Journal of Lie Theory
Volume **??** (??) ??–??

Version of October 12, 2017

On Jacquet modules of discrete series: the first inductive step

Ivan Matić

Abstract. The purpose of this paper is to determine Jacquet modules of discrete series which are obtained by adding a pair of consecutive elements to the Jordan block of an irreducible strongly positive representation such that the ϵ-function attains the same value on both elements. Such representations present the first inductive step in the realization of discrete series starting from the strongly positive ones. We are interested in determining Jacquet modules with respect to the maximal standard parabolic subgroups, with an irreducible essentially square-integrable representation on the general linear part.
Mathematics Subject Classification 2000: **primary 22E35; secondary 22E50.**
Key Words and Phrases: **discrete series, classical p-adic groups, Jacquet modules.**

1. Introduction

Discrete series representations present one of the most extensively studied parts of the unitary dual of a reductive group over p-adic field, with numerous applications in harmonic analysis and theory of automorphic forms. In the case of p-adic classical groups, this prominent class of representations has been classified in the work of Mœglin and Tadić ([11, 13]), under a natural hypothesis which now follows from the work of Arthur ([1]). Some further details on the completion of this classification can be found in [12]. According to this classification, discrete series are in bijective correspondence with the so-called admissible triples consisting of a Jordan block, an ϵ-function and a partial cuspidal support. Furthermore, each discrete series can be obtained as a result of an inductive procedure consisting of repeated adding of new consecutive pairs to the Jordan block, starting from the strongly positive discrete series.

Thus, the strongly positive discrete series serve as the cornerstone in such construction of discrete series. An algebraic classification of such representations is given in [6] and, based on that classification, a complete description of Jacquet modules of strongly positive discrete series has been determined in [7]. These

This work has been supported in part by Croatian Science Foundation under the project 9364.

results show that, if we denote by $\pi \otimes \sigma$ an irreducible representation contained in the Jacquet module of strongly positive representation with respect to the maximal standard parabolic subgroup, then σ is also a strongly positive discrete series, while π is a ladder representation of a particular type, in terminology of [4]. We note that such representations of general linear groups have lately been studied in detail in [2], [4] and [5].

On the other hand, it is clear that the classical group parts of Jacquet modules of non-strongly positive discrete series are representations belonging to different classes, ranging from discrete series to non-tempered ones. It is natural to initially extend the investigation of Jacquet modules of discrete series started in [7] to discrete series obtained by adding a pair of consecutive elements to the Jordan block of strongly positive discrete series such that the ϵ-function attains the same value on these elements. This class of representations already shows substantial differences from the strongly positive case and has played a fundamental role in determininig the occurrence indices for discrete series representations of metaplectic groups ([8]). In this paper, we are interested in deriving Jacquet modules of the representations of mentioned type with respect to the maximal standard parabolic subgroups, whose general linear part consists of an irreducible essentially square-integrable representation. Even in this case, a variety of different representations appear, which have to be considered separately.

Recently the main properties defining the ϵ-function attached to a discrete series representation have been rewritten in terms of Jacquet modules in [18], and these results are mainly expressed using Jacquet modules analogous to those studied in this paper.

To determine the Jacquet modules we use elementary but non-standard methods which are essentially different from the ones used in [7]. First, starting from appropriate embeddings of discrete series σ_{ds}, we apply the structural formula of Tadić ([16]), combined with results of [7], to obtain all (not necessarily irreducible) elements appearing in the Jacquet modules of the induced representation containing σ_{ds}. Then, using a description of composition series of certain generalized principle series, described in [10] and enhanced by [9, Proposition 3.2], we derive all possible candidates for Jacquet modules of discrete series σ_{ds}, together with their multiplicities. We note that the results of [10] also hold in the unitary case, since they are completely based on the Mœglin-Tadić classification and Jacquet modules method. To deduce whether an irreducible representation $\pi \otimes \tau$ appears in the Jacquet module of σ_{ds} or not, using a case-by-case consideration we choose an element $\pi' \otimes \tau'$ appearing in the Jacquet module of τ and, by means of transitivity of Jacquet modules, turn our attention to representations of general linear groups having $\pi \otimes \pi'$ in their Jacquet modules. This puts us in a position to deduce further information carried in the irreducible representation $\pi \otimes \tau$ and, consequently, [18] can be used to determine whether such a representation belongs to the Jacquet module of σ_{ds} or not.

Our results, besides being interesting by themselves, might have applications in the theory of automorphic forms, where both discrete series and their Jacquet modules play an important role. Also, one can use our results to identify discrete series subquotients of generalized principal series, similarly as in [9].

We now describe the content of the paper in more details. In the second section we recall the required notations and preliminaries. In the third section we begin our study of Jacquet modules by solving certain elementary cases which are used afterwards. The description of Jacquet modules in the most complicated case, divided in several subcases, is provided in the fourth section. The exceptional case is handled in Section 5.

The author would like to thank Marko Tadić for useful conversations on Jacquet modules of representations of general linear groups and to Goran Muić for his active interest in publication of this paper. Also, the author's thanks go to Šime Ungar for many useful suggestions and help with English language. The author would also like to thank the referee for helping to improve presentation style.

2. Notation and preliminaries

Let F denote a non-archimedean local field of characteristic different than two. We will fix one of the following series $\{G_n\}$ of classical groups over F.

In the case of odd orthogonal groups we fix an anisotropic orthogonal vector space Y_0 over F of odd dimension and consider the Witt tower based on Y_0. For each n satisfying $2n+1 \geq \dim Y_0$, there is exactly one space V_n in the tower of dimension $2n+1$. By G_n we denote the special orthogonal group of this space. Similarly, if V_n stands for the symplectic space of dimension $2n$ in the corresponding Witt tower, G_n will denote the symplectic group of this space. We also consider the case of unitary groups $U(n, F'/F)$, where F' denotes a separable quadratic extension of F. We have also an anisotropic unitary space Y_0 over F' and the Witt tower of unitary spaces V_n based on Y_0. The unitary group of the space V_n of dimension either $2n+1$ or $2n$ will be denoted by G_n.

A minimal parabolic subgroup in G_n will be fixed and we will consider only standard parabolic subgroups with respect to this fixed minimal parabolic subgroup. Abusing the notation, if we are working with unitary groups, then F' will denote a separable quadratic extension of F, and otherwise F' denotes F. If F' is a separable quadratic extension, we denote by θ the non-trivial element of the Galois group of F' over F. Otherwise, i.e., if $F' = F$, θ will denote the identity mapping on F. For an irreducible representation π of $GL(k, F')$, we denote by $\check{\pi}$ the mapping $g \mapsto \widetilde{\pi}(\theta(g))$, where $\widetilde{\pi}$ stands for the contragredient representation of π. We say that π is self-dual if $\check{\pi} \simeq \pi$.

We fix one of the series $\{G_n\}$ as above. Let n' be the Witt index of V_n if V_n is symplectic or even-unitary group, and $n' = n - \frac{1}{2}(\dim_{F'}(Y_0) - 1)$ otherwise. For $0 \leq k \leq n'$ there exists a standard parabolic subgroup $P_{(k)}$ having Levi factor naturally isomorphic to $GL(k, F') \times G_{n-k}$. For finite length representations δ of $GL(k, F')$ and τ of G_{n-k}, we denote by $\delta \rtimes \tau$ the representation parabolically induced by $\delta \otimes \tau$. For representations δ_i of $GL(n_i, F')$, $i = 1, 2, \ldots, k$, and a representation τ of $G_{n'}$, we write $\delta_1 \times \cdots \times \delta_k \rtimes \tau$ for the representation parabolically induced by $\delta_1 \otimes \cdots \otimes \delta_k \otimes \tau$.

The set of all irreducible admissible representations of G_n will be denoted by $Irr(G_n)$. Let $R(G_n)$ denote the Grothendieck group of admissible representations of finite length of G_n and set $R(G) = \oplus_{n\geq 0} R(G_n)$. In a similar way we define $Irr(GL(n,F'))$ and $R(GL) = \oplus_{n\geq 0} R(GL(n,F'))$. Furthermore, we denote by $Irr_{sc}(GL(n,F'))$ the set of all supercuspidal representations in $Irr(GL(n,F'))$. For $\sigma \in Irr(G_n)$ and $0 \leq k \leq n$ we denote by $r_{(k)}(\sigma)$ the normalized Jacquet module of σ with respect to the standard parabolic subgroup $P_{(k)}$. Then $r_{(k)}(\sigma)$ can be interpreted as an element of $R(GL) \otimes R(G)$. For $\sigma \in Irr(G_n)$ we introduce $\mu^*(\sigma) \in R(GL) \otimes R(G)$ by

$$\mu^*(\sigma) = \sum_{k=0}^{n} \text{s.s.}(r_{(k)}(\sigma))$$

(s.s. denotes the semisimplification), and extend μ^* linearly to the whole of $R(G)$.

Using Jacquet modules for the maximal standard parabolic subgroups of $GL(n,F')$ we can also define $m^*(\pi) = \sum_{k=0}^{n} \text{s.s.}(r_{(k)}(\pi)) \in R(GL) \otimes R(GL)$, for an irreducible representation π of $GL(n,F')$, and then extend m^* linearly to the whole of $R(GL)$. Here $r_k(\pi)$ denotes the normalized Jacquet module of π with respect to the standard parabolic subgroup having Levi factor equal to $GL(k,F') \times GL(n-k,F')$.

For an admissible representation $\sigma \in R(G)$ of finite length we write $\mu^*(\sigma) = \sum_{\tau,\sigma'} \tau \otimes \sigma'$ and introduce $(m\mu)^*(\sigma) \in R(GL) \otimes R(GL) \otimes R(G)$ by

$$(m\mu)^*(\sigma) = \sum_{\tau,\sigma'} m^*(\tau) \otimes \sigma'.$$

From now on, $Irr_{es}(GL(n,F'))$ stands for the set of all essentially square-integrable representations in $Irr(GL(n,F'))$. The results of [19] show that each representation $\delta \in Irr_{es}(GL(n,F'))$ is attached to a segment and we set $\delta = \delta([\nu^a\rho, \nu^b\rho])$, where $a, b \in \mathbb{R}$ such that $b-a$ is a nonnegative integer and ρ is a unitary element of $Irr_{sc}(GL(n_\rho, F'))$ (this defines n_ρ), while we denote by ν the character of $GL(n,F')$ defined by $|\det|_{F'}$. We recall that $\delta([\nu^a\rho, \nu^b\rho])$ is the unique irreducible subrepresentation of the induced representation $\nu^b\rho \times \nu^{b-1}\rho \times \cdots \times \nu^a\rho$.

We will frequently use the following equation:

$$m^*(\delta([\nu^a\rho, \nu^b\rho])) = \sum_{i=a-1}^{b} \delta([\nu^{i+1}\rho, \nu^b\rho]) \otimes \delta([\nu^a\rho, \nu^i\rho]).$$

Note that multiplicativity of m^* implies

$$m^*(\prod_{j=1}^{n} \delta([\nu^{a_j}\rho_j, \nu^{b_j}\rho_j])) = \prod_{j=1}^{n} (\sum_{i_j=a_j-1}^{b_j} \delta([\nu^{i_j+1}\rho_j, \nu^{b_j}\rho_j]) \otimes \delta([\nu^{a_j}\rho_j, \nu^{i_j}\rho_j])). \tag{1}$$

We take a moment to state a result, derived in [16], which presents a crucial structural formula for our calculations of Jacquet modules of representations of classical groups.

Lemma 2.1. *Let $\rho \in Irr_{sc}(GL(n_\rho, F'))$ and $k, l \in \mathbb{R}$ such that $k + l \in \mathbb{Z}_{\geq 0}$. Let $\sigma \in R(G)$ be an admissible representation of finite length. Write $\mu^*(\sigma) = \sum_{\tau,\sigma'} \tau \otimes \sigma'$. Then the following holds:*

$$\mu^*(\delta([\nu^{-k}\rho, \nu^l\rho]) \rtimes \sigma) = \sum_{i=-k-1}^{l} \sum_{j=i}^{l} \sum_{\tau,\sigma'} \delta([\nu^{-i}\widetilde{\rho}, \nu^k\widetilde{\rho}]) \times \delta([\nu^{j+1}\rho, \nu^l\rho]) \times \tau \otimes \otimes \delta([\nu^{i+1}\rho, \nu^j\rho]) \rtimes \sigma'. \quad (2)$$

We put $\delta([\nu^x\rho, \nu^y\rho]) = 1$ (the one-dimensional representation of the trivial group if $y = x - 1$, and $\delta([\nu^x\rho, \nu^y\rho]) = 0$ if $y < x - 1$.

We briefly recall the Langlands classification for general linear groups. As in [3], we favor the subrepresentation version of this classification over the quotient one.

For every $\delta \in Irr_{es}GL(n, F')$, there is a unique $e(\delta) \in \mathbb{R}$ such that $\nu^{-e(\delta)}\delta$ is unitarizable. Suppose that $\delta_1, \delta_2, \ldots, \delta_k$ are the representations belonging to $Irr_{es}(GL(n_1, F')), Irr_{es}(GL(n_2, F')), \ldots, Irr_{es}(GL(n_k, F'))$ respectively, with $e(\delta_1) \leq e(\delta_2) \leq \ldots \leq e(\delta_k)$. Then the induced representation $\delta_1 \times \delta_2 \times \cdots \times \delta_k$ has a unique irreducible subrepresentation, which we denote by $L(\delta_1 \times \delta_2 \times \cdots \times \delta_k)$. This irreducible subrepresentation is called the Langlands subrepresentation, and it appears with multiplicity one in $\delta_1 \times \delta_2 \times \cdots \times \delta_k$. Every irreducible representation π of $GL(n, F')$ is isomorphic to some $L(\delta_1 \times \delta_2 \times \cdots \times \delta_k)$. For a given π, the representations $\delta_1, \delta_2, \ldots, \delta_k$ are unique up to a permutation.

Similarly, throughout the paper we use the subrepresentation version of the Langlands classification for classical groups, which also happens to be more appropriate for our Jacquet module considerations. Thus, we realize a non-tempered irreducible representation π of G_n as the unique irreducible (Langlands) subrepresentation of the induced representation of the form $\delta_1 \times \delta_2 \times \cdots \times \delta_k \rtimes \tau$, where τ is the tempered representation of G_t (this defines t), δ_i is an element of $Irr_{es}(GL(n_{\delta_i}, F'))$ attached to the segment $[\nu^{a_i}\rho_i, \nu^{b_i}\rho_i]$ for $i = 1, 2, \ldots, k$, and $a_1 + b_1 \leq a_2 + b_2 \leq \cdots \leq a_k + b_k < 0$ (note that $e(\delta([\nu^{a_i}\rho_i, \nu^{b_i}\rho_i])) = (a_i + b_i)/2$). In this case, we write $\pi = L(\delta_1 \times \delta_2 \times \cdots \times \delta_k \rtimes \tau)$.

We will now recall the Mœglin-Tadić classification of discrete series for classical groups, which presents the framework for our study. Fix a certain tower of classical groups (symplectic, orthogonal or unitary). Also, in the sequel we fix an additive character ψ of F'. Every discrete series representation of such group is uniquely described by its three invariants: partial cuspidal support, Jordan block and ϵ-function.

The partial cuspidal support of a discrete series $\sigma \in Irr(G_n)$ is an irreducible cuspidal representation σ_{cusp} of some G_m such that there exists a representation $\pi \in R(GL(n_\pi, F'))$ such that σ is a subrepresenation of $\pi \rtimes \sigma_{cusp}$.

The Jordan block of σ, which we denote by $\mathrm{Jord}(\sigma)$, is the set of all pairs (c, ρ) where $\rho \in Irr_{sc}(GL(n_\rho, F'))$ is self-dual and c is a positive half-integer such that the following two conditions are satisfied:

1. c is not an integer if and only if $L(s, \rho, r)$ has a pole at $s = 0$. The local L-factor $L(s, \rho, r)$ is the one defined by Shahidi (see for instance [14], [15]),

where $r = \bigwedge^2 \mathbb{C}^{n_\rho}$ is the exterior square representation of the standard representation on $\mathbb{C}^{n_\rho}$ of $GL(n_\rho, \mathbb{C})$ if G_n is a symplectic or even-orthogonal group, and $r = \mathrm{Sym}^2\mathbb{C}^{n_\rho}$ is the symmetric-square representation of the standard representation on $\mathbb{C}^{n_\rho}$ of $GL(n_\rho, \mathbb{C})$ if G_n is an odd-orthogonal group. For unitary groups, the appropriate definitions are discussed in [11] and [13, Section 15].

2. The induced representation $\delta([\nu^{-c}\rho, \nu^c\rho]) \rtimes \sigma$ is irreducible.

To explain the notion of the ϵ-function, we first define Jordan triples. This are triples of the form $(\mathrm{Jord}, \sigma', \epsilon)$ where

- σ' is an irreducible cuspidal representation of some G_n.
- Jord is the finite set (possibly empty) of pairs (c, ρ), where $\rho \in Irr_{sc}(GL(n_\rho, F'))$ is a self-dual representation, and $c > 0$ is a half-integer such that c is an integer if and only if $L(s, \rho, r)$ does not have a pole at $s = 0$ (as above). For a self-dual representation $\rho \in Irr_{sc}(GL(n_\rho, F'))$ we write $\mathrm{Jord}_\rho = \{c : (c, \rho) \in \mathrm{Jord}\}$. If $\mathrm{Jord}_\rho \neq \emptyset$ and $c \in \mathrm{Jord}_\rho$, we put $c_- = \max\{d \in \mathrm{Jord}_\rho : d < c\}$, if it exists.
- We say that two functions $\epsilon_1, \epsilon_2 : \mathrm{Jord} \to \{1, -1\}$ are equivalent if for each $\rho \in Irr_{sc}(GL(n_\rho, F'))$ such that Jord_ρ consists of integers and $\mathrm{Jord}_\rho(\sigma') \neq \emptyset$, there is an $s_\rho \in \{1, -1\}$ such that $\epsilon_1((c, \rho)) = s_\rho \cdot \epsilon_2((c, \rho))$ for all $(c, \rho) \in \mathrm{Jord}$. ϵ is now an equivalence class with respect to this equivalence relation.

Suppose that, for the Jordan triple $(\mathrm{Jord}, \sigma', \epsilon)$, there is a $(c, \rho) \in \mathrm{Jord}$ such that $\epsilon((c_-, \rho)) = \epsilon((c, \rho))$. If we put $\mathrm{Jord}' = \mathrm{Jord} \setminus \{(c_-, \rho), (c, \rho)\}$ and consider the restriction ϵ' of ϵ to Jord', we obtain a new Jordan triple $(\mathrm{Jord}', \sigma', \epsilon')$, and we say that such a Jordan triple is subordinated to $(\mathrm{Jord}, \sigma', \epsilon)$.

We say that the Jordan triple $(\mathrm{Jord}, \sigma', \epsilon)$ is a triple of alternated type if $\epsilon((c_-, \rho)) \neq \epsilon((c, \rho))$ whenever c_- is defined and there is an increasing bijection $\phi_\rho : \mathrm{Jord}_\rho \to \mathrm{Jord}'_\rho(\sigma')$, where

$$\mathrm{Jord}'_\rho(\sigma') = \begin{cases} \mathrm{Jord}_\rho(\sigma') \cup \{0\} & \text{if } c \text{ is not an integer and } \epsilon(\min \mathrm{Jord}_\rho, \rho) = 1; \\ \mathrm{Jord}_\rho(\sigma') & \text{otherwise.} \end{cases}$$

A Jordan triple $(\mathrm{Jord}, \sigma', \epsilon)$ dominates the Jordan triple $(Jord', \sigma', \epsilon')$ if there is a sequence of Jordan triples $(\mathrm{Jord}_i, \sigma', \epsilon_i)$, $0 \leq i \leq k$, such that $(\mathrm{Jord}_0, \sigma', \epsilon_0) = (\mathrm{Jord}, \sigma', \epsilon)$, $(\mathrm{Jord}_k, \sigma', \epsilon_k) = (\mathrm{Jord}', \sigma', \epsilon')$, and $(\mathrm{Jord}_i, \sigma', \epsilon_i)$ is subordinated to $(\mathrm{Jord}_{i-1}, \sigma', \epsilon_{i-1})$ for $i \in \{1, 2, \ldots, k\}$. A Jordan triple $(\mathrm{Jord}, \sigma', \epsilon)$ is called an admissible triple if it dominates a triple of alternated type.

The classification given in [11] and [13] states that there is a one-to-one correspondence between the set of all discrete series in $Irr(G)$ and the set of all admissible triples $(\mathrm{Jord}, \sigma', \epsilon)$ given by $\sigma = \sigma_{(\mathrm{Jord}, \sigma', \epsilon)}$, such that $\sigma_{cusp} = \sigma'$ and $\mathrm{Jord}(\sigma) = \mathrm{Jord}$. Furthermore, if $(c, \rho) \in Jord$ is such that $\epsilon((c_-, \rho)) = \epsilon((c, \rho))$, we set $\mathrm{Jord}' = \mathrm{Jord} \setminus \{(c_-, \rho), (c, \rho)\}$ and consider the restriction of ϵ to Jord'. Let us denote this restriction by ϵ'. Then $(\mathrm{Jord}', \sigma', \epsilon')$ is an admissible triple and σ is a subrepresentation of $\delta([\nu^{-c_-}\rho, \nu^{c}\rho]) \rtimes \sigma_{(\mathrm{Jord}',\sigma',\epsilon')}$. Such induced representation has exactly two discrete series subrepresentations, which are non-isomorphic. Moreover, the induced representation $\delta([\nu^{-c_-}\rho, \nu^{c_-}\rho]) \rtimes \sigma_{(\mathrm{Jord}',\sigma',\epsilon')}$ is the direct sum of two non-isomorphic tempered representations τ_+ and τ_- and there is a unique tempered representation $\tau \in \{\tau_+, \tau_-\}$ such that σ is a subrepresentation of $\delta([\nu^{c_-+1}\rho, \nu^{c}\rho]) \rtimes \tau$ (by [13, Section 6]).

We shall also say that the discrete series σ and its corresponding admissible triple $(\mathrm{Jord}, \sigma', \epsilon)$ are attached to each other.

For $(c,\rho) \in \mathrm{Jord}$ such that (c_-,ρ) is defined, $\epsilon((c,\rho)) = \epsilon((c_-,\rho))$ if there is some irreducible representation $\pi \in R(G)$ such that

$$\sigma \hookrightarrow \delta([\nu^{c_-+1}\rho, \nu^{c}\rho]) \rtimes \pi.$$

Let us now assume that Jord_ρ consists of non-integers or $\mathrm{Jord}_\rho(\sigma') = \emptyset$. To define ϵ on Jord_ρ it suffices to define $\epsilon((c,\rho))$ for a single ordered pair $(c,\rho) \in \mathrm{Jord}$.

If $\mathrm{Jord}_\rho \neq \emptyset$ consists of non-integers, we denote by $c_{\min,\rho}$ the minimum of Jord_ρ, and set $\epsilon((c_{\min,\rho},\rho)) = 1$ if there exists an irreducible representation $\pi \in R(G)$ such that

$$\sigma \hookrightarrow \delta([\nu^{\frac{1}{2}}\rho, \nu^{c_{\min,\rho}}\rho]) \rtimes \pi.$$

Otherwise, let $\epsilon((c_{\min,\rho},\rho)) = -1$.

If $\mathrm{Jord}_\rho \neq \emptyset$ consists of odd integers and $\rho \rtimes \sigma'$ is reducible (equivalently, $\mathrm{Jord}_\rho(\sigma') = \emptyset$), then the induced representation $\rho \rtimes \sigma'$ decomposes into two non-isomorphic irreducible tempered representations, which we denote by $\tau_1^{(\sigma',\rho)}$ and $\tau_{-1}^{(\sigma',\rho)}$. We note that this labeling is completely determined by the choice of ψ (details can be seen in [11, Section 6]). We denote the maximum of Jord_ρ by $c_{\max,\rho}$ and set $\epsilon((c_{\max,\rho},\rho)) = 1$ if there exists an irreducible representation $\pi' \in R(GL)$ such that

$$\sigma \hookrightarrow \pi' \times \delta([\nu\rho, \nu^{c_{\max,\rho}}\rho]) \rtimes \tau_1^{(\sigma',\rho)}.$$

Otherwise, let $\epsilon((c_{\max,\rho},\rho)) = -1$.

An irreducible representation $\sigma \in R(G)$ is called strongly positive if for every embedding

$$\sigma \hookrightarrow \nu^{s_1}\rho_1 \times \nu^{s_2}\rho_2 \times \cdots \times \nu^{s_k}\rho_k \rtimes \sigma_{cusp},$$

such that $\rho_i \in R(GL)$, $i = 1, 2, \ldots, k$, are irreducible cuspidal unitary representations, and $\sigma_{cusp} \in R(G)$ is an irreducible cuspidal representation, we have $s_i > 0$ for $i = 1, 2, \ldots, k$.

In [11, Section 1] and [13, Proposition 7.1] it is proved that triples of alternated type correspond to strongly positive discrete series and the definition of such triples shows that the strongly positive discrete series are completely determined by their partial cuspidal support and the Jordan block. Since all strongly positive discrete series which appear in this paper share a common partial cuspidal support, it suffices to define only Jordan block when introducing these strongly positive discrete series. This procedure is also summarized in [10, Proposition 1.2]. For cuspidal representation $\pi_{cusp} \in Irr(G)$, we denote by $Irr_{sp}^{\pi_{cusp}}(G)$ the set of all strongly positive discrete series in $Irr(G)$ whose partial cuspidal support is π_{cusp}.

3. Some elementary cases

In this section, we begin to determine the Jacquet modules of discrete series representation σ with respect to the maximal standard parabolic subgroups by examining several elementary cases. Here and subsequently, we denote the admissible triple corresponding to σ by $(\mathrm{Jord}, \sigma_{cusp}, \epsilon)$.

Throughout the paper we assume that σ is a discrete series representation of G_n and there are $d_-, d \in \mathrm{Jord}_{\rho'}$ such that

$$\sigma \hookrightarrow \delta([\nu^{-d_-}\rho', \nu^d \rho']) \rtimes \sigma_{sp} \tag{3}$$

for strongly positive representation σ_{sp} such that $[d_-, d] \cap \mathrm{Jord}_{\rho'}(\sigma_{sp}) = \emptyset$. Let us denote the unique discrete series subrepresentation of $\delta([\nu^{-d_-}\rho', \nu^d \rho']) \rtimes \sigma_{sp}$ different than σ by σ' and the admissible triple corresponding to σ' by $(\mathrm{Jord}, \sigma_{cusp}, \epsilon')$.

We are interested in determining all irreducible constituents of $\mu^*(\sigma)$ of the form $\delta \otimes \pi$, for $\delta \in Irr_{es}(GL(n_\delta, F'))$. We write δ in the form $\delta([\nu^a\rho, \nu^b\rho])$. It is well know ([13, Proposition 2.1]) that this implies $2b+1 \in \mathrm{Jord}_\rho$. To keep the notation uniform, for $(c, \rho) \in \mathrm{Jord}$ we denote by $\mu^*(\sigma)_{(c,\rho)}$ the sum of all irreducible constituents of $\mu^*(\sigma)$ of the form

$$\delta([\nu^a\rho, \nu^c\rho]) \otimes \pi.$$

Let $\rho \in Irr_{sc}(GL(n_\rho, F'))$ (this defines n_ρ) denote a self-dual representation such that there is some $c \in \mathbb{R}$ such that $\nu^c\rho \otimes \pi$ is an irreducible constituent of $\mu^*(\sigma)$ for some irreducible representation π. Furthermore, let us denote the minimal element of $\mathrm{Jord}_\rho(\sigma)$ by $c_{\min}(\rho)$.

In the following sequence of propositions we determine $\mu^*(\sigma)_{(c,\rho)}$ in some elementary cases. First we recall the following result [18, Theorem 8.2].

Lemma 3.1. *If $c \neq c_{\min}(\rho)$ and $a \geq c_- + 2$, there is a unique discrete series representation $\pi_{(a,\rho)}$ such that σ is a subrepresentation of the induced representation*

$$\delta([\nu^a\rho, \nu^c\rho]) \rtimes \pi_{(a,\rho)}.$$

Using this result, we obtain a description of certain Jacquet modules of σ.

Proposition 3.2. *If an irreducible representation $\delta([\nu^a\rho, \nu^c\rho]) \otimes \pi$, for $c \neq c_{\min}(\rho)$ and $a \geq c_- + 2$, appears in $\mu^*(\sigma)$, then π is the unique discrete series representation such that σ is a subrepresentation of $\delta([\nu^a\rho, \nu^c\rho]) \rtimes \pi$, i.e., $\pi \simeq \pi_{(a,\rho)}$. Furthermore, such irreducible constituent appears in $\mu^*(\sigma)$ with multiplicity one.*

Proof. Using the previous lemma we obtain $\mu^*(\sigma) \geq \delta([\nu^a\rho, \nu^c\rho]) \otimes \pi_{(a,\rho)}$ for $a \geq c_- + 2$. Using the same result we get that there is also a discrete series π' such that $\mu^*(\sigma') \geq \delta([\nu^a\rho, \nu^c\rho]) \otimes \pi'$.

Let us prove that if an irreducible constituent of the form $\delta([\nu^a\rho, \nu^b\rho]) \otimes \pi''$ appears in $\mu^*(\sigma)$, then $\pi'' \simeq \pi_{(a,\rho)}$. Applying formula (2) for μ^* to the right-hand side of (3) we obtain that there are $-(d_- + 1) \leq i \leq j \leq d$ and an irreducible constituent $\delta \otimes \tau$ of $\mu^*(\sigma_{sp})$ such that

$$\delta([\nu^a\rho, \nu^c\rho]) \leq \delta([\nu^{-i}\rho', \nu^{d_-}\rho']) \times \delta([\nu^{j+1}\rho', \nu^d\rho']) \times \delta$$

and

$$\pi'' \leq \delta([\nu^{i+1}\rho', \nu^j\rho']) \rtimes \tau.$$

If $(c, \rho) = (d, \rho)$, since $a > d_-$, we obtain $i = -(d_- + 1)$ and $\tau \simeq \sigma_{sp}$. Similarly, if $(c, \rho) = (d_-, \rho')$, we obtain $j = d$ and again $\tau \simeq \sigma_{sp}$. Finally, if $(c, \rho) \notin \{(d, \rho'), (d_-, \rho')\}$ then, using [7, Theorem 4.6], we deduce that $i = -(d_- + 1)$, $j = d$ and τ is a unique strongly positive discrete series such that $\mathrm{Jord}(\tau) = \mathrm{Jord}(\sigma_{sp}) \setminus \{(c, \rho)\} \cup \{(2a - 1, \rho)\}$. In any case, by [10, Theorem 2.1], we obtain that $\delta([\nu^{i+1}\rho', \nu^j\rho']) \rtimes \tau$ is a length three representation and in $R(G)$ we have

$$\delta([\nu^{i+1}\rho', \nu^j\rho']) \rtimes \tau = \pi + \pi' + L(\delta([\nu^{-j}\rho', \nu^{-i-1}\rho']) \rtimes \tau).$$

It follows that both $\delta([\nu^a\rho, \nu^c\rho]) \otimes \pi$ and $\delta([\nu^a\rho, \nu^c\rho]) \otimes \pi'$ appear with multiplicity one in $\mu^*(\delta([\nu^{-c_-}\rho', \nu^c\rho']) \rtimes \sigma_{sp})$. Thus, since $\delta([\nu^a\rho, \nu^c\rho]) \otimes \pi'$ appears in $\mu^*(\sigma')$, it does not appear in $\mu^*(\sigma)$.

Let us now assume that $\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-j}\rho', \nu^{-i-1}\rho']) \rtimes \tau)$ appears in $\mu^*(\sigma)$. Then the transitivity of Jacquet modules implies that $\delta([\nu^a\rho, \nu^c\rho]) \otimes \delta([\nu^{-j}\rho', \nu^{-i-1}\rho']) \otimes \tau$ is contained in $\mu^*(\sigma)$ and it is now easy to obtain a contradiction with the square-integrability of σ. Thus, we obtain that $\pi'' \simeq \pi$ and that $\delta([\nu^a\rho, \nu^c\rho]) \otimes \pi$ appears in $\mu^*(\sigma)$ with multiplicity one.

If we assume that an irreducible constituent of the form $\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-a+1}\rho, \nu^{c_-}\rho]) \rtimes \sigma_{sp})$ appears in $\mu^*(\sigma)$, transitivity of Jacquet modules immediately provides a contradiction with the square-integrability of σ. This completes the proof. ∎

As a direct consequence of the previous proposition and [18, Proposition 7.2], we obtain the following:

Corollary 3.3. *Assume $c \neq c_{\min}(\rho)$ and $\epsilon((c_-, \rho)) \cdot \epsilon((c, \rho)) = -1$. Then*

$$\mu^*(\sigma)_{(c,\rho)} = \sum_{a=c_-+2}^{c} \delta([\nu^a\rho, \nu^c\rho]) \otimes \pi_{(a,\rho)}.$$

In a similar way we handle the case of the minimal element of Jord_ρ. Similarly to [18, Theorem 8.2], we obtain:

Lemma 3.4. *If $c_{\min}(\rho)$ is an integer, set $x = 1$, otherwise set $x = (2 - \epsilon((c_{\min}(\rho), \rho)))/2$. If there is some irreducible representation π such that $\mu^*(\sigma) \geq \nu^{c_{\min}(\rho)}\rho \otimes \pi$, then for $a \geq x$ there exists a unique discrete series representation $\pi_{(a,\rho)}$ such that σ is a subrepresentation of the induced representation*

$$\delta([\nu^a\rho, \nu^{c_{\min}(\rho)}\rho]) \rtimes \pi_{(a,\rho)}.$$

Proof. Let us first assume that σ is a subrepresentation of the induced representation of the same form as in the right-hand side of (3) with $(d_{\_},\rho') \neq (c_{\min}(\rho),\rho)$. Using [7], we see that there is an embedding

$$\sigma_{sp} \hookrightarrow \delta([\nu^a\rho, \nu^{c_{\min}(\rho)}\rho]) \rtimes \sigma'_{sp},$$

for an appropriate strongly positive discrete series σ'_{sp}. Thus, we obtain

$$\begin{aligned} \sigma &\hookrightarrow \delta([\nu^{-d_{\_}}\rho', \nu^d\rho']) \times \delta([\nu^a\rho, \nu^{c_{\min}(\rho)}\rho]) \rtimes \sigma'_{sp} \\ &\simeq \delta([\nu^a\rho, \nu^{c_{\min}(\rho)}\rho]) \times \delta([\nu^{-d_{\_}}\rho', \nu^d\rho']) \rtimes \sigma'_{sp}. \end{aligned}$$

Consequently, there is some irreducible representation $\pi_{(a,\rho)}$ such that σ is a subrepresentation of $\delta([\nu^a\rho, \nu^{c_{\min}(\rho)}\rho]) \rtimes \pi_{(a,\rho)}$. Frobenius reciprocity shows that $\mu^*(\sigma) \geq \delta([\nu^a\rho, \nu^{c_{\min}(\rho)}\rho]) \otimes \pi_{(a,\rho)}$. Lemma 2.1 implies that there are $-(d_{\_}+1) \leq i \leq j \leq d$ and an irreducible constituent $\delta \otimes \tau$ of $\mu^*(\sigma_{sp})$ such that

$$\delta([\nu^a\rho, \nu^{c_{\min}(\rho)}\rho]) \leq \delta([\nu^{-i}\rho', \nu^{d_{\_}}\rho']) \times \delta([\nu^{j+1}\rho', \nu^d\rho']) \times \delta$$

and

$$\pi_{(a,\rho)} \leq \delta([\nu^{i+1}\rho', \nu^j\rho']) \rtimes \tau.$$

It directly follows that $\delta \simeq \delta([\nu^a\rho, \nu^{c_{\min}(\rho)}\rho])$ and $\pi_{(a,\rho)}$ is an irreducible subquotient of $\delta([\nu^{-d_{\_}}\rho', \nu^d\rho']) \rtimes \tau$ for $\tau \in Irr_{sp}^{\sigma_{cusp}}(G)$ such that $[d_{\_}, d] \cap \mathrm{Jord}_{\rho'}(\tau) = \emptyset$. Using the square-integrability of σ we deduce that $\pi_{(a,\rho)} \neq L(\delta([\nu^{-d}\rho', \nu^{d_{\_}}\rho']) \rtimes \tau)$. Thus, [10, Theorem 2.1] implies that $\pi_{(a,\rho)}$ is a discrete series representation and that $\delta \otimes \pi_{(a,\rho)}$ appears with multiplicity one. Since the same discussion can be made for σ', the uniqueness of $\pi_{(a,\rho)}$ follows.

Let us now assume that σ is a subrepresentation of

$$\delta([\nu^{-c_{\min}(\rho)}\rho, \nu^d\rho]) \rtimes \sigma_{sp},$$

where $[c_{\min}(\rho), d] \cap \mathrm{Jord}_\rho(\sigma_{sp}) = \emptyset$, and $\epsilon(c_{\_},\rho)\cdot\epsilon(c,\rho) = -1$ for $c \neq d$. For $a > \frac{1}{2}$, using [10, Proposition 3.1], we get

$$\begin{aligned} \sigma &\hookrightarrow \delta([\nu^{-a+1}\rho, \nu^d\rho]) \times \delta([\nu^{-c_{\min}(\rho)}\rho, \nu^{-a}\rho]) \rtimes \sigma_{sp} \\ &\simeq \delta([\nu^{-a+1}\rho, \nu^d\rho]) \times \delta([\nu^a\rho, \nu^{c_{\min}(\rho)}\rho]) \rtimes \sigma_{sp} \\ &\simeq \delta([\nu^a\rho, \nu^{c_{\min}(\rho)}\rho]) \times \delta([\nu^{-a+1}\rho, \nu^d\rho]) \rtimes \sigma_{sp}. \end{aligned}$$

In the same way as before we conclude that there is a unique discrete series $\pi_{(a,\rho)}$ such that σ is a subrepresentation of $\delta([\nu^a\rho, \nu^{c_{\min}(\rho)}\rho]) \rtimes \pi_{(a,\rho)}$.

It remains to consider the case $a = \frac{1}{2}$ and $\epsilon(c_{\min}(\rho),\rho) = 1$. By definition, there is some irreducible representation $\pi_{(1/2,\rho)}$ such that σ is a subrepresentation of $\delta([\nu^{1/2}\rho, \nu^{c_{\min}(\rho)}\rho]) \rtimes \pi_{(1/2,\rho)}$. Again, Frobenius reciprocity implies $\mu^*(\sigma) \geq \delta([\nu^{1/2}\rho, \nu^{c_{\min}(\rho)}\rho]) \otimes \pi_{(1/2,\rho)}$, and using Lemma 2.1 we deduce that $\pi_{(1/2,\rho)}$ is an irreducible subquotient of $\delta([\nu^{1/2}\rho, \nu^d\rho]) \rtimes \sigma_{sp}$. Square-integrability of σ shows that $\pi_{(1/2,\rho)} \neq L(\delta([\nu^{-d}\rho, \nu^{-1/2}\rho]) \rtimes \sigma_{sp})$, and by [10, Theorem 5.1] it is the uniquely defined discrete series. This proves the lemma. ∎

Analogously to Proposition 3.2 we have:

Proposition 3.5. *If $c_{\min}(\rho)$ is an integer then set $x = 1$, otherwise set $x = (2 - \epsilon((c_{\min}(\rho), \rho)))/2$. If there is some irreducible representation π such that $\mu^*(\sigma) \geq \nu^{c_{\min}(\rho)}\rho \otimes \pi$, then the following equality holds in $R(GL) \otimes R(G)$:*

$$\mu^*(\sigma)_{(c_{\min}(\rho),\rho)} = \sum_{a=x}^{c_{\min}(\rho)} \delta([\nu^a\rho, \nu^{c_{\min}(\rho)}\rho]) \otimes \pi_{(a,\rho)}.$$

In the rest of the paper we are interested in determining $\mu^*(\sigma)_{(c,\rho)}$ for $c \neq c_{\min}(\rho)$ and $\epsilon((c_-, \rho)) = \epsilon((c, \rho))$. Thus, to keep things simple, we assume $c = d$ and $\rho \simeq \rho'$, so σ can be given as an irreducible subrepresentation of the induced representation of the form

$$\sigma \hookrightarrow \delta([\nu^{-c_-}\rho, \nu^c\rho]) \rtimes \sigma_{sp} \tag{4}$$

for $\sigma_{sp} \in Irr_{sp}^{\sigma_{cusp}}(G)$ such that $[c_-, c] \cap \text{Jord}_\rho(\sigma_{sp}) = \emptyset$. Let us denote by σ_{ind} the induced representation $\delta([\nu^{-c_-}\rho, \nu^c\rho]) \rtimes \sigma_{sp}$ and by σ' a unique discrete series subrepresentation of σ_{ind} different than σ. We note that σ_{ind} is a length three representation, by [10, Theorem 2.1].

Also, applying (2) to the right-hand side of (4) we see at once that if an irreducible constituent of the form $\delta([\nu^a\rho, \nu^c\rho]) \otimes \pi$ appears in $\mu^*(\sigma)$, then $a \geq c_-$.

Two possible cases shall be examined in separate sections.

4. Case $\text{Jord}_\rho(\sigma_{sp}) \neq \emptyset$ or c non-integral.

Since in the case which we consider in this section the classical-group part π of an irreducible constituent $\delta([\nu^a\rho, \nu^c\rho]) \otimes \pi$ appearing in $\mu^*(\sigma)_{(c,\rho)}$ heavily depends on the left end of the segment $[\nu^a\rho, \nu^c\rho]$, several subcases will be treated separately. To simplify the notation, we write $\mu^*(\sigma)_{(a,c,\rho)}$ (resp., $\mu^*(\sigma_{ind})_{(a,c,\rho)}$) for the formal sum of all irreducible constituents of $\mu^*(\sigma)_{(c,\rho)}$ (resp., $\mu^*(\sigma_{ind})$) of the form $\delta([\nu^a\rho, \nu^c\rho]) \otimes \pi$.

We can assume $a \leq c_- + 1$, since in the same way as in the proof of Proposition 3.2, one can see that $\mu^*(\sigma)_{(\rho,c,a)} = \delta([\nu^a\rho, \nu^c\rho]) \otimes \pi_{(a,\rho)}$ for $a \geq c_- + 2$.

Also, it follows from [13, Section 4], that there is a unique irreducible tempered subrepresentation τ of $\delta([\nu^{-c_-}\rho, \nu^{c_-}\rho]) \rtimes \sigma_{sp}$ such that

$$\mu^*(\sigma)_{(c_-+1,c,\rho)} = \delta([\nu^{c_-+1}\rho, \nu^c\rho]) \otimes \tau.$$

To determine of $\mu^*(\sigma)_{(a,c,\rho)}$ we begin with several elementary but useful technical results.

Lemma 4.1. *Suppose that $\delta([\nu^a\rho, \nu^c\rho]) \otimes \pi$ is an irreducible constituent of $\mu^*(\sigma_{ind})$, with $a \leq c_-$. Then $\delta([\nu^a\rho, \nu^c\rho]) \otimes \pi$ does not appear in $\mu^*(L(\delta([\nu^{-c}\rho, \nu^{c_-}\rho]) \rtimes \sigma_{sp}))$.*

Proof. Suppose, on the contrary, that $\delta([\nu^a\rho, \nu^c\rho]) \otimes \pi$ is an irreducible constituent of $\mu^*(L(\delta([\nu^{-c}\rho, \nu^{c_-}\rho]) \rtimes \sigma_{sp}))$, for $a \leq c_-$. Then transitivity of Jacquet modules implies that $\delta([\nu^{c_-+1}\rho, \nu^c\rho]) \otimes \pi' \leq \mu^*(L(\delta([\nu^{-c}\rho,\ \nu^{c_-}\rho]) \rtimes \sigma_{sp}))$ for some irreducible representation π'. But it is well-known ([13, Section 4]) that there are only two irreducible constituents of the form $\delta([\nu^{c_-+1}\rho,\ \nu^{c}\rho]) \otimes \pi'$ appearing in $\mu^*(\sigma_{ind})$ and each of them is contained either in $\mu^*(\sigma)$ or in $\mu^*(\sigma')$, a contradiction. ∎

Proposition 4.2. *For $-c_-+1 \leq a \leq c_-$, $\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp})$ appears in $\mu^*(\sigma)_{(a,c,\rho)}$ with multiplicity one.*

Proof. From (4) we get

$$\sigma \hookrightarrow \delta([\nu^a\rho, \nu^c\rho]) \times \delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}$$

and Frobenius reciprocity shows that the irreducible representation

$$\delta([\nu^a\rho, \nu^c\rho]) \otimes \delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \otimes \sigma_{sp}$$

appears in $(m\mu)^*(\sigma)$. By transitivity of Jacquet modules, there is some irreducible representation π such that $\mu^*(\sigma)_{(a,c,\rho)}$ contains $\delta([\nu^a\rho, \nu^c\rho]) \otimes \pi$ and $\mu^*(\pi) \geq \delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \otimes \sigma_{sp}$. We determine π by calculating μ^* of the right-hand side of (4). By Lemma 2.1, there are $-c_- + 1 \leq i \leq j \leq c$ and an irreducible constituent $\delta \otimes \pi'$ of $\mu^*(\sigma_{sp})$ such that

$$\delta([\nu^a\rho, \nu^c\rho]) \leq \delta([\nu^{-i}\rho, \nu^{c_-}\rho]) \times \delta([\nu^{j+1}\rho, \nu^c\rho]) \times \delta$$

and

$$\pi \leq \delta([\nu^{i+1}\rho, \nu^j\rho]) \rtimes \pi'.$$

Firstly, if $-i = a$ or $j + 1 = a$, using [7, Theorem 4.6] and the fact that $[c_-, c] \cap \mathrm{Jord}_\rho(\sigma_{sp}) = \emptyset$, we obtain $\pi \leq \delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}$.

Now we calculate the multiplicity of $\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \otimes \sigma_{sp}$ in $\mu^*(\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp})$.

Again, there are $-c_- + 1 \leq i_1 \leq j_1 \leq a - 1$ and an irreducible constituent $\delta_1 \otimes \pi_1 \leq \mu^*(\sigma_{sp})$ such that

$$\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \leq \delta([\nu^{-i_1}\rho, \nu^{c_-}\rho]) \times \delta([\nu^{j_1+1}\rho, \nu^{a-1}\rho]) \times \delta_1$$

and

$$\sigma_{sp} \leq \delta([\nu^{i_1+1}\rho, \nu^{j_1}\rho]) \rtimes \pi_1.$$

Since $a - 1 < c_-$, we get $i_1 = -c_- + 1$. Furthermore, the strong positivity of σ_{sp} implies that $j_1 = -c_- + 1$, so $\pi_1 = \sigma_{sp}$. Thus, $\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \otimes \sigma_{sp}$ appears with multiplicity one in $\mu^*(\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp})$. Since it obviously appears in $\mu^*(L(\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}))$, it follows that $L(\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp})$ is the unique irreducible subquotient π_2 of $\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}$ such that $\mu^*(\pi_2)$ contains $\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \otimes \sigma_{sp}$.

Secondly, if $-i > a$ and $j+1 > a$, it follows that $\delta = \delta([\nu^a\rho, \nu^b\rho])$ for some $a \leq b \leq c$. This further gives $\delta([\nu^{-i}\rho, \nu^{c_-}\rho]) \times \delta([\nu^{j+1}\rho, \nu^c\rho]) \simeq \delta([\nu^{b+1}\rho, \nu^c\rho])$ and it directly follows that $\pi \leq \delta([\nu^{b+1}\rho, \nu^{c_-}\rho]) \rtimes \pi'$, where, by [7, Theorem 4.6], π' is the unique element of $Irr_{sp}^{\sigma_{cusp}}(G)$ such that $\mathrm{Jord}(\pi') = \mathrm{Jord}(\sigma_{sp}) \setminus \{(2b+1, \rho)\} \cup \{(2a-1, \rho)\}$.

Since $b \geq a$ and π' is a strongly positive representation, it can be seen directly from (2) that each irreducible constituent of the form $\delta([\nu^{-c_-}\rho, \nu^{d}\rho]) \otimes \pi''$ appearing in $\mu^*(\delta([\nu^{b+1}\rho, \nu^{c_-}\rho]) \rtimes \pi')$ satisfies $d > a$.

Consequently, $\mu^*(\sigma)_{(a,c,\rho)} \geq \delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp})$. We have already seen that such irreducible constituent appears in $\mu^*(\sigma_{ind})$ with multiplicity two and it can be proved in completely analogous manner that it also appears in $\mu^*(\sigma')$. Thus, it appears in $\mu^*(\sigma)_{(a,c,\rho)}$ with multiplicity one. ∎

The following lemma illustrates our general method, and will be used several times in this paper.

Lemma 4.3. *Suppose that π is an irreducible subquotient of $\delta([\nu^{-c_-}\rho, \nu^c\rho]) \rtimes \sigma_{sp}$ for $\sigma_{sp} \in Irr_{sp}^{\sigma_{cusp}}(G)$ such that $[c_-, c] \cap Jord_\rho(\sigma_{sp}) = \emptyset$ and $c_- > \min(Jord_\rho(\sigma_{sp}))$. If there is an irreducible subquotient $\delta \otimes \pi'$ of $\mu^*(\pi)$ such that $m^*(\delta)$ contains $\delta([\nu^a\rho, \nu^c\rho]) \otimes \delta([\nu^{(c_-)_-+1}\rho, \nu^{c_-}\rho])$, for $a \leq c_-$, and $\min\{d \in Jord_\rho(\sigma_{sp}) \cup \{c_-, c\} : a \leq d\} \neq (c_-)_-$, then π is the unique discrete series subrepresentation of $\delta([\nu^{-c_-}\rho, \nu^c\rho]) \rtimes \sigma_{sp}$ such that for the corresponding admissible triple $(Jord(\pi), \sigma_{cusp}, \epsilon_\pi)$ we have $\epsilon_\pi((c_-, \rho)) = \epsilon_\pi((c, \rho))$.*

Proof. From Lemma 4.1 we deduce that π is a discrete series subrepresentation of $\delta([\nu^{-c_-}\rho, \nu^c\rho]) \rtimes \sigma_{sp}$. Let us denote the corresponding admissible triple by $(\mathrm{Jord}(\pi), \sigma_{cusp}, \epsilon_\pi)$. We will determine δ from $\mu^*(\delta([\nu^{-c_-}\rho, \nu^c\rho]) \rtimes \sigma_{sp})$. By Lemma 2.1, there are $-c_- + 1 \leq i \leq j \leq c$ and an irreducible constituent $\delta' \otimes \pi''$ of $\mu^*(\sigma_{sp})$ such that

$$\delta \leq \delta([\nu^{-i}\rho, \nu^{c_-}\rho]) \times \delta([\nu^{j+1}\rho, \nu^c\rho]) \times \delta'.$$

There are two possibilities to consider:

- $a \leq 0$.

Using [7, Theorem 4.6] we deduce that either $j+1 = a$ or $-i = a$. In the first case, $i = -((c_-)_- + 1)$ and δ is an irreducible subquotient of the induced representation $\delta([\nu^{(c_-)_-+1}\rho, \nu^{c_-}\rho]) \times \delta([\nu^a\rho, \nu^c\rho])$ which is irreducible ([19]). Thus, in this case

$$\delta \simeq \delta([\nu^{(c_-)_-+1}\rho, \nu^{c_-}\rho]) \times \delta([\nu^a\rho, \nu^c\rho]).$$

In the second case, $j = (c_-)_-$ and δ is an irreducible subquotient of the induced representation $\delta([\nu^a\rho, \nu^{c_-}\rho]) \times \delta([\nu^{(c_-)_-+1}\rho, \nu^c\rho])$, which is, by [19], length two representation which contains $\delta([\nu^{(c_-)_-+1}\rho, \nu^{c_-}\rho]) \times \delta([\nu^a\rho, \nu^c\rho])$ as an irreducible subquotient. Let us determine the multiplicity of $\delta([\nu^a\rho, \nu^c\rho]) \otimes \delta([\nu^{(c_-)_-+1}\rho, \nu^{c_-}\rho])$ in $m^*(\delta([\nu^a\rho, \nu^{c_-}\rho]) \times \delta([\nu^{(c_-)_-+1}\rho, \nu^c\rho]))$.

There are $a - 1 \leq i_1 \leq c_-$ and $(c_-)_- \leq j_1 \leq c$ such that

$$\delta([\nu^a\rho, \nu^c\rho]) \leq \delta([\nu^{i_1+1}\rho, \nu^{c_-}\rho]) \times \delta([\nu^{j_1+1}\rho, \nu^c\rho])$$

and

$$\delta([\nu^{(c_-)_-+1}\rho, \nu^{c_-}\rho]) \leq \delta([\nu^a\rho, \nu^{i_1}\rho]) \times \delta([\nu^{(c_-)_-+1}\rho, \nu^{j_1}\rho]).$$

Since $a < (c_-)_- + 1$, from the first inequality we obtain $i_1 = a - 1$ and $j_1 = c_-$. Therefore, $\delta([\nu^a\rho, \nu^c\rho]) \otimes \delta([\nu^{(c_-)_-+1}\rho, \nu^{c_-}\rho])$ appears with multiplicity one in

$m^*(\delta([\nu^a\rho, \nu^{c_-}\rho]) \times \delta([\nu^{(c_-)_-+1}\rho,\ \nu^c\rho]))$ and it obviously appears in $m^*(\delta([\nu^{(c_-)_-+1}\rho,$ $\nu^{c_-}\rho]) \times \delta([\nu^a\rho,\ \nu^c\rho]))$. Again, we conclude that δ is isomorphic to $\delta([\nu^{(c_-)_-+1}\rho,$ $\nu^{c_-}\rho]) \times \delta([\nu^a\rho, \nu^c\rho])$.

- $a > 0$.

If $j+1 = a$ or $-i = a$, in the same way as before we obtain $\delta \simeq \delta([\nu^{(c_-)_-+1}\rho, \nu^{c_-}\rho]) \times \delta([\nu^a\rho, \nu^c\rho])$. It remains to discuss the case when $\nu^a\rho$ appears in the cuspidal support of δ'. To shorten notation, we denote the minimum of the set $\{d \in \mathrm{Jord}_\rho(\sigma_{sp}) \cup \{c_-, c\} : a \leq d\}$ by x and the maximal d such that $\nu^d\rho$ appears in cuspidal support of δ' by y. Since $y \in \mathrm{Jord}_\rho(\sigma_{sp})$, we deduce $y < (c_-)_-$, since otherwise we would have $y > c$, which is impossible. Now from (1) and

$$\delta([\nu^a\rho, \nu^c\rho]) \otimes \delta([\nu^{(c_-)_-+1}\rho, \nu^{c_-}\rho]) \leq m^*(\delta([\nu^{-i}\rho, \nu^{c_-}\rho]) \times \delta([\nu^{j+1}\rho, \nu^c\rho]) \times \delta')$$

we see that there are $-i-1 \leq i_1 \leq c_-$ and $j \leq i_2 \leq c$ such that

$$\delta([\nu^a\rho, \nu^c\rho]) \leq \delta([\nu^{i_1+1}\rho, \nu^{c_-}\rho]) \times \delta([\nu^{i_2+1}\rho, \nu^c\rho]) \times \delta'.$$

Since neither $\nu^a\rho$ nor $\nu^y\rho$ appear in the cuspidal support of the representations $\delta([\nu^{i_1+1}\rho, \nu^{c_-}\rho])$ and $\delta([\nu^{i_2+1}\rho, \nu^c\rho])$, and $m^*(\delta([\nu^a\rho, \nu^c\rho])) \geq \delta([\nu^{y+1}\rho, \nu^c\rho]) \otimes \delta([\nu^a\rho, \nu^y\rho])$, we directly obtain that $\delta' \simeq \delta([\nu^a\rho, \nu^y\rho])$. [7, Theorem 4.6] now implies $x = y$. Further, it follows that $m^*(\delta([\nu^{-i}\rho, \nu^{c_-}\rho]) \times \delta([\nu^{j+1}\rho, \nu^c\rho]))$ contains $\delta([\nu^{x+1}\rho, \nu^c\rho]) \otimes \delta([\nu^{(c_-)_-+1}\rho, \nu^{c_-}\rho])$. Now it can be easily seen that δ is an irreducible subquotient of either or both of the following representations:

$$\pi_1 = \delta([\nu^a\rho, \nu^x\rho]) \times \delta([\nu^{x+1}\rho, \nu^c\rho]) \times \delta([\nu^{(c_-)_-+1}\rho, \nu^{c_-}\rho]),$$

$$\pi_2 = \delta([\nu^a\rho, \nu^x\rho]) \times \delta([\nu^{x+1}\rho, \nu^{c_-}\rho]) \times \delta([\nu^{(c_-)_-+1}\rho, \nu^c\rho]).$$

We note that both representations π_1 and π_2 contain $\delta([\nu^{(c_-)_-+1}\rho,\ \nu^{c_-}\rho]) \times \delta([\nu^a\rho, \nu^c\rho])$ as an irreducible subquotient. Since $x \neq (c_-)_-$, using formula (1) we directly obtain that the multiplicity of $\delta([\nu^a\rho, \nu^c\rho]) \otimes \delta([\nu^{(c_-)_-+1}\rho, \nu^{c_-}\rho])$ equals one in $m^*(\pi_1)$, $m^*(\pi_2)$ and in $m^*(\delta([\nu^a\rho, \nu^c\rho]) \times \delta([\nu^{(c_-)_-+1}\rho, \nu^{c_-}\rho]))$. Again we deduce that δ is isomorphic to $\delta([\nu^a\rho, \nu^c\rho]) \times \delta([\nu^{(c_-)_-+1}\rho, \nu^{c_-}\rho])$.

This enables us to conclude that $(m\mu)^*(\pi)$ contains

$$\delta([\nu^{(c_-)_-+1}\rho, \nu^{c_-}\rho]) \otimes \delta([\nu^a\rho, \nu^c\rho]) \otimes \pi'$$

and [18, Proposition 7.2] gives $\epsilon_\pi(((c_-)_-, \rho)) = \epsilon_\pi((c_-, \rho))$. This characterizes π uniquely and completes the proof. ∎

The Jacquet modules in the two main cases will be studied separately.

- Case $a \leq 0$.

Let us first consider the case $a \leq 0$. In the following lemma, applying (2) to the induced representation (4), we obtain all candidates for irreducible constituents of $\mu^*(\sigma)_{(a,c,\rho)}$.

Lemma 4.4. *Suppose $a \leq 0$. If $c_- > \min(Jord_\rho(\sigma))$ and $-a \leq (c_-)_-$ then the following equality holds in $R(GL) \otimes R$:*

$$\mu^*(\sigma_{ind})_{(a,c,\rho)} = 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}) + \\ + 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-(c_-)_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma'_{sp}),$$

for the unique $\sigma'_{sp} \in Irr_{sp}^{\sigma_{cusp}}(G)$ such that $Jord(\sigma'_{sp}) = Jord(\sigma_{sp}) \setminus \{((c_-)_-, \rho)\} \cup \{(c_-, \rho)\}$. Otherwise, in $R(GL) \otimes R$ we have

$$\mu^*(\sigma_{ind})_{(a,c,\rho)} = 2\ \delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}).$$

Proof. Let us comment only the case $c_- > \min(\mathrm{Jord}_\rho(\sigma))$ and $-a \leq (c_-)_-$ case. In other cases $\mu^*(\sigma_{ind})_{(a,c,\rho)}$ can be obtained in the same way but more easily. Using Lemma 2.1, we deduce that there are $-c_- + 1 \leq i \leq j \leq c$ and an irreducible constituent $\delta \otimes \pi'$ of $\mu^*(\sigma_{sp})$ such that

$$\delta([\nu^a\rho, \nu^c\rho]) \leq \delta([\nu^{-i}\rho, \nu^{c_-}\rho]) \times \delta([\nu^{j+1}\rho, \nu^c\rho]) \times \delta$$

and

$$\pi \leq \delta([\nu^{i+1}\rho, \nu^j\rho]) \rtimes \pi'.$$

Since $a \leq 0$, the strong positivity of σ_{sp} implies that either $-i = a$ or $j + 1 = a$. Also, $\pi' \simeq \sigma_{sp}$. If $-i = a$, it directly follows that $j = c_-$ and if $j + 1 = a$ we have $i = -c_- + 1$. Now [10, Proposition 3.1.(i)] shows that in $R(G)$

$$\delta([\nu^{i+1}\rho, \nu^j\rho]) \rtimes \sigma_{sp} = L(\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}) + L(\delta([\nu^{-(c_-)_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma'_{sp}),$$

holds, and the lemma is proved. ■

We are now ready to describe $\mu^*(\sigma)_{(a,c,\rho)}$ for $a \leq 0$.

Theorem 4.5. *Suppose $a \leq 0$. If $c_- > \min(Jord_\rho(\sigma))$, $-a \leq (c_-)_-$, and $\epsilon(((c_-)_-, \rho)) = \epsilon((c_-, \rho))$ then in $R(GL) \otimes R$ we have:*

$$\mu^*(\sigma)_{(a,c,\rho)} = \delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}) + \\ + 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-(c_-)_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma'_{sp}),$$

for the unique $\sigma'_{sp} \in Irr_{sp}^{\sigma_{cusp}}(G)$ such that $Jord(\sigma'_{sp}) = Jord(\sigma_{sp}) \setminus \{((c_-)_-, \rho)\} \cup \{(c_-, \rho)\}$. Otherwise, in $R(GL) \otimes R$ we have

$$\mu^*(\sigma)_{(a,c,\rho)} = \delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}).$$

Proof. We have already seen that $\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp})$ appears in $\mu^*(\sigma)_{(a,c,\rho)}$ with multiplicity one. Previous lemma enables us to assume that $c_- > \min(\mathrm{Jord}_\rho(\sigma))$ and $-a \leq (c_-)_-$. Lemma 4.1 shows that $\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-(c_-)_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma'_{sp})$ can appear only in $\mu^*(\sigma)$ or in $\mu^*(\sigma')$.

It is not hard to see that there is a unique $\pi \in Irr_{sp}^{\sigma_{cusp}}(G)$ such that σ'_{sp} is a subrepresentation of the induced representation

$$\delta([\nu^{(c_-)_-+1}\rho, \nu^{c_-}\rho]) \rtimes \pi.$$

This provides an embedding

$$L(\delta([\nu^{-(c_-)_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma'_{sp}) \hookrightarrow \delta([\nu^{-(c_-)_-}\rho, \nu^{a-1}\rho]) \times \delta([\nu^{(c_-)_-+1}\rho, \nu^{c_-}\rho]) \rtimes \pi.$$

Since $a \leq 0$, we have

$$\delta([\nu^{-(c_-)_-}\rho, \nu^{a-1}\rho]) \times \delta([\nu^{(c_-)_-+1}\rho, \nu^{c_-}\rho]) \simeq \delta([\nu^{(c_-)_-+1}\rho, \nu^{c_-}\rho]) \times \delta([\nu^{-(c_-)_-}\rho, \nu^{a-1}\rho])$$

and by [13, Lemma 3.2] there exists an irreducible representation π' such that $L(\delta([\nu^{-(c_-)_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma'_{sp})$ is a subrepresentation of $\delta([\nu^{(c_-)_-+1}\rho, \nu^{c_-}\rho]) \rtimes \pi'$. Frobenius reciprocity and transitivity of Jacquet modules show that if $\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-(c_-)_-}\rho,\ \nu^{a-1}\rho]) \rtimes \sigma'_{sp})$ appears in $\mu^*(\sigma)$, then $(m\mu)^*(\sigma)$ contains

$$\delta([\nu^a\rho, \nu^c\rho]) \otimes \delta([\nu^{(c_-)_-+1}\rho, \nu^{c_-}\rho]) \otimes \pi'.$$

Using transitivity of Jacquet modules again, we deduce that there is some irreducible constituent $\delta \otimes \pi'$ of $\mu^*(\sigma)$ such that $m^*(\delta)$ contains $\delta([\nu^a\rho, \nu^c\rho]) \otimes \delta([\nu^{(c_-)_-+1}\rho,\ \nu^{c_-}\rho])$. By Lemma 4.3, we have $\epsilon(((c_-)_-, \rho)) = \epsilon((c_-, \rho))$.

On the other hand, since discrete series σ and σ' are not isomorphic, by definition of σ we have $\epsilon(((c_-)_-, \rho)) = \epsilon((c_-, \rho))$ if and only if $\epsilon'(((c_-)_-, \rho)) \neq \epsilon'((c_-, \rho))$.

Consequently, if $\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-(c_-)_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma'_{sp})$ appears in $\mu^*(\sigma)$ then it is not an irreducible constituent of $\mu^*(\sigma')$, and the theorem is proved. ■

- Case $a \geq 1$.

We shall now consider the case $a \geq 1$. Let us begin with a technical lemma. From now on, x stands for the minimum of the set $\{d \in \mathrm{Jord}_\rho : a \leq d\}$.

Lemma 4.6. *Suppose $a \geq 1$. If $x = c_-$ or $a = x_- + 1$ then for any irreducible constituent $\delta([\nu^a\rho, \nu^c\rho]) \otimes \pi$ of $\mu^*(\sigma_{ind})$ we have*

$$\pi \leq \delta([\nu^{-a+1}\rho, \nu^{c_-}\rho]) \rtimes \sigma_{sp}.$$

Otherwise,

$$\pi \leq \delta([\nu^{-a+1}\rho, \nu^{c_-}\rho]) \rtimes \sigma_{sp} \oplus \delta([\nu^{-x}\rho, \nu^{c_-}\rho]) \rtimes \sigma'_{sp},$$

for the unique $\sigma'_{sp} \in Irr_{sp}^{\sigma_{cusp}}(G)$ such that $Jord(\sigma'_{sp}) = Jord(\sigma_{sp}) \setminus \{(x, \rho)\} \cup \{(2a-1, \rho)\}$.

Proof. Similarly as in the previous case, we see that there are $-c_- + 1 \leq i \leq j \leq c$ and an irreducible constituent $\delta \otimes \pi'$ of $\mu^*(\sigma_{sp})$ such that

$$\delta([\nu^a\rho, \nu^c\rho]) \leq \delta([\nu^{-i}\rho, \nu^{c_-}\rho]) \times \delta([\nu^{j+1}\rho, \nu^c\rho]) \times \delta$$

and

$$\pi \leq \delta([\nu^{i+1}\rho, \nu^j\rho]) \rtimes \pi'.$$

If $-i = a$ or $j + 1 = a$, we deduce that $\pi \leq \delta([\nu^{-a+1}\rho, \nu^{c_-}\rho]) \rtimes \sigma_{sp}$. Otherwise, [7, Theorem 4.6] implies that $a \neq x_- + 1$ and $x < c$. This further gives $x < c_-$, $\delta \simeq \delta([\nu^a\rho, \nu^x\rho])$, and $\pi' \simeq \sigma'_{sp}$. Consequently,

$$\delta([\nu^{x+1}\rho, \nu^c\rho]) \leq \delta([\nu^{-i}\rho, \nu^{c_-}\rho]) \times \delta([\nu^{j+1}\rho, \nu^c\rho])$$

and in the same way as before we conclude that $\pi \leq \delta([\nu^{-x}\rho, \nu^{c_-}\rho]) \rtimes \sigma'_{sp}$. ■

In the following sequence of propositions, considering case by case, we provide a complete description of $\mu^*(\sigma)_{(a,c,\rho)}$ for $a \geq 1$.

Proposition 4.7. *If $a \geq 1$, $x = c_-$, and $a \neq x_- + 1$, then there is a unique discrete series subrepresentation π of $\delta([\nu^{-a+1}\rho, \nu^{c_-}\rho]) \rtimes \sigma_{sp}$ such that in $R(GL) \otimes R$ we have:*

$$\mu^*(\sigma)_{(a,c,\rho)} = \delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}) + \\ 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes \pi.$$

Proof. We have already seen that $\mu^*(\sigma)_{(a,c,\rho)}$ contains an irreducible constituent $\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp})$. Furthermore, the following equality holds in $R(GL) \otimes R$:

$$\mu^*(\sigma_{ind})_{(a,c,\rho)} = 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}) + \\ + 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes \sigma_1 + 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes \sigma_2,$$

where σ_1 and σ_2 are mutually non-isomorphic discrete series subrepresentations of the induced representations $\delta([\nu^{-a+1}\rho, \nu^{c_-}\rho]) \rtimes \sigma_{sp}$.

Since $\mathrm{Jord}_\rho(\sigma_{sp})$ is non-empty, let us first assume that there is a $y \in \mathrm{Jord}_\rho$ such that $y_- = c$.

We denote by σ_1 the unique discrete series subrepresentation of the induced representation $\delta([\nu^{-a+1}\rho, \nu^{c_-}\rho]) \rtimes \sigma_{sp}$ which is also a subrepresentation of the induced representation $\delta([\nu^{-c_-}\rho, \nu^y\rho]) \rtimes \sigma'_{sp}$, for the unique $\sigma'_{sp} \in Irr_{sp}^{\sigma_{cusp}}(G)$ such that $\mathrm{Jord}(\sigma'_{sp}) = \mathrm{Jord}(\sigma_{sp}) \setminus \{(y,\rho)\} \cup \{(2a-1,\rho)\}$. If an irreducible subquotient π' of σ_{ind} contains $\delta([\nu^a\rho, \nu^c\rho]) \otimes \sigma_1$ in $\mu^*(\pi')$, then transitivity of Jacquet modules shows that $(m\mu)^*(\pi')$ contains $\delta([\nu^a\rho, \nu^c\rho]) \otimes \delta([\nu^{-c_-}\rho, \nu^y\rho]) \otimes \sigma'_{sp}$. Thus, there is some irreducible representation δ such that $\mu^*(\pi') \geq \delta \otimes \sigma'_{sp}$ and $m^*(\delta) \geq \delta([\nu^a\rho, \nu^c\rho]) \otimes \delta([\nu^{-c_-}\rho, \nu^y\rho])$. Since $\pi' \leq \sigma_{ind}$ there are $-c_- + 1 \leq i \leq j \leq c$ and an irreducible constituent $\delta' \otimes \pi''$ of $\mu^*(\sigma_{sp})$ such that

$$\delta \leq \delta([\nu^{-i}\rho, \nu^{c_-}\rho]) \times \delta([\nu^{j+1}\rho, \nu^c\rho]) \times \delta'.$$

It directly follows that either $-i = -c_-$ or $j + 1 = -c_-$. If $j + 1 = -c_-$, then $i = -c_- + 1$ and $\delta' \simeq \delta([\nu^a\rho, \nu^y\rho])$, which implies

$$\delta \simeq \delta([\nu^{-c_-}\rho, \nu^c\rho]) \times \delta([\nu^a\rho, \nu^y\rho]).$$

If $-i = -c_-$, we get $j + 1 = c_- + 1$ and again $\delta' \simeq \delta([\nu^a\rho, \nu^y\rho])$. Thus, in this case δ is an irreducible subquotient of

$$\delta([\nu^{-c_-}\rho, \nu^{c_-}\rho]) \times \delta([\nu^{c_-+1}\rho, \nu^c\rho]) \times \delta([\nu^a\rho, \nu^y\rho]). \tag{5}$$

Since $\delta([\nu^{-c_-}\rho, \nu^c\rho]) \times \delta([\nu^a\rho, \nu^y\rho])$ is an irreducible subquotient of the induced representation (5), and since it can easily be seen that $\delta([\nu^a\rho, \nu^c\rho]) \otimes \delta([\nu^{-c_-}\rho, \nu^y\rho])$ appears with multiplicity one in both $m^*(\delta([\nu^{-c_-}\rho,\ \nu^c\rho]) \times \delta([\nu^a\rho, \nu^y\rho]))$ and $m^*(\delta([\nu^{-c_-}\rho,\ \nu^{c_-}\rho]) \times \delta([\nu^{c_-+1}\rho,\ \nu^c\rho]) \times \delta([\nu^a\rho, \nu^y\rho]))$, we again obtain $\delta \simeq \delta([\nu^{-c_-}\rho,\ \nu^c\rho]) \times \delta([\nu^a\rho,\ \nu^y\rho])$.

Lemma 4.1 shows that $\pi' \in \{\sigma, \sigma'\}$. Furthermore, by [18, Proposition 7.2], $\pi' \simeq \sigma$ implies that $\epsilon((c,\rho)) = \epsilon((y,\rho))$. If $\epsilon((c,\rho)) = \epsilon((y,\rho))$ then by the definition of σ' we have $\epsilon'((c,\rho)) \neq \epsilon'((y,\rho))$ and, consequently, $2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes \sigma_1 \leq \mu^*(\sigma)_{(a,c,\rho)}$. Otherwise, in the same way we conclude that $2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes \sigma_1 \leq \mu^*(\sigma')_{(a,c,\rho)}$.

We also denote by σ_2 the unique discrete series subrepresentation of the induced representation $\delta([\nu^{-a+1}\rho, \nu^{c_-}\rho]) \rtimes \sigma_{sp}$ different than σ_1. Using Lemma 2.1 we see that $\delta([\nu^a\rho, \nu^c\rho]) \otimes \sigma_2$ does not appear as an irreducible constituent of $\mu^*(\delta([\nu^{-c}\rho, \nu^y\rho]) \rtimes \sigma''_{sp})$ for a unique $\sigma''_{sp} \in Irr_{sp}^{\sigma_{cusp}}(G)$ such that $\mathrm{Jord}(\sigma'_{sp}) = \mathrm{Jord}(\sigma_{sp}) \setminus \{(y,\rho)\} \cup \{(c_-,\rho)\}$. Thus, if $\epsilon((c,\rho))$ equals $\epsilon((y,\rho))$, then $\mu^*(\sigma)_{(a,c,\rho)}$ does not contain $\delta([\nu^a\rho, \nu^c\rho]) \otimes \sigma_2$, since σ is a subrepresentation of $\delta([\nu^{-c}\rho, \nu^y\rho]) \rtimes \sigma''_{sp}$. Analogously, if $\epsilon'((c,\rho)) = \epsilon'((y,\rho))$ then $\mu^*(\sigma')_{(a,c,\rho)}$ does not contain $\delta([\nu^a\rho, \nu^c\rho]) \otimes \sigma_2$, so Lemma 4.1 implies $2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes \sigma_2 \leq \mu^*(\sigma)_{(a,c,\rho)}$.

Now we assume $c = \max(\mathrm{Jord}_\rho)$ and denote by z the element in Jord_ρ such that $(c_-)_- = z$.

Similarly as in the previous case, let us denote by σ_3 the unique discrete series subrepresentation of $\delta([\nu^{-a+1}\rho, \nu^{c_-}\rho]) \rtimes \sigma_{sp}$ which is also a subrepresentation of $\delta([\nu^{-z}\rho, \nu^{a-1}\rho]) \rtimes \sigma'_{sp}$, for the unique $\sigma'_{sp} \in Irr_{sp}^{\sigma_{cusp}}(G)$ such that $\mathrm{Jord}(\sigma'_{sp}) = \mathrm{Jord}(\sigma_{sp}) \setminus \{(z,\rho)\} \cup \{(2a-1,\rho)\}$. Also, we denote by σ_4 the unique discrete series subrepresentation of $\delta([\nu^{-a+1}\rho, \nu^{c_-}\rho]) \rtimes \sigma_{sp}$ different than σ_3. If we let π' stand for any irreducible subquotient of σ_{ind} such that $\delta([\nu^a\rho, \nu^c\rho]) \otimes \sigma_3$ appears in $\mu^*(\pi')$, then it follows that $(m\mu)^*(\pi')$ contains

$$\delta([\nu^a\rho, \nu^c\rho]) \otimes \delta([\nu^{-z}\rho, \nu^{a-1}\rho]) \otimes \delta([\nu^{z+1}\rho, \nu^{c_-}\rho]) \otimes \tau,$$

for some irreducible representation τ. Consequently, there is an irreducible constituent $\delta' \otimes \tau$ appearing in $\mu^*(\pi')$ such that Jacquet module of δ with respect to the appropriate standard parabolic subgroup contains $\delta([\nu^a\rho, \nu^c\rho]) \otimes \delta([\nu^{-z}\rho, \nu^{a-1}\rho]) \otimes \delta([\nu^{z+1}\rho, \nu^{c_-}\rho])$. Again, there are $-c_- + 1 \leq i \leq j \leq c$ and an irreducible constituent $\delta' \otimes \pi''$ of $\mu^*(\sigma_{sp})$ such that

$$\delta \leq \delta([\nu^{-i}\rho, \nu^{c_-}\rho]) \times \delta([\nu^{j+1}\rho, \nu^c\rho]) \times \delta'.$$

From the cuspidal support of δ we obtain $-z \in \{-i, j+1\}$ and $\delta' \simeq \delta([\nu^{z+1}\rho, \nu^{c_-}\rho])$. In the same manner as previously, we get $\delta \simeq \delta([\nu^{-z}\rho, \nu^c\rho]) \times \delta([\nu^{z+1}\rho, \nu^{c_-}\rho])$.

This allows us to conclude that $\epsilon((z,\rho)) = \epsilon((c_-,\rho))$ implies that $\mu^*(\sigma)_{(a,c,\rho)} \geq 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes \sigma_3$ and $\mu^*(\sigma')_{(a,c,\rho)} \geq 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes \sigma_4$. On the other hand, $\epsilon((z,\rho)) \neq \epsilon((c_-,\rho))$ leads to $\mu^*(\sigma)_{(a,c,\rho)} \geq 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes \sigma_4$ and $\mu^*(\sigma')_{(a,c,\rho)} \geq 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes \sigma_3$. This proves the proposition. ■

Proposition 4.8. *Suppose $a \geq 1$, $x = c_-$, and $a = x_- + 1$. If $\epsilon((x_-,\rho)) = \epsilon((c_-,\rho))$ then in $R(GL) \otimes R$ we have:*

$$\mu^*(\sigma)_{(a,c,\rho)} = \delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}) + \\ + 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes \tau_{temp},$$

where τ_{temp} stands for the unique irreducible tempered subquotient of $\delta([\nu^{-a+1}\rho, \nu^{c_-}\rho]) \rtimes \sigma_{sp}$.

If $\epsilon((x_-,\rho)) \neq \epsilon((c_-,\rho))$ *then in* $R(GL) \otimes R$ *we have:*

$$\mu^*(\sigma)_{(a,c,\rho)} = \delta([\nu^a\rho,\nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho,\nu^{a-1}\rho]) \rtimes \sigma_{sp}).$$

Proof. In $R(GL) \otimes R$ we have

$$\mu^*(\sigma_{ind})_{(a,c,\rho)} = 2\,\delta([\nu^a\rho,\nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho,\nu^{a-1}\rho]) \rtimes \sigma_{sp}) + \\ + 2\,\delta([\nu^a\rho,\nu^c\rho]) \otimes \tau_{temp},$$

so it is enough to consider $\delta([\nu^a\rho,\nu^c\rho]) \otimes \tau_{temp}$. Since τ_{temp} is a subrepresentation of $\delta([\nu^{-a+1}\rho,\nu^{c_-}\rho]) \rtimes \sigma_{sp}$, using Frobenius reciprocity we conclude that an irreducible subquotient π of σ_{ind} such that $\delta([\nu^a\rho,\nu^c\rho]) \otimes \tau_{temp} \leq \mu^*(\pi)$ also contains $\delta([\nu^a\rho,\nu^c\rho]) \otimes \delta([\nu^{-a+1}\rho,\nu^{c_-}\rho]) \otimes \sigma_{sp}$ in $(m\mu)^*(\pi)$. Therefore, there is an irreducible constituent $\delta \otimes \sigma_{sp}$ of $\mu^*(\pi)$ such that $m^*(\delta)$ contains $\delta([\nu^a\rho,\nu^c\rho]) \otimes \delta([\nu^{-a+1}\rho,\nu^{c_-}\rho])$.

Using the same procedure as in the proof of the previous proposition, we get $\delta \leq \delta([\nu^a\rho,\nu^c\rho]) \times \delta([\nu^{-a+1}\rho,\nu^{c_-}\rho])$.

Since $\delta([\nu^a\rho,\nu^{c_-}\rho]) \times \delta([\nu^{-a+1}\rho,\nu^c\rho])$ is an irreducible subquotient of the induced representation $\delta([\nu^a\rho,\nu^c\rho]) \times \delta([\nu^{-a+1}\rho,\nu^{c_-}\rho])$ and multiplicity of $\delta([\nu^a\rho,\nu^c\rho]) \otimes \delta([\nu^{-a+1}\rho,\nu^{c_-}\rho])$ equals two in both $m^*(\delta([\nu^a\rho,\nu^{c_-}\rho]) \times \delta([\nu^{-a+1}\rho,\nu^c\rho]))$ and $m^*(\delta([\nu^a\rho,\nu^c\rho]) \times \delta([\nu^{-a+1}\rho,\nu^{c_-}\rho]))$, we conclude that

$$\delta \simeq \delta([\nu^a\rho,\nu^{c_-}\rho]) \times \delta([\nu^{-a+1}\rho,\nu^c\rho]).$$

Using [18, Proposition 7.2] and the fact that $\epsilon((x_-,\rho)) = \epsilon((c_-,\rho))$ if and only if $\epsilon'((x_-,\rho)) \neq \epsilon'((c_-,\rho))$, we deduce that $\mu^*(\sigma)$ contains $\delta([\nu^a\rho,\nu^c\rho]) \otimes \tau_{temp}$ if and only if $\epsilon((x_-,\rho)) = \epsilon((c_-,\rho))$, and it directly follows that $\mu^*(\sigma)$ contains either both or no copies of $\delta([\nu^a\rho,\ \nu^c\rho]) \otimes \tau_{temp}$. This completes the proof. ■

Proposition 4.9. *Suppose* $a \geq 1$, $x \neq c_-$, *and* $a = x_- + 1$. *If* $\epsilon(((c_-)_-,\rho)) = \epsilon((c_-,\rho))$ *then in* $R(GL) \otimes R$ *we have:*

$$\mu^*(\sigma)_{(a,c,\rho)} = \delta([\nu^a\rho,\nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho,\nu^{a-1}\rho]) \rtimes \sigma_{sp}) + \\ + 2\,\delta([\nu^a\rho,\nu^c\rho]) \otimes L(\delta([\nu^{-(c_-)_-}\rho,\nu^{a-1}\rho]) \rtimes \sigma'_{sp}),$$

for the unique $\sigma'_{sp} \in\in Irr_{sp}^{\sigma_{cusp}}(G)$ *such that* $Jord(\sigma'_{sp}) = Jord(\sigma_{sp}) \setminus \{((c_-)_-,\rho)\} \cup \{(c_-,\rho)\}$.
If $\epsilon(((c_-)_-,\rho)) \neq \epsilon((c_-,\rho))$ *then in* $R(GL) \otimes R$ *we have:*

$$\mu^*(\sigma)_{(a,c,\rho)} = \delta([\nu^a\rho,\nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho,\nu^{a-1}\rho]) \rtimes \sigma_{sp}).$$

Proof. Since the following equality holds in $R(GL) \otimes R$:

$$\mu^*(\sigma_{ind})_{(a,c,\rho)} = 2\,\delta([\nu^a\rho,\nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho,\nu^{a-1}\rho]) \rtimes \sigma_{sp}) + \\ + 2\,\delta([\nu^a\rho,\nu^c\rho]) \otimes L(\delta([\nu^{-(c_-)_-}\rho,\nu^{a-1}\rho]) \rtimes \sigma'_{sp}),$$

we discuss only the irreducible constituent $\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-(c_-)_-}\rho,\ \nu^{a-1}\rho]) \rtimes \sigma'_{sp})$. First, we note the following embeddings and isomorphism:

$$\begin{aligned} L(\delta([\nu^{-(c_-)_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma'_{sp}) &\hookrightarrow \delta([\nu^{-(c_-)_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma'_{sp} \\ &\hookrightarrow \delta([\nu^{-(c_-)_-}\rho, \nu^{a-1}\rho]) \times \delta([\nu^{(c_-)_-+1}\rho, \nu^{c_-}\rho]) \rtimes \sigma_{sp} \\ &\simeq \delta([\nu^{(c_-)_-+1}\rho, \nu^{c_-}\rho]) \times \delta([\nu^{-(c_-)_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}. \end{aligned}$$

Consequently, if π is an irreducible subquotient of σ_{ind} such that $\mu^*(\pi)$ contains $\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-(c_-)_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma'_{sp})$, then there is some irreducible representation π' such that $\delta([\nu^a\rho, \nu^c\rho]) \otimes \delta([\nu^{(c_-)_-+1}\rho,\ \nu^{c_-}\rho]) \otimes \pi'$ is contained in $(m\mu)^*(\pi)$. Applying (2) to $\mu^*(\sigma_{ind})$, we conclude that $m^*(\pi)$ contains the irreducible representation

$$\delta([\nu^a\rho, \nu^c\rho]) \times \delta([\nu^{(c_-)_-+1}\rho, \nu^{c_-}\rho]) \otimes \pi'.$$

Now the rest of the proof runs as before. ∎

Proposition 4.10. *Suppose $a \geq 1$, $x \neq c_-$, and $a \neq x_- + 1$. We denote $(c_-)_-$ by y and suppose $x = y$. If $\epsilon((x, \rho)) = \epsilon((c_-, \rho))$ then the following equality holds in $R(GL) \otimes R$:*

$$\begin{aligned} \mu^*(\sigma)_{(a,c,\rho)} = {} & \delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}) + \\ & + 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^x\rho]) \rtimes \sigma_{sp}^{(1)}) + \\ & + 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-x}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}^{(2)}) + \\ & + 4\,\delta([\nu^a\rho, \nu^c\rho]) \otimes \sigma_1 + \delta([\nu^a\rho, \nu^{c_-}\rho]) \otimes \sigma_2, \end{aligned}$$

for unique $\sigma_{sp}^{(1)}, \sigma_{sp}^{(2)} \in Irr_{sp}^{\sigma_{cusp}}(G)$ such that $Jord(\sigma_{sp}^{(1)}) = Jord(\sigma_{sp}) \setminus \{(x, \rho)\} \cup \{(2a-1, \rho)\}$ and $Jord(\sigma_{sp}^{(2)}) = Jord(\sigma_{sp}) \setminus \{(x, \rho)\} \cup \{(c_-, \rho)\}$, while σ_1 and σ_2 are mutually non-isomorphic discrete series subrepresentations of $\delta([\nu^{-x}\rho, \nu^{c_-}\rho]) \rtimes \sigma_{sp}^{(1)}$ and σ_1 is also a subrepresentation of $\delta([\nu^{-a+1}\rho, \nu^x\rho]) \rtimes \sigma_{sp}^{(2)}$.
If $\epsilon((x, \rho)) \neq \epsilon((c_-, \rho))$ then the following equality holds in $R(GL) \otimes R$:

$$\begin{aligned} \mu^*(\sigma)_{(a,c,\rho)} = {} & \delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}) + \\ & + 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^x\rho]) \rtimes \sigma_{sp}^{(1)}) + \\ & + \delta([\nu^a\rho, \nu^{c_-}\rho]) \otimes \sigma_2, \end{aligned}$$

for $\sigma_{sp}^{(1)}$ and σ_2 as above.

Proof. First, in $R(GL) \otimes R$ we have (we note that here is also used [9, Proposition 3.2])

$$\begin{aligned} \mu^*(\sigma_{ind})_{(a,c,\rho)} = {} & 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}) + \\ & +4\,\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^x\rho]) \rtimes \sigma_{sp}^{(1)}) + \\ & +2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-x}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}^{(2)}) + \\ & +4\,\delta([\nu^a\rho, \nu^c\rho]) \otimes \sigma_1 + \\ & +2\,\delta([\nu^a\rho, \nu^{c_-}\rho]) \otimes \sigma_2. \end{aligned}$$

Again, we have already seen that $\delta([\nu^a\rho,\nu^c\rho])\otimes L(\delta([\nu^{-c_-}\rho,\nu^{a-1}\rho])\rtimes\sigma_{sp})$ appears in both $\mu^*(\sigma)$ and $\mu^*(\sigma')$ with multiplicity one.

It directly follows that both σ and σ' are irreducible subrepresentations of the induced representation $\delta([\nu^{-c_-}\rho,\nu^c\rho])\times\delta([\nu^a\rho,\nu^x\rho])\rtimes\sigma_{sp}^{(1)}$. It can be easily seen that $\delta([\nu^a\rho,\nu^c\rho])\otimes\delta([\nu^{-c_-}\rho,\nu^x\rho])$ appears in $m^*(\delta([\nu^{-c_-}\rho,\nu^c\rho])\times\delta([\nu^a\rho,\nu^x\rho]))$ with multiplicity two. Now transitivity of Jacquet modules shows that there is some irreducible constituent $\delta([\nu^a\rho,\nu^c\rho])\otimes\pi'$ appearing in both $\mu^*(\sigma)$ and $\mu^*(\sigma')$ such that $\mu^*(\pi')\geq\delta([\nu^{-c_-}\rho,\nu^x\rho])\otimes\sigma_{sp}^{(1)}$.

The description of $\mu^*(\sigma_{ind})_{(a,c,\rho)}$ implies that $\pi'\simeq L(\delta([\nu^{-c_-}\rho,\nu^x\rho])\rtimes\sigma_{sp}^{(1)})$ and, since $\mu^*(L(\delta([\nu^{-c_-}\rho,\nu^x\rho])\rtimes\sigma_{sp}^{(1)}))$ contains $\delta([\nu^{-c_-}\rho,\nu^x\rho])\otimes\sigma_{sp}^{(1)}$ with multiplicity one, $\delta([\nu^a\rho,\nu^c\rho])\otimes L(\delta([\nu^{-c_-}\rho,\nu^x\rho])\rtimes\sigma_{sp}^{(1)})$ appears in both $\mu^*(\sigma)$ and $\mu^*(\sigma')$ with multiplicity two.

On the other hand, $L(\delta([\nu^{-x}\rho,\nu^{a-1}\rho])\rtimes\sigma_{sp}^{(2)})$ is a subrepresentation of

$$\delta([\nu^{-x}\rho,\nu^{a-1}\rho])\times\delta([\nu^{x+1}\rho,\nu^{c_-}\rho])\rtimes\sigma_{sp}^{(1)}.$$

Since $a-1<x$, Frobenius reciprocity implies that $\mu^*(L(\delta([\nu^{-x}\rho,\ \nu^{a-1}\rho])\rtimes\sigma_{sp}^{(2)}))$ contains $\delta([\nu^{-x}\rho,\nu^{a-1}\rho])\times\delta([\nu^{x+1}\rho,\nu^{c_-}\rho])\otimes\sigma_{sp}^{(1)}$. Consequently, if π is an irreducible subquotient of σ_{ind} such that $\delta([\nu^a\rho,\nu^c\rho])\otimes L(\delta([\nu^{-x}\rho,\nu^{a-1}\rho])\rtimes\sigma_{sp}^{(2)})$ is contained in $\mu^*(\pi)$, then $(m\mu)^*(\pi)$ contains $\delta([\nu^a\rho,\nu^c\rho])\otimes\delta([\nu^{-x}\rho,\nu^{a-1}\rho])\times\delta([\nu^{x+1}\rho,\nu^{c_-}\rho])\otimes\sigma_{sp}^{(1)}$. Therefore, there is some irreducible constituent $\delta\otimes\sigma_{sp}^{(1)}$ of $\mu^*(\pi)$ such that $m^*(\delta)$ contains $\delta([\nu^a\rho,\nu^c\rho])\otimes\delta([\nu^{-x}\rho,\nu^{a-1}\rho])\times\delta([\nu^{x+1}\rho,\nu^{c_-}\rho])$. From $\mu^*(\sigma_{ind})$ we obtain that δ is an irreducible subquotient of $\delta([\nu^{-x}\rho,\nu^{c_-}\rho])\times\delta([\nu^{x+1}\rho,\nu^c\rho])$ and, in a standard way, we conclude that

$$\delta\simeq\delta([\nu^{-x}\rho,\nu^c\rho])\times\delta([\nu^{x+1}\rho,\nu^{c_-}\rho]).$$

[18, Proposition 7.2] and definition of representations σ and σ' show that $\mu^*(\sigma)$ contains $\delta([\nu^a\rho,\nu^c\rho])\otimes L(\delta([\nu^{-x}\rho,\nu^{a-1}\rho])\rtimes\sigma_{sp}^{(2)})$ if and only if $\epsilon((x,\rho))=\epsilon((c_-,\rho))$.

Now we consider the irreducible constituent $\delta([\nu^a\rho,\nu^c\rho])\otimes\sigma_1$. From [9, Proposition 3.2] we obtain the following embeddings and isomorphism:

$$\begin{aligned}\sigma_1 &\hookrightarrow \delta([\nu^{-a+1}\rho,\nu^{c_-}\rho])\rtimes\sigma_{sp}^{(2)}\\ &\hookrightarrow \delta([\nu^{-a+1}\rho,\nu^{c_-}\rho])\times\delta([\nu^a\rho,\nu^x\rho])\rtimes\sigma_{sp}^{(1)}\\ &\simeq \delta([\nu^a\rho,\nu^x\rho])\times\delta([\nu^{-a+1}\rho,\nu^{c_-}\rho])\rtimes\sigma_{sp}^{(1)}\\ &\hookrightarrow \delta([\nu^a\rho,\nu^x\rho])\times\delta([\nu^a\rho,\nu^{c_-}\rho])\times\delta([\nu^{-a+1}\rho,\nu^{a-1}\rho])\rtimes\sigma_{sp}^{(1)}.\end{aligned}$$

Since the representation $\delta([\nu^a\rho,\nu^x\rho])\times\delta([\nu^a\rho,\nu^{c_-}\rho])$ is irreducible, by [13, Lemma 3.2] there is an irreducible representation τ such that σ_1 is a subrepresentation of $\delta([\nu^a\rho,\nu^x\rho])\times\delta([\nu^a\rho,\nu^{c_-}\rho])\rtimes\tau$, and Frobenius reciprocity implies $\mu^*(\sigma_1)\geq\delta([\nu^a\rho,\nu^x\rho])\times\delta([\nu^a\rho,\nu^{c_-}\rho])\otimes\tau$. Consequently, for an irreducible subquotient π of σ_{ind} such that $\mu^*(\pi)\geq\delta([\nu^a\rho,\nu^c\rho])\otimes\sigma_1$ there is some irreducible constituent $\delta\otimes\tau$ of $\mu^*(\pi)$ such that $m^*(\delta)$ contains $\delta([\nu^a\rho,\nu^c\rho])\otimes\delta([\nu^a\rho,\nu^x\rho])\times\delta([\nu^a\rho,\nu^{c_-}\rho])$.

It follows at once from $\mu^*(\sigma_{ind})$ and from the cuspidal support of δ that δ is isomorphic to $\delta([\nu^a\rho,\nu^c\rho])\times\delta([\nu^a\rho,\nu^x\rho])\times\delta([\nu^a\rho,\nu^{c_-}\rho])$. In the same way as before we conclude that $\mu^*(\sigma)$ contains $\delta([\nu^a\rho,\nu^c\rho])\otimes\sigma_1$ if and only if $\epsilon((x,\rho)) = \epsilon((c_-,\rho))$.

What is left is to show that $\delta([\nu^a\rho,\nu^c\rho])\otimes\sigma_2$ appears in both $\mu^*(\sigma)$ and $\mu^*(\sigma')$.

First, we have already seen that $\delta([\nu^{-x}\rho,\nu^c\rho])\otimes L(\delta([\nu^{-c_-}\rho,\ \nu^{-(x+1)}\rho])\rtimes\sigma_{sp})$ appears in both $\mu^*(\sigma)$ and $\mu^*(\sigma')$. Furthermore, we have the following embeddings and isomorphisms (note that the representation $\delta([\nu^{-c_-}\rho,\ \nu^{-x+1}\rho])\rtimes\sigma_{sp}^{(1)}$ is irreducible by the results of [10]):

$$\begin{aligned} L(\delta([\nu^{-c_-}\rho,\nu^{-x+1}\rho])\rtimes\sigma_{sp}) &\hookrightarrow \delta([\nu^{-c_-}\rho,\nu^{-x+1}\rho])\rtimes\sigma_{sp} \\ &\hookrightarrow \delta([\nu^{-c_-}\rho,\nu^{-x+1}\rho])\times\delta([\nu^a\rho,\nu^x\rho])\rtimes\sigma_{sp}^{(1)} \\ &\simeq \delta([\nu^a\rho,\nu^x\rho])\times\delta([\nu^{-c_-}\rho,\nu^{-x+1}\rho])\rtimes\sigma_{sp}^{(1)} \\ &\simeq \delta([\nu^a\rho,\nu^x\rho])\times\delta([\nu^{x+1}\rho,\nu^{c_-}\rho])\rtimes\sigma_{sp}^{(1)}. \end{aligned}$$

Using the description of the composition series of representation $\delta([\nu^{-c_-}\rho,\nu^{-x+1}\rho])\rtimes\sigma_{sp}$, given in [10, Proposition 3.1], we deduce that there is no irreducible constituent of the form $\delta([\nu^{x+1}\rho,\nu^{c_-}\rho])\otimes\pi$ appearing in $\mu^*(L(\delta([\nu^{-c_-}\rho,\nu^{-x+1}\rho])\rtimes\sigma_{sp}))$. Thus, $L(\delta([\nu^{-c_-}\rho,\ \nu^{-x+1}\rho])\rtimes\sigma_{sp})$ is contained in the kernel of an intertwining operator

$$\delta([\nu^a\rho,\nu^x\rho])\times\delta([\nu^{x+1}\rho,\nu^{c_-}\rho])\rtimes\sigma_{sp}^{(1)}\to\delta([\nu^{x+1}\rho,\nu^{c_-}\rho])\times\delta([\nu^a\rho,\nu^x\rho])\rtimes\sigma_{sp}^{(1)},$$

which is, according to [19], isomorphic to

$$L(\delta([\nu^a\rho,\nu^x\rho])\times\delta([\nu^{x+1}\rho,\nu^{c_-}\rho]))\rtimes\sigma_{sp}^{(1)}.$$

In this way we conclude that both $(m\mu)^*(\sigma)$ and $(m\mu)^*(\sigma')$ contain the irreducible representation

$$\delta([\nu^{-x}\rho,\nu^c\rho])\otimes L(\delta([\nu^a\rho,\nu^x\rho])\times\delta([\nu^{x+1}\rho,\nu^{c_-}\rho]))\otimes\sigma_{sp}^{(1)}.$$

If $\delta\otimes\sigma_{sp}^{(1)}$ is an irreducible constituent of $\mu^*(\sigma)$ or $\mu^*(\sigma')$ such that $m^*(\delta)$ contains $\delta([\nu^{-x}\rho,\nu^c\rho])\otimes L(\delta([\nu^a\rho,\nu^x\rho])\times\delta([\nu^{x+1}\rho,\nu^{c_-}\rho]))$, in the same fashion as before we get that δ is an irreducible subquotient of

$$\delta([\nu^{-x}\rho,\nu^{c_-}\rho])\times\delta([\nu^{c_-+1}\rho,\nu^c\rho])\times\delta([\nu^a\rho,\nu^x\rho])\times\delta([\nu^{x+1}\rho,\nu^{c_-}\rho]).$$

It is not hard to see that the unique irreducible subquotient of this induced representation which contains $\delta([\nu^{-x}\rho,\nu^c\rho])\otimes L(\delta([\nu^a\rho,\nu^x\rho])\times\delta([\nu^{x+1}\rho,\ \nu^{c_-}\rho]))$ in the Jacquet module with respect to the appropriate standard parabolic subgroup, is isomorphic to

$$\delta([\nu^{-x}\rho,\nu^c\rho])\times L(\delta([\nu^a\rho,\nu^x\rho])\times\delta([\nu^{x+1}\rho,\nu^{c_-}\rho]))$$

(which is irreducible by [3, Lemma 1.3.3]). Thus, $\delta([\nu^{-x}\rho,\nu^c\rho])\times L(\delta([\nu^a\rho,\nu^x\rho])\times\delta([\nu^{x+1}\rho,\nu^{c_-}\rho]))\otimes\sigma_{sp}^{(1)}$ appears in both $\mu^*(\sigma)$ and $\mu^*(\sigma')$. By (2), such irreducible constituent appears with multiplicity two in $\mu^*(\sigma_{ind})$, so it appears with multiplicity one in both $\mu^*(\sigma)$ and $\mu^*(\sigma')$.

Using [19] and [3, Lemma 1.3.3]), we can assert that in $R(GL)$ we have

$$\begin{aligned}\delta([\nu^{-x}\rho,\nu^{c}\rho])\times\delta([\nu^{a}\rho,\nu^{x}\rho])\times\delta([\nu^{x+1}\rho,\nu^{c_-}\rho])\\ =\delta([\nu^{-x}\rho,\nu^{c}\rho])\times\delta([\nu^{a}\rho,\nu^{c_-}\rho])+\\ +\delta([\nu^{-x}\rho,\nu^{c}\rho])\times L(\delta([\nu^{a}\rho,\nu^{x}\rho])\times\delta([\nu^{x+1}\rho,\nu^{c_-}\rho])).\end{aligned}$$

Since $\delta([\nu^{a}\rho,\nu^{c}\rho])\otimes\delta([\nu^{-x}\rho,\nu^{c_-}\rho])$ appears with multiplicity three in

$$m^*(\delta([\nu^{-x}\rho,\nu^{c}\rho])\times\delta([\nu^{a}\rho,\nu^{x}\rho])\times\delta([\nu^{x+1}\rho,\nu^{c_-}\rho]))$$

and with multiplicity two in $m^*(\delta([\nu^{-x}\rho,\nu^{c}\rho])\times\delta([\nu^{a}\rho,\ \nu^{c_-}\rho]))$ it follows that $\delta([\nu^{a}\rho,\nu^{c}\rho])\otimes\delta([\nu^{-x}\rho,\nu^{c_-}\rho])$ appears with multiplicity one in

$$m^*(\delta([\nu^{-x}\rho,\nu^{c}\rho])\times L(\delta([\nu^{a}\rho,\nu^{x}\rho])\times\delta([\nu^{x+1}\rho,\nu^{c_-}\rho]))).$$

Using (2), we also obtain that if $\delta\otimes\sigma_{sp}^{(1)}$ is an irreducible constituent of $\mu^*(\sigma_{ind})$ such that $m^*(\delta)\geq\delta([\nu^{a}\rho,\nu^{c}\rho])\otimes\delta([\nu^{-x}\rho,\nu^{c_-}\rho])$, then δ is either isomorphic to $\delta([\nu^{-x}\rho,\nu^{c}\rho])\times\delta([\nu^{a}\rho,\ \nu^{c_-}\rho])$ or to $\delta([\nu^{-x}\rho,\nu^{c}\rho])\times L(\delta([\nu^{a}\rho,\nu^{x}\rho])\times\delta([\nu^{x+1}\rho,\nu^{c_-}\rho]))$.

Transitivity of Jacquet modules now shows that for $\pi\in\{\sigma,\sigma'\}$ there is an irreducible constituent $\delta([\nu^{a}\rho,\nu^{c}\rho])\otimes\pi'$ of $\mu^*(\pi)$ such that $\mu^*(\pi')$ contains $\delta([\nu^{-x}\rho,\nu^{c_-}\rho])\otimes\sigma_{sp}^{(1)}$. The description of $\mu^*(\sigma_{ind})_{(a,c,\rho)}$ given in the beginning of the proof leads to $\pi'\in\{\sigma_1,\sigma_2\}$.

Let us denote by τ the element of $\{\sigma,\sigma'\}$ which is not an irreducible subrepresentation of $\delta([\nu^{-x}\rho,\nu^{c_-}\rho])\rtimes\sigma'_{sp}$, for the unique $\sigma'_{sp}\in Irr_{sp}^{\sigma_{cusp}}(G)$ such that $\mathrm{Jord}(\sigma'_{sp})=\mathrm{Jord}(\sigma_{sp})\setminus\{(x,\rho)\}\cup\{(c,\rho)\}$. Then, as we have already proved, $\delta([\nu^{a}\rho,\nu^{c}\rho])\otimes\sigma_1$ does not appear in $\mu^*(\tau)$, and hence $\delta([\nu^{a}\rho,\nu^{c}\rho])\otimes\sigma_2\leq\mu^*(\tau)$. Also, such irreducible constituent appears in $\mu^*(\tau)$ with multiplicity one, since otherwise $\delta([\nu^{a}\rho,\nu^{c}\rho])\otimes\delta([\nu^{-x}\rho,\nu^{c_-}\rho])\otimes\sigma_{sp}^{(1)}$ would appear in $(m\mu)^*(\tau)$ with multiplicity two and, consequently, $\delta([\nu^{-x}\rho,\nu^{c}\rho])\times L(\delta([\nu^{a}\rho,\nu^{x}\rho])\times\delta([\nu^{x+1}\rho,\nu^{c_-}\rho]))\otimes\sigma_{sp}^{(1)}$ would appear in $\mu^*(\tau)$ with multiplicity two, which is impossible.

Therefore, $\delta([\nu^{a}\rho,\nu^{c}\rho])\otimes\sigma_2$ appears in both $\mu^*(\sigma)$ and $\mu^*(\sigma')$ with multiplicity one and the proposition is proved. ■

Proposition 4.11. *Suppose $a\geq1$, $x\neq c_-$, and $a\neq x_-+1$. We denote $(c_-)_-$ by y and suppose $x=y_-$. If $\epsilon((y,\rho))=\epsilon((c_-,\rho))$ then the following equality holds in $R(GL)\otimes R$:*

$$\begin{aligned}\mu^*(\sigma)_{(a,c,\rho)}=&\ \delta([\nu^{a}\rho,\nu^{c}\rho])\otimes L(\delta([\nu^{-c_-}\rho,\nu^{a-1}\rho])\rtimes\sigma_{sp})+\\ &+2\,\delta([\nu^{a}\rho,\nu^{c}\rho])\otimes L(\delta([\nu^{-c_-}\rho,\nu^{x}\rho])\rtimes\sigma_{sp}^{(1)})+\\ &+2\,\delta([\nu^{a}\rho,\nu^{c}\rho])\otimes L(\delta([\nu^{-y}\rho,\nu^{a-1}\rho])\rtimes\sigma_{sp}^{(2)})+\\ &+4\,\delta([\nu^{a}\rho,\nu^{c}\rho])\otimes L(\delta([\nu^{-y}\rho,\nu^{x}\rho])\rtimes\sigma_{sp}^{(3)})+\\ &+\delta([\nu^{a}\rho,\nu^{c}\rho])\otimes L(\delta([\nu^{-c_-}\rho,\nu^{y}\rho])\rtimes\sigma_{sp}^{(4)})+\\ &+2\,\delta([\nu^{a}\rho,\nu^{c}\rho])\otimes\sigma_1,\end{aligned}$$

for unique $\sigma_{sp}^{(1)},\sigma_{sp}^{(2)},\sigma_{sp}^{(3)},\sigma_{sp}^{(4)}\in Irr_{sp}^{\sigma_{cusp}}(G)$ such that $Jord(\sigma_{sp}^{(1)})=Jord(\sigma_{sp})\setminus\{(x,\rho)\}\cup\{(2a-1,\rho)\}$, $Jord(\sigma_{sp}^{(2)})=Jord(\sigma_{sp})\setminus\{(y,\rho)\}\cup\{(c_-,\rho)\}$, $Jord(\sigma_{sp}^{(3)})=$

$Jord(\sigma_{sp}^{(2)}) \setminus \{(x,\rho)\} \cup \{(2a-1,\rho)\}$ and $Jord(\sigma_{sp}^{(4)}) = Jord(\sigma_{sp}) \setminus \{(y,\rho)\} \cup \{(2a-1,\rho)\}$, while σ_1 denotes the unique discrete series subrepresentation of $\delta([\nu^{-x}\rho, \nu^{c_-}\rho]) \rtimes \sigma_{sp}^{(1)}$.
If $\epsilon((y,\rho)) \neq \epsilon((c_-,\rho))$ then the following equality holds in $R(GL) \otimes R$:

$$\begin{aligned}\mu^*(\sigma)_{(a,c,\rho)} = {} & \delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}) + \\ & + 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{x}\rho]) \rtimes \sigma_{sp}^{(1)}) + \\ & + \delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{y}\rho]) \rtimes \sigma_{sp}^{(4)}),\end{aligned}$$

for $\sigma_{sp}^{(1)}$ and $\sigma_{sp}^{(4)}$ as above.

Proof. We start with the following equality (again, we obtain σ_1 by using [9, Proposition 3.2]):

$$\begin{aligned}\mu^*(\sigma_{ind})_{(a,c,\rho)} = {} & 4\,\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}) + \\ & + 4\,\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{x}\rho]) \rtimes \sigma_{sp}^{(1)}) + \\ & + 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-y}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}^{(2)}) + \\ & + 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-y}\rho, \nu^{x}\rho]) \rtimes \sigma_{sp}^{(3)}) + \\ & + 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{y}\rho]) \rtimes \sigma_{sp}^{(4)}) + \\ & + 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes \sigma_1.\end{aligned}$$

In Proposition 4.2 we have seen that $\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho,\ \nu^{a-1}\rho]) \rtimes \sigma_{sp})$ appears in both $\mu^*(\sigma)$ and $\mu^*(\sigma')$ with multiplicity one.

For $\pi \in \{L(\delta([\nu^{-y}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}^{(2)}), L(\delta([\nu^{-y}\rho, \nu^{x}\rho]) \rtimes \sigma_{sp}^{(3)}),\ \sigma_1\}$ it can be directly seen that $\mu^*(\pi)$ contains an irreducible constituent of the form $\delta([\nu^{y+1}\rho, \nu^{c_-}\rho]) \otimes \pi'$ for some irreducible representation π'. Thus, if τ is an irreducible subquotient of σ_{ind} such that $\delta([\nu^a\rho, \nu^c\rho]) \otimes \pi$ appears in $\mu^*(\tau)$ for $\pi \in \{L(\delta([\nu^{-y}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}^{(2)}), L(\delta([\nu^{-y}\rho, \nu^{x}\rho]) \rtimes \sigma_{sp}^{(3)}), \sigma_1\}$, then there is an irreducible constituent $\delta \otimes \pi'$ of $\mu^*(\tau)$ such that $m^*(\delta)$ contains $\delta([\nu^a\rho, \nu^c\rho]) \otimes \delta([\nu^{y+1}\rho, \nu^{c_-}\rho])$. Lemma 4.3 shows that $\delta([\nu^a\rho, \nu^c\rho]) \otimes \pi$ appears in $\mu^*(\sigma)$ for $\pi \in \{L(\delta([\nu^{-y}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}^{(2)}), L(\delta([\nu^{-y}\rho,\ \nu^{x}\rho]) \rtimes \sigma_{sp}^{(3)}), \sigma_1\}$ if and only if $\epsilon((y,\rho)) = \epsilon((c_-,\rho))$.

On the other hand, both discrete series representations σ and σ' are subrepresentations of

$$\delta([\nu^{-c_-}\rho, \nu^c\rho]) \times \delta([\nu^a\rho, \nu^x\rho]) \rtimes \sigma_{sp}^{(1)}.$$

Irreducibility of $\delta([\nu^{-c_-}\rho, \nu^c\rho]) \times \delta([\nu^a\rho, \nu^x\rho])$ and Frobenius reciprocity imply that $\mu^*(\sigma)$ contains $\delta([\nu^{-c_-}\rho, \nu^c\rho]) \times \delta([\nu^a\rho, \nu^x\rho]) \otimes \sigma_{sp}^{(1)}$.

Since $m^*(\delta([\nu^{-c_-}\rho, \nu^c\rho]) \times \delta([\nu^a\rho, \nu^x\rho]))$ contains irreducible constituent $\delta([\nu^a\rho, \nu^c\rho]) \otimes \delta([\nu^{-c_-}\rho, \nu^x\rho])$ (with multiplicity two), transitivity of Jacquet modules shows that there is some irreducible constituent $\delta([\nu^a\rho, \nu^c\rho]) \otimes \pi$ of $\mu^*(\sigma)$ such that $\mu^*(\pi)$ contains $\delta([\nu^{-c_-}\rho,\ \nu^x\rho]) \otimes \sigma_{sp}^{(1)}$.

Directly from the description of $\mu^*(\sigma_{ind})$ given at the beginning of our proof, we conclude that π has to be isomorphic to $L(\delta([\nu^{-c_-}\rho, \nu^x\rho]) \rtimes \sigma_{sp}^{(1)})$. Furthermore, one easily verify that $\mu^*(L(\delta([\nu^{-c_-}\rho, \nu^x\rho]) \rtimes \sigma_{sp}^{(1)}))$ contains $\delta([\nu^{-c_-}\rho, \nu^x\rho]) \otimes \sigma_{sp}^{(1)}$

with multiplicity one. Consequently, $\delta([\nu^a\rho,\ \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho,\ \nu^x\rho]) \rtimes \sigma_{sp}^{(1)})$ appears in $\mu^*(\sigma)$ with multiplicity at least two.

In analogous way we deduce that $\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho,\ \nu^x\rho]) \rtimes \sigma_{sp}^{(1)})$ also appears in $\mu^*(\sigma')$ with multiplicity at least two, so $\delta([\nu^a\rho,\ \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^x\rho]) \rtimes \sigma_{sp}^{(1)})$ appears in both $\mu^*(\sigma)$ and $\mu^*(\sigma')$ with multiplicity two.

Similarly, one can see that $\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho,\ \nu^y\rho]) \rtimes \sigma_{sp}^{(4)})$ appears in both $\mu^*(\sigma)$ and $\mu^*(\sigma')$ with multiplicity one. ∎

Proposition 4.12. *Suppose $a \geq 1$, $x \neq c_-$, and $a \neq x_- + 1$. We denote $(c_-)_-$ by y and suppose $x < y_-$. If $\epsilon((y,\rho)) = \epsilon((c_-,\rho))$ then the following equality holds in $R(GL) \otimes R$:*

$$\begin{aligned}
\mu^*(\sigma)_{(a,c,\rho)} = {} & \delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}) + \\
& + 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^x\rho]) \rtimes \sigma_{sp}^{(1)}) + \\
& + 4\,\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-y}\rho, \nu^{x-1}\rho]) \rtimes \sigma_{sp}^{(2)}) + \\
& + 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-y}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}^{(3)}) + \\
& + \delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^z\rho]) \rtimes \sigma_{sp}^{(4)}) + \\
& + 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-y}\rho, \nu^z\rho]) \rtimes \sigma_{sp}^{(5)}),
\end{aligned}$$

for unique $\sigma_{sp}^{(1)}, \sigma_{sp}^{(2)}, \sigma_{sp}^{(3)}, \sigma_{sp}^{(4)}, \sigma_{sp}^{(5)} \in Irr_{sp}^{\sigma_{cusp}}(G)$ such that $Jord(\sigma_{sp}^{(1)}) = Jord(\sigma_{sp}) \setminus \{(x,\rho)\} \cup \{(2a-1,\rho)\}$, $Jord(\sigma_{sp}^{(2)}) = Jord(\sigma_{sp}^{(2)}) \setminus \{(y,\rho)\} \cup \{(c_-,\rho)\}$, $Jord(\sigma_{sp}^{(3)}) = Jord(\sigma_{sp}) \setminus \{(y,\rho)\} \cup \{(c_-,\rho)\}$, $Jord(\sigma_{sp}^{(4)}) = Jord(\sigma_{sp}) \setminus \{(z,\rho)\} \cup \{(2a-1,\rho)\}$ and $Jord(\sigma_{sp}^{(5)}) = Jord(\sigma_{sp}^{(3)}) \setminus \{(z,\rho)\} \cup \{(2a-1,\rho)\}$.
If $\epsilon((y,\rho)) \neq \epsilon((c_-,\rho))$ then the following equality holds in $R(GL) \otimes R$:

$$\begin{aligned}
\mu^*(\sigma)_{(a,c,\rho)} = {} & \delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}) + \\
& + 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^x\rho]) \rtimes \sigma_{sp}^{(1)}) + \\
& + \delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^z\rho]) \rtimes \sigma_{sp}^{(4)}),
\end{aligned}$$

for $\sigma_{sp}^{(1)}$ and $\sigma_{sp}^{(4)}$ as above.

Proof. We just note the equality

$$\begin{aligned}
\mu^*(\sigma_{ind})_{(a,c,\rho)} = {} & 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}) + \\
& + 4\,\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^x\rho]) \rtimes \sigma_{sp}^{(1)}) + \\
& + 4\,\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-y}\rho, \nu^{x-1}\rho]) \rtimes \sigma_{sp}^{(2)}) + \\
& + 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-y}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}^{(3)}) + \\
& + 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^z\rho]) \rtimes \sigma_{sp}^{(4)}) + \\
& + 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-y}\rho, \nu^z\rho]) \rtimes \sigma_{sp}^{(5)}).
\end{aligned}$$

The rest of the proof proceeds in a completely analogous manner as the proof of Proposition 4.11, details being left to the reader. ∎

- Case $a = \frac{1}{2}$

Now we discuss the remaining case $a = \frac{1}{2}$. Throughout the rest of this section we denote $\min(\mathrm{Jord}_\rho)$ by $c_{\min}$. Again, we start with a technical lemma, the proof of which we omit, followed by some elementary situations.

Lemma 4.13. *If $c_- = c_{\min}$ or $\epsilon((c_{\min}, \rho)) = -1$ then for any irreducible constituent $\delta([\nu^{\frac{1}{2}}\rho, \nu^c\rho]) \otimes \pi$ of $\mu^*(\sigma_{ind})$ we have*

$$\pi \leq \delta([\nu^{\frac{1}{2}}\rho, \nu^{c_-}\rho]) \rtimes \sigma_{sp},$$

Otherwise,

$$\pi \leq \delta([\nu^{\frac{1}{2}}\rho, \nu^{c_-}\rho]) \rtimes \sigma_{sp} \oplus \delta([\nu^{-c_{\min}}\rho, \nu^{c_-}\rho]) \rtimes \sigma'_{sp},$$

for the unique $\sigma'_{sp} \in Irr_{sp}^{\sigma_{cusp}}(G)$ such that $Jord(\sigma'_{sp}) = Jord(\sigma_{sp}) \setminus \{(c_{\min}, \rho)\}$, i.e., σ'_{sp} is the unique strongly positive discrete series such that σ_{sp} embeds in $\delta([\nu^{\frac{1}{2}}\rho, \nu^{c_{\min}}\rho]) \rtimes \sigma'_{sp}$. Also, $\epsilon_{\sigma'_{sp}}((\min(Jord_\rho(\sigma'_{sp})), \rho)) = -1$, where the admissible triple attached to σ'_{sp} is denoted by $(Jord(\sigma'_{sp}), \sigma_{cusp}, \epsilon_{\sigma'_{sp}})$.

Proposition 4.14. *Suppose that $c_- = c_{\min}$. If $\epsilon((c_-, \rho)) = -1$ then in $R(GL) \otimes R$ we have*

$$\mu^*(\sigma)_{(\frac{1}{2}, c, \rho)} = \delta([\nu^{\frac{1}{2}}\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{-\frac{1}{2}}\rho]) \rtimes \sigma_{sp}).$$

If $\epsilon((c_-, \rho)) = 1$ then in $R(GL) \otimes R$ we have

$$\mu^*(\sigma)_{(\frac{1}{2}, c, \rho)} = \delta([\nu^{\frac{1}{2}}\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{-\frac{1}{2}}\rho]) \rtimes \sigma_{sp}) + \\ + 2\,\delta([\nu^{\frac{1}{2}}\rho, \nu^c\rho]) \otimes \sigma_{ds},$$

where σ_{ds} denotes the unique discrete series subquotient of $\delta([\nu^{-c_-}\rho, \nu^{-\frac{1}{2}}\rho]) \rtimes \sigma_{sp}$.

Proof. One readily sees that

$$\mu^*(\sigma_{ind})_{(\frac{1}{2}, c, \rho)} = 2\,\delta([\nu^{\frac{1}{2}}\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{-\frac{1}{2}}\rho]) \rtimes \sigma_{sp}) + \\ + 2\,\delta([\nu^{\frac{1}{2}}\rho, \nu^c\rho]) \otimes \sigma_{ds}.$$

It can be deduced from [10, Theorem 5.1] that $\mu^*(\sigma_{ds}) \geq \delta([\nu^{\frac{1}{2}}\rho, \nu^{c_-}\rho]) \otimes \sigma_{sp}$. Thus, if π is an irreducible subquotient of σ_{ind} such that $\mu^*(\pi)$ contains $\delta([\nu^{\frac{1}{2}}\rho, \nu^c\rho]) \otimes \sigma_{ds}$, then π is a discrete series representation (by Lemma 4.1) and there is an irreducible constituent $\delta \otimes \sigma_{sp}$ of $\mu^*(\pi)$ such that $m^*(\delta) \geq \delta([\nu^{\frac{1}{2}}\rho, \nu^c\rho]) \otimes \delta([\nu^{\frac{1}{2}}\rho, \nu^{c_-}\rho])$. Standard arguments show that

$$\delta \simeq \delta([\nu^{\frac{1}{2}}\rho, \nu^c\rho]) \times \delta([\nu^{\frac{1}{2}}\rho, \nu^{c_-}\rho])$$

and [18, Proposition 7.4] shows that $\mu^*(\sigma) \geq \delta([\nu^{\frac{1}{2}}\rho, \nu^c\rho]) \otimes \sigma_{ds}$ if and only if $\epsilon((c_-, \rho)) = 1$. This finishes the proof. ∎

Proposition 4.15. *Suppose $c_- \neq c_{\min}$ and $\epsilon((c_{\min},\rho)) = -1$. If $\epsilon(((c_-)_-,\rho)) \neq \epsilon((c_-,\rho))$ then in $R(GL)\otimes R$ we have*

$$\mu^*(\sigma)_{(\frac{1}{2},c,\rho)} = \delta([\nu^{\frac{1}{2}}\rho,\nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho,\nu^{-\frac{1}{2}}\rho]) \rtimes \sigma_{sp}).$$

If $\epsilon(((c_-)_-,\rho)) = \epsilon((c_-,\rho))$ then in $R(GL)\otimes R$ we have

$$\mu^*(\sigma)_{(\frac{1}{2},c,\rho)} = \delta([\nu^{\frac{1}{2}}\rho,\nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho,\nu^{-\frac{1}{2}}\rho]) \rtimes \sigma_{sp}) + \\ + 2\,\delta([\nu^{\frac{1}{2}}\rho,\nu^c\rho]) \otimes \tau,$$

where τ is the unique irreducible subquotient of $\delta([\nu^{-c_-}\rho,\nu^{-\frac{1}{2}}\rho]) \rtimes \sigma_{sp}$ different than $L(\delta([\nu^{-c_-}\rho,\nu^{-\frac{1}{2}}\rho]) \rtimes \sigma_{sp})$.

Proof. The following equality holds in $R(GL)\otimes R$:

$$\mu^*(\sigma_{ind})_{(\frac{1}{2},c,\rho)} = 2\,\delta([\nu^{\frac{1}{2}}\rho,\nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho,\nu^{-\frac{1}{2}}\rho]) \rtimes \sigma_{sp}) + \\ + 2\,\delta([\nu^{\frac{1}{2}}\rho,\nu^c\rho]) \otimes \tau,$$

and the first part of [10, Theorem 5.1] implies that there is some irreducible constituent of the form $\delta([\nu^{(c_-)_-+1}\rho,\nu^{c_-}\rho]) \otimes \pi$ appearing in $\mu^*(\tau)$. Now one can see in the same fashion as in the proof of previous proposition that $\mu^*(\sigma) \geq \delta([\nu^{\frac{1}{2}}\rho,\nu^c\rho]) \otimes \tau$ if and only if $\epsilon(((c_-)_-,\rho)) = \epsilon((c_-,\rho))$. ■

In the rest of this section we assume $c_- \neq c_{\min}$ and $\epsilon((c_{\min},\rho)) = 1$. Let us denote by x the element of Jord_ρ such that $x_- = c_{\min}$ and by y the element of Jord_ρ such that $(c_-)_- = y$.

Proposition 4.16. *Suppose $x = c_-$, i.e. $y = c_{\min}$. If $\epsilon((c_{\min},\rho)) = \epsilon((c_-,\rho))$ then the following equality holds in $R(GL)\otimes R$:*

$$\mu^*(\sigma)_{(\frac{1}{2},c,\rho)} = \delta([\nu^{\frac{1}{2}}\rho,\nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho,\nu^{-\frac{1}{2}}\rho]) \rtimes \sigma_{sp}) + \\ + 2\,\delta([\nu^{\frac{1}{2}}\rho,\nu^c\rho]) \otimes L(\delta([\nu^{-c_{\min}}\rho,\nu^{-\frac{1}{2}}\rho]) \rtimes \sigma_{sp}^{(1)}) + \\ + 2\,\delta([\nu^{\frac{1}{2}}\rho,\nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho,\nu^{c_{\min}}\rho]) \rtimes \sigma_{sp}^{(2)}) + \\ + 4\,\delta([\nu^{\frac{1}{2}}\rho,\nu^c\rho]) \otimes \sigma_1 + \\ + \delta([\nu^{\frac{1}{2}}\rho,\nu^c\rho]) \otimes \sigma_2,$$

for unique $\sigma_{sp}^{(1)},\sigma_{sp}^{(2)} \in Irr_{sp}^{\sigma_{cusp}}(G)$ such that $Jord(\sigma_{sp}^{(1)}) = Jord(\sigma_{sp}) \setminus \{(c_{\min},\rho)\} \cup \{(c_-,\rho)\}$ and $Jord(\sigma_{sp}^{(2)}) = Jord(\sigma_{sp}) \setminus \{(c_{\min},\rho)\}$, while σ_1 and σ_2 are mutually non-isomorphic discrete series subrepresentations of $\delta([\nu^{-c_{\min}}\rho,\nu^{c_-}\rho]) \rtimes \sigma_{sp}^{(2)}$ and $\mu^*(\sigma_1)$ contains an irreducible constituent of the form $\delta([\nu^{\frac{1}{2}}\rho,\nu^{c_{\min}}\rho]) \otimes \tau$.
If $\epsilon((c_{\min},\rho)) \neq \epsilon((c_-,\rho))$ then the following equality holds in $R(GL)\otimes R$:

$$\mu^*(\sigma)_{(\frac{1}{2},c,\rho)} = \delta([\nu^{a}\rho,\nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho,\nu^{-\frac{1}{2}}\rho]) \rtimes \sigma_{sp}) + \\ + 2\,\delta([\nu^{\frac{1}{2}}\rho,\nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho,\nu^{c_{\min}}\rho]) \rtimes \sigma_{sp}^{(2)}) + \\ + \delta([\nu^{\frac{1}{2}}\rho,\nu^c\rho]) \otimes \sigma_2,$$

for $\sigma_{sp}^{(2)}$ and σ_2 as above.

Proof. In $R(GL) \otimes R$ we have

$$\begin{aligned}\mu^*(\sigma_{ind})_{(\frac{1}{2},c,\rho)} = {} & 2\,\delta([\nu^{\frac{1}{2}}\rho,\nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho,\nu^{-\frac{1}{2}}\rho]) \rtimes \sigma_{sp}) + \\ & + 2\,\delta([\nu^{\frac{1}{2}}\rho,\nu^c\rho]) \otimes L(\delta([\nu^{-c_{\min}}\rho,\nu^{-\frac{1}{2}}\rho]) \rtimes \sigma_{sp}^{(1)}) + \\ & + 4\,\delta([\nu^{\frac{1}{2}}\rho,\nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho,\nu^{c_{\min}}\rho]) \rtimes \sigma_{sp}^{(2)}) + \\ & + 4\,\delta([\nu^{\frac{1}{2}}\rho,\nu^c\rho]) \otimes \sigma_1 + \\ & + 2\,\delta([\nu^{\frac{1}{2}}\rho,\nu^c\rho]) \otimes \sigma_2.\end{aligned}$$

We have already seen that $\delta([\nu^a\rho,\nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho,\nu^{-\frac{1}{2}}\rho]) \rtimes \sigma_{sp})$ appears in $\mu^*(\sigma)$ with multiplicity one.

Furthermore, it follows directly that $\mu^*(L(\delta([\nu^{-c_{\min}}\rho,\nu^{-\frac{1}{2}}\rho]) \rtimes \sigma_{sp}^{(1)}))$ contains some irreducible constituent of the form

$$\delta([\nu^{-c_{\min}}\rho,\nu^{-\frac{1}{2}}\rho]) \times \delta([\nu^{c_{\min}+1}\rho,\nu^{c_-}\rho]) \otimes \pi.$$

Thus, if π_1 is an irreducible subquotient of σ_{ind} with the property $\mu^*(\pi_1) \geq \delta([\nu^{\frac{1}{2}}\rho,\nu^c\rho]) \otimes L(\delta([\nu^{-c_{\min}}\rho,\nu^{-\frac{1}{2}}\rho]) \rtimes \sigma_{sp}^{(1)})$, then there is some irreducible constituent $\delta \otimes \pi$ of $\mu^*(\pi_1)$ such that

$$m^*(\delta) \geq \delta([\nu^{\frac{1}{2}}\rho,\nu^c\rho]) \otimes \delta([\nu^{-c_{\min}}\rho,\nu^{-\frac{1}{2}}\rho]) \times \delta([\nu^{c_{\min}+1}\rho,\nu^{c_-}\rho]).$$

From $\mu^*(\sigma_{ind})$ we deduce that

$$\delta \simeq \delta([\nu^{c_{\min}+1}\rho,\nu^{c_-}\rho]) \times \delta([\nu^{-c_{\min}}\rho,\nu^c\rho])$$

and it follows that $\mu^*(\sigma) \geq \delta([\nu^{\frac{1}{2}}\rho,\nu^c\rho]) \otimes L(\delta([\nu^{-c_{\min}}\rho,\nu^{-\frac{1}{2}}\rho]) \rtimes \sigma_{sp}^{(1)})$ if and only if $\epsilon((c_{\min},\rho)) = \epsilon((c_-,\rho))$.

Since, by [10, Theorem 5.1], $\mu^*(\sigma_1) \geq \delta([\nu^{\frac{1}{2}}\rho,\nu^{c_{\min}}\rho]) \times \delta([\nu^{\frac{1}{2}}\rho,\ \nu^{c_-}\rho]) \otimes \tau'$, for some irreducible representation τ', in the same way we obtain that $\mu^*(\sigma) \geq \delta([\nu^{\frac{1}{2}}\rho,\nu^c\rho]) \otimes \sigma_1$ if and only if $\epsilon((c_{\min},\rho)) = \epsilon(c_-,\rho))$.

From the definition of σ we obtain

$$\sigma \hookrightarrow \delta([\nu^{-c_-}\rho,\nu^c\rho]) \times \delta([\nu^{\frac{1}{2}}\rho,\nu^{c_{\min}}\rho]) \rtimes \sigma_{sp}^{(2)}.$$

Since $m^*(\delta([\nu^{-c_-}\rho,\nu^c\rho]) \times \delta([\nu^{\frac{1}{2}}\rho,\nu^{c_{\min}}\rho])) \geq 2\ \delta([\nu^{\frac{1}{2}}\rho,\nu^c\rho]) \otimes \delta([\nu^{-c_-}\rho,\nu^{c_{\min}}\rho])$, using Frobenius reciprocity we obtain that

$$\delta([\nu^{\frac{1}{2}}\rho,\nu^c\rho]) \otimes \delta([\nu^{-c_-}\rho,\nu^{c_{\min}}\rho]) \otimes \sigma_{sp}^{(2)}$$

appears with multiplicity two in $(m\mu)^*(\sigma)$.

Transitivity of Jacquet modules implies that there is some irreducible constituent $\delta([\nu^{\frac{1}{2}}\rho,\nu^c\rho]) \otimes \pi$ appearing in $\mu^*(\sigma)$ such that $\mu^*(\pi) \geq \delta([\nu^{-c_-}\rho,\ \nu^{c_{\min}}\rho]) \otimes \sigma_{sp}^{(2)}$. Description of $\mu^*(\sigma_{ind})$ given at the beginning of the proof shows that $\pi \simeq L(\delta([\nu^{-c_-}\rho,\nu^{c_{\min}}\rho]) \rtimes \sigma_{sp}^{(2)})$. Also, since $\mu^*(L(\delta([\nu^{-c_-}\rho,\ \nu^{c_{\min}}\rho]) \rtimes \sigma_{sp}^{(2)}))$ contains $\delta([\nu^{-c_-}\rho,\nu^{c_{\min}}\rho]) \otimes \sigma_{sp}^{(2)}$ with multiplicity one, $\delta([\nu^{\frac{1}{2}}\rho,\ \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho,\nu^{c_{\min}}\rho]) \rtimes \sigma_{sp}^{(2)})$ is contained in $\mu^*(\sigma)$ with multiplicity at least two. Examining analogous properties of $\mu^*(\sigma')$ we get that such irreducible constituent is contained in $\mu^*(\sigma)$ with multiplicity exactly two.

It remains to consider $\delta([\nu^{\frac{1}{2}}\rho, \nu^{c}\rho]) \otimes \sigma_2$. First we note that, by [10, Proposition 3.1], in $R(G)$ we have

$$\delta([\nu^{c_{\min}+1}\rho, \nu^{c_-}\rho]) \rtimes \sigma_{sp} = L(\delta([\nu^{-c_-}\rho, \nu^{-c_{\min}+1}\rho]) \rtimes \sigma_{sp}) + \sigma_{sp}^{(1)}.$$

Since $\mu^*(\delta([\nu^{c_{\min}+1}\rho, \nu^{c_-}\rho]) \rtimes \sigma_{sp})$ contains $L(\delta([\nu^{\frac{1}{2}}\rho, \nu^{c_{\min}}\rho]) \times \delta([\nu^{c_{\min}+1}\rho, \nu^{c_-}\rho])) \otimes \sigma_{sp}^{(2)}$ and, by [7, Theorem 4.6], such irreducible constituent does not appear in $\mu^*(\sigma_{sp}^{(1)})$, it has to appear in $\mu^*(L(\delta([\nu^{-c_-}\rho, \nu^{-c_{\min}+1}\rho]) \rtimes \sigma_{sp}))$.

Now from $\mu^*(\sigma) \geq \delta([\nu^{\frac{1}{2}}\rho, \nu^{c}\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{-c_{\min}+1}\rho]) \rtimes \sigma_{sp})$, we obtain that there is some irreducible constituent $\delta \otimes \sigma_{sp}^{(2)}$ of $\mu^*(\sigma)$ such that $m^*(\delta) \geq \delta([\nu^{\frac{1}{2}}\rho, \nu^{c}\rho]) \otimes L(\delta([\nu^{\frac{1}{2}}\rho, \nu^{c_{\min}}\rho]) \times \delta([\nu^{c_{\min}+1}\rho, \nu^{c_-}\rho]))$. From $\mu^*(\sigma_{ind})$ it is not hard to see that

$$\delta \simeq \delta([\nu^{\frac{1}{2}}\rho, \nu^{c}\rho]) \times L(\delta([\nu^{\frac{1}{2}}\rho, \nu^{c_{\min}}\rho]) \times \delta([\nu^{c_{\min}+1}\rho, \nu^{c_-}\rho])).$$

This gives $m^*(\delta) \geq \delta([\nu^{\frac{1}{2}}\rho, \nu^{c}\rho]) \otimes \delta([\nu^{-c_{\min}+1}\rho, \nu^{c_-}\rho])$ and, by transitivity of Jacquet modules, there is some irreducible constituent $\delta([\nu^{\frac{1}{2}}\rho, \nu^{c}\rho]) \otimes \pi$ of $\mu^*(\sigma)$ such that $\mu^*(\pi) \geq \delta([\nu^{-c_{\min}+1}\rho, \nu^{c_-}\rho]) \otimes \sigma_{sp}^{(2)}$. It can be seen directly from the description of $\mu^*(\sigma_{ind})$ that $\pi \in \{\sigma_1, \sigma_2\}$.

The same conclusion holds for $\mu^*(\sigma')$, and in the same way as in the proof of Proposition 4.10 we obtain that $\delta([\nu^{\frac{1}{2}}\rho, \nu^{c}\rho]) \otimes \sigma_2$ appears in $\mu^*(\sigma)$ with multiplicity one. ■

Proposition 4.17. *Suppose $x = y$. If $\epsilon((x,\rho)) = \epsilon((c_-,\rho))$ then the following equality holds in $R(GL) \otimes R$:*

$$\begin{aligned}\mu^*(\sigma)_{(\frac{1}{2},c,\rho)} = {} & \delta([\nu^{\frac{1}{2}}\rho, \nu^{c}\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{-\frac{1}{2}}\rho]) \rtimes \sigma_{sp}) + \\ & + 2\,\delta([\nu^{\frac{1}{2}}\rho, \nu^{c}\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{c_{\min}}\rho]) \rtimes \sigma_{sp}^{(1)}) + \\ & + 2\,\delta([\nu^{\frac{1}{2}}\rho, \nu^{c}\rho]) \otimes L(\delta([\nu^{-x}\rho, \nu^{-\frac{1}{2}}\rho]) \rtimes \sigma_{sp}^{(2)}) + \\ & + 4\,\delta([\nu^{\frac{1}{2}}\rho, \nu^{c}\rho]) \otimes L(\delta([\nu^{-x}\rho, \nu^{c_{\min}}\rho]) \rtimes \sigma_{sp}^{(3)}) + \\ & + \delta([\nu^{\frac{1}{2}}\rho, \nu^{c}\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{x}\rho]) \rtimes \sigma_{sp}^{(4)}) + \\ & + 2\,\delta([\nu^{\frac{1}{2}}\rho, \nu^{c}\rho]) \otimes \sigma_1,\end{aligned}$$

for unique $\sigma_{sp}^{(1)}, \sigma_{sp}^{(2)}, \sigma_{sp}^{(3)}, \sigma_{sp}^{(4)} \in Irr_{sp}^{\sigma_{cusp}}(G)$ such that $Jord(\sigma_{sp}^{(1)}) = Jord(\sigma_{sp}) \setminus \{(c_{\min},\rho)\}$, $Jord(\sigma_{sp}^{(2)}) = Jord(\sigma_{sp}) \setminus \{(x,\rho)\} \cup \{(c_-,\rho)\}$, $Jord(\sigma_{sp}^{(3)}) = Jord(\sigma_{sp}) \setminus \{(c_{\min},\rho),(x,\rho)\} \cup \{(c_-,\rho)\}$ and $Jord(\sigma_{sp}^{(4)}) = Jord(\sigma_{sp}^{(1)}) \setminus \{(x,\rho)\} \cup \{(c_{\min},\rho)\}$, while σ_1 is the unique discrete series subrepresentation of both induced representations $\delta([\nu^{-c_{\min}}\rho, \nu^{x}\rho]) \rtimes \sigma_{sp}^{(2)}$ and $\delta([\nu^{-x}\rho, \nu^{c_-}\rho]) \rtimes \sigma_{sp}^{(4)}$.
If $\epsilon((x,\rho)) \neq \epsilon((c_-,\rho))$ then the following equality holds in $R(GL) \otimes R$:

$$\begin{aligned}\mu^*(\sigma)_{(\frac{1}{2},c,\rho)} = {} & \delta([\nu^{a}\rho, \nu^{c}\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{-\frac{1}{2}}\rho]) \rtimes \sigma_{sp}) + \\ & + 2\,\delta([\nu^{\frac{1}{2}}\rho, \nu^{c}\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{c_{\min}}\rho]) \rtimes \sigma_{sp}^{(1)}) + \\ & + \delta([\nu^{\frac{1}{2}}\rho, \nu^{c}\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{x}\rho]) \rtimes \sigma_{sp}^{(4)}),\end{aligned}$$

for $\sigma_{sp}^{(1)}$ *and* $\sigma_{sp}^{(4)}$ *as above.*

Proof. We provide only the main details of the proof since it mostly parallels the proof of previous proposition. Using the structural formula (2), [10, Theorem 5.1], and [9, Proposition 3.2], we get

$$\begin{aligned}\mu^*(\sigma_{ind})_{(\frac{1}{2},c,\rho)} = {} & 2\,\delta([\nu^{\frac{1}{2}}\rho,\nu^{c}\rho])\otimes L(\delta([\nu^{-c_-}\rho,\nu^{-\frac{1}{2}}\rho])\rtimes\sigma_{sp})+\\ & + 4\,\delta([\nu^{\frac{1}{2}}\rho,\nu^{c}\rho])\otimes L(\delta([\nu^{-c_-}\rho,\nu^{c_{\min}}\rho])\rtimes\sigma_{sp}^{(1)})+\\ & + 2\,\delta([\nu^{\frac{1}{2}}\rho,\nu^{c}\rho])\otimes L(\delta([\nu^{-x}\rho,\nu^{-\frac{1}{2}}\rho])\rtimes\sigma_{sp}^{(2)})+\\ & + 4\,\delta([\nu^{\frac{1}{2}}\rho,\nu^{c}\rho])\otimes L(\delta([\nu^{-x}\rho,\nu^{c_{\min}}\rho])\rtimes\sigma_{sp}^{(3)})+\\ & + 2\,\delta([\nu^{\frac{1}{2}}\rho,\nu^{c}\rho])\otimes L(\delta([\nu^{-c_-}\rho,\nu^{x}\rho])\rtimes\sigma_{sp}^{(4)})+\\ & + 2\,\delta([\nu^{\frac{1}{2}}\rho,\nu^{c}\rho])\otimes\sigma_1.\end{aligned}$$

For $\pi\in\{L(\delta([\nu^{-x}\rho,\nu^{-\frac{1}{2}}\rho])\rtimes\sigma_{sp}^{(2)}), L(\delta([\nu^{-x}\rho,\nu^{c_{\min}}\rho])\rtimes\sigma_{sp}^{(3)}),\sigma_1\}$ there is some irreducible constituent of the form $\delta([\nu^{x+1}\rho,\nu^{c_-}\rho])\otimes\pi'$ appearing in $\mu^*(\pi)$. If $\delta\otimes\pi'$ is an irreducible constituent of $\mu^*(\sigma_{ind})$ such that $m^*(\delta)\geq\delta([\nu^{\frac{1}{2}}\rho,\nu^{c}\rho])\otimes\delta([\nu^{x+1}\rho,\nu^{c_-}\rho])$, then it can be seen that $m^*(\delta)$ also contains $\delta([\nu^{x+1}\rho,\nu^{c_-}\rho])\otimes\delta([\nu^{\frac{1}{2}}\rho,\nu^{c}\rho])$. Consequently, $\mu^*(\sigma)$ contains $\delta([\nu^{\frac{1}{2}}\rho,\nu^{c}\rho])\otimes\pi$ for $\pi\in\{L(\delta([\nu^{-x}\rho,\nu^{-\frac{1}{2}}\rho])\rtimes\sigma_{sp}^{(2)}), L(\delta([\nu^{-x}\rho,\nu^{c_{\min}}\rho])\rtimes\sigma_{sp}^{(3)}),\sigma_1\}$ if and only if $\epsilon((x,\rho))=\epsilon((c_-,\rho))$.

Since both $\mu^*(\sigma)$ and $\mu^*(\sigma')$ contain $\delta([\nu^{-c_-}\rho,\nu^{c}\rho])\times\delta([\nu^{\frac{1}{2}}\rho,\ \nu^{c_{\min}}\rho])\otimes\sigma_{sp}^{(1)}$, and $\delta([\nu^{\frac{1}{2}}\rho,\nu^{c}\rho])\otimes\delta([\nu^{-c_-}\rho,\nu^{c_{\min}}\rho])$ appears with multiplicity two in $m^*(\delta([\nu^{-c_-}\rho,\nu^{c}\rho])\times\delta([\nu^{\frac{1}{2}}\rho,\ \nu^{c_{\min}}\rho]))$, in the same way as in the proof of Proposition 4.16 we get that $\mu^*(\sigma)$ contains $\delta([\nu^{\frac{1}{2}}\rho,\nu^{c}\rho])\otimes L(\delta([\nu^{-c_-}\rho,\nu^{c_{\min}}\rho])\rtimes\sigma_{sp}^{(1)})$ with multiplicity two.

Similarly, both σ and σ' are irreducible subrepresentations of

$$\delta([\nu^{-c_-}\rho,\nu^{c}\rho])\times L(\delta([\nu^{\frac{1}{2}}\rho,\nu^{c_{\min}}\rho])\times\delta([\nu^{c_{\min}+1}\rho,\nu^{x}\rho]))\rtimes\sigma_{sp}^{(4)}.$$

[3, Lemma 1.3.3] shows that the induced representation

$$\delta([\nu^{-c_-}\rho,\nu^{c}\rho])\times L(\delta([\nu^{\frac{1}{2}}\rho,\nu^{c_{\min}}\rho])\times\delta([\nu^{c_{\min}+1}\rho,\nu^{x}\rho]))$$

is irreducible and it is not hard to see, using Frobenius reciprocity and transitivity of Jacquet modules, that both $(m\mu)^*(\sigma)$ and $(m\mu)^*(\sigma')$ contain

$$\delta([\nu^{\frac{1}{2}}\rho,\nu^{c}\rho])\otimes\delta([\nu^{-c_-}\rho,\nu^{x}\rho])\otimes\sigma_{sp}^{(4)}.$$

Now it can be seen in the same way as before that $\delta([\nu^{\frac{1}{2}}\rho,\nu^{c}\rho])\otimes L(\delta([\nu^{-c_-}\rho,\nu^{x}\rho])\rtimes\sigma_{sp}^{(4)})$ appears in $\mu^*(\sigma)$ with multiplicity one. This finishes the proof. ∎

The remaining case is settled in the following proposition. We omit the proof, since it can be deduced applying the same arguments as in the proof of previous proposition.

Proposition 4.18. *Suppose $x < y$. If $\epsilon((y,\rho)) = \epsilon((c_-,\rho))$ then the following equality holds in $R(GL) \otimes R$:*

$$\begin{aligned}
\mu^*(\sigma)_{(\frac{1}{2},c,\rho)} = {} & \delta([\nu^{\frac{1}{2}}\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{-\frac{1}{2}}\rho]) \rtimes \sigma_{sp}) + \\
& + 2\,\delta([\nu^{\frac{1}{2}}\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{c_{\min}}\rho]) \rtimes \sigma_{sp}^{(1)}) + \\
& + 2\,\delta([\nu^{\frac{1}{2}}\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-y}\rho, \nu^{-\frac{1}{2}}\rho]) \rtimes \sigma_{sp}^{(2)}) + \\
& + 4\,\delta([\nu^{\frac{1}{2}}\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-y}\rho, \nu^{c_{\min}}\rho]) \rtimes \sigma_{sp}^{(3)}) + \\
& + \delta([\nu^{\frac{1}{2}}\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{x}\rho]) \rtimes \sigma_{sp}^{(4)}) + \\
& + 2\,\delta([\nu^{\frac{1}{2}}\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-y}\rho, \nu^{x}\rho]) \rtimes \sigma_{sp}^{(5)}).
\end{aligned}$$

for unique $\sigma_{sp}^{(1)}, \sigma_{sp}^{(2)}, \sigma_{sp}^{(3)}, \sigma_{sp}^{(4)}, \sigma_{sp}^{(5)} \in Irr_{sp}^{\sigma_{cusp}}(G)$ such that $Jord(\sigma_{sp}^{(1)}) = Jord(\sigma_{sp}) \setminus \{(c_{\min},\rho)\}$, $Jord(\sigma_{sp}^{(2)}) = Jord(\sigma_{sp}) \setminus \{(y,\rho)\} \cup \{(c_-,\rho)\}$, $Jord(\sigma_{sp}^{(3)}) = Jord(\sigma_{sp}) \setminus \{(c_{\min},\rho),(y,\rho)\} \cup \{(c_-,\rho)\}$, $Jord(\sigma_{sp}^{(4)}) = Jord(\sigma_{sp}^{(1)}) \setminus \{(x,\rho)\} \cup \{(c_{\min},\rho)\}$ and $Jord(\sigma_{sp}^{(5)}) = Jord(\sigma_{sp}^{(4)}) \setminus \{(y,\rho)\} \cup \{(c_-,\rho)\}$.
If $\epsilon((y,\rho)) \neq \epsilon((c_-,\rho))$ then the following equality holds in $R(GL) \otimes R$:

$$\begin{aligned}
\mu^*(\sigma)_{(\frac{1}{2},c,\rho)} = {} & \delta([\nu^{a}\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{-\frac{1}{2}}\rho]) \rtimes \sigma_{sp}) + \\
& + 2\,\delta([\nu^{\frac{1}{2}}\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{c_{\min}}\rho]) \rtimes \sigma_{sp}^{(1)}) + \\
& + \delta([\nu^{\frac{1}{2}}\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{x}\rho]) \rtimes \sigma_{sp}^{(4)}),
\end{aligned}$$

for $\sigma_{sp}^{(1)}$ and $\sigma_{sp}^{(4)}$ as before.

5. Case $\mathrm{Jord}_\rho(\sigma_{sp}) = \emptyset$ and c integral.

The purpose of this section is to provide a description of $\mu^*(\sigma)_{(c,\rho)}$ in an exceptional case. Throughout this section we assume that c is odd and $\mathrm{Jord}_\rho(\sigma_{sp}) = \emptyset$. Consequently, $c = \max(\mathrm{Jord}_\rho(\sigma_{sp}))$ and $\epsilon((c,\rho))$ is defined. Furthermore, the induced representation $\rho \rtimes \sigma_{cusp}$ is the direct sum of two nonisomorphic tempered representations, which we denote by τ_1 and τ_{-1}. Also, $\epsilon((c,\rho)) = i$ if and only if there is some irreducible representation π such that σ is a subrepresentation of $\pi \times \delta([\nu\rho, \nu^c\rho]) \rtimes \tau_i$. Also, $\epsilon((c,\rho)) \neq \epsilon'((c,\rho))$.

Obviously, we only need to consider $\mu^*(\sigma)_{(a,c,\rho)}$ for $a \leq c_-$. The following theorem provides a description of Jacquet modules in question:

Theorem 5.1. *For $-c_- \leq a \leq 0$, in $R(GL) \otimes R$ we have*

$$\mu^*(\sigma)_{(a,c,\rho)} = \delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}),$$

while for $1 \leq a \leq c_-$ we have:

$$\begin{aligned}
\mu^*(\sigma)_{(a,c,\rho)} = {} & \delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}) + \\
& + 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes \sigma_{ds}^{(a)},
\end{aligned}$$

where $\sigma_{ds}^{(a)}$ is the unique discrete series subrepresentation of $\delta([\nu^{-a+1}\rho, \nu^{c_-}\rho]) \rtimes \sigma_{sp}$ such that for the corresponding admissible triple $(Jord^{(a)}, \sigma_{cusp}, \epsilon^{(a)})$, $\epsilon^{(a)}((c_-,\rho)) = \epsilon((c,\rho))$ holds.

Proof. We only comment the case $a \geq 1$. In this case it is easy to obtain

$$\begin{aligned}\mu^*(\sigma_{ind})_{a,c,\rho} = & \, 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes L(\delta([\nu^{-c_-}\rho, \nu^{a-1}\rho]) \rtimes \sigma_{sp}) + \\ & + 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes \sigma_1 + \\ & + 2\,\delta([\nu^a\rho, \nu^c\rho]) \otimes \sigma_{-1},\end{aligned}$$

where σ_1 and σ_{-1} denote mutually non-isomorphic discrete series subrepresentations of the induced representation $\delta([\nu^{-a+1}\rho,\ \nu^{c_-}\rho]) \rtimes \sigma_{sp}$. Furthermore, we denote by $(\mathrm{Jord}^{(i)}, \sigma_{cusp}, \epsilon^{(i)})$ the admissible triple corresponding to σ_i, $i \in \{1, -1\}$, and assume $\epsilon^{(i)}((c_-, \rho)) = i$.

Proposition 4.2 shows that it is enough to consider $\delta([\nu^a\rho, \nu^c\rho]) \otimes \sigma_1$ and $\delta([\nu^a\rho, \nu^c\rho]) \otimes \sigma_{-1}$. Thus, suppose that $\delta([\nu^a\rho, \nu^c\rho]) \otimes \sigma_i$ is an irreducible constituent of $\mu^*(\sigma)$, for some $i \in \{1, -1\}$. By [17], there is an irreducible representation π such that $(m\mu)^*(\sigma_i)$ contains

$$\pi \otimes \delta([\nu\rho, \nu^{c_-}\rho]) \otimes \tau_i.$$

Consequently, there is some irreducible constituent $\delta \otimes \tau_i$ of $\mu^*(\sigma_i)$ such that $m^*(\delta)$ contains $\pi \otimes \delta([\nu\rho, \nu^{c_-}\rho])$. Calculating $\mu^*(\delta([\nu^{-a+1}\rho, \nu^{c_-}\rho]) \rtimes \sigma_{sp})$ we deduce that

$$\delta \simeq \pi' \times \delta([\nu\rho, \nu^{c_-}\rho]) \times \delta([\nu\rho, \nu^{a-1}\rho])$$

where π' stands for the unique irreducible representation such that $\pi' \otimes \sigma_{cusp} \leq \mu^*(\sigma_{sp})$ (uniqueness is proved in [7, Theorem 4.6]). Since there are no twists or ρ appearing in the cuspidal support of π', it easily follows that $\pi \simeq \pi' \times \delta([\nu\rho, \nu^{a-1}\rho]) \simeq \delta([\nu\rho, \nu^{a-1}\rho]) \times \pi'$.

Transitivity of Jacquet modules shows that

$$\delta([\nu^a\rho, \nu^c\rho]) \otimes \delta([\nu\rho, \nu^{a-1}\rho]) \times \pi' \otimes \delta([\nu\rho, \nu^{c_-}\rho]) \otimes \tau_i$$

appears in Jacquet module of σ with respect to an appropriate standard parabolic subgroup. Hence, there is some irreducible constituent $\delta' \otimes \tau_i$ of $\mu^*(\sigma)$ such that Jacquet module of δ' with respect to an appropriate standard parabolic subgroup contains $\delta([\nu^a\rho, \nu^c\rho]) \otimes \delta([\nu\rho, \nu^{a-1}\rho]) \times \pi' \otimes \delta([\nu\rho, \nu^{c_-}\rho])$. In the same way as before we conclude that

$$\delta' \simeq \delta([\nu\rho, \nu^c\rho]) \times \pi' \times \delta([\nu\rho, \nu^{c_-}\rho])$$

so $(m\mu)^*(\sigma)$ contains $\pi' \times \delta([\nu\rho, \nu^{c_-}\rho]) \otimes \delta([\nu\rho, \nu^c\rho]) \otimes \tau_i$. It follows that $\mu^*(\sigma)$ contains some irreducible constituent $\pi' \times \delta([\nu\rho, \nu^{c_-}\rho]) \otimes \tau$ such that $\mu^*(\tau) \geq \delta([\nu\rho, \nu^c\rho]) \otimes \tau_i$. From $\mu^*(\sigma_{ind})$ we directly obtain $\tau \leq \delta([\rho, \nu^c\rho]) \rtimes \sigma_{cusp}$. In $R(G)$ we have

$$\delta([\rho, \nu^c\rho]) \rtimes \sigma_{cusp} \leq \delta([\nu\rho, \nu^c\rho]) \rtimes \tau_1 \oplus \delta([\nu\rho, \nu^c\rho]) \rtimes \tau_{-1}$$

and it follows immediately that $\delta([\nu\rho, \nu^c\rho]) \otimes \tau_i$ appears with multiplicity one in Jacquet module of the right-hand side of the previous inequality and, by Frobenius reciprocity, it also appears in $\mu^*(\delta([\nu\rho, \nu^c\rho]) \rtimes \tau_i)$. Thus, τ is the unique irreducible subrepresentation of $\delta([\nu\rho, \nu^c\rho]) \rtimes \tau_i$ and [18, Proposition 7.5] shows $\epsilon((c, \rho)) = i$. Consequently, $\mu^*(\sigma)$ contains $\delta([\nu^a\rho, \nu^c\rho]) \otimes \sigma_i$ if and only if $\epsilon((c, \rho)) = \epsilon^{(i)}((c_-, \rho))$. This finishes the proof. ∎

References

[1] Arthur, J., "The endoscopic classification of representations" American Mathematical Society, Providence, RI, 2013.

[2] Barbasch, D., and D. Ciubotaru, *Ladder representations of* $\mathbb{Q}_p$, to appear in "Representations of Lie Groups: In Honor of David A Vogan, Jr. on his 60th Birthday" Progress in Mathematics Birkhäuser, Boston, CT, 2015.

[3] Jantzen, C., *Jacquet modules of p-adic general linear groups*, Representation Theory **11** (2007), 45–83.

[4] Kret, A., and E. Lapid, *Jacquet modules of ladder representations*, Comptes Rendus Mathématique. Académie des Sciences. Paris **350** (2012), 937–940.

[5] Lapid, E., and A. Minguez, *On a determinantal formula of Tadić*, American Journal of Mathematics **136** (2014), 111–142.

[6] Matić, I., *Strongly positive representations of metaplectic groups*, Journal of Algebra **334** (2011), 255–274.

[7] Matić, I., *Jacquet modules of strongly positive representations of the metaplectic group* $\widetilde{Sp(n)}$, Transactions of the American Mathematical Society **365** (2013), 2755–2778.

[8] Matić, I., *The conservation relation for discrete series representations of metaplectic groups*, International Mathematics Research Notices. IMRN **22** (2013), 5227–5269.

[9] Matić, I., *On discrete series subrepresentations of generalized principal series*, Glasnik Matematički, 51(71)(2016), 125–152.

[10] Muić, G., *Composition series of generalized principal series; the case of strongly positive discrete series*, Israel Journal of Mathematics **140** (2004), 157–202.

[11] Mœglin, C., *Sur la classification des séries discrètes des groupes classiques p-adiques: paramètres de Langlands et exhaustivité*, Journal of the European Mathematical Society (JEMS) **4** (2002), 143–200.

[12] —, *Paquets stables des séries discrètes accessibles par endoscopie tordue; leur paramètre de Langlands*, preprint (2012).

[13] —, and M. Tadić, *Construction of discrete series for classical p-adic groups*, Journal of the American Mathematical Society **15** (2002), 715–786.

[14] Shahidi F., *A proof of Langlands' conjecture on Plancherel measures; complementary series for p-adic groups*, Annals of Mathematics. Second Series **132** (1990), 273–330.

[15] —, *Twisted endoscopy and reducibility of induced representations for p-adic groups*, Duke Mathematical Journal **66** (1992), 1–41.

[16] Tadić, M., *Structure arising from induction and Jacquet modules of representations of classical p-adic groups*, Journal of Algebra **177** (1995), 1–33.

[17] —, *On invariants of discrete series representations of classical p-adic groups*, Manuscripta Mathematica **135** (2011), 417–435.

[18] —, *On tempered and square integrable representations of classical p-adic groups*, Science China. Mathematics **56** (2013), 2273–2313.

[19] Zelevinsky, A.V., *Induced representations of reductive p-adic groups. II. On irreducible representations of $GL(n)$*, Annales Scientifiques de l'École Normale Supérieure. Quatrième Série **13** (1980), 165–210.

Ivan Matić
Department of Mathematics,
University of Osijek,
Trg Ljudevita Gaja 6,
Osijek, Croatia
imatic@mathos.hr