M051	Obligatory - Semester 5	**Vector Spaces and Unitary Spaces**	L+P+S 2+2+0	ECTS 5

Course objectives. This course generalizes the concepts and results students have met in linear algebra courses during the first two years of their study. Through a more abstract algebraic approach, followed by detailed proofs of given results related to vector and unitary spaces, our aim is to understand better and more clearly the material used in most modern mathematical disciplines.

Course prerequisites. Geometry of Plane and Space, Linear Algebra I and II.

Syllabus.
1. Finite-dimensional vector spaces. Basis and dimension. Subspaces. Sum of the spaces. Quotient spaces.
2. Linear operators. Space L(V,W). Matrix of an linear operator in a pair of bases. Rank-nullity theorem. Dual operator and dual space.
3. Minimal polynomial and spectrum. Polynomial of the linear operator. Minimal polynomial. Spectrum. Characteristic polynomial. Hamilton-Cayley theorem.
4. Invariant subspaces. Projectors.
5. Nilpotent operators. Fitting decomposition. Nilpotency index. Nilpotent operators of the maximal index. Elementary Jordan chain. Decomposition of a nilpotent operator.
6. Reduction of the nilpotent operator. The greatest common divisor of polynomials and relatively prime polynomials. Decomposition of the kernel of a polynomial of the linear operator. Jordan form of the matrix of an operator.
7. Operator functions. Convergence in space L(V). Definition of f(A) for the entire function f. Representation of f(A) in Jordan basis. General definition of the function of an operator. Lagrange-Sylvester theorem. Operator f(A) as a polynomial.
8. Unitary spaces. Inner product. The Cauchy-Schwartz-Buniakowsky inequality. Ortonormed basis. Gram-Schmidt theorem. Orthogonal projection theorem. Hermitian, antihermitian, unitary and normal operators. Diagonalisation.

Expected learning outcomes.
After completing the course, students are expected to:
- adopt the concept of vector space;
- distinguish between the concept of the matrix and the concept of the linear operator;
- determine the spectrum of an operator;
- deterrmine the Jordan form of the matrix of an operator and functions of an operator;
- study properties of an inner product and recognize unitary spaces and normed spaces;
- understand the importance of ortonormed basis and be able to apply Gram-Schmidt theorem;
- adopt the concept and properties of hermitian operators.

Teaching methods and student assessment. Attendance at lectures and exercises is required. The exam consists of a written and an oral part, and it can be taken after the completion of lectures and exercises. During the semester students can take mid-term exams that replace the written examination.

Can the course be taught in English? Yes.

Basic literature:

1. H. Kraljević, Vektorski prostori, Odjel za matematiku, Sveučilište u Osijeku, 2005.
2. S. Kurepa, Konačno dimenzionalni vektorski prostori i primjene, Liber, Zagreb, 1992.

Recommended literature:

1. D. M. Bloom, Linear algebra and geometry, Cambridge Univ. Press, 1988.
2. S. Lang, Linear algebra, Springer Verlag, Berlin- Heidelberg-New York, 2004.

S. Lang, Algebra, Springer Verlag, Berlin-Heidelberg-New York, 2002.