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On the univalence of integral operations involving meromorphic functions

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Abstract. The main object of this paper is to give sufficient conditions for integral operators $\mathcal{H}_{\alpha,\beta,\gamma}$ and $\mathcal{G}_{\lambda,\mu}$, which are defined here by means of the meromorphic functions, to be univalent in the open unit disk. In particular cases, we find the corresponding simpler conditions for these integral operators.
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1. Introduction and preliminaries

Let $\mathcal{A}$ be the class the functions $f(z)$ which are analytic in the open unit disk

$$\mathcal{U} = \{z : |z| < 1\} \text{ and } f(0) = f'(0) - 1 = 0.$$

We show by $\mathcal{S}$ the subclass of $\mathcal{A}$ consisting of functions $f \in \mathcal{A}$ which are univalent in $\mathcal{U}$. Let $\widetilde{\sum}$ denote the class of functions $F(z)$ of the form

$$F(z) = \frac{1}{z} + \sum_{n=0}^{\infty} b_n z^n \tag{1}$$

which are *analytic* and *univalent* in the punctured unit disk

$$\mathcal{U}_0 = \{z :\ 0 < |z| < 1\} = \mathcal{U} - \{0\}.$$

We denote by $\widetilde{\sum}_0$ the class of functions G in $\widetilde{\sum}$ such that $G(z) \neq 0$ for all z in $\mathcal{U}_0$.

Let α, β and γ be any complex numbers. Let us denote by $\mathcal{H}_{\alpha,\beta,\gamma}$ the analytic function in $\mathcal{U}$ defined by the formula:

$$\mathcal{H}_{\alpha,\beta,\gamma}(z) = \left(\gamma \int_0^z t^{\gamma-1}(f'(t))^{\alpha}(t^{-1}g(t))^{\beta} dt\right)^{1/\gamma}, \tag{2}$$

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where f and g are functions of the class $\mathcal{S}$ or one of its subclasses. The problem of univalence of the function $\mathcal{H}_{\alpha,\beta,\gamma}$ in $\mathcal{U}$ for special cases of parameters $\alpha,\ \beta,\ \gamma$ and functions f and g were discussed by many authors, such as [2, 6, 7, 10, 12, 15, 17, 18, 20]. Furthermore, many authors recently (see [1, 3, 8, 9, 14, 16, 19]) have obtained various sufficient conditions for the univalence of generalized integral operators of type (2) for $f, g \in \mathcal{A}$.

In the present investigation, we introduce integral operators $\mathcal{F}_{\alpha,\beta,\gamma}$ and $\mathcal{G}_{\lambda,\mu}$ as follows;

$$\mathcal{F}_{\alpha,\beta,\gamma}(z) = \left(\gamma \int_0^z t^{\gamma-1}(-t^2F'(t))^{\alpha}(tG(t))^{\beta}dt\right)^{1/\gamma} \tag{3}$$

and

$$\mathcal{G}_{\lambda,\mu}(z) = \left(\mu \int_0^z t^{\mu-1}\left(e^{t^2G(t)}\right)^{\lambda} dt\right)^{1/\mu}, \tag{4}$$

where $F \in \widetilde{\sum}$ and $G \in \widetilde{\sum}_0$ and $\alpha,\ \beta,\ \gamma,\ \lambda$ and μ are complex numbers such that integrals (3) and (4) exist.

Remark 1. *In its special cases when $\gamma = 1$, the integral operator in (3) would obviously reduce to the integral operator $\mathcal{F}_{\alpha,\beta}$ which is defined in [20] as follows*

$$\mathcal{F}_{\alpha,\beta}(z) = \int_0^z (-t^2F'(t))^{\alpha}(tG(t))^{\beta}dt, \tag{5}$$

where $\alpha, \beta \in \mathbb{C}$, $F \in \widetilde{\sum}$ and $G \in \widetilde{\sum}_0$.

In our paper, we are mainly interested in some integral operators of types (3) and (4) which involve meromorphic functions. More precisely, we would like to show that by using some inequalities for the functions belonging to the class $\mathcal{S}$, the univalence of some integral operators which involve meromorphic functions can be derived easily via a well-known univalence criterion. In particular, we obtain simple sufficient conditions for some integral operators which involve special cases of parameters $\alpha,\ \beta,\ \gamma,\ \lambda$ and μ. We also extend and improve the aforementioned result of Wesolowski [20]. At least in some cases, our main results are stronger than the result obtained in [20].

In the proofs of our main results we need the following interesting univalence criteria.

Lemma 1 (See [13])**.** *Let $h \in \mathcal{A}$ and $\gamma \in \mathbb{C}$. If $\Re(\gamma) > 0$ and*

$$\frac{1-|z|^{2\Re(\gamma)}}{\Re(\gamma)}\left|\frac{zh''(z)}{h'(z)}\right| \leq 1, \qquad z \in \mathcal{U},$$

then the function $H_{\gamma}(z)$ given by

$$\mathcal{H}_{\gamma}(z) = \left(\gamma \int_0^z t^{\gamma-1}h'(t)dt\right)^{1/\gamma}, \quad z \in \mathcal{U}, \tag{6}$$

is in the class $\mathcal{S}$.

Lemma 2 (See [4]). *Let γ and c be complex numbers such that*

$$|\gamma-1|<1 \text{ and } |c|\leqslant 1 \ \ (c\neq -1)\,.$$

If the function $h\in\mathcal{A}$ satisfies the following inequality:

$$\left|c\,|z|^2+(1-|z|^2)\left[(\gamma-1)+\frac{zh''(z)}{h'(z)}\right]\right|\leqslant 1, \quad z\in\mathcal{U},$$

then the function $\mathcal{H}_\gamma$ defined by (6) is in the class $\mathcal{S}$.

We note that $\mathcal{H}_{1,0,\gamma}(z)=\mathcal{H}_\gamma(z)$.

The following lemma is of fundamental importance to our investigation.

Lemma 3 (See [11]). *For each function $f\in\mathcal{S}$ and a fixed $z,\ z\in\mathcal{U}$; the inequality*

$$\left|\frac{z}{f(z)}-1\right|\leq 2\,|z|+|z|^2$$

holds.

2. Univalence condition associated with the integral operator (3)

Our first main result is an application of Lemma 1 and it contains sufficient conditions for a general integral operator $\mathcal{F}_{\alpha,\beta,\gamma}$ of type (3).

Theorem 1. *Let $\alpha,\ \beta$ and γ with $\Re(\gamma)>0$ be any complex numbers. Also let $F\in\widetilde{\sum}$ and $G\in\widetilde{\sum}_0$. Moreover, suppose that the following inequalities*

$$\begin{aligned} 10\,|\alpha|+4\,|\beta|&\leq 1, \quad \text{for} \quad \Re(\gamma)\geq 1 \\ \frac{10\,|\alpha|+4\,|\beta|}{\Re(\gamma)}&\leq 1, \quad \text{for} \quad 0<\Re(\gamma)\leq 1 \end{aligned} \tag{7}$$

are satisfied. Then the function $\mathcal{F}_{\alpha,\beta,\gamma}(z)$ defined by (3) is in the class $\mathcal{S}$.

Proof. From (3) we begin by setting

$$h(z)=\int_0^z(-t^2F'(t))^\alpha(tG(t))^\beta dt, \tag{8}$$

so that, obviously,

$$h'(z)=(-z^2F'(z))^\alpha(zG(z))^\beta \tag{9}$$

and from the logarithmic differential of equality (9), we obtain

$$\frac{zh''(z)}{h'(z)}=\alpha\left(2+\frac{zF''(z)}{F'(z)}\right)+\beta\left(1+\frac{zG'(z)}{G(z)}\right). \tag{10}$$

It is known that $\varphi \in \mathcal{S}$ and $0 < |z| < 1$, then $\varphi^*(z) = \frac{1}{\varphi(z)}$ is in $\widetilde{\sum}$. Hence expression (10) can be rewritten as

$$\frac{zh''(z)}{h'(z)} = \alpha\left(2 + \frac{zf''(z)}{f'(z)} - 2\frac{zf'(z)}{f(z)}\right) + \beta\left(1 - \frac{zg'(z)}{g(z)}\right), \tag{11}$$

where $f,\, g \in \mathcal{S}$.

From a well-known transformation of Bieberbach preserving the class of univalent functions $f \in \mathcal{S}$ (see [5])

$$f(z) = \frac{k\left(\frac{z+z_0}{1+z\bar{z}_0}\right) - k(z_0)}{k'(z_0)(1-|z_0|^2)}, \quad z \in \mathcal{U},\ k \in \mathcal{S},$$

z_0 is a fixed point of the unit disk $\mathcal{U}$, we obtain the value of the functional at the point $z = -z_0$

$$\frac{-z_0 f''(-z_0)}{f'(-z_0)} = \frac{2|z_0|^2 - 2a_2 z_0}{1-|z_0|^2} \tag{12}$$

and

$$\frac{-z_0 f'(-z_0)}{f(-z_0)} = \frac{z_0}{k(z_0)(1-|z_0|^2)}, \tag{13}$$

where a_2 is the second coefficient in Maclaurin expansion of the function k.
From (11), (12) and (13) by putting $z_0 = -z$ we have

$$\frac{zh''(z)}{h'(z)} = \frac{1}{1-|z|^2}\left\{2\alpha a_2 z - \beta|z|^2 + 2\alpha\left[1 - \frac{z}{-k(-z)}\right] + \beta\left[1 - \frac{z}{-l(-z)}\right]\right\} \tag{14}$$

and

$$\begin{aligned}\frac{1-|z|^{2\Re(\gamma)}}{\Re(\gamma)}\left|\frac{zh''(z)}{h'(z)}\right| =& \frac{1-|z|^{2\Re(\gamma)}}{1-|z|^2}\frac{1}{\Re(\gamma)}\left|2\alpha a_2 z - \beta|z|^2 + 2\alpha\left[1 - \frac{z}{-k(-z)}\right]\right. \\ &\left. + \beta\left[1 - \frac{z}{-l(-z)}\right]\right|,\end{aligned} \tag{15}$$

where $k, l \in \mathcal{S}$.
Now, we investigate the following cases:
1. It is easy to observe that the function $\phi : (0,\infty) \to \mathbb{R}$, $\phi(x) = \frac{1-a^{2x}}{x}$ $(0 < a < 1)$ is a decreasing function. If $x = \Re(\gamma)$ with $\Re(\gamma) \geq 1,\ z \in \mathcal{U},\ a = |z|$, then

$$\frac{1-|z|^{2\Re(\gamma)}}{\Re(\gamma)(1-|z|^2)} \leq 1, \quad z \in \mathcal{U}. \tag{16}$$

In equation (15), putting inequality (16) and using the Lemma 3 $(-k(-z), -l(-z) \in \mathcal{S})$, we obtain

$$\begin{aligned}\frac{1-|z|^{2\Re(\gamma)}}{\Re(\gamma)}\left|\frac{zh''(z)}{h'(z)}\right| &\leq \left|2\alpha a_2 z - \beta|z|^2 + 2\alpha\left[1 - \frac{z}{-k(-z)}\right] + \beta\left[1 - \frac{z}{-l(-z)}\right]\right| \\ &\leq 2|\alpha||a_2||z| + |\beta||z|^2 + 2|\alpha|\left|1 - \frac{z}{-k(-z)}\right| + |\beta|\left|1 - \frac{z}{-l(-z)}\right| \\ &\leq 10|\alpha| + 4|\beta|\end{aligned}$$

which, in the light of hypothesis (7), yields

$$\frac{1-|z|^{2\Re(\gamma)}}{\Re(\gamma)}\left|\frac{zh''(z)}{h'(z)}\right| \leq 1, \quad z \in \mathcal{U}.$$

2. Now we consider the function $\vartheta : (0,\infty) \to \mathbb{R}$, $\vartheta(x) = 1 - a^{2x}$ $(0 < a < 1)$ which is an increasing function. Then, for $0 < \Re(\gamma) \leq 1$ we have

$$\frac{1-|z|^{2\Re(\gamma)}}{1-|z|^2} \leq 1, \quad z \in \mathcal{U}. \tag{17}$$

Hence, using inequality (17) and Lemma 3, we obtain

$$\begin{aligned}
&\frac{1-|z|^{2\Re(\gamma)}}{\Re(\gamma)}\left|\frac{zh''(z)}{h'(z)}\right| \\
&\quad\leq \frac{1}{\Re(\gamma)}\left|2\alpha a_2 z - \beta|z|^2 + 2\alpha\left[1-\frac{z}{-k(-z)}\right] + \beta\left[1-\frac{z}{-l(-z)}\right]\right| \\
&\quad\leq \frac{1}{\Re(\gamma)}\left\{2|\alpha||a_2||z| + |\beta||z|^2 + 2|\alpha|\left|1-\frac{z}{-k(-z)}\right| + |\beta|\left|1-\frac{z}{-l(-z)}\right|\right\} \\
&\quad\leq \frac{10|\alpha| + 4|\beta|}{\Re(\gamma)}
\end{aligned}$$

which, in the light of hypothesis (7), yields

$$\frac{1-|z|^{2\Re(\gamma)}}{\Re(\gamma)}\left|\frac{zh''(z)}{h'(z)}\right| \leq 1, \quad z \in \mathcal{U}.$$

Finally, by applying Lemma 1 we conclude that the function $\mathcal{F}_{\alpha,\beta,\gamma}(z)$ defined by (3) is in the univalent function class $\mathcal{S}$. This evidently completes the proof of Theorem 1. □

Putting $\gamma = 1$ in Theorem 1, we immediately arrive at the following application of Theorem 1.

Corollary 1. *Let $F \in \widetilde{\sum}$ and $G \in \widetilde{\sum}_0$. Also let α and β be any complex numbers. Moreover, suppose that these numbers satisfy the following inequality*

$$10|\alpha| + 4|\beta| \leq 1.$$

Then the function $\mathcal{F}_{\alpha,\beta}(z)$ defined by (5) is in the univalent function class $\mathcal{S}$.

Choosing $\alpha = 0$ in Theorem 1, we obtain the following interesting consequence of Theorem 1.

Corollary 2. *Let β and γ with $\Re(\gamma) > 0$ be any complex numbers. Moreover, suppose that the function $G \in \widetilde{\sum}_0$ and the following inequalities*

$$|\beta| \leq \frac{1}{4}, \quad \text{for} \quad \Re(\gamma) \geq 1$$

$$|\beta| \leq \frac{\Re(\gamma)}{4}, \quad \text{for} \quad 0 < \Re(\gamma) \leq 1$$

are valid. Then the function $\mathcal{F}_{\beta,\gamma}$ *defined by*

$$\mathcal{F}_{\beta,\gamma}(z)=\left(\gamma\int_0^z t^{\gamma-1}(tG(t))^{\beta}dt\right)^{1/\gamma}$$

is in the univalent function class $\mathcal{S}$.

Theorem 2. *Let* $F\in\widetilde{\sum}$ *and* $G\in\widetilde{\sum}_0$. *Suppose also that* $\alpha,\ \beta$ *and* γ *are any complex numbers and that these numbers satisfy the following inequality*

$$\left|\gamma-1\right|+10\left|\alpha\right|+3\left|\beta\right|\leq 1. \tag{18}$$

Then the function $\mathcal{F}_{\alpha,\beta,\gamma}(z)$ *defined by (3) is in the class* $\mathcal{S}$.

Proof. We consider the function h defined by (8). Then from (9)-(13) we have

$$\frac{zh''(z)}{h'(z)}=\frac{1}{1-\left|z\right|^2}\left\{2\alpha a_2 z-\beta\left|z\right|^2+2\alpha\left[1-\frac{z}{-k(-z)}\right]+\beta\left[1-\frac{z}{-l(-z)}\right]\right\}$$

and

$$\begin{aligned}c\left|z\right|^2+(1-\left|z\right|^2)\left[(\gamma-1)+\frac{zh''(z)}{h'(z)}\right]=&(c-\gamma+1-\beta)|z|^2+2\alpha a_2 z+(\gamma-1)\\&+2\alpha\left[1-\frac{z}{-k(-z)}\right]+\beta\left[1-\frac{z}{-l(-z)}\right],\end{aligned} \tag{19}$$

where $k,l\in\mathcal{S}$. Putting $c=\gamma-1+\beta$ in (19) and using Lemma 3 we obtain

$$\begin{aligned}&\left|(\gamma-1+\beta)|z|^2+(1-|z|^2)\left[(\gamma-1)+\frac{zh''(z)}{h'(z)}\right]\right|\\&\quad\leq\left|\gamma-1\right|+2\left|\alpha\right|\left[\left|a_2\right|\left|z\right|+\left|1-\frac{z}{-k(-z)}\right|\right]+\left|\beta\right|\left|1-\frac{z}{-l(-z)}\right|\\&\quad\leq\left|\gamma-1\right|+10\left|\alpha\right|+3\left|\beta\right|.\end{aligned}$$

Finally, by applying Lemma 2, we conclude that the function $\mathcal{F}_{\alpha,\beta,\gamma}(z)$ defined by (3) is in the univalent function class $\mathcal{S}$. This evidently completes the proof of Theorem 2. □

Remark 2. *For* $\gamma=1$, *from Theorem 2, we see that our result is stronger than the Wesolowski's result* $(12\left|\alpha\right|+4\left|\beta\right|\leq 1)$ *for the same integral operator (for details, see [20]).*

3. Univalence condition associated with the integral operator (4)

Finally, the following result contains another sufficient conditions for a general integral operator $\mathcal{G}_{\lambda,\mu}$ of type (4) to be univalent in the unit disk $\mathcal{U}$.

Theorem 3. *Let $G \in \widetilde{\sum}_0$ and $\mu,\ \lambda \in \mathbb{C}$. Moreover, suppose that these numbers satisfy the following inequalities*

$$\begin{aligned} 16\,|\lambda| &\le 1, \quad \text{for} \quad \Re(\mu) \ge 1, \\ \frac{16\,|\lambda|}{\Re(\mu)} &\le 1, \quad \text{for} \quad 0 < \Re(\mu) \le 1. \end{aligned} \tag{20}$$

Then the function $\mathcal{G}_{\lambda,\mu}(z)$ defined by (4) is in the univalent function class $\mathcal{S}$.

Proof. Let us consider the function $\mathcal{G}_\lambda : \mathcal{U} \to \mathbb{C}$, defined by

$$\mathcal{G}_\lambda(z) = \int_0^z \left(e^{t^2 G(t)}\right)^\lambda dt.$$

First observe that, since $G \in \widetilde{\sum}_0$, clearly $\mathcal{G}_\lambda \in \mathcal{A}$, i.e. $\mathcal{G}_\lambda(0) = \mathcal{G}'_\lambda(0) - 1 = 0$. On the other hand, it can be seen easily that

$$\mathcal{G}'_\lambda(z) = \left(e^{z^2 G(z)}\right)^\lambda$$

and

$$\frac{z\mathcal{G}''_\lambda(z)}{\mathcal{G}'_\lambda(z)} = \lambda z^2 G(z) \left(2 + \frac{zG'(z)}{G(z)}\right). \tag{21}$$

Hence expression (21) can be rewritten as

$$\frac{z\mathcal{G}''_\lambda(z)}{\mathcal{G}'_\lambda(z)} = \lambda \frac{z^2}{g(z)} \left(2 - \frac{zg'(z)}{g(z)}\right),$$

where $g \in \mathcal{S}$.

Now, by using equality (13) and Lemma 3 and by putting $z_0 = -z$ in (13) , we obtain

$$\begin{aligned} \left|\frac{z\mathcal{G}''_\lambda(z)}{\mathcal{G}'_\lambda(z)}\right| &= \left|\lambda \frac{z^2}{g(z)} \left(2 - \frac{-z}{k(-z)(1-|z|^2)}\right)\right| \\ &= \left|\frac{\lambda}{(1-|z|^2)} \frac{z^2}{g(z)} \left(2(1-|z|^2) - \frac{z}{-k(-z)}\right)\right| \\ &\le \frac{|\lambda|\,|z|^2}{(1-|z|^2)\,|g(z)|} \left\{\left|1 - 2\,|z|^2\right| + \left|1 - \frac{z}{-k(-z)}\right|\right\} \\ &\le \frac{4\,|\lambda|\,|z|^2}{(1-|z|^2)} \frac{1}{|g(z)|}. \end{aligned}$$

In the last inequality if we again apply Lemma 3 for $g \in \mathcal{S}$, we have

$$\left|\frac{z\mathcal{G}''_\lambda(z)}{\mathcal{G}'_\lambda(z)}\right| \le \frac{16\,|\lambda|}{1-|z|^2},$$

for all $z \in \mathcal{U}$.

Now, by using Lemma 1 and the hypothesis, we obtain

$$\frac{1-|z|^{2\Re(\mu)}}{\Re(\mu)}\left|\frac{z\mathcal{G}_\lambda''(z)}{\mathcal{G}_\lambda'(z)}\right| \leq \frac{1-|z|^{2\Re(\mu)}}{1-|z|^2}\frac{16\,|\lambda|}{\Re(\mu)} \leq 1,$$

which in Lemma 1 implies that $\mathcal{G}_{\lambda,\mu} \in \mathcal{S}$. This completes the proof. $\square$

Now, by choosing $\mu = 1$ in Theorem 3, we have the following result.

Corollary 3. *Let $G \in \widetilde{\sum}_0$ and $\lambda \in \mathbb{C}$. Moreover, suppose that this number satisfies the inequality $|\lambda| \leq \frac{1}{16}$. Then, the function $\mathcal{G}_\lambda(z)$, defined by*

$$\mathcal{G}_\lambda(z) = \int_0^z \left(e^{t^2G(t)}\right)^\lambda dt,$$

is in $\mathcal{S}$, i.e. it is univalent in $\mathcal{U}$.

Example 1. *Let us consider the function $G(z) = -\frac{\ln(1-z)}{z^2} \in \widetilde{\sum}_0$. If we choose for some λ, $|\lambda| > 1$, then*

$$\mathcal{G}_\lambda(z) = \int_0^z \left(e^{-\ln(1-t)}\right)^\lambda dt = \frac{1}{\lambda-1}\left((1-z)^{1-\lambda}-1\right) \tag{22}$$

is not a univalent function in $\mathcal{U}$.

Proof. From the property of the function e^w and from the fact that $|\arg(1-z)| < \frac{\pi}{2}$ it follows that the function $f_\lambda(z) = (1-z)^{1-\lambda}$ is not univalent in $\mathcal{U}$ if there exists λ such that $|1-\lambda|\frac{\pi}{2} > \pi$ or $|\lambda| > 1$. This proves that there exists a function G in $\widetilde{\sum}_0$ such that for some λ, $|\lambda| > 1$ the function $\mathcal{G}_\lambda$ given by (22) is not univalent in $\mathcal{U}$. $\square$

References

[1] D. Breaz, N. Breaz, H. M. Srivastava, *An extension of the univalent condition for a family of integral operators*, Appl. Math. Lett. **22**(2009), 41–44.

[2] W. M. Causey, *The close-to-convexity and univalence of an integral*, Math. Z. **99**(1967), 207–212.

[3] E. Deniz, H. Orhan, *An extension of the univalence criterion for a family of integral operators*, Ann. Univ. Mariae Curie-Skłodowska Sect. A **64**(2010), 29–35.

[4] E. Deniz, H. Orhan, *Univalence criteria related with Ruscheweyh and Sălăgean derivatives and Loewner chains*, arXiv:1103.1802v1 [math.CV].

[5] P. L. Duren, *Univalent functions*, Springer-Verlag, New York, 1983.

[6] P. L. Duren, H. S. Shapiro, A. L. Shields, *Singular measures and domains not of the Smirnov type*, Duke Math. J. **33**(1966), 247–254.

[7] J. Godula, *On univalence of a certain integral*, Ann. Univ. Mariae Curie-Sklodowska Sec. A **33**(1979), 69–76.

[8] B. A. Frasin, *Univalence of two general integral operators*, Filomat **23**(2009), 223–229.

[9] B. A. Frasin, *Univalence criteria for general integral operator*, Math. Commun. **16**(2011), 115–124.

[10] J. Krzyz, Z. Lewandowski, *On the integral of univalent functions*, Bull. Polon. Sci. Ser. Math. Astr. Phys. **11**(1963),447–448.

[11] J. Miazga, A. Wesolowski, *On the univalence of a certain integral*, Zeszyty Nauk. Politech. Rzeszow **60**(1989), 25–31.

[12] S. Moldoveanu, N. N. Pascu, *Integral operators which preserve the univalence*, Mathematica (Cluj) **32**(1990), 159–166.

[13] N. N. Pascu, *On a univalence criterion II*, Itinerant Seminar on Functional Equations, Approximation and Convexity (Cluj-Napoca, 1985), Preprint, Vol. 85, Universitatea Babes-Bolyai, ClujNapoca, 1985, 153–154.

[14] V. Pescar, *On the univalence of an integral operator*, Appl. Math. Lett. **23**(2010), 615-619.

[15] V. Pescar, *New criteria for univalence of certain integral operators,* Demonstratio Math. **33**(2000), 51–54.

[16] V. Pescar, D. Breaz, *On an integral operator*, Appl. Math. Lett. **23**(2010), 625–629.

[17] J. A. Pfaltzgraff, *Univalence of the integral of* $(f'(z))^{\lambda}$, Bull. London Math. Soc. **7**(1975), 254–256.

[18] W. C. Royster, *On the univalence of a certain integral,* Michigan Math. J. **12**(1965), 385–387.

[19] H. M. Srivastava, E. Deniz, H. Orhan, *Some general univalence critera for a family of integral operators,* Appl. Math. Comput. **215**(2010), 3696–3701.

[20] A. Wesolowski, *On the univalence of certain integrals of meromorphic functions*, Glas. Mat. Ser. III **25**(1990), 43–47.