M070	FIN, MR, IPM - elective – Year 2	**Introduction to Algebraic Topology**	L+P+S 2+0+2	ECTS 6

Course objectives. Topology, quite an abstract branch of mathematics, has recently become important in the theory of dynamical systems, the qualitative theory of partial differential equations, physics, robotics, the theory of nets, and elsewhere. The purpose of this course is to teach students about manifolds, in particular surfaces and their classification, and to introduce the fundamental group - one of the basic topological invariants.

Course prerequisites. Bachelor degree in mathematics or Bachelor degree in mathematics and computer science education.

Desirable prerequisites. Metric Spaces and/or Real Analysis, Algebra

Syllabus.

1. Manifolds. Examples of surfaces, i.e., compact connected 2-manifolds.
2. Classification theorem for compact surfaces.
3. Euler characteristics. Manifolds with boundary. Classification of compact 2-manifolds with boundary.
4. Definition of the fundamental group of a topological space. Fundamental group of the circle.
5. Applications: Brouwer's fixed point theorem in dimension two. $\mathbf{R}^2$ is not homeomorphic to $\mathbf{R}^n$ for $n\neq 2$.
6. Homotopy type and homotopy equivalence.
7. Free groups and the free product of groups. Generators and relators.
8. Seifert-van Kampen's theorem.
9. Covering spaces.

Expected learning outcomes.

After completing the course, students should be able to:

- classify orientable and nonorientable closed surfaces;
- provide a valid argument that $\mathbf{R}^2$ and $\mathbf{R}^n$ are not homeomorphic for $n\neq 2$;
- find the fundamental group of the circle, torus and projective plain;
- use the Seifert-van Kampen's theorem to recognize homotopy non-equivalent spaces;
- create covering spaces of closed graphs;
- present the acquired knowledge to a wide audience as well as to experts.

Teaching methods and student assessment.

All students are obliged to attend lectures and actively participate in seminars. Each student will have to study one or two particular topics which expand upon the material covered in lectures, or prove some of the assertions which have not been proved in lectures, and present them to other students in seminars. The final grade is based upon these seminar presentations.

Can the course be taught in English: Yes.

Basic literature:

1. J.Gallier and D.Xu, A guide to the Classification Theorem for Compact Surfaces
http://www.cis.upenn.edu/~jean/gbooks/surftop.html
2. W.S.Massey, Algebraic Topology: An Introduction, Springer-Verlag, 1977

Recommended literature:

1. A.Hatcher: Algebraic Topology, Cambridge University Press, Cambridge, 2002
http://www.math.cornell.edu/~hatcher/AT/ATpage.html
2. J.R.Munkres: Topology, Second Edition, Prentice Hall, 2000