Implementations of the Indefinite Block–Jacobi Method

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October 9, 2006

One-sided indefinite Jacobi method for computing hyperbolic singular values of rectangular matrices can be used as an accurate eigensolver for Hermitian indefinite matrices, when combined with the Hermitian indefinite factorization.

This method is traditionally regarded as slow for serial computation. In recent times block-Jacobi methods have been proposed for parallel computation. We have the opposite goal: instead of forcing data independency for different processors, we will try to reuse data in cache memory on a single processor.

The main idea of our block algorithm is to work with block-columns to take advantage of the cache memory. We propose two generalizations of the indefinite Jacobi algorithm

- block oriented algorithm,
- full block algorithm.

Both types of algorithms are performed in cycles, just like the ordinary non-blocked algorithm.

In each cycle of a block oriented algorithm we annihilate single elements of the working matrix, but the order of annihilation respects the block structure of A (for example, in block-cyclic order).

In full block algorithms we diagonalize pivotal submatrices

$$\widehat{A} = \begin{bmatrix} A_{ii} & A_{ij} \\ A_{ij}^* & A_{jj} \end{bmatrix}$$

of the working matrix A in some cyclic order.

After a thorough testing, we present some strategies which retain high relative accuracy, but run up to 50% faster then the non-blocked algorithm.