The relation of Connected Set Cover and Group Steiner Tree

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Introduction

- NP-hard combinatorial optimizations problem
- efficient algorithm not known
- polynomial approximation

$$OPT \leq ALG \leq \rho * OPT$$

- ρ - approximation algorithm
- $\rho = \rho(\cdot)$ - function of input size = approximation ratio

Set Cover

- U - set of elements (universe)
- $\mathcal{S}$ family of subsets of U
  - $\bigcup_{S \in \mathcal{S}} S = U$
- **Problem:** Find $\mathcal{R} \subseteq \mathcal{S}$
  - each $u \in U$ is contained in at least one set from $\mathcal{R}$
  - among all such subfamilies, $\mathcal{R}$ has minimal cardinality

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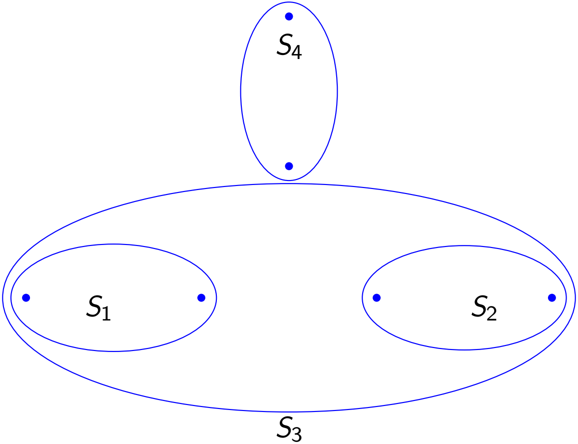

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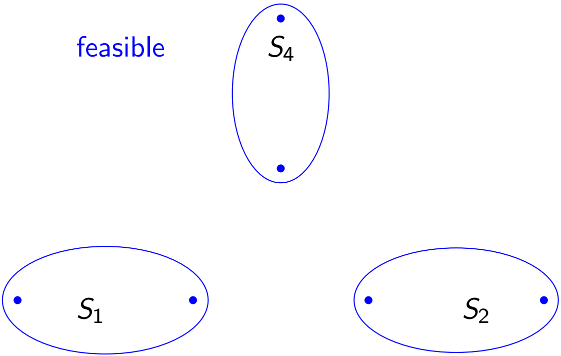

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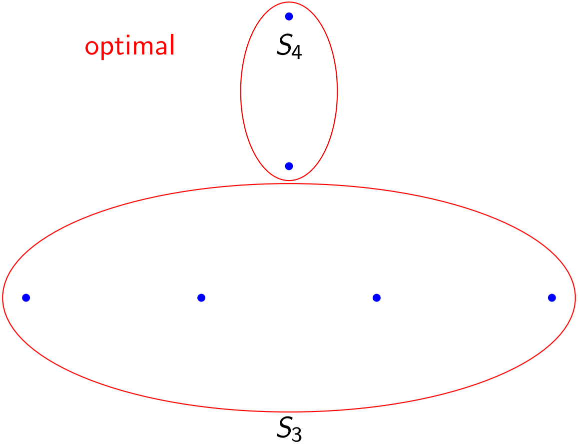

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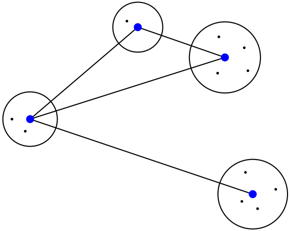

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  - NWCSC - Node Weighted Connected Set Cover

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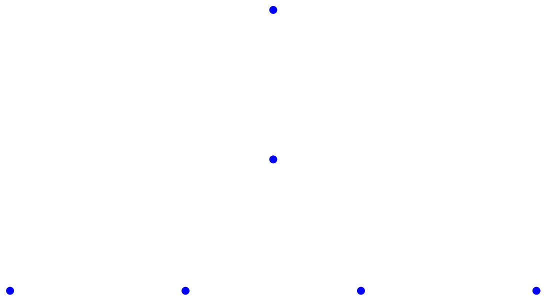

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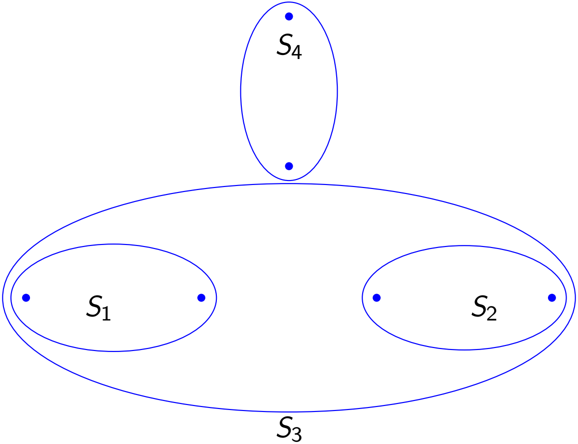

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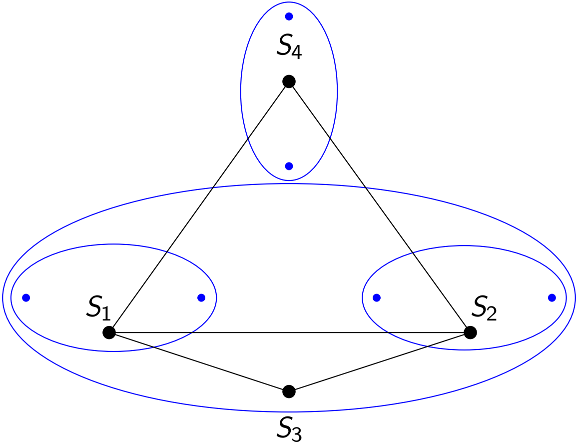

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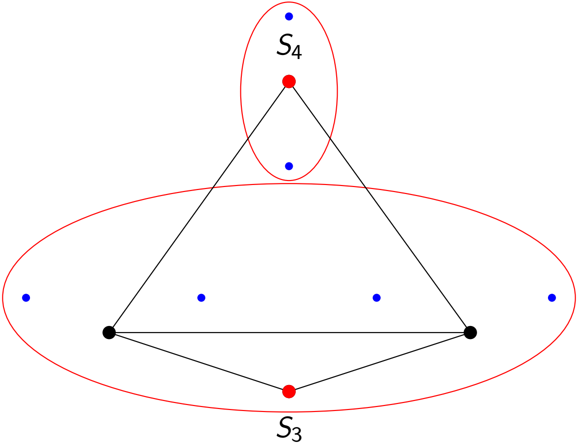

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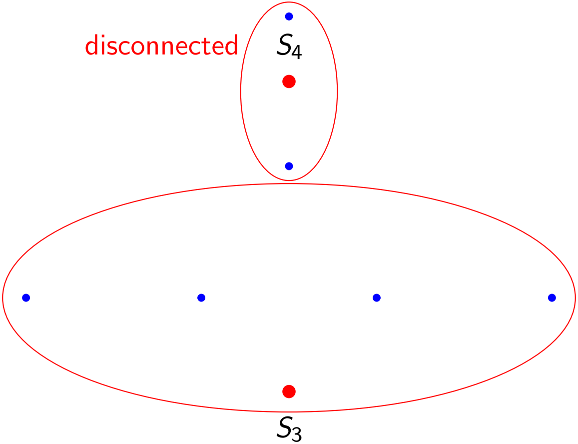

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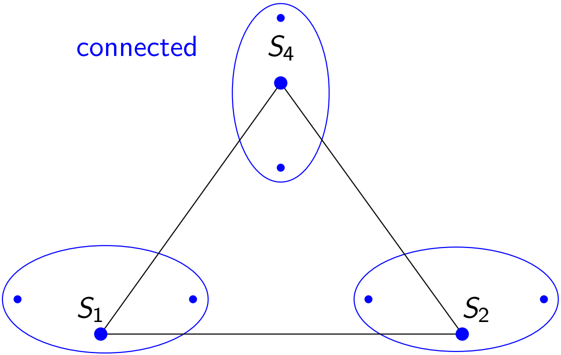

Applications of CSC problems

- constructing optimal biodiversity reserve system
  - set of species live on protected areas - reserves
  - choose subset of reserves which represents all species
  - overall number (cost) of reserves has minimized
- Debinski and Holt, *A survey and overview of habitat fragmentation experiments*, Conservation Biology, (2000)
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- Zhang, Gao and Wu, *Algorithms for connected set cover problem and fault-tolerant connected set cover problem*, Theoretical Computer Science (2009)
  - first greedy approximation algorithm for CSC on general graph
  - approximation ratio: $1 + D_c(G)H(\gamma - 1)$
    - $D_c(G)$ - length of the longest path in graph G between two non-disjoint sets
    - $\gamma = \max\{|S| : S \in \mathcal{S}\}$
  - NWCSC presented as an open problem

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Our improvements on CSC and NWCSC

- $D_c(G) \in O(m)$
- improvement on CSC:
  - approximation ratio $O(\log^2 m \log\log m \log n)$
- results on NWCSC:
  - we are not aware of any previous algorithm for NWCSC
  - our algorithm: $O(\sqrt{m} \log m)$

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Bad example for greedy strategy

- $n \in \mathbb{N}$ is even, $n \geq 6$

$$U = \{1, 2, \ldots, n\}$$

- $\mathcal{S} = \{S_1, \ldots, S_n\}$
    - $S_1 = \{1, 2, \ldots, n/2\}$
    - $S_n = \{n/2 + 1, \ldots, n\}$
    - $S_i = \{i - 1, i\}$ for $2 \leq i \leq n - 1$

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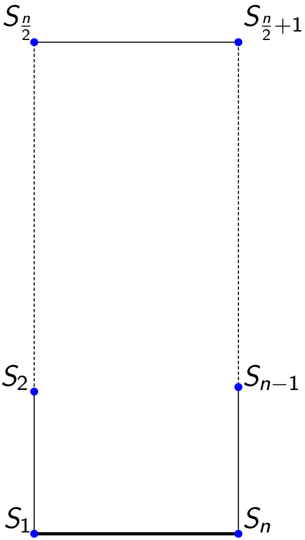
greedy: $\mathcal{S}$

optimal: $\{S_1, S_n\}$



Group Steiner Tree (GST)

- introduced by Reich and Widmayer (1990).
- motivation: wire routing with multiport terminals in physical VLSI design
- definition:
  - graph G
  - edge weight function $w : E(G) \to \mathbb{R}^+$
  - $\mathcal{G} = \{g_1, g_2, \ldots, g_k\}, \quad g_i \subset V$
  - objective: find subtree T
    - $V(T) \cap g_i \neq \emptyset$ for all $i \in \{1, \ldots, k\}$
    - minimize $\sum_{e \in E(T)} w(e)$
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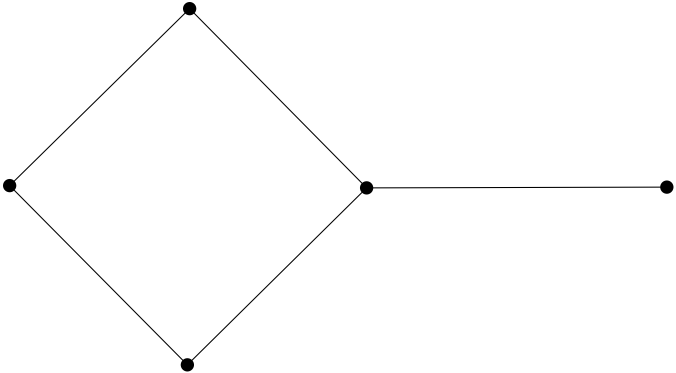

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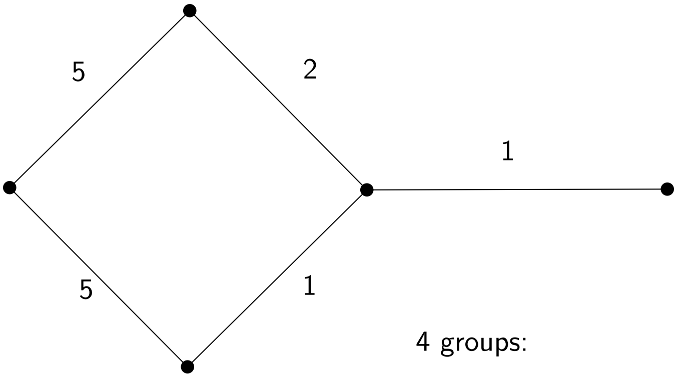
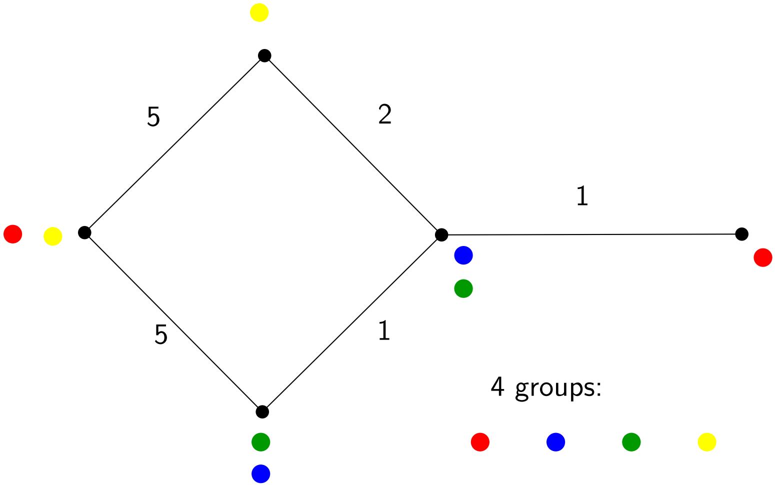


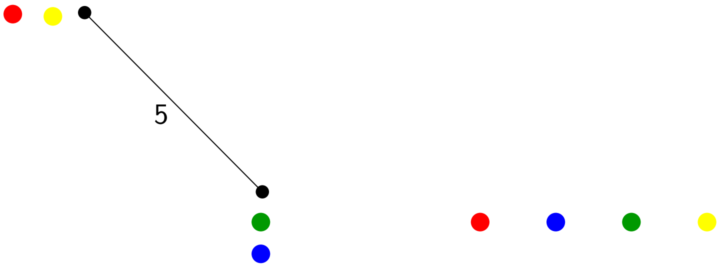
4 groups:

Group Steiner Tree



Group Steiner Tree

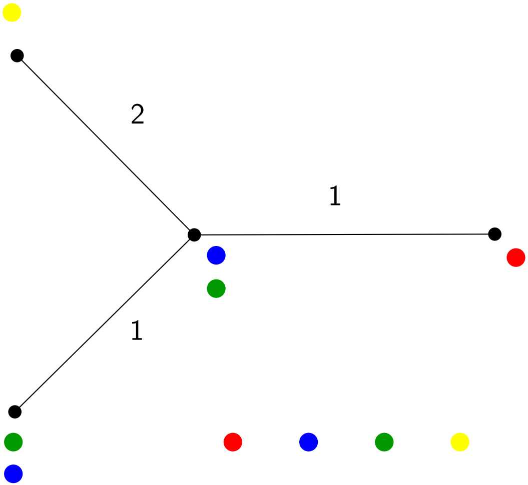
total weight $= 5$



Group Steiner Tree

total weight $= 4$

optimal



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  - first polylogarithmic approximation algorithm for GST
  - solving problem on trees in approximation ratio $O(\log k \log N)$
  - technique: LP relaxation + randomized rounding
  - generalization: stretch $= O(\log n \log \log n)$
  - Bartal probabilistic approximation of metric spaces
- result on general graph: $O(\log N \log n \log \log n \log k)$

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  - $O(\sqrt{n}\log n)$ - approximation algorithm

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Main Result: Equivalence of NWCSC and NWGST

- NWCSC and NWGST are equivalent
  - NWCSC reducible to NWGST
  - NWGST reducible to NWCSC
- NWCSC algorithm:
  - reduce NWCSC to NWGST
  - solve NWGST by Khandekar et al. algorithm
- CSC algorithm:
  - reduce CSC to GST with edge weights equal to 1
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- Zhang, Gao and Wu, *Algorithms for connected set cover problem and fault-tolerant connected set cover problem*, Theoretical Computer Science (2009)
  - solution subgraph has to be k - connected
  - each element covered by at least m sets
- (k, m) - CSC
- generalization of (1,m)-CSC
  - each element u has requirement $r_u \in \mathbb{N}$
  - element u has to be covered by at least r_u sets
- Connected Set Cover with requirements (CSC-R)

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Covering Steiner Tree problem

- generalization of GST
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- Konjevod, Ravi and Srinivasan, *Approximation algorithms for the covering steiner problem*, Random Structures and Algorithms (2002)
  - polylog approximation for CST
  - approximation ratio: $O(\log n \log \log n \log N \log(K \cdot k))$

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  - polylog approximation for CST
  - approximation ratio: $O(\log n \log\log n \log N \log(K \cdot k))$

Our algorithm for CSC-R

- CSC-R equivalent to CST with mutually equal edge weights ($w(e) = 1$ for all $e \in E(G)$)
- algorithm for CSC-R:
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  - $R = \max_{u \in U} r_u$

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NWCSC to NWGST reduction

- reduction from NWCSC to NWGST
  - fixed element $u \in U$

$$g_u = \{S \in \mathcal{S} : u \in S\}$$

  - graph G, node weights $w_N(\cdot)$
  - NWGST instance
  - let T is solution of NWGST
  - take $\mathcal{R} = V(T)$ as solution of NWCSC
    - $G[\mathcal{R}]$ is connected since T is connected
    - $\mathcal{R}$ is set cover since T touches each group
- CSC to GST reduction: $w(e) = 1$ for all $e \in E(G)$
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Inapproximability results on NWCSC and open problems

- Halperin and Krauthgamer, *Polylogarithmic inapproximability*, STOC,(2003)
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- open problems:
    - polylog approximation of NWCSC
    - log approximation of CSC (or proof of nonexistence)

Thank you for attention!!!