Multifractality of products of geometric
Ornstein-Uhlenbeck type processes

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Abstract

This is joint work with V.V. Anh (Queensland University of Technology, Brisbane) and N.-R. Shieh (National Taiwan University, Taipei).

We consider multifractal products of stochastic processes as defined in Mannersalo et al. (2002), but we provide a new interpretation of the conditions on the mean, variance and covariance functions of the resulting cumulative processes in terms of the moment generating functions. We show that the logarithms of the corresponding limiting processes have an infinitely divisible distribution such as the gamma and variance gamma distributions (resulting in the log-gamma and log-variance gamma scenarios respectively), the inverse Gaussian and normal inverse Gaussian distributions (yielding the log-inverse Gaussian and log-normal inverse Gaussian scenarios respectively). We describe the behavior of their q-th order moments and Rényi functions, which are nonlinear, hence displaying their multifractality. A property on the dependence structure of the limiting processes, leading to their possible long-range dependence, is also obtained.

We consider very different scenarios such as the log-gamma and log-inverse Gaussian scenarios as typical examples of our approach. We should also note some related results by Barndorff-Nielsen and Schmiegel (2004) who introduced some Lévy-based spatiotemporal models for parametric modelling of turbulence. Log-infinitely divisible scenarios related to independently scattered random measures were introduced in Bacry and Muzy (2003) and others. We should note that Chris Heyde (1999) proposed to use a multifractality into risky asset model with fractal activity time (see also Heyde and Leonenko (2005)).

References


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