

M022	Obligatory - Semester 6	Introduction to Set Theory and Mathematical Logic	L+P+S 2+2+0	ECTS 5
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Course objectives. Introduce students to the basic ideas of the two most fundamental mathematical theories: *naïve set theory*, and motivate the need for axiomatization of the theory, and *mathematical logic*. Although the students have seen and used the *language* of set theory and of mathematical logics for quite a while, certain mathematical maturity is needed for a deeper understanding and formalization of these theories.

Course prerequisites. Elementary Mathematics I and II, Differential Calculus.

Syllabus.

1. Introduction. The notion of a set. Basic operations on sets.
2. Infinite sets. Natural numbers. Transitive sets. Axiom of substitution.
3. Cartesian product. Relations and functions. Order relations.
4. Equipotent sets. Cardinal number. Axiom of choice. Countable sets. Uncountable sets. Comparison and arithmetic of cardinal numbers.
5. Ordered sets. Ordering characterization of the sets of rational and of real numbers. Zorn's lemma. Well ordered sets.
6. Ordinals. Well ordering theorem. Cardinal numbers. Continuum hypothesis.
7. Language of propositional logic. Interpretations.
8. Normal forms. Semantic trees.
9. Propositional calculus (Frege-Lukasyczev system). Consistency. Completeness and compactness theorems.

Expected learning outcomes.

After completing the course, students are expected to:

- distinguish between finite, countably infinite and uncountable sets;
- understand the essence of the basic paradoxes in set theory;
- recognize the problems of the naïve set theory which call for an axiomatization;
- identify those facts, veracity of which depends on the axiom of choice;
- understand ordinals;
- do the arithmetic with cardinal numbers;
- understand and can use the language of propositional logic.

Teaching methods and student assessment.

Attending lectures and problem sessions is compulsory for all students. The final exam, which consists of a written and an oral part, has to be taken upon the completion of all lectures and problem sessions. Acceptable mid-term exam scores replace the written examination.

Can the course be taught in English: Yes.

Basic literature:

1. P. Papić, Uvod u teoriju skupova, HMD, Zagreb, 2000.
2. M. Vuković, Matematička logika 1, lecture notes, PMF-Matematički odjel, Zagreb, 2000.

Recommended literature:

1. M. Vuković, Matematička logika, Element, Zagreb, 2009.
2. F. R. Drake, D. Singh, Intermediate Set Theory, John Wiley & Sons, 1996.
3. S. Lipschutz, Schaum's Outline of Set Theory and Related Topics, McGraw-Hill, New York, 1998.