1
A • B	0	0	0	1

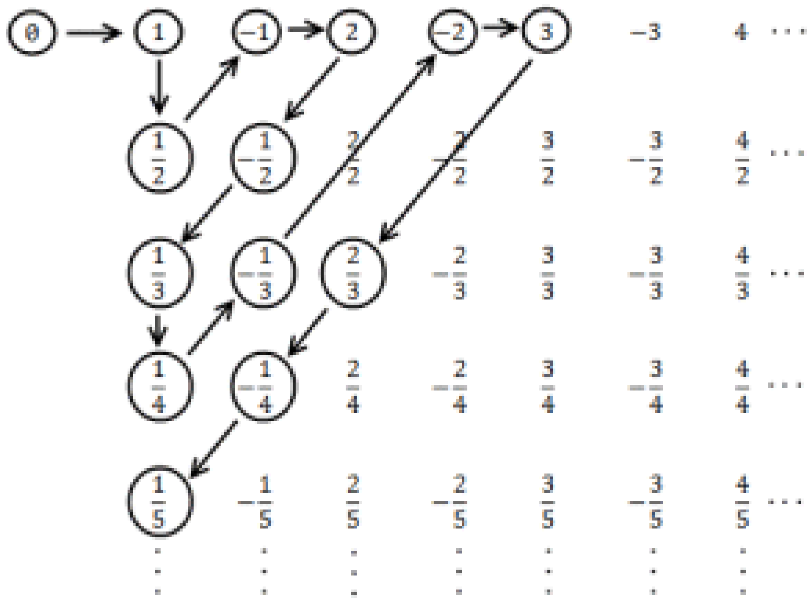
A	0	0	1	1
B	0	1	0	1
A + B	0	1	1	1



Cantor (1845-1918)

- German mathematician
- number theory
- solving a problem on the uniqueness of the representation of a function by trigonometric series

Cantor compared the infinite set of rational numbers with the infinite set of natural numbers by a procedure of listing and enumerating all the rationals…



…and then pairing each rational in this list with the successive natural numbers:

$$
\begin{array}{lccccccc}
\mathbb{N}: \{ & 1 & 2 & 3 & 4 & 5 & 6 & \ldots\} \\
 & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \\
\mathbb{Q}: \{ & 0 & 1 & \frac{1}{2} & -1 & 2 & -\frac{1}{2} & \ldots\}
\end{array}
$$

In this way, he showed that the rational numbers are denumerable (or countable), and that the infinity of rational numbers is the same size as the infinity of natural numbers.

Cantor imagined an infinite set of numbers made up of an infinite pattern of just two digits (e.g. 1 and 0, 5 and 4, or m and w in the example below).

He showed how a new number could always be created by making sure that the first digit of the new number is different from the first digit of the first number in the set, the second digit is different from the second digit of the second number, etc, etc.

It is known as the "diagonal argument" because the new number (blue) is different in every place from the diagonal digits (red).

In this way, the new number could never be a duplicate of any number in the infinitely long set, and Cantor proved through this that even an infinite set of numbers cannot contain all possible numbers, and indeed that there are more sets of numbers than there are numbers!

$$
\begin{array}{l}
E_0 = m\ m\ m\ m\ m\ m\ m\ m\ m\ m\ m\ m\ \cdots \\
E_1 = w\ w\ w\ w\ w\ w\ w\ w\ w\ w\ w\ w\ \cdots \\
E_2 = m\ w\ m\ w\ m\ w\ m\ w\ m\ w\ m\ w\ \cdots \\
E_3 = w\ m\ w\ m\ w\ m\ w\ m\ w\ m\ m\ w\ \cdots \\
E_4 = w\ m\ m\ w\ w\ m\ m\ w\ m\ w\ m\ w\ \cdots \\
E_5 = m\ w\ m\ w\ w\ m\ w\ m\ w\ m\ w\ m\ \cdots \\
E_6 = m\ w\ m\ w\ w\ m\ w\ w\ m\ w\ m\ w\ \cdots \\
E_7 = w\ m\ m\ w\ m\ w\ m\ w\ m\ w\ m\ w\ \cdots \\
E_8 = m\ m\ w\ m\ w\ m\ w\ m\ w\ m\ w\ m\ \cdots \\
E_9 = w\ m\ w\ m\ m\ w\ w\ m\ w\ w\ m\ w\ \cdots \\
E_{10} = w\ w\ m\ w\ m\ w\ m\ w\ m\ m\ w\ m\ \cdots \\
E_{11} = m\ w\ m\ w\ w\ m\ w\ m\ m\ w\ m\ m\ \cdots \\
\vdots \quad \vdots\ \vdots\ \vdots\ \vdots\ \vdots\ \vdots\ \vdots\ \vdots\ \vdots\ \vdots\ \vdots\ \vdots\ \ddots \\
\\
E_u \neq w\ m\ w\ w\ m\ w\ m\ m\ m\ m\ m\ w\ \cdots
\end{array}
$$

Some of the notation of basic set theory includes:

$\mathbf{N}$ = the set of natural numbers
$\mathbf{Q}$ = the set of rational numbers
$\mathbf{R}$ = the set of real numbers
$\mathbf{P}$ = the set of prime numbers
$\mathbf{Z}$ = the set of integers
$\mathbf{E}$ = the set of even integers
$\mathbf{O}$ = the set of odd integers

The symbol for showing that one set is a member of another set is $\subset$, so that:

$$\mathbf{P} \subset \mathbf{N} \subset \mathbf{Z} \subset \mathbf{Q}$$

The symbol for showing that one set is equivalent to another set is $\sim$, so that:

$$\mathbf{Q} \sim \mathbf{Z} \sim \mathbf{N} \sim \mathbf{P} \sim \mathbf{E} \sim \mathbf{O}$$

The number of members in a set is shown as $\#$, while the cardinality of a set (a measure of the size of a set, whether finite or infinite) is denoted by $\aleph$:

$$\#\mathbf{N} = \aleph_0 \qquad \#\mathbf{R} = 2^{\aleph_0}$$

POINCARÉ (1854-1912)

- ”Last Universalist”
- "it is by logic that we prove, but by intuition that we discover"
- ”three-body problem”
- science of topology

"THREE-BODY PROBLEM"

- Computer representation of the paths generated by Poincaré's analysis of the three body problem

Match the mathematicians with facts!

- Galois
- Gauss
- Bolyai and Lobachevsky
- Riemann
- Boole
- Cantor
- Poincare

- Hiperbolic geometry
- Topology
- Set of numbers
- Group
- a type of linguistic algebra
- Zeta function
- “Prince of Mathematicians”
- Procedure of bijection
- The theory of polynomial
- AND,OR,NOT
- “three-body problem”
- Multi-dimensional space
- Euclid’s fifth postulate
- Fundamental theorem of Algebra