Reducibility of collections of endomorphisms

A collection C of endomorphisms of a vector space V is reducible if there is a proper nonzero subspace U invariant under C. The reason to consider reducible vector spaces is the famous invariant subspace problem: If V is an infinite dimensional separable Hilbert space over the field of complex numbers and C consists of only one bounded operator, it is not known whether C is necessarily topologically reducible or not. Other motivations come from Algebra, where the most interesting case is a finite-dimensional space over a field F. However, if F is not algebraically closed, then even a collection of one endomorphism may be irreducible as soon as the dimension of V is greater than one. Many famous names of mathematicians such as Schur, Engel, Levitzki, Kaplansky made contributions to these problems, considering different collections of endomorphisms such as groups, semigroups or Lie algebras, possibly having some additional properties. In this talk, we will present history and some newer developments in the area.