Second order shape derivatives in optimal design problems

Petar Kunštek
University of Zagreb, Faculty of Science Department of Mathematics, petar@math.hr

Optimal design problems for stationary diffusion in the case of two isotropic phases is considered. The goal is to find an optimal distribution of the phases which maximizes the energy functional. Assuming that the interface between phases is regular, one deals with transmission problems for which the first and second order shape derivatives are derived. Shape derivatives can be formulated in two equivalent ways as a domain integral or as an integral over the interface.

By using the first and second order shape derivatives in the form of interface integral, we study the question whether critical stable shapes are local maxima for smooth perturbations.

The domain integral representation of the second order shape derivative is used to construct a novel quasi-Newton method. We have tested it in classes of problems for which classical solutions exist and can be explicitly calculated from optimality conditions. We have observed a more stable and faster convergence compared to the first order shape derivative methods.