M066	Obligatory - Semester 5	**Vector Spaces**	L+P+S 2+2+0	ECTS 5

Course objectives. This course generalizes the concepts and results students have met in linear algebra courses during the first two years of their study. Through a more abstract algebraic approach, followed by detailed proofs of given results related to vector spaces, our aim is to understand better and more clearly the material used in most modern mathematical disciplines.

Course prerequisites. Geometry of Plane and Space, Linear Algebra I and II.

Syllabus.
1. Finite-dimensional vector spaces. Basis and dimension. Subspaces. Sum of the spaces.
2. Unitary spaces. Inner product. The Cauchy-Schwartz-Buniakowsky inequality. Ortonormed basis. Gram-Schmidt theorem.
3. Linear operators. Space L(V,W). Matrix of a linear operator in a pair of bases. Rank-nullity theorem. Dual operator and dual space.
4. Minimal polynomial and spectrum. Polynomial of the linear operator. Minimal polynomial. Spectrum. Characteristic polynomial. Hamilton-Cayley theorem.
5. Invariant subspaces. Projectors.
6. Nilpotent operators. Fitting decomposition. Nilpotency index. Nilpotent operators of the maximal index. Elementary Jordan chain. Decomposition of a nilpotent operator.
7. Reduction of the nilpotent operator. The greatest common divisor of polynomials and relatively prime polynomials. Decomposition of the kernel of a polynomial of linear operator. Jordan form of the matrix of an operator.
8. Structure of bilinear forms. Symmetric, alternating and hermitian forms. Symmetric forms and orthogonal basis. Sylvester theorem for symmetric forms over real vector spaces.

Expected learning outcomes.
After completing the course, students are expected to:
- adopt the concept of vector space and unitary spaces;
- examine linear independence of a set of vectors;
- distinguish between the concept of the matrix and the concept of a linear operator;
- determine the spectrum of an operator;
- determine the Jordan form of the matrix of an operator and functions of an operator;
- understand the concept and importance of bilinear forms.

Teaching methods and student assessment. Attendance at lectures and exercises is required. The exam consists of a written and an oral part, and it can be taken after the completion of lectures and exercises. During the semester students can take mid-term exams that replace the written examination.

Can the course be taught in English? Yes.

Basic literature:
1. H. Kraljević, Vektorski prostori, Odjel za matematiku, Sveučilište u Osijeku, 2005.
2. S. Kurepa, Konačno dimenzionalni vektorski prostori i primjene, Liber, Zagreb, 1992.

Recommended literature:
1. D. M. Bloom, Linear algebra and geometry, Cambridge Univ. Press, 1988.
2. S. Lang, Linear algebra, Springer Verlag, Berlin- Heidelberg-New York, 2004.
S. Lang, Algebra, Springer Verlag, Berlin-Heidelberg-New York, 2002.