M128	**Linear optimization**	L	E	S	ECTS
		3	2	1	8

Course objectives.. Introduce students to modeling, solving, and interpreting real problems that can be reduced to linear optimization. To process and analyze known numerical methods for solving linear optimization problems as well as present appropriate geometric interpretations.

Prerequisites. Undergraduate study programme in mathematics, computer science and similar study programmes

Course content.

1. Introductory part: Definition of linear programming problem. Examples of linear programming problems. By parts linear convex function. Graphic solution of two-dimensional linear programming problem.
2. Linear programming geometry: Polyhedron and convex sets. Extreme points, vertices and basic feasible solution. Polyhedron in standard form. Degeneration. Extreme point existence and optimality
3. Simplex method: Optimality condition. Derivation and implementation of the simplex method. Bland's rule. Determining the initial basic solution. Complexity analysis of the simplex method.
4. Dual problem: Dual problem. Theorems of weak and strong duality. Farkas' lemma and linear inequalities. Theorems and separation. Dual simplex method.
5. Sensitivity analysis: Local sensitivity analysis. Global sensitivity analysis. Interpretation.
6. Ellipsoidal method: Geometric meaning and complexity.
7. Network Flow Problems: Definitions, Formulation of Network Flow Problems and Properties. The law of conservation of flow. Equivalent problems: transport problem, join problem, various variants of network flow problems. Simplex algorithm for network flow problem: trees and basic permissible solutions, base change, simplex method for capacity problems.
8. Maximum flow problem: Definitions, formulation of maximum flow problem, properties, Ford-Fulkerson algorithm, magnifying path search, graph cut, Max-flow min-cut theorem.
9. Problems of integer programming (backpack problem, packing, partitioning, coverage, merchant passenger problem, scheduling problems, etc.) Modeling techniques. Strong formulation of the problem. Modeling with exponentially many conditions.

LEARNING OUTCOMES

No.	LEARNING OUTCOMES
1.	Create a goal function and minimization area based on a real problem that can be reduced to linear optimization problems.
2.	Implement numerical methods to solve linear optimization problems and interpret results.
3.	Prove mathematically the soundness of the procedures and formulas used in the inference.
4.	Use the mathematical literature of different sources and apply at least one programming tool to illustrate different examples.
5.	Experts and laymen make their conclusions clear and unambiguous.

RELATING THE LEARNING OUTCOMES, ORGANIZATION OF THE EDUCATIONAL PROCESS AND ASSESSMENT OF THE LEARNING OUTCOMES

TEACHING ACTIVITY	ECTS	LEARNING OUTCOME **	STUDENT ACTIVITY*	EVALUATION METHOD	POINTS	
					min	max
Attending lectures and exercises	1	1-5	Lecture attendance, discussion, team work and independent work on given tasks	Attendance lists, tracking activities	0	4
Homework	1	1-5	Solving of programming tasks	Checking the correct solutions (rating)	0	4
Written exam (Mid-terms)	2	1-5	Preparing for written exam	Evaluation	25	46
Final exam	4	1-5	Revision	Oral exam	25	46
TOTAL	8				50	100

Teaching methods and student assessment. Lectures and exercises are obligatory. The exam consists of a written and an oral part. Upon completion of the course, students can take the exam. Successful midterm exam scores replace the written exam.

Can the course be taught in English: Yes

Basic literature:

1. D. Bertsimas, J. N. Tsitsiklis, Introduction to Linear Optimization, Athena Scientific, 1997.
2. I. Kuzmanović, K. Sabo, Linearno programiranje, Sveučilište J.J. Strossmayera u Osijeku - Odjel za matematiku, 2016.

Recommended literature:

1. K. G. Murty, Linear and Combinatorial Programming, John Wiley & Sons, Inc., 1983.
2. L. Neralić, Uvod u matematicko programiranje 1, Element, Zagreb, 2003.
3. G. Sierksma, Linear and Integer Programming, Marcel Dekker, Inc., Nemhauser, 1999.
4. D. Kincaid, W.Cheney, Numerical Analysis, Brooks/Cole Publishing Company, New York, 1996.
   A. Schrijver, Theory of Linear and Integer Programming, John Wiley & Sons, Inc., NY, SAD, 1999.