Multiple ellipse detection by using RANSAC and DBSCAN methods

Kristian Sabo, Rudolf Scitovski

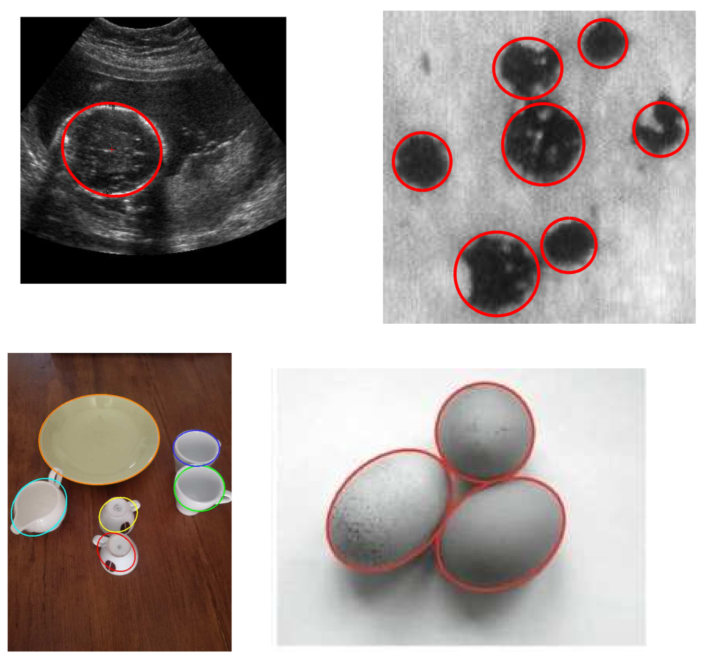
Department of Mathematics, J.J. Strossmayer University of Osijek, Croatia

This work was supported by the Croatian Science Foundation through research grant IP-2016-06-6545

Key words
Multiple ellipse detection; Clustering; RANSAC; DBSCAN

Problem statement

Real-world images



We consider a one ellipse and a multiple ellipse detection problem on the basis of data points:

$$\mathcal{A} = \{a^i = (x_i, y_i)^T : i = 1, \dots, m\} \subset \Delta,$$

$\Delta = [a, b] \times [c, d] \subset \mathbb{R}^2$, coming from one or several ellipses not known in advance.

Some methods known for solving this problem in the literature are:

- Hough transform (Mukhopadhyay and Chaudhuri, 2015)
- Center based clustering (Marošević and Scitovski, 2015; Moshtaghi et al., 2011; Morales-Esteban et al., 2014)
- Geometric methods (Isack and Boykov, 2012; Prasad et al., 2013; Akinlar and Topal (2013))

The EDCircles method proposed in Akinlar and Topal (2013) can be used in real-time applications.

Multiple Ellipse Detection problem can be formulated as a global optimization problem:

$$\underset{p,q,\xi,\eta,\vartheta}{\operatorname{argmin}} F(p, q, \xi, \eta, \vartheta),$$

$$F(p, q, \xi, \eta, \vartheta) = \sum_{i=1}^{m} \min_{1 \le j \le k} \mathfrak{D}(a^i, E_j(p_j, q_j, \xi_j, \eta_j, \vartheta_j)),$$

where $\mathfrak{D}$ is some distance - like function defining the distance from a point $a \in \mathcal{A}$ to the ellipse $E_j \equiv E_j(p_j, q_j, \xi_j, \eta_j, \vartheta_j)$

$$E_j(p_j, q_j, \xi_j, \eta_j, \vartheta_j) = \left\{ \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}, t \in [0, 2\pi] \right\}, j = 1, \dots, k,$$

where

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} p_j \\ q_j \end{bmatrix} + U(\vartheta) \begin{bmatrix} \xi_j \cos t \\ \eta_j \sin t \end{bmatrix}, t \in [0, 2\pi],$$

$S_j = (p_j, q_j)^T$ are the centers, $\xi_j, \eta_j > 0$ are the lengths of semiaxes and ϑ_j are the angles, $U(\vartheta) = \begin{bmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{bmatrix}$.

The objective function F is nonconvex and nondifferentiable and this problem represents a complex global optimization problem.

Some assumptions about the data

A subset of data points $\pi(E) \subset \mathcal{A}$ coming from some ellipse E satisfies the homogeneity property, i.e. we assume that the set $\pi(E)$ is uniformly scattered around the ellipse E, and the number

$$\rho(\pi) = \tfrac{|\pi(E)|}{|E|},$$

where $|E|$ is the length of the ellipse E, will be called the *local density* of the data point set $\pi(E)$.
Using the parameters from the DBSCAN method (Ester et al., 1996), the lower bound of the local density can be approximated in the following way:

$$\tfrac{MinPts}{2\epsilon(\mathcal{A})} \lessapprox \rho(\mathcal{A}),$$

where $MinPts = \lfloor \log|\mathcal{A}| \rfloor$ (Scitovski and Sabo, 2020), $\epsilon(\mathcal{A})$ is the 99.5% quantile of the set $\{\epsilon_a : a \in \mathcal{A}\}$ and $\epsilon_a > 0$ is a radius of the smallest disc centered at a and containing at least $MinPts$ elements of the set $\mathcal{A}$.

One Ellipse Detection problem (OED)

An ellipse as a Mahalanobis circle

An ellipse $E(S, \xi, \eta, \vartheta)$ can be written as a Mahalanobis circle:

$$E(S, r, \Sigma) = \{u \in \mathbb{R}^2 : d_M(S, u; \Sigma) = r^2\},$$

where

$$d_M(u, v; \Sigma) := \sqrt{\det\Sigma}\,(u - v)^T \Sigma^{-1} (u - v) = \|u - v\|_\Sigma^2,$$

$\Sigma = (\sigma_{ij}) \in \mathbb{R}^{2\times2}$ is a positive definite matrix with eigenvalues ξ^2, η^2 and $r^2 = \sqrt{\det\Sigma} = \xi\eta$

$$\operatorname{diag}(\xi^2, \eta^2) = U\big(\tfrac{r^2}{\sqrt{\det\Sigma}}\Sigma\big)U^T,\ U(\vartheta) = \begin{bmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{bmatrix}, \quad \vartheta = \tfrac{1}{2}\arctan\tfrac{2\sigma_{12}}{\sigma_{11} - \sigma_{22}}.$$

The algebraic distance-like function from the point $a \in \mathbb{R}^2$ to the ellipse E is defined as (Morales-Esteban et al., 2014):

$$\mathfrak{D}(a, E) = (\|S - a\|_\Sigma^2 - r^2)^2.$$

One Ellipse Detection problem (OED) can be formulated as a global optimization problem:

$$\underset{S,r,\Sigma}{\operatorname{argmin}} F(S, r, \Sigma), \quad F(S, r, \Sigma) = \sum_{i=1}^{m} \mathfrak{D}(a^i, E(S, r, \Sigma)). \tag{1}$$

Method 1 for OED: the local optimization method

1) According to (Scitovski and Sabo, 2020), define an initial approximation:

$$S_0 = Mean[\mathcal{A}],\ \Sigma_0 = \tfrac{1}{m}\sum_{a\in\mathcal{A}}(S_0 - a)(S_0 - a)^T$$

and $r_0 = \frac{1}{m}\sum_{a\in\mathcal{A}} \|S_0 - a\|_{\Sigma_0}^2$, since

$$\sum_{a\in\mathcal{A}} \big(\|S_0 - a\|_{\Sigma_0}^2 - r^2\big)^2 \ge \sum_{a\in\mathcal{A}} \Big(\|S_0 - a\|_{\Sigma_0}^2 - \tfrac{1}{m}\sum_{a\in\mathcal{A}}\|S_0 - a\|_{\Sigma_0}^2\Big)^2.$$

2) Apply some local optimization methods (Newton or Quasi-Newton) to problem (1).

Method 2 for OED: using the RANSAC and the DBSCAN method

1) Using the main idea of the RANSAC-method (Fischler and Bolles (1981)), randomly choose 5 non-collinear points $(x_1, y_1)^T, \dots, (x_5, y_5)^T \in \mathcal{A}$. Then there exists a unique ellipse $E(S, r, \Sigma)$ that contains these points. If $E \subset \Delta$, we assume that we have found an acceptable candidate for the ellipse.
2) In the $\epsilon(\mathcal{A})$-neighborhood of the acceptable ellipse determine the number of points from the set $\mathcal{A}$.
3) Repeat the procedure N times (say, 10) and keep the ellipse $\hat{E}$ for which the corresponding set of points is the largest.
4) Ellipse $\hat{E}(\hat{S}, \hat{r}, \hat{\Sigma})$, is a good initial approximation for the ellipse which will be searched for by solving local optimization problem (1).

Multiple Ellipse Detection problem

Method description

1) Using the main idea of the RANSAC-method (Fischler and Bolles (1981)), randomly choose 5 non-collinear points from the set $\mathcal{A}$. The ellipse $E(S, r, \Sigma)$ determined on the basis of these points and contained in rectangle Δ is an acceptable candidate for the searched ellipse. By repeating the procedure, we assume that we have found N candidates.
2) The best ellipse $\hat{E}$ has the largest local density of points in its $\epsilon(\mathcal{A})$-neighborhood. The cluster $\hat{\pi} := \{a \in \mathcal{A} : \mathfrak{D}(a, \hat{E}) < \epsilon(\mathcal{A})\} \subset \mathcal{A}$ of points from this $\epsilon(\mathcal{A})$-neighborhood should be dropped from the set $\mathcal{A}$ and the procedure should be repeated on the rest of the set $\mathcal{A} \setminus \hat{\pi}$.
3) Repeat the whole procedure until the number of the remaining sets becomes smaller than some number given in advance (for example, $5\,MinPts$). In that way, we obtain κ ellipses $\hat{E}_j$, $j = 1, \dots, \kappa$.
4) Determine the local density $\hat{\rho}_j(\hat{E}_j) = \frac{|\hat{\pi}_j|}{|\hat{E}_j|}$ for each pair $(\hat{\pi}_j, \hat{E}_j)$, where $|\hat{\pi}_j|$ is the number of points in the cluster $\hat{\pi}_j$, and $|\hat{E}_j|$ is the length (circumference) of the ellipse $\hat{E}_j$ which can be estimated using the well-known *Ramanujan approximation*

$$|\hat{E}_j| \approx \pi(\hat{\xi}_j + \hat{\eta}_j)\big(1 + \tfrac{3h}{10 + \sqrt{4 - 3h}}\big),$$

where $h = \frac{(\hat{\xi}_j - \hat{\eta}_j)^2}{(\hat{\xi}_j + \hat{\eta}_j)^2}$. Using the lower bound for the local density of the set $\mathcal{A}$, the ellipses, for which

$$\hat{\rho}_j(\hat{E}_j) < \tfrac{MinPts}{2\epsilon(\mathcal{A})},$$

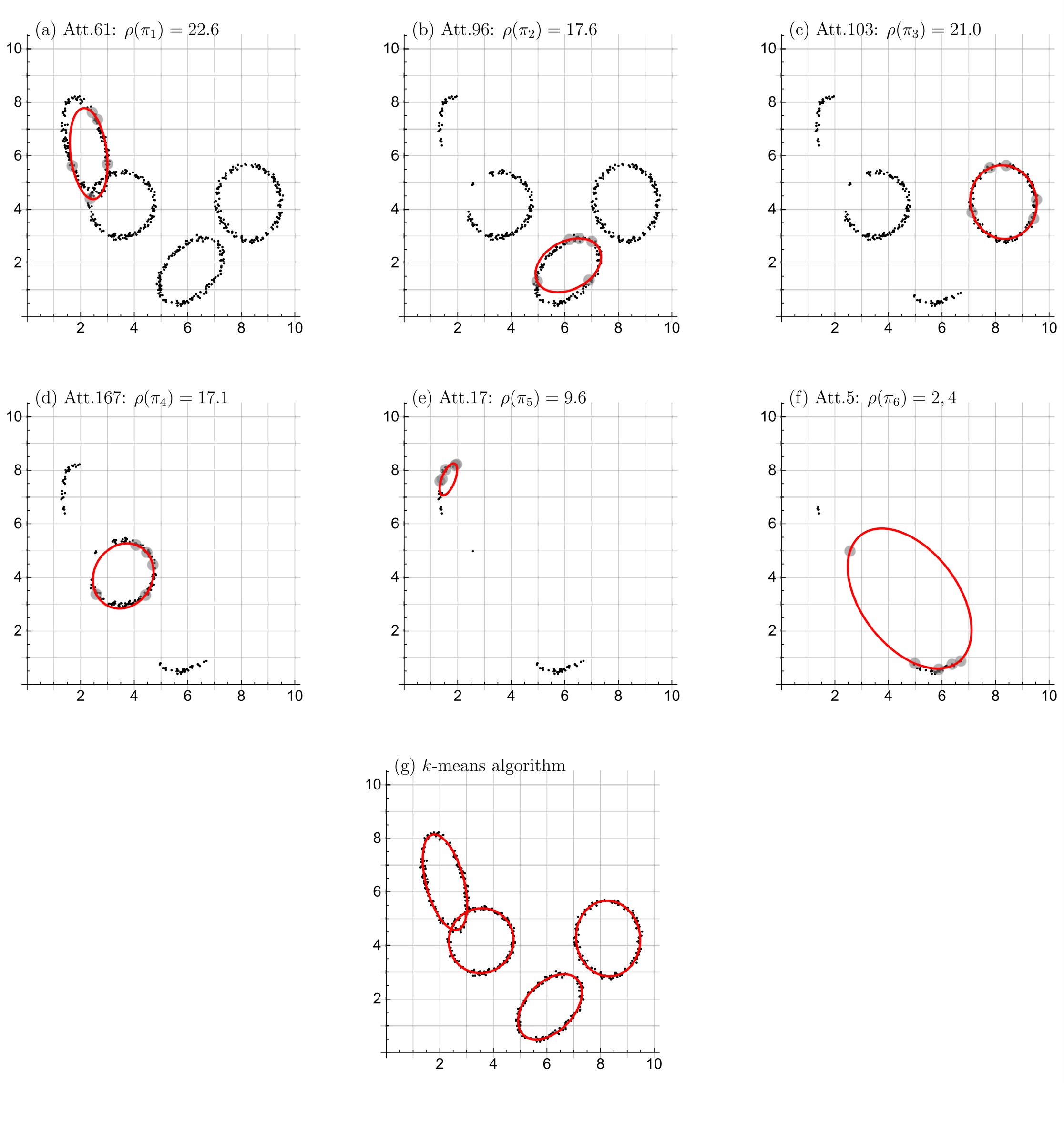
will be dropped.
5) We apply the Adaptive Mahalanobis k-means algorithm to all remaining ellipses (Grbić et al., 2016). The algorithm can be described in the following two steps which are repeated iteratively:

Step A: (Assignment step) For each set of mutually different M-circles $E_1(S_1, r_1, \Sigma_1)$, …, $E_k(S_k, r_k, \Sigma_k)$, the set $\mathcal{A}$ should be divided into k disjoint nonempty clusters $\pi_1, \dots, \pi_k$ by using the minimal distance principle;

Step B: (Update step) Given a partition $\Pi\{\pi_1, \dots, \pi_k\}$ of the set $\mathcal{A}$, one can define the corresponding M-circle-centers $\hat{E}_j(\hat{S}_j, \hat{r}_j, \hat{\Sigma}_j)$ $j = 1, \dots, k$ by using Method 1 or Method 2 for OED
Set $E_j(S_j, r_j, \Sigma_j) = \hat{E}_j(\hat{S}_j, \hat{r}_j, \hat{\Sigma}_j)$ for $j = 1, \dots, k$;

Numerical example

Example *Let us consider the data point set $\mathcal{A}$ shown in Fig.(a) which comes from four ellipses. The number of points is $|\mathcal{A}| = 669$ and DBSCAN-parameters are $MinPts = 6$ and $\epsilon(\mathcal{A}) = 0.284$. The lower bound for the local density is in that case* 10.6.



Conclusions and further research

Solving the multiple ellipse detection problem is important in many applications. We considered a one ellipse and a multiple ellipse detection problem on the basis of a data point set coming from a number of ellipses with noisy edges in the plane. We supposed that the subset of data points coming from some ellipse satisfies the homogeneity property. For that situation, a method based on the RANSAC method is proposed, whereby the DBSCAN parameters $MinPts$ and ϵ play a significantly important role.
It is important to note that our method does not require the use of indexes for recognizing the most appropriate partition with ellipse-cluster-centers. This is the basic advantage of this method in comparison to the EDCircles method given in Akinlar and Topal (2013) and the method given in Grbić et al. (2016). Unlike our method, EDCircles does not recognize an ellipse with semi-axes (ξ, η), $\frac{\xi}{\eta} \ge 4$ and cannot detect a single ellipse with a clear edge if its shape departs significantly from a circular shape. However, our method requires more computing time than EDCircles.
The method proposed in our paper could be applied to the case of other geometrical objects too, but its application is also possible in $3D$.

Selected references

1. Akinlar, C. and Topal, C. (2013). *Edcircles: A real-time circle detector with a false detection control.* Pattern Recognition, 46: 725–740.
2. Ester, M., Kriegel, H., and Sander, J. (1996). *A density based algorithm for discovering clusters in large spatial databases with noise.* In 2nd International Conference on Knowledge Discovery and Data Mining (KDD-96), pp. 226–231, Portland.
3. Fischler, M. and Bolles, R. (1981). *Random sample consensus: a paradigm for model fitting with applications to image analysis and automated cartography.* Communications of the ACM, 24: 381–395.
4. Grbić, R., Grahovac, D., and Scitovski, R. (2016). *A method for solving the multiple ellipses detection problem.* Pattern Recognition, 60: 824–834.
5. Isack, H. and Boykov, Y. (2012). *Energy-based geometric multi-model fitting.* International Journal of Computer Vision, 97: 123–147.
6. Marošević, T. and Scitovski, R. (2015). *Multiple ellipse fitting by center-based clustering.* Croatian Operational Research Review, 6: 43–53.
7. Morales-Esteban, A., Martínez-Álvarez, F., Scitovski, S., and Scitovski, R. (2014). *A fast partitioning algorithm using adaptive Mahalanobis clustering with application to seismic zoning.* Computers & Geosciences, 73: 132–141.
8. Moshtaghi, M., Havens, T. C., Bezdek, J. C., Park, L., Leckie, C., Rajasegarar, S., Keller, J. M., and Palaniswami, M. (2011). *Clustering ellipses for anomaly detection.* Pattern Recognition, 44: 55–69.
9. Mukhopadhyay, P. and Chaudhuri, B. B. (2015). *A survey of Hough transform.* Pattern Recognition, 48:9931010.
10. Prasad, D. K., Leung, M. K. H., and Quek, C. (2013). *ElliFit: An unconstrained, non-iterative, least squares based geometric ellipse fitting method.* Pattern Recognition, 46: 1449-1465.
11. Scitovski, R. and Sabo, K. (2019). *Application of the DIRECT algorithm to searching for an optimal $k-$partition of the set $\mathcal{A}$ and its application to the multiple circle detection problem.* Journal of Global Optimization, 74(1): 63-77.
12. Scitovski, R. and Sabo, K. (2020). *DBSCAN-like clustering method for various data densities.* Pattern Analysis and Applications. DOI: 10.1007/s10044-019-00809-z.