

M087	<b>Linear Algebra II</b>	L	S	E	ECTS 7
		3	0	3	

**The aim of the course.** Adoption of concepts of vector and inner spaces. Introducing the students with the concepts of linear operators over finite dimensional vector spaces. Capability of applying the matrix calculation, adopted in Linear algebra I course, when dealing with linear operators over finite dimensional vector spaces.

**Prerequisites.** Linear algebra I.

**Course content.**

1. Concept of finite dimensional vector space and vector subspace. Examples of vector spaces and subspaces. Basis and dimension. Sum of subspaces. Direct sum and direct complement. Quotient space.
2. Concept of the linear operator. Basic properties of linear operators. Linear operators in a plane. Examples of linear operators: central symmetry, axial symmetry, homotety, orthogonal projection, rotation. Rank-nullity theorem. Space of linear operators and product of linear operators. Matrix of a linear operator and transfer matrices. Similar matrices. Spectrum of linear operator. Contraction and dilatation of plane. Minimal polynomial of an operator. Diagonalization. Hamilton-Cayley theorem. Characteristic polynomial of an operator. Jordan form. Exponential function of an operator.
3. Inner product and inner space. Norm. Orthogonality, orthonormal basis and Gram-Schmidt orthogonalization procedure. Orthogonal complement. Operators on inner spaces. Hermitian adjoint operators, Hermitian operators and unitary operators. Diagonalization of symmetric matrices. Quadratic forms and second order curves.

**EXPECTED LEARNING OUTCOMES**

No.	LEARNING OUTCOMES
	Determine the basis and dimension of finite dimensional vector spaces.
	Distinguish properties of linear operators.
	Determine matrix of a linear operator.
1.	Determine the characteristic and minimal polynomial of an operator, determine the spectrum of an operator.
	Determine the Jordan form of the matrix of an operator.
2.	Describe the structure of inner space.
3.	Use vector norms and distinguish inner and normed spaces.
4.	Construct the orthonormal basis of inner space.
5.	Describe the diagonalization procedure.

**RELATING THE LEARNING OUTCOMES, ORGANIZATION OF THE EDUCATIONAL PROCESS AND ESTIMATION OF THE LEARNING OUTCOMES**

Organization of the educational process	ECTS	Learning outcomes **	Student activities*	The method of estimate	Points	
					Min	max
Lecture attendance	1	1-9	Lecture attendance, discussion, team work and independent work on given tasks	Attendance sheets, tracking activities	0	4
Written exam (preliminary exam)	3	1-9	Preparing for written exam	Evaluation	25	48
Final exam	3	1-9	Repetition of the subject matter	Oral exam	25	48
Total	7				50	100

**Teaching and evaluation of knowledge.** Attending lectures and exercises is required. The exam consists of written and oral part, and can be taken after completion of lectures and exercises. During the semester students can take preliminary exams that replace the written examination.

**Can the course be taught in English:** Yes.

**Basic literature:**

1. D. Bakić, Linearna algebra, Školska knjiga, Zagreb, 2008.

**Additional literature:**

1. H. Kraljević, Vektorski prostori, reviewed materials available at the web pages of Department of mathematics, University of Osijek, 2008.
2. R. Scitovski, Geometrija ravnine i prostora, reviewed materials available at the web pages of Department of mathematics, University of Osijek, 2011.
3. N. Bakić, A. Milas, Zbirka zadataka iz linearne algebre, PMF-Matematički odjel Sveučilišta u Zagrebu, 1995.
4. N. Elezović, A. Aglič, Linearna algebra: zbirka zadataka, Element, Zagreb, 1999.
5. H. Anton, R. Rorres, Elementary linear algebra, John Wiley & Sons, Danvers, 2000.