M001	Obligatory - Semester 6	**Algebra**	L+P+S 2+2+0	ECTS 6

Course objectives. The objective of this course is to familiarize students with basic algebraic structures and their properties. During the lectures we will introduce and analyze the basic concepts and prove their properties and mutual links, accompanied by numerous examples, with special emphasis on the examples seen in the previously taken course. During the exercises students should adopt techniques of examining the properties of abstract algebraic structures and problem solving.

Course prerequisites. Geometry of Plane and Space, Linear Algebra I and II, Vector Spaces / Vector Spaces and Unitary Spaces

Syllabus.

1. Groups. Gruopoid, semigroup, monoid, group. Group homomorphisms and isomorphisms. Finite groups, Lagrange theorem. Normal subgroups and quotient groups, isomorphism theorems. Groups of permutations, action of groups, Cayley theorem. Cyclic groups. Sylow theorems. Solvable groups.
2. Rings and module. Rings and ideals. Quotient rings. Ring homomorphisms and isomorphisms. Skew fields and fields. Ring of polynomials. Modules and vector spaces.
3. Integral domains. Quotient fields. Maximal ideals and prime ideals. Principal ideal domains.
4. Factorial rings. Prime and irreducible elements in the rings. Factorisation in rings of polynomials, Gauss lemma and Eisenstein criterium.
5. Field extensions. Degree of an extension and finite extensions. Algebraic extensions. Minimal polynomial. Splitting fields. Finite fields. Algebraic closure. Ruler and compass constructions.
6. Galois theory. Field automorphisms. Galois group of an extension. Galois group of a polynomial. Separable polynomials and separable extensions. Normal extensions. Fundamental theorem of Galois theory. Solvability in radicals.

Expected learning outcomes.
After passing the course students are expected to:

- recognize fundamental algebraic structures;
- distinguish basic properties of certain algebraic structures;
- be able to construct an algebraic structure with the desired properties;
- identify the basis of the structural theory of finite groups;
- examine the reducibility of polynomials over a given ring;
- analyze field extensions and determine the corresponding Galois group.

Teaching methods and student assessment. Attending lectures and exercises is obligatory. The exam consists of a written and an oral part, and it can be taken after the completion of lectures and exercises. During the semester students can take mid-term exams that replace the written examination.

Can the course be taught in English? Yes.

Basic literature:

1. T.W. Hungerford, Algebra, Springer-Verlag, New York, 1974.
2. H. Kraljević, Algebra, lecture notes, 2007.

Recommended literature:

1. S. Lang, Algebra, Springer-Verlag, New York, 2002.
2. I. Stewart, Galois Theory, Chapmann and Hall, London, 2004.