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Modified indices of political power - a case study of a few parliaments

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Abstract According to yes-no voting systems, players (e.g., parties in a parliament) have some influence on making some decisions. In formal voting situations, taking into account that a majority vote is needed for making a decision, the question of political power of parties can be considered. There are some well-known indices of political power e.g., the Shapley-Shubik index, the Banzhaf index, the Johnston index, the Deegan-Packel index.

In order to take into account different political nature of the parties, as the main factor for forming a winning coalition i.e., a parliamentary majority, we give a modification of the power indices. For the purpose of comparison of these indices of political power from the empirical point of view, we consider the indices of power in some cases, i.e., in relation to a few parliaments.

Keywords yes-no voting systems · index of political power · the Shapley-Shubik index · the Banzhaf index · the Johnston index · the Deegan-Packel index

1 Introduction

One of the important concepts of political science is power. We consider a voting power in a yes-no voting systems, where voters (i.e., players) have some influence on making some decisions. In the sense of formal voting situations,

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one takes into account that a majority vote is needed for making a decision. So, for passage of some decision, the winning coalition should be observed.

For the purpose of measuring a voting power of corresponding players (e.g., parties in a parliament), some voting or political power indices have been proposed. We consider some well-known indices of political power: the Shapley-Shubik index (SSI), the Banzhaf index (BI), the Johnston index (JI), the Deegan-Packel index (DPI), (see [1,5,6,9]). Let us describe main properties of these indices.

Let $X = \{p_1, p_2, \ldots, p_n\}$ be the set of n players in yes-no weighted voting systems, where the corresponding weights s_k of the player p_k, $k = 1, \ldots, n$ are given, such that s_k is a number of votes of the player p_k. *A coalition* C is any subset of the set X. Also, assume that the *quota* q needed for coalition to win is given. If the inequality $\sum_{p_i \in C} s_i \geq q$ holds, a coalition $C \subset X$ is called a *winning coalition.*

Shapley-Shubik index has been proposed in 1954 (see [5]). One looks at orderings (permutations) of players. For an ordering $p_{\sigma_1} p_{\sigma_2} \ldots p_{\sigma_n}$, one of the player p_{σ_k} is called a *pivot* (i.e., *a pivotal player*), if the non-winning coalition $p_{\sigma_1} p_{\sigma_2} \ldots p_{\sigma_{k-1}}$ by joining of the player p_{σ_k} becomes the winning coalition $p_{\sigma_1} p_{\sigma_2} \ldots p_{\sigma_{k-1}} p_{\sigma_k}$.

Then the Shapley-Shubik index of the player p_k is fraction of orderings for which p_k is the pivotal player (see [9]). More formally, the definition runs as follows.

Definition 1 Let $p_k \in X$. The Shapley-Shubik index of the player p_k is given by

$$SSI(p_k) = \frac{\text{the number of orderings of } X \text{ for which } p_k \text{ is pivotal}}{\text{the total number of possible orderings of the set } X}. \quad (1)$$

Let us notice that denominator in the above expression is equal to $n!$.

The Banzhaf index has been proposed in 1965 (see [5]). Firstly, one can define so-called *total Banzhaf power of the player* p_k, denoted by the symbol $TBP(p_k)$, as the number of coalitions C satisfying following conditions:

a) $p_k \in C$;
b) C is a winning coalition;
c) $C \setminus \{p_k\}$ is a non-winning coalition.

So, in that case deletion of p_k results that the coalition $C \setminus \{p_k\}$ is a non-winning, i.e., one says that p_k's defection from C is *critical* (see [9]).

Definition 2 Let $p_k \in X$. The Banzhaf index of the player p_k is given by

$$BI(p_k) = \frac{TBP(p_k)}{\sum_{l=1}^{n} TBP(p_l)}. \quad (2)$$

By Banzhaf index of power, critical defections from winning coalitions are taken into account. However, one can also consider the total number of players whose defection from a given coalition is critical. So, Johnston index of power is defined as follows (see [9].

Assume that $C_1, \ldots, C_m$ are winning coalitions where the player p_k is critical. Let n_1 be the number of players whose defection from C_1 is critical, n_2 be the number of players whose defection from C_2 is critical, and so on, up to n_m be the number of players whose defection from C_m is critical. Then *total Johnston power of the player* p_k is given by:

$$TJP(p_k) = \frac{1}{n_1} + \frac{1}{n_2} + \ldots + \frac{1}{n_m}. \tag{3}$$

Definition 3 The Johnston index of the player p_k is given by

$$JI(p_k) = \frac{TJP(p_k)}{\sum_{l=1}^{n} TJP(p_l)}. \tag{4}$$

In 1978, Deegan and Packel (see [5]) introduced a power index very similar to Johnston index, but based on the minimal winning coalitions. (A *minimal winning coalition* is a coalition which will become a non-winning coalition if its any member is deleted from it). So, Deegan-Packel index of power is defined as follows (see [9]).

Assume that $C_1, \ldots, C_j$ are the minimal winning coalitions to which the player p_k belongs. Let n_1 be the number of players in C_1, n_2 be the number of players in C_2, ..., n_j be the number of players in C_j. Then *total Deegan-Packel power of the player* p_k is given by:

$$TDPP(p_k) = \frac{1}{n_1} + \frac{1}{n_2} + \ldots + \frac{1}{n_j}. \tag{5}$$

Definition 4 The Deegan-Packel index of the player p_k is given by

$$DPI(p_k) = \frac{TDPP(p_k)}{\sum_{l=1}^{n} TDPP(p_l)}. \tag{6}$$

From the above definitions it is obvious that power indices are fractions between 0 and 1. One can express these indices in percents.

In previously mentioned voting power indices of political parties, the approach is based on the technical formation of winning coalitions, but political distance between parties are ignored. In order to take into account different political nature of the parties, as the main factor for forming a parliamentary majority, we give a modification of power indices.

Computation of power indices is a special topic, that we are not going to consider here. There are numerous methods for calculating particular power indices, by dynamic programming, enumeration algorithms (see [2,5]). One can also find online power index calculator on the internet homepages (e.g., [3,8]).

In Section 2, the indices of power are illustrated by some empirical cases of a few parliaments. In Section 3, we propose modified power indices and the modified indices are compared in some examples, i.e., in relation to a few parliaments. In Section 4, we give concluding remarks about presented modified power indices.

2 Power indices of a few parliaments

In order to illustrate indices of power by examples, in this section we consider indices of voting power in relation to a few parliaments. In each example, the calculated value of indices of power is presented in the corresponding table. For the convenience of the reader, we give the graphic illustration of obtained values, so one can easy compare the values calculated by using mentioned types of indices of power.

Example 1 The data origins from results of elections for Croatian Parliament in 2015. The electoral system in Croatia is considered in [4]. There are $S = 151$ members of Croatian Parliament, so quota $q = 76$ represents the majority votes. In Table 1, it is shown the number s_k of seats (i.e., members) that parties or political group have obtained in the Parliament ($k = 1, \ldots, n = 11$) and its percents. Further, corresponding calculated indices of particular parties are given in percents.

Table 1 *Composition of Croatian Parliament after elections in 2015, and corresponding power indices*

party (group)	DK	HR	Most	Minorities
s_k	59	56	15	8
(%)	39.07	37.09	9.93	5.93
SSI (%)	33.66	26.51	22.63	4.21
BI (%)	31.91	25.82	24.92	3.95
JI (%)	38.48	26.97	25.55	2.60
DPI (%)	11.82	10.68	14.26	7.46

<table>
<tr><th>party (group)</th><th>HRID</th><th>IDS</th><th>HDSSB</th><th>BM365</th><th>RE</th><th>ŽZ</th><th>NLSP</th></tr>
<tr><td>s_k</td><td>3</td><td>3</td><td>2</td><td>2</td><td>1</td><td>1</td><td>1</td></tr>
<tr><td>(%)</td><td colspan="2">1.99</td><td colspan="2">1.32</td><td colspan="3">0.66</td></tr>
<tr><td>SSI (%)</td><td colspan="2">2.94</td><td colspan="2">2.23</td><td colspan="3">0.88</td></tr>
<tr><td>BI (%)</td><td colspan="2">3.04</td><td colspan="2">2.14</td><td colspan="3">1.02</td></tr>
<tr><td>JI (%)</td><td colspan="2">1.59</td><td colspan="2">1.01</td><td colspan="3">0.40</td></tr>
<tr><td>DPI (%)</td><td colspan="2">8.83</td><td colspan="2">8.14</td><td colspan="3">7.28</td></tr>
</table>

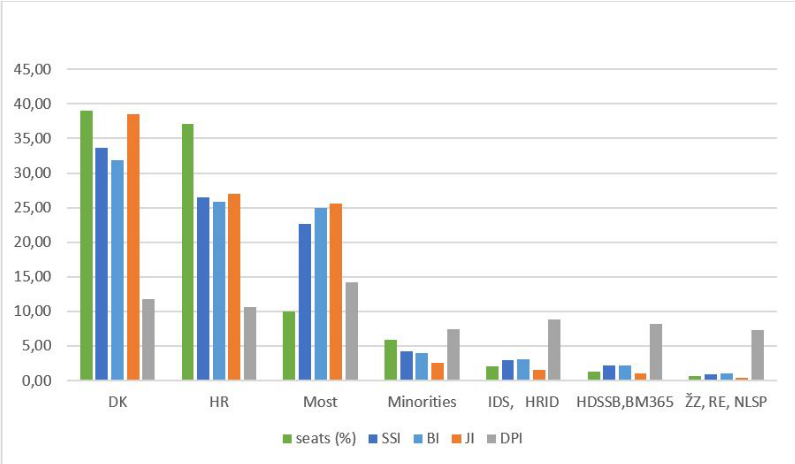

Fig. 1 *Graphic representation of data from Table 1*

Example 2 We consider at the 8th European Parliament, that consists of 750 members, which one can divide in $n = 9$ political groups (although these are subject to change, [7]). So quota $q = 376$ represents the majority votes. Corresponding number s_k of seats (i.e., members) that political groups have got in the Parliament ($k = 1, \ldots, n = 9$) and its percents are shown in Table 2. Moreover, in percents we give the corresponding calculated indices of particular parties.

Table 2 *Composition of 8th European Parliament by political groups in 2015, and corresponding power indices*

political group	EPP	S&D	ECR	ALDE	GUE-NGL
s_k (%)	215 28.63	189 25.17	74 9.85	70 9.32	52 6.92
SSI (%)	31.98	24.84	9.84	9.13	6.98
BI (%)	29.36	21.43	11.11	10.32	7.94
JI (%)	42.82	25.45	8.95	7.04	4.77
DPI (%)	13.57	11.00	13.14	12.43	12.57

political group	Greens-EFA	EFDD	ENF	NA
s_k (%)	50 6.66	46 6.12	39 5.19	16 2.13
SSI (%)	6.27	5.56	4.13	1.27
BI (%)	7.14	6.35	4.76	1.59
JI (%)	4.09	3.54	2.55	0.79
DPI (%)	11.86	11.14	9.71	4.57

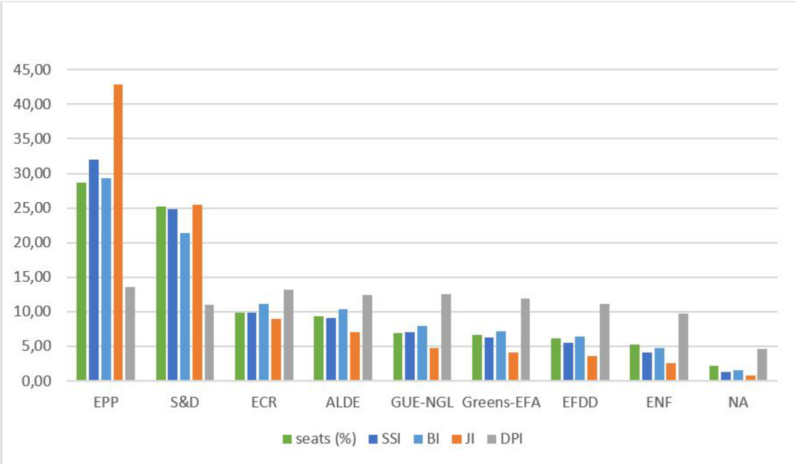

Fig. 2 *Graphic representation of data from Table 2*

Example 3 We look at Congress of Deputies in Spain after elections in June 2016, that consists of 350 members. For the sake of simplicity, let us divide them in $n = 7$ political groups (although it can be subject to change). So quota $q = 176$ represents the majority votes. In Table 3, we present corresponding number s_k of seats (i.e., members) that political groups have got in the Congress of Deputies ($k = 1, \ldots, n = 9$) and its percents. Further, corresponding calculated indices of particular groups are given in percents.

Table 3 *Composition of Congress of Deputies in Spain in 2016, and corresponding power indices*

party (group)	PP	PSOE	POD	CI	CAT	BAS	CAN
s_k	137	85	71	32	17	7	1
(%)	39.14	24.29	20.29	9.14	4.86	2	0.29
SSI (%)	43.33	20.00	20.00	10.00	3.33	3.33	0
BI (%)	42.31	19.23	19.23	11.54	3.85	3.85	0
JI (%)	56.04	15.79	15.79	8.05	2.17	2.17	0
DPI (%)	27.78	18.06	18.06	16.67	9.72	9.72	0

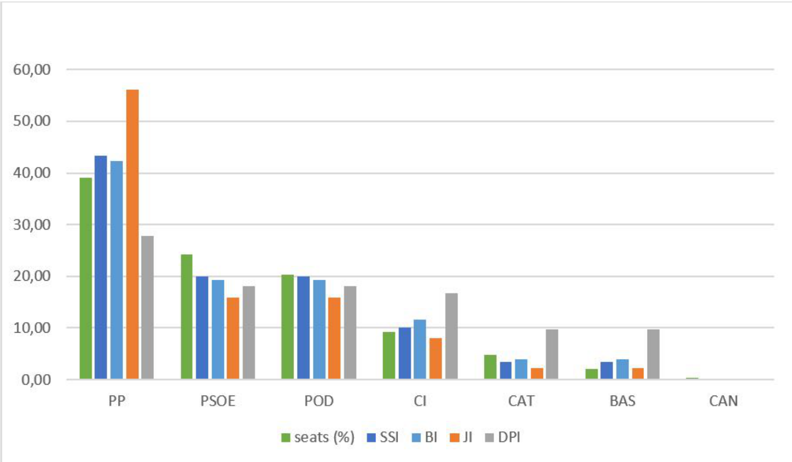

Fig. 3 *Graphic representation of data from Table 3*

From Tables 1, 2, 3, and corresponding figures, one can see that indices SSI, BI, JI are monotone functions with respect to the number of seats that each political group represents, but they are not a linear function of the seats. By Example 1 and Example 2, one can observe that DPI index is not monotone with respect to the number of seats that each political group has, since by DPI one takes into account only minimal winning coalitions.

3 Modified power indices

With respect to the winning coalitions, the power indices of political parties that are mentioned above do not take into account political nature of the parties. Furthermore, it is considered that winning coalitions which are taken into account, will have the same probability to be formed. However, the main factor for forming a winning coalition (i.e., a parliamentary majority) is usually political distance between parties.

So, another possible approach is to take into account different political features, and to look at a set of *politically feasible winning coalitions*. In such case, one should use possible combinations of parties, with respect to political or ideological distance between the parties. For example, if two parties are considered as competing and incompatible for a coalition, then all coalitions that involve both of these two parties must be excluded from consideration. In that case, one can observe only winning coalitions with possible allies.

Remark 1 There exist various measures of political and ideological distance between parties. For example, the Leiserson distance between two parties is based on the order of the different parties, from the left to the right political axis (see [1]). Then one can use a restricted set of a winning coalitions with respect to the distance threshold that parties in a winning coalition should satisfied.

Remark 2 In [1], a voting power index that takes into account political distance between parties is given by

$$mi(p_k) = \frac{n_k v_k s_k}{\sum_{l=1}^{n} n_l v_l s_l}, \tag{7}$$

where:

n_k is the number of distinct coalitions in the restricted set $\{C_1, \ldots, C_r\}$ to which party p_k belongs,
v_k is the sum of inverses of numbers of parties that belongs to the same coalitions as p_k,
s_k is the share of seats that party p_k has.

According to differences between parties and for the purpose of considering only feasible winning coalitions, we can modify a well-known indices of power in the following way.

Assume that m feasible winning coalitions $C_1 \subset X, \ldots, C_m \subset X$ have been determined, by means of expert. Then, using only the set C_j one can calculate a corresponding index of power of each party in that set C_j $(j = 1, \ldots, m)$. After that, modified index of power of the party p_k is obtained as the sum of the weighted indices of power of the party p_k, along those sets C_j, where the party p_k belongs (otherwise, if the party $p_k \notin C_i$, that contribution to the sum is zero).

We suppose that the weight w_j, is proportional to the inverse of the number of parties that belongs to the winning coalition C_j and it is given by

$$w_j = \frac{\frac{1}{|C_j|}}{\sum_{i=1}^{m} \frac{1}{|C_i|}}, \quad j = 1, \ldots, m. \tag{8}$$

Therefore, we obtain the following definitions of modified indices of voting power.

Definition 5 Let m feasible winning coalitions $C_1, \ldots, C_m$ be given. Using only the set C_j, let SSI_j be the Shapley-Shubik index in the set C_j of the player $p_k \in C_j$ (otherwise, $SSI_i(p_k) = 0$, if $p_k \notin C_i$). Then the modified Shapley-Shubik index (MSSI) of the player $p_k \in X$, $k = 1, \ldots, n$, is given by

$$MSS(p_k) = \sum_{j=1}^{m} w_j SSI_j(p_k). \tag{9}$$

Definition 6 Let m feasible winning coalitions $C_1, \ldots, C_m$ be given. Using only the set C_j, let BI_j be the Banzhaf index in the set C_j of the player $p_k \in C_j$ (otherwise, $BI_i(p_k) = 0$, if $p_k \notin C_i$). Then the modified Banzhaf index (MBI) of the player $p_k \in X$, $k = 1, \ldots, n$, is given by

$$MB(p_k) = \sum_{j=1}^{m} w_j BI_j(p_k). \tag{10}$$

Definition 7 Let m feasible winning coalitions $C_1, \ldots, C_m$ be given. Using only the set C_j, let JI_j be the Johnston index in the set C_j of the player $p_k \in C_j$ (otherwise, $JI_i(p_k) = 0$, if $p_k \notin C_i$). Then the modified Johnston index (MJI) of the player $p_k \in X$, $k = 1, \ldots, n$, is given by

$$MJ(p_k) = \sum_{j=1}^{m} w_j JI_j(p_k). \tag{11}$$

Definition 8 Let m feasible winning coalitions $C_1, \ldots, C_m$ be given. Using only the set C_j, let DPI_j be the Deegan-Packel index in the set C_j of the player $p_k \in C_j$ (otherwise, $DPI_i(p_k) = 0$, if $p_k \notin C_i$). Then the modified Deegan-Packel index (MDPI) of the player $p_k \in X$, $k = 1, \ldots, n$, is given by

$$MDP(p_k) = \sum_{j=1}^{m} w_j DPI_j(p_k). \tag{12}$$

From the above definitions, it is easy to see that the sum over all parties for a modified index is equal to 1, too. One can also express these indices in percents.

Now, we consider the modified indices of voting power in relation to a few parliaments and want to illustrate them by examples. Calculated value of each modified index of power is presented in the corresponding table, and in reasonable case accompanied with the graphic illustration. For the purpose of comparison, the voting power index (7) is taken into account, too. In relation to previous definitions, in the following examples we take $m = 2$.

Example 4 We use data as in Example 1, from results of elections for Croatian Parliament in 2015. There are $S = 151$ members of Croatian Parliament, so quota $q = 76$ represents the majority votes. Let us assume that there are two feasible winning coalitions:

I. feasible winning coalition:
nI={DK,Most,Minorities, BM365, HDSSB, RE}$\equiv\{59, 15, 8, 2, 2, 1\}$, $n_1 = 6$;
II. feasible winning coalition:
nII={HR,Most,Minorities, IDS, HRID, RE, ŽZ, NLSP}$\equiv\{56, 15, 8, 3, 3, 1, 1, 1\}$, $n_2 = 8$.

In Table 4, it is shown the number s_k of seats (i.e., members) that parties or political group have obtained in the Parliament ($k = 1, \ldots, n = 11$) and its percents. Furthermore, corresponding calculated modified indices of particular parties are given in percents, as well as the index (7).

In Table 4 one can see that the modified indices MSS, MB and MJ are not monotone functions with respect to the number of seats that each political group represents. These modified indices have the largest values for the political group Most (43.57%, 38.22%, 40.74%), which has only 9.93% of seats. It was in accordance with the real political situation, since Most had the crucial role in forming a winning coalition, i.e., in forming a parliamentary majority.

Table 4 *Composition of Croatian Parliament after elections in 2015, and corresponding modified power indices, and index* (7)

coalition (party)	DK	HR	Most	Minorities	HRID	IDS
s_k (%)	59 39.07	56 37.09	15 9.93	8 5.93	3 1.99	3 1.99
MSS (%)	25.71	17.86	43.57	4.56	1.33	1.33
MB (%)	23.53	14.69	38.22	8.26	2.45	2.45
MJ (%)	24.49	16.25	40.74	7.48	1.73	1.73
MDP (%)	19.05	9.55	28.60	8.13	4.55	4.55
mi by (7), (%)	29.14	20.74	25.93	13.83	2.96	2.96

coalition (party)	HDSSB	BM365	RE	ŽZ	NLSP
s_k (%)	2 1.32	2 1.32	1 0.66	1 0.66	1 0.66
MSS (%)	1.90	1.90	0.61	0.61	0.61
MB (%)	3.36	3.36	1.22	1.22	1.22
MJ (%)	2.72	2.72	0.71	0.71	0.71
MDP (%)	6.35	6.35	4.29	4.29	4.29
mi by (7), (%)	0.988	0.988	1.73	0.37	0.37

This above conclusion holds for the modified index MDP, too. However, the index MDP has got overlarge values for parties with very small number of seats. This observation and the graphic representation of values from Table 4 can be seen in Figure 4, too.

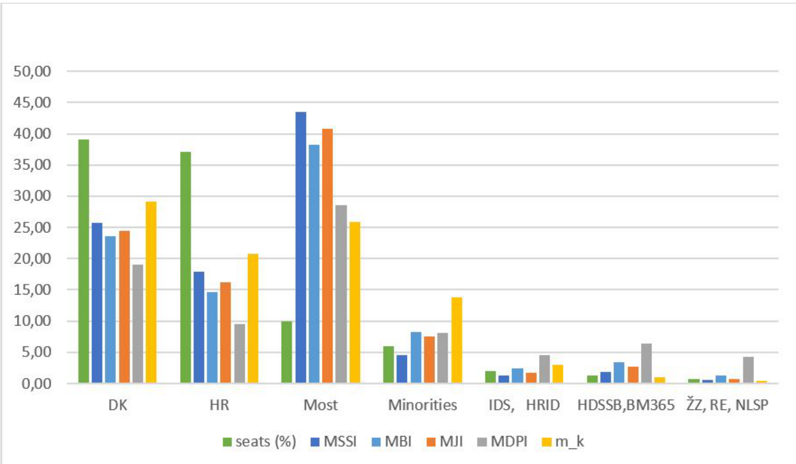

Fig. 4 *Graphic representation of data from Table 4*

Example 5 We use data as in Example 2 from the European Parliament, that consists of 751 members organized in $n = 9$ political groups. So quota $q = 376$ represents the majority votes.

Let us assume hypothetical situation, that there are two feasible winning coalitions (although they could be subject of change):

I. feasible winning coalition:
nI={EPP, ECR, EFDD, ENF, NA} $\equiv \{215, 74, 46, 39, 16\}$, $n_1 = 5$;
II. feasible winning coalition:
nII={S&D, ALDE, GUE-NGL, Greens-EFA, NA} $\equiv \{189, 70, 52, 50, 16\}$, $n_2 = 5$.

In Table 5, it is shown the number s_k of seats (i.e., members) that parties or political group have obtained in the Parliament ($k = 1, \dots, n = 11$) and its percents. Further, corresponding calculated modified indices of particular parties are given in percents, as well as the index (7).

Table 5 *Composition of European Parliament by political groups in 2015, and corresponding modified power indices, and index* (7)

political group	EPP	S&D	ECR	ALDE	GUE-NGL
s_k	215	189	74	70	52
(%)	28.63	25.17	9.85	9.32	6.92
MSS (%)	10	10	10	10	10
MB (%)	10	10	10	10	10
MJ (%)	10	10	10	10	10
MDP (%)	10	10	10	10	10
mi by (7), (%)	26.84	23.6	9.24	8.74	6.49

political group	Greens-EFA	EFDD	ENF	NA
s_k	50	46	39	16
(%)	6.66	6.12	5.19	2.13
MSS (%)	10	10	10	20
MB (%)	10	10	10	20
MJ (%)	10	10	10	20
MDP (%)	10	10	10	20
mi by (7), (%)	6.24	5.99	4.87	7.99

From Table 5, one can see that this is a special case, when it is assumed that feasible winning coalitions are two minimal coalitions with the equal number of parties ($n_1 = n_2 = 5$). So, modified indices have got the equal values (10%) for all those parties that are members of only one feasible winning coalition. Therefore, in this case we will exclude the graphic representation. However, the modified indices of the political group NA have the larger value (20.00%), because it is assumed that only the group NA takes part in both feasible winning coalitions.

Example 6 We use data as in Example 3, for Congress of Deputies in Spain after elections in June 2016, that consists of 350 members, which one could divide in $n = 7$ political groups. So quota $q = 176$ represents the majority votes. Corresponding number s_k of seats (i.e., members) that political groups have got in the Congress of Deputies ($k = 1, \dots, n = 9$) and its percents are shown in Table 6.

In Spain there is a problem of forming a parliamentary majority. In spite of that, let us assume hypothetical situation, that there are two feasible winning coalitions (although they do not correspond with real situation):

I. feasible winning coalition:
   nI={PP, CI, CAT, BAS, CAN} $\equiv \{137, 32, 17, 7, 1\}$, $n_1 = 5$,
II. feasible winning coalition:
   nII={PSOE, UP, CAT, BAS, CAN} $\equiv \{85, 71, 17, 7, 1\}$, $n_2 = 5$.

In Table 6, it is shown the number s_k of seats (i.e., members) that parties or political group have obtained in the Parliament ($k = 1, \ldots, n = 7$) and its percents. Further, corresponding calculated modified indices of particular parties are given in percents, as well as the index (7).

Table 6 *Composition of Congress of Deputies in Spain in 2016, and corresponding modified power indices, and index* (7)

party (group)	PP	PSOE	POD	CI	CAT	BAS	CAN
s_k	137	85	71	32	17	7	1
(%)	39.14	24.29	20.29	9.14	4.86	2	0.29
MSS (%)	20.83	12.5	12.5	20.83	16.67	16.67	0
MB (%)	18.75	12.5	12.5	18.75	18.75	18.75	0
MJ (%)	19.44	12.50	12.50	19.44	18.06	18.06	0
MDP (%)	16.67	12.50	12.50	16.67	20.83	20.83	0
mi by (7), (%)	32.23	20	16.71	7.53	16	6.59	0.94

From Table 6, one can observe that parties, which take part in both feasible winning coalitions, have proportionally greater modified indices in comparation to parties that take part only in one coalition. The graphic representation of corresponding values from Table 6 can be seen in Figure 5.

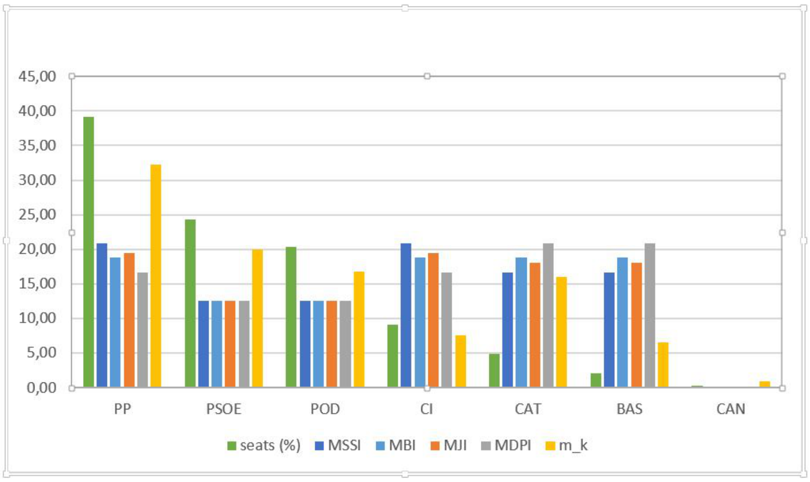

Fig. 5 *Graphic representation of data from Table 6*

4 Concluding remarks

Considering some well-known indices of political power (the Shapley-Shubik index, the Banzhaf index, the Johnston index, the Deegan-Packel index) in a political context of making some decisions in a parliament, one can observe that they use formal winning coalitions and do not take into account political distance between the parties. However, one can consider differences between parties and use only possible coalitions with respect to political features of parties.

In cases where parties have not got a majority votes, there appears the problem of forming a parliamentary majority. Then one should search for a stable governing coalition. In such a case one should consider those winning coalitions that consist of possible allies. In general, the parties with a *greater capability for coalition* will have a greater influence to form a winning coalition that represents a parliamentary majority. Therefore we give the modifications (9), (10), (11), and (12) of the mentioned indices, in order to consider a feasible winning coalitions. Corresponding modified indices are not monotone functions of number of seats that parties have got in a parliament. It is illustrated by a few examples, that the proposed modified indices well reflect a voting power of parties to form a winning coalition, i.e., to form a parliamentary majority.

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