Topology and computability in Euclidean space

A compact subset S of $\mathbf{R}^n$ is said to be *recursive* if there exists an algorithm which for every $k \in \mathbf{N}$ gives finitely many points $x_0, \dots, x_m$ in $\mathbf{R}^n$ such that the set $\{x_0, \dots, x_m\}$ approximates S with precision 2^{-k} in sense of the Hausdorff metric. On the other hand, a compact subset S of $\mathbf{R}^n$ is said to be *co-recursively enumerable* if $\mathbf{R}^n \setminus S$ can be effectively covered by open balls. Each recursive subset of $\mathbf{R}^n$ is co-recursively enumerable, while converse does not hold in general. However, it turns out that under some assumptions the implication

$$S \text{ co-recursively enumerable } \Rightarrow S \text{ recursive} \tag{1}$$

does hold. An important role here play certain topological properties of the set S. In this talk it will be shown that (1) holds in the case when S is a continuum which is not chainable, but which is circularly chainable. This for example means that (1) holds whenever S has topological type of the Warsaw circle or the dyadic solenoid.