Optimal control of parabolic equations by spectral decomposition

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Abstract. We consider the constrained minimisation problem

$$(\mathcal{P}) \qquad \min_{u \in L^2_{T,\mathcal{U}}} \left\{ J(u) : \quad y(T) \in \overline{B_\varepsilon\left(x^T\right)} \right\}, \tag{1}$$

where y^T is some given target state, J is a given cost functional and y is the solution of

$$(\mathcal{E}) \qquad \begin{cases} \frac{d}{dt} y(t) + \mathcal{A} y(t) = 0 & \text{for } t \in (0,T) \\ y(0) = u. \end{cases} \tag{2}$$

Here $\mathcal{A}$ is an unbounded linear operator allowing for spectral decomposition.

In the paper [1] we suggest a new methodology based on the spectral decomposition in terms of eigenfunctions of the operator $\mathcal{A}$. Surprisingly, the problem reduces to a non-linear equation for a scalar unknown, representing a Lagrangian multiplier. The method leads us to an explicit expression of the optimal final state in terms of the given problem data and the multiplier. The obtained expression, combined with standard optimal control arguments enable construction of a one-shot algorithm providing an approximate solution. In the talk basic steps of the method will be explained, followed by numerical examples demonstrating its efficiency.

Application of the method to a distributed control problem will be discussed as well. As can be expected, in this case one has to consider the associated dual problem which makes the calculation more complicated, although the algorithm steps follow a similar structure as in [1].

References

[1] Lazar, M, Molinari C, J. Peypouquet: *Optimal control of parabolic equations by spectral decomposition*, Optimisation, (2017)

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