Functional limit theorem with the M_1 topology

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(joint work with Bojan Basrak[1] and Johan Segers[2])

We consider a strictly stationary sequence of random variables $(X_n)_{n\geq 1}$ with infinite second moments. Under the properties of weak dependence and regular variation with index $\alpha \in (0,2)$, the partial sum stochastic process

$$V_n(t) = a_n^{-1}(S_{\lfloor nt \rfloor} - \lfloor nt \rfloor b_n), \qquad t \in [0,1],$$

converges in distribution to an α–stable Lévy process in the space $D[0,1]$ endowed with Skorohod's M_1 topology, where $S_n = X_1 + \cdots + X_n$, $(a_n)_n$ is a sequence of positive real numbers such that $n \, \mathrm{P}(|X_1| > a_n) \to 1$ as $n \to \infty$, and $b_n = \mathrm{E}\big(X_1 \, 1_{\{|X_1| \leq a_n\}}\big)$. Here, $D[0,1]$ is the space of real-valued right continuous functions on $[0,1]$ with left limits. The limiting process is characterized in terms of its characteristic triple. This result is then applied to moving average processes.

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