On question of equivalence of two descriptions of boundary conditions related to Friedrichs systems

The Friedrichs system is a class of boundary value problems which admits the study of a wide range of differential equations in an unified framework. The were introduced by K. O. Friedrichs in 1958. as an attempt to treat equations of mixed type (such as Tricomi equation). They consist of first order system of partial differential equations (of specific type) and the *admissible* boundary condition enforced by matrix valued boundary field.

In paper [EGC] a new view on the theory of Friedrichs systems has been given, as the theory is written in terms of Hilbert spaces, and a new way of representation of boundary conditions was introduced. Here, the admissible boundary conditions are characterize by two intrinsic geometric conditions in graph space, which avoids invoking traces at the boundary. They also introduce another representation of boundary conditions via boundary operator, and show that this representation is equivalent with intrinsic one (those enforced by two geometric conditions) if sum of two specific subspaces V and $\tilde{V}$ of graph space is closed, but are not sure whether this requirement is always achieved.

We have noted that these two geometric conditions can be naturally written in terminology of indefinite inner product on graph space, and use of classical results in Krein spaces allowed us to construct the counter–example, which shows that $V + \tilde{V}$ does not need to be always closed in graph space. In case of one space dimension we will give complete classification of admissible boundary conditions (those that satisfy two geometric conditions).

The relation between *classical* representation of admissible boundary conditions (via matrix fields on boundary), and those given by boundary operator will be addressed as well.

References:

[EGC] A. Ern, J.-L. Guermond, G. Caplain: *An Intrinsic Criterion for the Bijectivity Of Hilbert Operators Related to Friedrichs' Systems*, *Comunications in Partial Differential Equations*, **32**, (2007), 317–341.