Construction of high-rank elliptic curves

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The group of an elliptic curve over the rationals is the product of the torsion group and r copies of infinite cyclic group. By the famous theorem of Mazur, there are exactly 15 possible torsion groups. On the other hand, very little is known about which values of rank r are possible. The conjecture is that rank can be arbitrary large, but at present only an example of elliptic curve over $\mathbb{Q}$ with rank ≥ 24 is known. There is even a stronger conjecture that for any of 15 possible torsion groups T we have $B(T) = \infty$, where

$$B(T) = \sup\{\text{rank}\,(E(\mathbb{Q})) \,:\, \text{torsion group of } E \text{ over } \mathbb{Q} \text{ is } T\}.$$

It follows from results of Montgomery and Atkin & Morain (motivated by finding curves suitable for the elliptic curve method of factorization) that $B(T) \geq 1$ for all admissible torsion groups T. We improved this result by showing that $B(T) \geq 3$ for all T.

The information about current records for all admissible torsion groups can be found at `http://www.math.hr/~duje/tors/tors.html`.