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## Numerička integracija dinamike više tijela na mnogostrukostima i Lievim grupama

## Numerical integration of multibody system dynamics on manifolds and Lie-groups

Geometric modeling of dynamical systems is a field of extensive ongoing research during last two decades. It has proven to be an approach that provides very usefull insights into dynamics and control of mechanical systems, from theoretical as well as from the practical computation point of view. However, although numerous algorithms, that have been developed on this ground, have found application in many important areas of engineering (specially in the field of non-linear control), a more widespread implementation of these models within the engineering communities are yet to happen. However, the situation is changing rapidly and within research communities of various focusses (in the spectrum from computational multibody dynamics to CFD), there is a growing interest in exploring geometric background with the aim of revisiting some 'classical' algorithms as well as finding new solutions.

Specially, in the field of multibody system (MBS) dynamics, there are several reasons for adopting a geometric point of view within the synthesis of mathematical models. The numerical integration procedures that are widely used in the computational multibody dynamics domain (from odinary differential equations (ODE) on manifolds to differential-algebraic (DAE) problems) can be succesfully studied on the geometric objects and connection between analytical mechanics (which is a starting point for contemporary MBS) and geometrical insights is deep and well-rooted in the history of mechanics.

Moreover, some important issues of computational mechanics that certanly lie in-the-heart of MBS, namely modeling of rigid body motion and numerical treatment of holonomic and non-holonomic constraints, call naturally for a geometric framework. These are some of the reasons why the geometric aspects of kinematical and dynamical modeling are increasingly recognized to play a significant role within research of the novel computational concepts in MBS. By operating on manifolds, and Lie-groups in particular, instead of linear vector spaces, geometric algorithms respect the geometric structure underlying many technical systems and hence offer attractive features such as numerical robustness and efficiency as well as avoidance of the kinematical singularities.

The lecture will review issues of geometric modeling of dynamics of discrete mechanical systems with the special focus on mathematical models and numerical procedures for computational forward dynamics of constrained multibody systems. Starting with the configuration space of rigid body motion and analysis of it's Lie-group structure, the elements of the respective Lie-algebra will be addressed and relations pertinent to geometric formulations and integration of multibody system dynamics surveyed. The aspects of dynamical modeling of unconstrained multibody systems on manifolds are introduced, along with the outline of geometric characteristics of holonomic and non-holonomic kinematical constraints. Time integration numerical algorithms for constrained multibody systems in the form of ODE on manifolds, expressed in the local integration coordinates, as well as DAE systems via redundant coordinates, will be reviewed and numerical stabilization algorithms for kinematical constraints violation discussed. The generalized coordinates partitioning algorithm is analysed in the context of the issue of optimal partitioning of system coordinates within the framework of the local ODE integration as well as projective stabilization algorithms for holonomic and non-holonomic multibody systems. Geometric modeling and integration procedures in Lie-group setting for constrained MBS will be discussed together with the issues of structure-preserving dynamical integration based on the geometric concepts.