Complete Asymptotic Analysis of Second-Order Differential Equations of Emden-Fowler Type in the Framework of Regular Variation

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We present a survey of results that have been obtained over the past years on asymptotic analysis of positive solutions of second order differential equations of Emden-Fowler type

$$x'' = q(t)|x|^{\gamma-1}x, \qquad \gamma > 0, \;\; \gamma \neq 1, \tag{1}$$

in the framework of regular variation in the sense of Karamata. Both sublinear ($0 < \gamma < 1$) and superlinear ($\gamma > 1$) case of (1) will be considered.

Classification of all possible positive solutions according to their asymptotic behavior will be done and necessary and sufficient conditions for the existence of all such solutions will be established. It will be shown that with the effective use of Karamata theory of regular variation, that all positive increasing and all positive decreasing solutions of (1) are regularly varying, providing the coefficient q is regularly varying as well as that the asymptotic behavior at infinity of these solutions can be determined explicitly and accurately.