Damping optimization of vibrational systems using dimension reduction

Zoran Tomljanović
Department of Mathematics,
J.J. Strossmayer University of Osijek
ztomljan@mathos.hr

We consider two problems which arise in damping optimization.

The first problem corresponds to the system described with the second-order differential equation

$$M\ddot{x} + D\dot{x} + Kx = 0,$$

where M and K are positive definite matrices of order n, called mass and stiffness, respectively. The damping matrix is $D = C_u + C$, where C_u represents the internal damping and C the external damping which is positive semidefinite matrix.

In optimization problem our aim is to determine the "best" damping matrix D which will insure optimal evanescence of each component of x. In this case the damping optimization is equivalent to a minimization of the trace of the solution of the corresponding Lyapunov equation.

In the second problem we consider the system given by

$$\begin{aligned} M\ddot{x}(t) + D\dot{x} + Kx(t) &= B_2u(t) + E_2w(t), \\ y &= C_2\dot{x}(t), \\ z &= H_1x(t) + H_2x(t), \end{aligned}$$

where M and K are again mass and stiffness, respectively, similar as in the first problem. The vectors $u \in \mathbb{R}^p$ and w are the control and the primary excitation inputs, respectively, while the vector $y \in \mathbb{R}^p$ contains the output velocities and z is the performance output.

A linear semi-active damper can be modeled as $u(t) = -G(t)y(t)$, where $G(t) \in \mathbb{R}^{p\times p}$ is a diagonal matrix with positive diagonal elements that corresponds to the coefficients of the dampers.

In the above system we would like to minimize the effect of the input w on the output z by optimizing the coefficients in the matrix G. The influence of the input w on the output z can be measured using the impulse response energy. This criterion leads to the problem of solving corresponding Lyapunov matrix equations.

In both problems optimization procedure is a very demanding, since a large number of Lyapunov equations has to be solved. Thus, we propose a techniques that uses dimension reduction which significantly accelerates the optimization process and allow us to efficiently determine the optimal damping.

Joint work with Peter Benner [1] and Ninoslav Truhar [2]

[1] Max Planck Institute for Dynamics of Complex Technical Systems, Magdeburg, Germany, benner@mpi-magdeburg.mpg.de

[2] Department of Mathematics, J.J. Strossmayer University of Osijek, ntruhar@mathos.hr