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Searching for a best LAD-solution of an overdetermined system of linear equations
motivated by searching for a best LAD-hyperplane on the basis of given data

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Abstract. We consider the problem of searching for a best LAD-solution of an overdetermined
system of linear equations \( \mathbf{X} \mathbf{a} = \mathbf{z}, \ \mathbf{X} \in \mathbb{R}^{m \times n}, \ m \geq n, \ \mathbf{a} \in \mathbb{R}^n, \ \mathbf{z} \in \mathbb{R}^m. \) This problem is
equivalent to the problem of determining a best LAD-hyperplane \( \mathbf{x} \mapsto \mathbf{a}^T \mathbf{x} \), \( \mathbf{x} \in \mathbb{R}^n \) on the basis
of given data \( (\mathbf{x}_i, z_i) \), \( \mathbf{x}_i = (x_1^{(i)}, \ldots, x_n^{(i)})^T \in \mathbb{R}^n, \ z_i \in \mathbb{R}, \ i = 1, \ldots, m, \) whereby the minimizing
functional is of the form

\[
F(\mathbf{a}) = \|\mathbf{z} - \mathbf{X} \mathbf{a}\|_1 = \sum_{i=1}^{m} |z_i - \mathbf{a}^T \mathbf{x}_i|.
\]

An iterative procedure is constructed as a sequence of weighted median problems, which gives
the solution in finitely many steps. A criterion of optimality follows from the fact that the
minimizing functional \( F \) is convex, and therefore the point \( \mathbf{a}^* \in \mathbb{R}^n \) is the point of a global
minimum of the functional \( F \) if and only if \( \mathbf{0} \in \partial F(\mathbf{a}^*). \)

Motivation for the construction of the algorithm was found in a geometrically visible algo-
rithm for determining a best LAD-plane \( (x, y) \mapsto \alpha x + \beta y, \) passing through the origin of the
coordinate system, on the basis of the data \( (x_i, y_i, z_i), \ i = 1, \ldots, m. \)