Krein spaces applied to Friedrichs systems

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Friedrichs systems are a class of boundary value problems which allows the study of a wide range of differential equations in a unified framework. They were introduced by K. O. Friedrichs in 1958 in an attempt to treat equations of mixed type (such as the Tricomi equation). The Friedrichs system consists of a first order system of partial differential equations (of a specific type) and an *admissible* boundary condition enforced by a matrix-valued boundary field.

In a recent paper: A. Ern, J.-L. Guermond, G. Caplain: *An Intrinsic Criterion for the Bijectivity Of Hilbert Operators Related to Friedrichs' Systems*, *Commun. Part. Diff. Equat.* **32** (2007) 317–341 a new approach to the theory of Friedrichs systems has been proposed, rewritting them in terms of Hilbert spaces, and a new way of representing the boundary conditions has been introduced. The admissible boundary conditions have been characterised by two intrinsic geometric conditions in the graph space, thus avoiding the traces at the boundary. Another representation of boundary conditions via boundary operators has been introduced as well, which is equivalent to the intrinsic one (those enforced by two geometric conditions) if two specific operators P and Q on the graph space exist. However, the validity of the last condition was left open. The authors have also shown that their admissible (geometric) conditions imply maximality of the boundary condition.

We note that these two geometric conditions can be naturally written in the terms of an indefinite inner product on the graph space, and by use of simple geometric properties of Krein spaces we show that maximality of boundary condition is equivalent to its admissibility. An aplication of classical results on Krein spaces also allow us to construct a counter–example, which shows that the operators P and Q do not necessarily exist. In the case of one space dimension we give complete classification of admissible boundary conditions (those satisfying the two geometric conditions).

The relation between the *classical* representation of admissible boundary conditions (via matrix fields on the boundary), and those given by the boundary operator will be addressed as well: sufficient conditions on the matrix boundary field in order to define boundary operator will be given and tested on examples.