LeArEst - The Software for Border and Area Estimation of Data Measured with Additive Error

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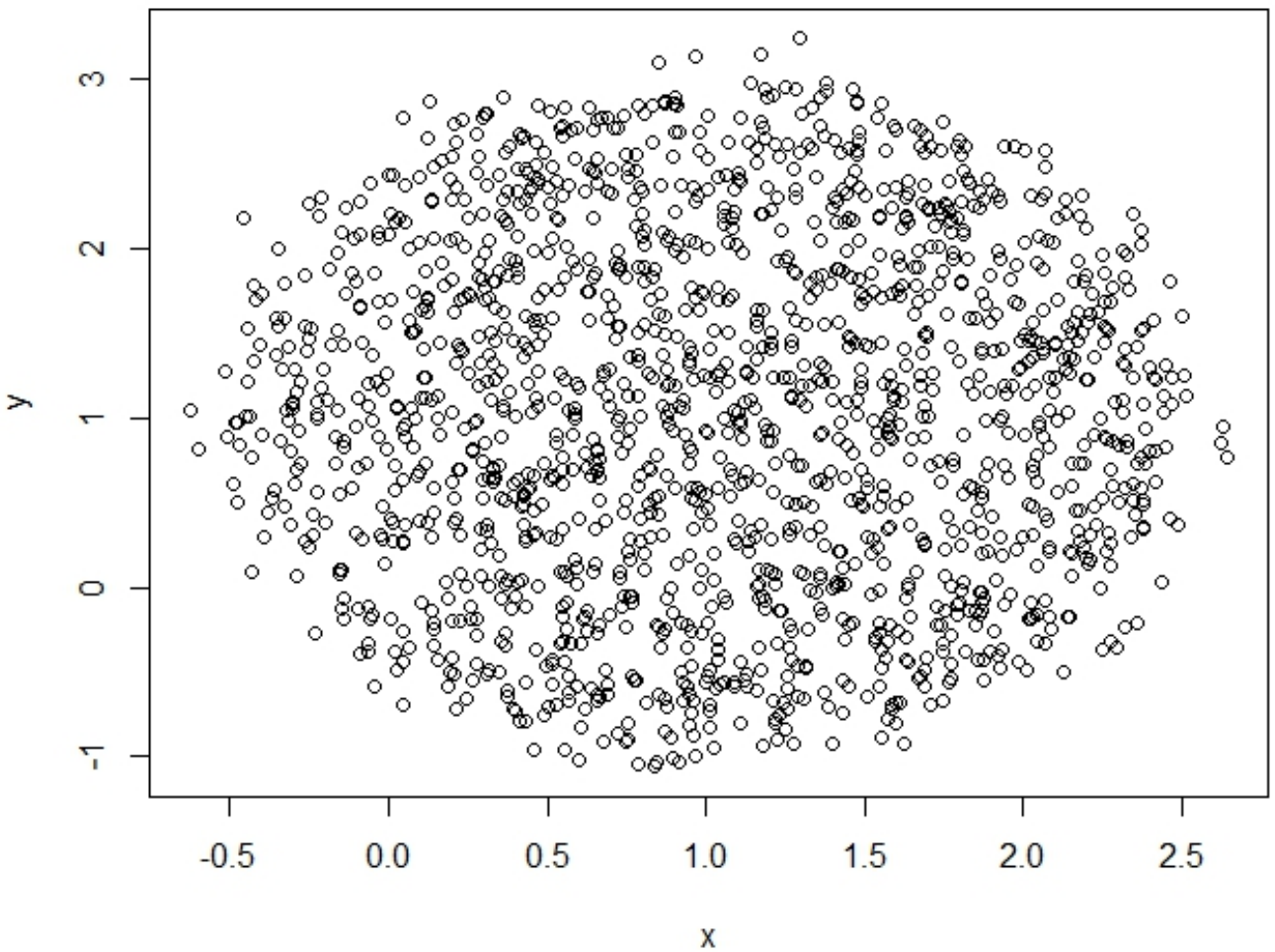
Example



Problem

$$X = U + \varepsilon$$

- U and ε independent
- U uniform on some region
- ε error

Model

$$X = U + \varepsilon$$

- U and ε independent random variables
  - U uniform on some region
    One-dimensional example:

$$f_U(x; a) = \begin{cases} \frac{1}{2a}, & x \in [-a, a] \\ 0, & otherwise \end{cases}$$

  - One-dimensional examples for ε
    - normal

$$f_\varepsilon^{(\mathcal{N})}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/(2\sigma^2)}$$

    - Laplace

$$f_\varepsilon^{(\mathcal{L})}(t) = \frac{1}{2\lambda} e^{-|x|/\lambda}$$

General one-dimensional model

- $X = U + \varepsilon$
- $$\Rightarrow \quad f_X(x; a) = \frac{1}{2a} \left(F_\varepsilon(x + a) - F_\varepsilon(x - a) \right)$$
- Data x_i, $i = 1, \ldots, m$ from independent replications of the model variable

Goals

To estimate:

- $a > 0$ (or $2a$, a length of the uniform support)
- $\sigma > 0$, error variance.

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Two-dimensional model

- reduce the original problem to several corresponding one-dimensional problems
- set of the data points: $D = \{(x_i, y_i),\ i = 1, \ldots, n\}$

Algorithm - Transformation through the y-axis

Step 1 separating through y-axis

Choose an integer $m < n$ and real numbers $\eta_1 < \eta_2 < \cdots < \eta_m$ such that

(i) $\eta_1 \leq \min\{y_i : i = 1, \ldots n\},\ \max\{y_i : i = 1, \ldots n\} \leq \eta_m$ and

(ii) $C_k := \{(x_i, y_i) \in D : y_i \in [\eta_k, \eta_{k+1}]\}$ is a nonempty set.

Two-dimensional model

Algorithm - Transformation through the y-axis; cont'd

Step 2 centering through y-axis

Let us denote

$$c_k := \frac{1}{|C_k|} \sum_{(x_i,y_i)\in C_k} x_i, \quad d_k := \frac{1}{|C_k|} \sum_{(x_i,y_i)\in C_k} y_i,$$

$$k = 1, \ldots, m-1$$

For $k = 1, \ldots, m-1$ define $\overline{C}_k := \{x_i - c_k : (x_i, y_i) \in C_k\}$.

- sets $\overline{C_k}$ represent centered tiny strips $\rightarrow$ one-dimensional model
- $\Rightarrow$ border of the domain

Some references

- H. Schneeweiss, *Estimating the endpoint of a uniform distribution under measurement errors*, CEJOR 12 (2004), 221–231.
- M. Benšić, K. Sabo, *Estimating the width of a uniform distribution when data are measured with additive normal errors with known variance*, Computational Statistics and Data Analysis, 51(2007), 4731-4741
- M. Benšić, K. Sabo, *Border estimation of a Two-dimensional Uniform Distribution if Data are Measured with Additive Error*, Statistics, 41 (2007), 4, 311–319.
- K. Sabo, M. Benšić, *Border estimation of a disc observed with random errors solved in two steps*, Journal of Computational and Applied Mathematics, 229 (2009)
- M. Benšić, K. Sabo, *Estimating a uniform distribution when data are measured with a normal additive error with unknown variance*, Statistics, 44 (2010), 235–246.
- M. Benšić, K. Sabo, *Uniform distribution width estimation from data observed with Laplace additive error*, Journal of the Korean Statistical Society, 45 (2016), 505–517.

LeArEst package

- software for border and area estimation of data measured with additive error
- package for R programming language, available on CRAN: `https://cran.r-project.org/package=LeArEst`
- border and area estimation
- objects may be defined numerically, or recorded in picture

LeArEst package - border estimation, function *lengthest()*

Function input

- vector of input data,
- error distribution (normal, Laplace, Student with 5 degrees of freedom),
- error variance or estimation method (Method of Moments, Maximum Likelihood Method),
- confidence level.

Function output

- estimated half-width of uniform distribution,
- error variance, estimated or given,
- used method for computing a confidence interval (asymptotic distribution of ML or likelihood ratio statistic).

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LeArEst package - border estimation, object on the picture

- function *startweb.esttest()* starts a web application for border estimation
- demo

LeArEst package - area estimation, function *areaest()*

Function input

- vector of two-dimensional input data,
- number of *slices* for plain data cutting,
- error distribution (normal, Laplace, Student with 5 degrees of freedom),
- error variance or estimation method (Method of Moments, Maximum Likelihood Method),
- whether to plot input data, calculated edge points and the resulting ellipse.

Function output

- estimated area of the object,
- set of calculated object's edge points,
- resulting ellipse's semi-axes.

LeArEst package - area estimation, function *areaest()*

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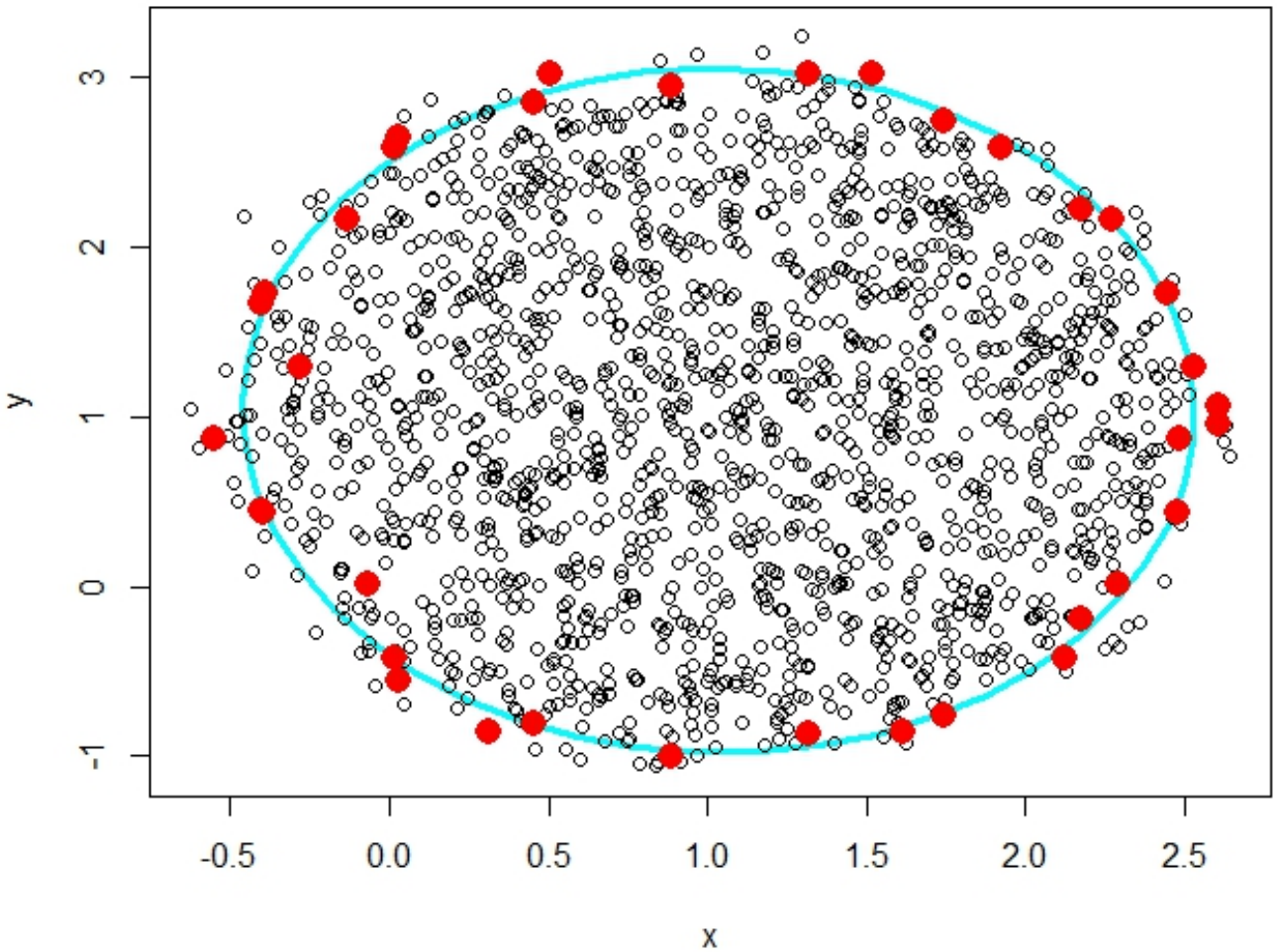

LeArEst package - area estimation, object on the picture

- function *startweb.area()* starts a web application for area estimation
- demo

Thank you for your attention!