Nonlinear Eigenvalue Problems from Maximizing Sum of Trace Ratios

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Abstract

We are concerned with the maximization of

$$\frac{\mathrm{trace}(V^{\mathrm{T}}AV)}{\mathrm{trace}(V^{\mathrm{T}}BV)} + \mathrm{trace}(V^{\mathrm{T}}CV)$$

over the Stiefel manifold $\{V \in \mathbb{R}^{m\times \ell} | V^{\mathrm{T}}V = I_\ell\}$ $(\ell < m)$, where B is a given symmetric and positive definite matrix, A and C are symmetric matrices, and $\mathrm{trace}(\,\cdot\,)$ is the trace of a square matrix. Applications that give rise to such a problem include the the problem of balancing individual capacities in a multi-user MIMO downlink channel $(\ell = 1)$ and the sparse Fisher discriminant analysis in pattern recognition $(C = 0)$. We establish necessary conditions for both the local and global maximizers and connect the problem with a nonlinear extreme eigenvalue problem which can be solved by a self-consistent-field (SCF) iteration. We analyze the global and local convergence of the SCF iteration, and show that the necessary condition for the global maximizers is fulfilled at any convergent point of the sequences of approximations generated by the SCF iteration. This is one of the advantages of the SCF iteration over optimization-based methods. Numerical tests show that the SCF iteration is much more efficient than manifold-based optimization methods.

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