

ZADACI ZA SAMOSTALNO RJEŠAVANJE: LINEARNA ALGEBRA

MATRICE

ZADATAK 1. Zbrojite matrice

(a)

$$\mathbf{A} = \begin{bmatrix} 15 & 3 & 5 \\ 4 & 5 & 9 \\ 3 & -4 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 25 & 4 & 3 \\ -4 & -2 & 3 \\ 2 & 5 & 1 \end{bmatrix},$$

(b)

$$\mathbf{C} = \begin{bmatrix} 5 & -3 & 2 \\ 7 & 4 & 3 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} 7 & 8 & 5 \\ -2 & -9 & 7 \end{bmatrix}.$$

Rješenje.

$$(a) \quad \mathbf{A} + \mathbf{B} = \begin{bmatrix} 40 & 7 & 8 \\ 0 & 3 & 12 \\ 5 & 1 & 2 \end{bmatrix}.$$

$$(b) \quad \mathbf{C} + \mathbf{D} = \begin{bmatrix} 12 & 5 & 7 \\ 5 & -5 & 10 \end{bmatrix}.$$

ZADATAK 2. Skalarima $\alpha = -1$ i $\alpha = 5$ pomnožite matricu

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 5 & -4 & 9 \\ 54 & 67 & 7 \end{bmatrix}.$$

Rješenje.

$$-1 \cdot \mathbf{A} = \begin{bmatrix} -1 & -2 & -3 \\ -5 & 4 & -9 \\ -54 & -67 & -7 \end{bmatrix}, \quad 5 \cdot \mathbf{A} = \begin{bmatrix} 5 & 10 & 15 \\ 25 & -20 & 45 \\ 270 & 335 & 35 \end{bmatrix}.$$

ZADATAK 3. Za matrice

$$\mathbf{A} = \begin{bmatrix} 3 & 9 \\ 1 & -3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 10 & 4 \\ 19 & 22 \end{bmatrix},$$

izračunajte:

a) $\mathbf{A} + \mathbf{B}$, b) $\mathbf{A} - \mathbf{B}$, c) \mathbf{AB} , d) \mathbf{BA} .

Rješenje.

$$a) \quad \mathbf{A} + \mathbf{B} = \begin{bmatrix} 13 & 12 \\ 20 & 19 \end{bmatrix}.$$

$$b) \quad \mathbf{A} - \mathbf{B} = \begin{bmatrix} -7 & 5 \\ -18 & -25 \end{bmatrix}.$$

$$c) \mathbf{AB} = \begin{bmatrix} 201 & 210 \\ -47 & -62 \end{bmatrix}.$$

$$d) \mathbf{BA} = \begin{bmatrix} 34 & 78 \\ 79 & 105 \end{bmatrix}.$$

ZADATAK 4. Za matrice

$$\mathbf{A} = \begin{bmatrix} 4 & 3 & 5 \\ -1 & 55 & 5 \\ 5 & 2 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -2 & 5 & 1 \\ 7 & -9 & 7 \\ 7 & 3 & 2 \end{bmatrix},$$

izračunajte:

a) $\mathbf{A} + \mathbf{B}$, b) $\mathbf{A} - \mathbf{B}$, c) \mathbf{AB} i d) \mathbf{BA} .

Rješenje.

$$a) \mathbf{A} + \mathbf{B} = \begin{bmatrix} 2 & 8 & 6 \\ 6 & 46 & 12 \\ 12 & 5 & 3 \end{bmatrix}.$$

$$b) \mathbf{A} - \mathbf{B} = \begin{bmatrix} 6 & -2 & 4 \\ -8 & 64 & -2 \\ -2 & -1 & -1 \end{bmatrix}.$$

$$c) \mathbf{AB} = \begin{bmatrix} 48 & 8 & 35 \\ 422 & -485 & 394 \\ 11 & 10 & 21 \end{bmatrix}.$$

$$d) \mathbf{BA} = \begin{bmatrix} -8 & 271 & 16 \\ 72 & -460 & -3 \\ 35 & 190 & 52 \end{bmatrix}.$$

ZADATAK 5. Izračunajte sljedeće produkte matrica

$$a) \begin{bmatrix} 4 & 5 & 3 \\ 9 & -7 & -3 \end{bmatrix} \begin{bmatrix} 12 & 21 \\ 5 & -1 \\ 2 & 1 \end{bmatrix},$$

$$b) [3 \quad -4 \quad 5] \begin{bmatrix} 1 \\ 1 \\ 10 \end{bmatrix} \text{ i}$$

$$c) \begin{bmatrix} 2 & 11 & 1 \\ 0 & 9 & -3 \\ 1 & 0 & -7 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 19 & -2 & 1 \\ -1 & 3 & 4 \end{bmatrix}.$$

Rješenje.

$$a) \begin{bmatrix} 4 & 5 & 3 \\ 9 & -7 & -3 \end{bmatrix} \begin{bmatrix} 12 & 21 \\ 5 & -1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 79 & 82 \\ 67 & 193 \end{bmatrix}.$$

$$\text{b) } [3 \quad -4 \quad 5] \begin{bmatrix} 1 \\ 1 \\ 10 \end{bmatrix} = [49].$$

$$\text{c) } \begin{bmatrix} 2 & 11 & 1 \\ 0 & 9 & -3 \\ 1 & 0 & -7 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 19 & -2 & 1 \\ -1 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 210 & -19 & 13 \\ 174 & -27 & -3 \\ 8 & -21 & -29 \end{bmatrix}.$$

NEKE SPECIJALNE MATRICE

ZADATAK 6. Izračunajte trag sljedećih matrica:

$$\text{a) } \mathbf{A} = \begin{bmatrix} 10 & 12 \\ 3 & -11 \end{bmatrix}, \text{ b) } \mathbf{B} = \begin{bmatrix} 14 & 3 & -4 \\ 6 & 65 & 6 \\ 5 & 56 & -33 \end{bmatrix}, \text{ c) } \mathbf{C} = \begin{bmatrix} -1 & 4 & 54 & 3 \\ 8 & 4 & 6 & 7 \\ 7 & 4 & -3 & 66 \\ -2 & 13 & -98 & 1 \end{bmatrix}.$$

Rješenje.

- a) $\text{tr}\mathbf{A} = -1$,
- b) $\text{tr}\mathbf{B} = 46$,
- c) $\text{tr}\mathbf{C} = 1$.

ZADATAK 7. Provjerite jesu li sljedeće matrice simetrične, antisimetrične ili niti jedno od navedenog.

$$\text{a) } \mathbf{A} = \begin{bmatrix} 10 & 4 & -9 \\ 4 & 3 & 1 \\ -9 & 1 & -15 \end{bmatrix}, \text{ b) } \mathbf{B} = \begin{bmatrix} 0 & -2 & 4 \\ 2 & 0 & -15 \\ -4 & 15 & 0 \end{bmatrix}, \text{ c) } \mathbf{C} = \begin{bmatrix} -1 & 45 & -8 \\ -6 & 0 & 15 \\ 4 & 1 & 0 \end{bmatrix}.$$

Rješenje.

- a) Matrica \mathbf{A} je simetrična.
- b) Matrica \mathbf{B} je antisimetrična.
- c) Matrica \mathbf{C} nije niti simetrična, niti antisimetrična.

REGULARNE MATRICE

ZADATAK 8. Gaussovom metodom nađite inverz sljedećih matrica:

$$\text{a) } \mathbf{A} = \begin{bmatrix} 3 & 2 \\ -3 & 5 \end{bmatrix}, \text{ b) } \mathbf{B} = \begin{bmatrix} 1 & -3 \\ 6 & 4 \end{bmatrix}, \text{ c) } \mathbf{C} = \begin{bmatrix} 4 & 5 \\ 1 & -6 \end{bmatrix},$$

$$\text{d) } \mathbf{D} = \begin{bmatrix} 10 & 0 & -2 \\ 1 & 3 & 6 \\ 5 & 7 & 2 \end{bmatrix}, \text{ e) } \mathbf{E} = \begin{bmatrix} 1 & 5 & 4 \\ 6 & 3 & 2 \\ 1 & -2 & 1 \end{bmatrix}.$$

Rješenje.

$$\begin{aligned} \text{a) } \mathbf{A}^{-1} &= \frac{1}{21} \begin{bmatrix} 5 & -2 \\ 3 & 3 \end{bmatrix}. \\ \text{b) } \mathbf{B}^{-1} &= \frac{1}{22} \begin{bmatrix} 4 & 3 \\ -6 & 1 \end{bmatrix}. \\ \text{c) } \mathbf{C}^{-1} &= \frac{1}{19} \begin{bmatrix} 6 & -5 \\ -1 & 4 \end{bmatrix}. \\ \text{d) } \mathbf{D}^{-1} &= \frac{1}{172} \begin{bmatrix} 18 & 7 & -3 \\ -14 & -15 & 31 \\ 4 & 35 & -15 \end{bmatrix}. \\ \text{e) } \mathbf{E}^{-1} &= \frac{1}{73} \begin{bmatrix} -7 & 13 & 2 \\ 4 & 3 & -22 \\ 15 & -7 & 27 \end{bmatrix}. \end{aligned}$$

SUSTAV LINEARNIH ALGEBARSKIH JEDNAŽBI

ZADATAK 9. Sljedeće sustave linearnih algebarskih jednadžbi prikažite grafički, te iz slike odredite rješenja i objasnite njihov geometrijski smisao:

$$\text{a) } \begin{cases} 3x_1 + x_2 = 10 \\ x_1 + x_2 = 2 \end{cases}, \text{ b) } \begin{cases} -4x_1 + 2x_2 = 1 \\ -8x_1 + 4x_2 = 7 \end{cases}.$$

Rješenje.

a) $(x_1, x_2) = (4, -2)$.

b) Sustav nema rješenje.

GAUSSOVA METODA ELIMINACIJE ZA RJEŠAVANJE SUSTAVA LINEARNIH ALGEBARSKIH JEDNAŽBI

ZADATAK 10. Gaussovom metodom eliminacije riješite sljedeće sustave:

$$\text{a) } \begin{cases} 2x_1 + 3x_2 + 4x_3 = 1 \\ x_1 + 4x_2 + 5x_3 = -2 \\ 3x_1 - 2x_2 + 4x_3 = 5 \end{cases}, \text{ b) } \begin{cases} -x_1 + 2x_2 + 8x_3 = 4 \\ 6x_1 + 3x_2 + 2x_3 = 1 \\ 4x_1 + 6x_2 + 9x_3 = 2 \end{cases}.$$

Rješenje.

a) $(x_1, x_2, x_3) = \left(\frac{61}{29}, \frac{-11}{29}, \frac{-15}{29}\right)$.

b) $(x_1, x_2, x_3) = \left(\frac{10}{17}, \frac{-25}{17}, \frac{16}{17}\right)$.

DETERMINANTE

ZADATAK 11. Izračunajte sljedeće determinante matrica prvog i drugog reda:

$$\text{a) } \mathbf{A} = \begin{bmatrix} -9 \end{bmatrix}, \text{ b) } \mathbf{B} = \begin{bmatrix} 25 \end{bmatrix}, \text{ c) } \mathbf{C} = \begin{bmatrix} 3 & -9 \\ 15 & -4 \end{bmatrix},$$

$$\text{d) } \mathbf{D} = \begin{bmatrix} 1 & -34 \\ 12 & 19 \end{bmatrix}, \text{ e) } \mathbf{E} = \begin{bmatrix} -2 & 4 \\ 7 & 9 \end{bmatrix}.$$

Rješenje.

$$\text{a) } \det \mathbf{A} = -9.$$

$$\text{b) } \det \mathbf{B} = 25.$$

$$\text{c) } \det \mathbf{C} = 123.$$

$$\text{d) } \det \mathbf{D} = 427.$$

$$\text{e) } \det \mathbf{E} = -46.$$

ZADATAK 12. Izračunajte sljedeće determinante matrica trećeg reda:

$$\text{a) } \mathbf{A} = \begin{bmatrix} 4 & 5 & 6 \\ 0 & 8 & -5 \\ 0 & 0 & 10 \end{bmatrix}, \text{ b) } \mathbf{B} = \begin{bmatrix} 12 & 12 & 8 \\ 123 & 123 & 3 \\ 3 & 3 & 156 \end{bmatrix}, \text{ c) } \mathbf{C} = \begin{bmatrix} -3 & -9 & 4 \\ 9 & 27 & 9 \\ 6 & 18 & -15 \end{bmatrix},$$

$$\text{d) } \mathbf{D} = \begin{bmatrix} 1 & 22 & -3 \\ 5 & 8 & 9 \\ -2 & 0 & 1 \end{bmatrix}, \text{ e) } \mathbf{E} = \begin{bmatrix} 7 & 0 & 3 \\ -5 & -22 & 19 \\ 2 & 5 & -1 \end{bmatrix}, \text{ f) } \mathbf{F} = \begin{bmatrix} 4 & 3 & 2 \\ 1 & 0 & 9 \\ 8 & 7 & 6 \end{bmatrix}.$$

Rješenje.

$$\text{a) } \det \mathbf{A} = 320.$$

$$\text{b) } \det \mathbf{B} = 0.$$

$$\text{c) } \det \mathbf{C} = 0.$$

$$\text{d) } \det \mathbf{D} = -546.$$

$$\text{e) } \det \mathbf{E} = -454.$$

$$\text{f) } \det \mathbf{F} = -40.$$

ZADATAK 13. Izračunajte

$$\text{a) } \det \mathbf{AB}, \text{ b) } \det \mathbf{CD}, \text{ c) } \det \mathbf{EF},$$

pri čemu su \mathbf{A} , \mathbf{B} , \mathbf{C} , \mathbf{D} , \mathbf{E} i \mathbf{F} , matrice iz prethodnog zadatka.

Rješenje.

$$\text{a) } \det \mathbf{AB} = 0.$$

$$\text{b) } \det \mathbf{CD} = 0.$$

$$\text{c) } \det \mathbf{EF} = 18160.$$

CRAMEROVO PRAVILO

ZADATAK 14. Primjenom Cramerovog pravila diskutirajte rješenja sljedećih sustava jednačbi u ovisnosti o parametru $\lambda \in \mathbb{R}$

$$(a) \begin{cases} \lambda x_1 + x_2 = \lambda \\ x_1 + \lambda x_2 = 1 \end{cases},$$

$$(b) \begin{cases} \lambda x_1 + 5x_2 = 1 \\ 5x_1 + \lambda x_2 = 1 \end{cases}.$$

Rješenje.

(a) Za $\lambda \in \mathbb{R} \setminus \{-1, 1\}$ sustav ima jedinstveno rješenje, a za $\lambda \in \{-1, 1\}$ sustav ima beskonačno rješenja.

(b) Za $\lambda \in \mathbb{R} \setminus \{-5, 5\}$ sustav ima jedinstveno rješenje, za $\lambda = 5$ sustav ima beskonačno rješenja, a za $\lambda = -5$ sustav nema rješenje.

ZADATAK 15. Cramerovim pravilom riješite sljedeće sustave:

$$a) \begin{cases} 3x_1 + 4x_2 - 3x_3 = 0 \\ -2x_1 + x_2 + 4x_3 = 5 \\ 7x_1 + 8x_2 + 9x_3 = 2 \end{cases}, \quad b) \begin{cases} x_1 - x_2 + 6x_3 = 4 \\ 2x_1 + x_3 = 1 \\ 4x_1 + 2x_2 - 5x_3 = 3 \end{cases}.$$

Rješenje.

$$a) (x_1, x_2, x_3) = \left(\frac{-131}{92}, \frac{57}{46}, \frac{21}{92} \right).$$

$$b) (x_1, x_2, x_3) = \left(\frac{-1}{2}, \frac{15}{2}, 2 \right).$$