Polynomial root-finding using companion matrices

Fernando De Terán Vergara

Universidad Carlos III de Madrid Departamento de Matemáticas, Madrid

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Abstract

The use of companion matrices to compute the roots of scalar polynomials is a standard approach (it is for instance, the one followed by the command 'roots' in MATLAB). It essentially consists in computing the roots of a scalar polynomial as the eigenvalues of the companion matrix. In this talk, I will review several numerical and theoretical issues on this topic. I will pay special attention to the backward stability of solving the polynomial root-finding problem using companion matrices. More precisely, to this question: Even if the computed eigenvalues of the companion matrix are the eigenvalues of a nearby matrix, does this guarantee that they are the roots of a nearby polynomial? Typically, the companion matrix methods focus on monic polynomials, since one can always divide by the leading coefficient, if necessary. But, is it enough for the backward stability issue to focus on monic polynomials? I will also pay attention to some other (more theoretical) questions like: How many companion matrices are there and what do they look like?