Reconstruction of signals from frame coefficients with erasures

A sequence $(x_n)_{n=1}^{\infty}$ in a real or complex Hilbert space H is a *frame* for H if there exist positive constants A and B, that are called frame bounds, such that

$$A\|x\|^2 \leq \sum_{n=1}^{\infty} |\langle x, x_n \rangle|^2 \leq B\|x\|^2, \quad \forall x \in H.$$

In general, frames are linearly dependent (hence, redundant) generating systems. They are often used in process of encoding and decoding signals. It is the redundancy property of frames that makes them useful in applications where some of the coefficients could be corrupted or erased during the data transmission.

We shall discuss a new approach to the problem of recovering signal from frame coefficients with erasures. Provided that the erasure set satisfies the minimal redundancy condition, we construct a suitable synthesizing dual frame which enables us to perfectly reconstruct the original signal without recovering the lost coefficients. Such dual frames which compensate for erasures will be described from various viewpoints.

The second part of the talk will be concerned with frames that are robust with respect to finitely many erasures. We shall characterize all full spark frames for finite-dimensional Hilbert spaces. In particular, we will show that each full spark frames is generated by a matrix whose all square submatrices are nonsingular. In addition, we shall briefly discuss a method for constructing totally positive matrices.