Non-isothermal fluid flow through a thin pipe

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Abstract

The aim of this talk is to present some results about the asymptotic approximation of the non–isothermal fluid flow through a thin (or long) pipe. We first consider the problem in a thin straight pipe with circular cross-section and suppose that the fluid inside of the pipe is cooled by the exterior medium. Such situation appears naturally in many applications, especially in heat exchanging devices. The flow is described by the Navier–Stokes equations (with temperature–dependent viscosity) coupled with heat conducting equation. Using the asymptotic analysis with respect to the small parameter (being the ratio between pipe's thickness and its length) we derive the explicit formulae of Poiseuille type for the velocity and pressure and a simple constitutive law for the temperature.
Curved pipes are frequently used to improve the properties of heat exchanging devices. Therefore, similar problem is studied in a thin curved pipe with a smooth central curve and constant circular cross-section. Starting from the classical Boussinesq system written in curvilinear coordinates, the asymptotic expansion of the solution in terms of the pipe's thickness is built. By doing that, we find a simple approximation, easy to compute, showing explicitly the effects of pipe's geometry in their nature and magnitude. In both cases we rigorously justify obtained approximations by proving the corresponding error estimate.

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