

$$\begin{aligned}
 q_x &= \frac{d_x}{l_x} = \frac{l_x - l_{x+1}}{l_x}, & p_x &= 1 - q_x = \frac{l_{x+1}}{l_x} \\
 {}_n p_x &= \frac{l_{x+n}}{l_x} = \frac{l_{x+1}}{l_x} \frac{l_{x+2}}{l_{x+1}} \cdots \frac{l_{x+n}}{l_{x+n-1}} = p_x p_{x+1} \cdots p_{x+n-1} \\
 {}_n q_x &= 1 - {}_n p_x \\
 {}_{m|n} q_x &= {}_m p_x - {}_{m+n} p_x = \frac{l_{x+m} - l_{x+m+n}}{l_x} \\
 \mu_x &= -\frac{d}{dx} \ln(l_x) = -\frac{1}{l_x} \frac{d}{dx} l_x \\
 l_x &= \int_x^\omega l_t \mu_t dt, & l_x &= l_0 \exp\left\{-\int_0^x \mu_t dt\right\} \\
 {}_n p_x &= \exp\left\{-\int_x^{x+n} \mu_t dt\right\} = \exp\left\{-\int_0^n \mu_{x+t} dt\right\}, \quad \forall n \\
 {}_n q_x &= \int_0^n {}_t p_x \mu_{x+t} dt, \quad \forall n
 \end{aligned}$$


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$$\begin{aligned}
 D_x &= v^x l_x, & N_x &= \sum_{t=0}^{\omega-1} D_x, & N_x - D_x &= N_{x+1} \\
 a_x &= \frac{N_{x+1}}{D_x}, & \ddot{a}_x &= \frac{N_x}{D_x} \\
 a_{x:\bar{n}} &= \frac{N_{x+1} - N_{x+n+1}}{D_x}, & \ddot{a}_{x:\bar{n}} &= \frac{N_x - N_{x+n}}{D_x} \\
 {}_m | a_x &= \frac{N_{x+m+1}}{D_x}, & {}_m | \ddot{a}_x &= \frac{N_{x+m}}{D_x} \\
 {}_{m|n} | a_x &= \frac{N_{x+m+1} - N_{x+m+n+1}}{D_x}, & {}_{m|n} | \ddot{a}_x &= \frac{N_{x+m} - N_{x+m+n}}{D_x} \\
 S_{x:\bar{n}} &= \frac{D_x}{D_{x+n}} a_{x:\bar{n}}, & \ddot{S}_{x:\bar{n}} &= \frac{D_x}{D_{x+n}} \ddot{a}_{x:\bar{n}} \\
 (Ia)_x &= \frac{S_{x+1}}{D_x}, & (I\ddot{a})_x &= \frac{S_x}{D_x} \\
 a_x^{(m)} &\approx a_x + \frac{m-1}{2m}, & \ddot{a}_x^{(m)} &\approx \ddot{a}_x - \frac{m-1}{2m} \\
 a_{x:\bar{n}}^{(m)} &\approx a_{x:\bar{n}} + \frac{m-1}{2m} \left(1 - \frac{D_{x+n}}{D_x}\right), & \ddot{a}_{x:\bar{n}}^{(m)} &\approx \ddot{a}_{x:\bar{n}} - \frac{m-1}{2m} \left(1 - \frac{D_{x+n}}{D_x}\right) \\
 {}_n | a_x^{(m)} &\approx {}_n | a_x + \frac{m-1}{2m} \frac{D_{x+n}}{D_x}, & {}_n | \ddot{a}_x^{(m)} &\approx {}_n | \ddot{a}_x - \frac{m-1}{2m} \frac{D_{x+n}}{D_x} \\
 {}_n | a_x^{(m)} &\approx \frac{N_{x+n+1}}{D_x} + \frac{m-1}{2m} \cdot \frac{D_{x+n}}{D_x}, & \ddot{a}_x^{(m)} &\approx \frac{N_{x+n}}{D_x} - \frac{m-1}{2m} \cdot \frac{D_{x+n}}{D_x}
 \end{aligned}$$


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$$A_{x:\bar{n}}^1 = \frac{D_{x+n}}{D_x}, \quad A_{x:\bar{n}}^1 = \frac{M_x - M_{x+n}}{D_x}, \quad A_{x:\bar{n}} = A_{x:\bar{n}}^1 + A_{x:\bar{n}}^1 = \frac{M_x - M_{x+n} + D_{x+n}}{D_x}$$