M139	**Calculus of variations**	L	P	S	ECTS
		2	2	0	6

Course objectives. To familiarize students with the calculus of variations, with special attention on its applications.

Prerequisites. Knowledge of basic results of the real analysis and differential equations.

Course content.

1. Introduction: basics of the calculus of variations and motivational problems for its introduction. Historical background.
2. Necessary conditions for the existence of the extreme point of the corresponding functionals (classical methods): Euler-Lagrange equation and its generalizations to vector functions and functions of multiple variables, Erdmann necessary condition, transversality conditions, the Legendre necessary condition, the Weierstrass necessary condition, Jacobi's necessary condition.
3. Sufficient conditions, global extreme points and convexity of the Lagrangian.
4. Direct methods of the calculus of variations, Dirichlet integral, a general existence theorem, Euler-Lagrange equations for direct methods.
5. Applications of the calculus of variations.

LEARNING OUTCOMES

No.	LEARNING OUTCOMES
1.	Identify the real world problems which can be solved by usage of the calculus of variations.
2.	Explain reasons for introduction of calculus of variations, its basic idea and historical background.
3.	Explain fundamental concepts, such as Lagrangian, as well as the first and the second variation of the functional.
4.	Investigate the existence of the extreme point of the given functional, and conclude is it local or global extreme point.
5.	Explain classical and direct methods of the calculus of variations, and their similarities and differences.
6.	Model a real world problems and solve them by application of the calculus of variations.
7.	Formulate conjectures related to the subject, and prove or deny them.

RELATING THE LEARNING OUTCOMES, ORGANIZATION OF THE EDUCATIONAL PROCESS AND ASSESSMENT OF THE LEARNING OUTCOMES

TEACHING ACTIVITY	ECTS	LEARNING OUTCOME **	STUDENT ACTIVITY*	EVALUATION METHOD	POINTS	
					min	max
Attending lectures	1	1-7	Lecture attendance, discussion, team work and independent work on given tasks	Attendance lists, tracking activities	0	4
Written exam (Mid-terms)	2	1-7	Preparing for written exam	Evaluation	25	48
Final exam	3	1-7	Revision	Oral exam	25	48
TOTAL	6				50	100

Teaching methods and student assessment. Lectures and exercises are obligatory. The exam consists of a written and an oral part. Upon completion of the course, students can take the exam. Successful midterm exam scores replace the written exam.

Can the course be taught in English: Yes

Basic literature:

1. U. Brechtken - Manderscheid, Introduction to the calculus of variations, First Edition, CRC Press, 1991.
2. B. Dacorogna, Introduction to the calculus of variations, third ed., Imperial College Press, London, 2015.

Recommended literature:

1. H. Brezis, Functional analysis, Sobolev spaces and partial differential equations, Universitext, Springer, New York, 2011.
2. J. A. Burns, Introduction to the calculus of variations and control with modern applications, CRC Press, 2013.
3. B. Dacorogna, Direct methods in the calculus of variations, 2ed., Springer, 2008.
4. L. C. Evans, Partial differential equations, AMS, 1998.
5. I. M. Gelfand, S. V. Fomin, Calculus of variations, 2ed., Courier Corporation, 2012.
6. J. L. Troutman, Variational calculus and optimal control, 2ed., Springer, New York, 1995.