Boundary oscillations in linear Dirichlet problems

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Abstract

A second-order linear differential equation (P): $y'' + f(x)y = 0$, $x \in I$, where $I = (0,1)$ and $f \in C(I)$, is said to be two-point oscillatory on I, if all its nontrivial solutions $y \in C(\bar{I}) \cap C^2(I)$, oscillate both at $x = 0$ and $x = 1$, i.e. having sequences of infinite zeros converging to $x = 0$ and $x = 1$. It necessarily implies that all solutions $y(x)$ of (P) must satisfy the Dirichlet boundary conditions and that $f(x)$ must be singular at both end points of $\bar{I}$. We first describe a class of two-point oscillatory equations of (P). Secondly, we prove that (P) is two-point oscillatory if $f(x)$ satisfies certain Hartman-Wintner type asymptotic conditions. Next, we study Dirichlet problem for linear elliptic equation (L): $-\Delta u = f(|x|)u$ in $B \setminus \{0\}$, and $u = 0$ on ∂B, where $B = \{x \in \mathbf{R}^N : |x| < 1\}$, $N \geq 2$, ∂B is the boundary of B, and $u \in C^2(B \setminus \{0\})$. The function $f(r)$ is positive and smooth in $(0,1)$, and singular at $r = 1$. We give a sufficient condition on $f(r)$ such that all radially symmetric solutions of (L) are oscillatory near ∂B.

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