| | | | | |
|---|---|---|---|---|
| M065 | Elective (Semester 1 MR and Fin+Stat, Semester 7, MEd study programme) | **Convex Sets** | L+P+S 1+0+1 | ECTS 4 |

**Course objective.** The main goal of the course is to give an introduction to the subjects of convexity, linear programming and convex optimization.

**Course prerequisites.** Linear Algebra I, Linear Algebra II, Differential Calculus, Integral Calculus.

**Syllabus.**

1. Introduction. Motivating examples from linear programming, approximation theory and mathematical statistics.
2. Geometry of convex sets. Definition and operations with convex sets. Convex cone. Convex polyhedron. Linear programming (LP) and polyhedra. Polytopes. LP and polytopes. Convex hull. LP and convex hull. Affine hull. Simplex. Conical hull. LP and conical hull. Carathéodory's theorem for convex sets. Carathéodory's theorem for convex cone.
3. Convex sets and topology. Relative interior. Relative boundary.
4. Projection and separation. Projection onto a convex set. The linear least squares problem. Supporting hyperplane. Strong and weak separation of convex sets. Farkas' lemma.
5. Representation of convex sets. Faces. Edges. Extreme rays. Extreme points. The recession cone. Minkowski's theorem for compact convex sets. Polytopes and polyhedra. *Euler's formula for convex* polyhedra.

**Expected learning outcomes:**

After completing the course, students are expected to:

- know the basic concepts and properties of convex sets;,
- understand well Carathéodory's theorems for convex sets and convex cones, topology of convex sets, theorems of projection onto a convex set, the separation theorems and the representation of convex sets;
- become familiar with the concepts of linear programming, convex optimization and the least squares method;
- gain knowledge required for many other courses like Linear Programming, Convex Functions and Optimization Methods.

**Teaching methods and student assessment.** The course is delivered in form of lectures and seminars. Participation in lectures and seminars is obligatory. Each student must write a seminar paper and present it to other students. The written exam follows after a successful presentation of the seminar paper.

**Can the course be taught in English:** No.

**Basic literature**:

1. G. Dahl, An Introduction to Convexity, University of Oslo, Oslo, 2010. Available at: **http://heim.ifi.uio.no/~geird/conv.pdf**

**Recommended literature:**

1. R.T. Rockafellar, Convex Analysis, Princeton University Press, New York, 1997.
2. J.-B. Hiriart-Urruty and C. Lemaréchal, Convex Analysis and Minimization Algorithms I, Springer, 1993.
3. R. J. Vanderbei, Linear Programming - Foundations and Extensions, Kluwer Academic Publ, 2001.