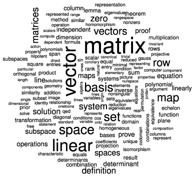

M018 Linearna algebra 1

Vježbe 13

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Zadatak 2.

Primjenom Cramerove metode u ovisnosti o parametru $\lambda \in \mathbb{R}$ diskutirajte sustave jednadžbi:

a)

$$\begin{aligned} \lambda x_1 + 5x_2 &= 1 \\ 5x_1 + \lambda x_2 &= 1, \end{aligned}$$

b)

$$\begin{array}{rcrcrcr} 2x_1 & - & \lambda x_2 & + & 2x_3 & = & -2 \\ 4x_1 & + & x_2 & + & \lambda x_3 & = & 2 \\ -2x_1 & - & x_2 & & & = & -2. \end{array}$$

c)

$$\begin{array}{rcrcrcl} -\lambda x_1 & + & x_2 & + & 2x_3 & = & 1 \\ 2x_1 & + & x_2 & + & 4x_3 & = & 2 \\ -2x_1 & + & x_2 & + & \lambda x_3 & = & 1. \end{array}$$

SUSTAVI LINEARNIH JEDNADŽBI

GAUSS - JORDANOVA METODA

Zadatak 1.

Gauss - Jordanovom metodom nađite opće rješenje sljedećih sustava linearnih jednadžbi:

a) $$\begin{aligned} 3x_1 - x_2 + 2x_3 &= 0 \\ 2x_1 + 3x_2 - 5x_3 &= 0 \\ x_1 + x_2 + x_3 &= 0, \end{aligned}$$

b) $$\begin{aligned} 2x_1 - x_2 + 3x_3 &= 0 \\ x_1 + 2x_2 - 5x_3 &= 0 \\ 3x_1 + x_2 - 2x_3 &= 0, \end{aligned}$$

c) $x_1 + 3x_2 + x_3 + x_4 = 0$
$7x_1 + 5x_2 - x_3 + 5x_4 = 0$
$3x_1 + x_2 - x_3 + 2x_4 = 0$
$5x_1 + 7x_2 + x_3 + 4x_4 = 0,$

Rješenje: $x = x_0 + \lambda_1 u_1 + \lambda_2 u_2, x_0 = (0,0,0,0)^T, u_1 = (\frac{1}{2}, -\frac{1}{2}, 1, 0)^T, u_2 = (-\frac{5}{8}, -\frac{1}{8}, 0, 1)^T, \lambda_1, \lambda_2 \in \mathbb{R}$

d) $x_1 + 2x_2 + 3x_3 = 3$
$-2x_1 + x_3 = -2$
$x_1 + 2x_2 - x_3 = 3$
$-x_1 + 2x_2 + 12x_3 = 1,$

e) $x_1 + 2x_2 + 3x_3 = 5$
$2x_1 - x_2 - x_3 = 1$
$x_1 + 3x_2 + 4x_3 = 6,$

Rješenje: $x = (1, -1, 2)^T$

f) $3x_1 - x_2 + 3x_3 = 4$
$6x_1 - 2x_2 + 6x_3 = 1$
$5x_1 + 4x_2 = 2,$

g) $x_1 - 2x_2 + x_3 = 4$
$2x_1 + 3x_2 - x_3 = 3$
$4x_1 - x_2 + x_3 = 11,$

Rješenje:
$x = x_0 + \lambda_1 u_1, x_0 = (\frac{18}{7}, -\frac{5}{7}, 0)^T, u_1 = (-\frac{1}{7}, \frac{3}{7}, 1)^T, \lambda_1 \in \mathbb{R}$

h) $x_1 + 2x_2 - x_3 + x_4 = -1$
$2x_1 + 5x_2 - x_3 + 2x_4 = -2$
$3x_1 - x_2 - 2x_3 + x_4 = 5$
$x_1 - x_2 + 3x_3 - 5x_4 = 6,$

Rješenje: $x = (2, -1, 1, 0)^T$

i) $4x_1 - 2x_2 - 3x_3 - 2x_4 = 1$
$2x_1 + 2x_2 + 3x_3 - 4x_4 = 5$
$3x_1 + 2x_2 - 2x_3 - 5x_4 = 1$
$2x_1 - 5x_2 - 3x_3 + 3x_4 = -1,$

Rješenje:

$x = x_0 + \lambda_1 u_1, x_0 = (1,0,1,0)^T, u_1 = (1,1,0,1)^T, \lambda_1 \in \mathbb{R}$

j) $x_1 + x_2 - x_3 - 3x_4 + 4x_5 = 2$
$3x_1 + x_2 - x_3 - x_4 = 2$
$9x_1 + x_2 - 2x_3 - x_4 - 2x_5 = 5$
$x_1 - x_2 - x_4 + 2x_5 = 1.$