How to split optimization and reduction cost in $\mathcal{H}_2$-optimal model order reduction

Alessandro Castagnotto*and Boris Lohmann
Chair of Automatic Control, Technical University of Munich
Boltzmannstr. 15, D-85748 Garching, Germany

Abstract

In recent years, several interpolation-based model reduction techniques have been developed to produce high quality reduced order models that minimize (locally) the approximation error, measured in the $\mathcal{H}_2$-norm [1, 2, 3, 4]. These methods are numerically efficient even for large-scale models and have been extended to different system classes, such as *differential-algebraic equations* [5], *bilinear* systems [6, 7] as well as *irrational* (and even *data-driven*) models [8], testifying to their validity.

In order to obtain an optimal set of reduction parameters (*shifts, tangential directions*), $\mathcal{H}_2$-optimal methods require the repeated reduction of the full order model to evaluate *gradients* and *Hessians* [2, 4] or *Newton steps* [1]. Therefore, the cost involved in optimizing the reduction parameters is weighted with the full cost of one reduction at every step.

In this contribution, we present a reduction framework that allows the separation of *optimization* and *reduction* cost [9]. We prove that the reduced order models obtained through this strategy are indeed $\mathcal{H}_2$-optimal and show that the cost of $\mathcal{H}_2$-reduction can be significantly reduced. The proposed procedure is not a single reduction algorithm but rather a *general framework* that can be applied to several $\mathcal{H}_2$-optimal reduction algorithms and different model classes, as the ones mentioned above.

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*Corresponding author: Tel: +49 89 289 15654, email: a.castagnotto@tum.de

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