Polynomial root separation

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Abstract

We consider the question how close to each other can be two distinct roots of an integer polynomial $P(X)$ of degree d. We compare the distance between two distinct roots of $P(X)$ with its height $H(P)$, defined as the maximum of the absolute values of its coefficients. The first important result concerning this question in due to Mahler, who proved that the distance is $> c(d)H(P)^{-d+1}$, for an explicit constant $c(d)$, depending only on d. We will present some results in the opposite direction, obtained by constructing explicit parametric families of polynomials having two roots very close to each other. We also consider the absolute variant of the problem (the minimal nonzero distance between absolute values of the roots), and give tight bounds for the case of real roots. This is joint work with Yann Bugeaud, Tomislav Pejković and Bruno Salvy.