Irreducibility criteria for the generalized principal series of unitary groups

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Abstract

We present an algebraic proof of the irreducibility criteria for the generalized principal series of unitary groups over a non-archimedean local field.

1 Introduction

The aim of this paper is to provide a uniform and simple irreducibility criteria for the induced representations of the form $\delta \rtimes \sigma$, where δ stands for an irreducible essentially square-integrable representation of the general linear group and σ stands for a discrete series representation of the unitary group over a non-archimedean local field. Induced representations of such a form are called the generalized principal series, and play an important role in the representation theory of reductive p-adic groups.

We note that reducibility of the generalized principal series of symplectic and odd orthogonal groups has been described in [8], in terms of the Mœglin-Tadić classification, using an approach mostly based on the intertwining operators method. On the other hand, we use purely algebraic methods and all our proofs are also valid in the symplectic and odd orthogonal group case, so the results of this paper can also be regarded as a shorter and algebraic version of [8].

The main strategy follows the one initiated in [8] and [4]: to prove the observed representation is irreducible we show that every irreducible subquotient is isomorphic to its Langlands quotient, and to prove the reducibility we

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construct an irreducible subquotient non-isomorphic to the Langlands quotient. Our approach is based on the calculation of the Jacquet modules of induced representations, and uses embeddings of discrete series provided in Sections 7 and 8 of [10]. An advantage of this approach is that one could expect to extend our results to the case of odd GSpin and metaplectic groups, as soon as one extends the discrete series classification there.

In the following section we present some preliminaries, and in the next three sections we provide a description of the reducibility for the generalized principal series, using a case-by-case consideration.

2 Preliminaries

Let F denote a non-archimedean local field and let F' a separable quadratic extension of F. Let us denote by θ the non-trivial F-automorphism of F'. We fix an anisotropic unitary space Y_0 over F' and consider the Witt tower of unitary spaces V_n based on Y_0.

If $\dim_{F'}(Y_0)$ is odd, for each $2n+1 \geqslant \dim_{F'}(Y_0)$ there is a unique space V_n in the Witt tower of dimension $2n+1$, and we denote the unitary group of this space by G_n.

If $\dim_{F'}(Y_0)$ is even, for each $2n \geqslant \dim_{F'}(Y_0)$ there is a unique space V_n in the Witt tower of dimension $2n$, and we denote the unitary group of this space by G_n.

We fix a minimal parabolic subgroup in G_n and consider standard parabolic subgroups with respect to this minimal parabolic subgroup. The Levi factors are naturally isomorphic to $GL(n_1, F') \times \cdots \times GL(n_k, F') \times G_{n'}$, where $GL(m, F')$ denotes the general linear group of rank m over F'. If δ_i is a representation of $GL(n_i, F')$, for $i = 1, 2, \ldots, k$, τ a representation of $G_{n'}$, and $M \cong GL(n_1, F') \times \cdots \times GL(n_k, F') \times G_{n'}$ we denote by $\delta_1 \times \cdots \times \delta_k \rtimes \tau$ the normalized parabolically induced representation $\mathrm{Ind}_M^{G_n}(\delta_1 \otimes \cdots \otimes \delta_k \otimes \tau)$. We use a similar notation to denote a parabolically induced representation of $GL(m, F')$.

By $\mathrm{Irr}(G_n)$ we denote the set of all irreducible admissible representations of G_n. Let $R(G_n)$ denote the Grothendieck group of admissible representations of finite length of G_n and define $R(G) = \oplus_{n \geqslant 0} R(G_n)$. In a similar way we define $\mathrm{Irr}(GL(n, F'))$ and $R(GL) = \oplus_{n \geqslant 0} R(GL(n, F'))$.

For $\sigma \in \mathrm{Irr}(G_n)$ and $1 \leqslant k \leqslant n'$, where n' denotes the Witt index of V_n, we denote by $r_{(k)}(\sigma)$ the normalized Jacquet module of σ with respect to the parabolic subgroup having the Levi subgroup isomorphic to $GL(k,F')\times G_{n-k}$. We identify $r_{(k)}(\sigma)$ with its semisimplification in $R(GL(k,F'))\otimes R(G_{n-k})$ and consider

$$\mu^*(\sigma)=1\otimes\sigma+\sum_{k=1}^{n'} r_{(k)}(\sigma)\in R(GL)\otimes R(G).$$

We denote by ν a composition of the determinant mapping with the normalized absolute value on F'. Let $\rho\in R(GL)$ denote an irreducible cuspidal representation. By a segment we mean a set of the form $[\rho,\nu^m\rho]:=\{\rho,\nu\rho,\dots,\nu^m\rho\}$, for a non-negative integer m. The induced representation $\rho\times\nu\rho\times\cdots\times\nu^m\rho$ has a unique irreducible subrepresentation, denoted by $\delta([\rho,\nu^m\rho])$, which is essentially square-integrable.

For an irreducible smooth representation $\pi\in R(GL)$, let $\check{\pi}$ denote the representation $g\mapsto\widetilde{\pi}(\theta(g))$, where $\widetilde{\pi}$ stands for the contragredient representation of π. The representation π is called F'/F-selfdual if $\pi\cong\check{\pi}$.

Note that, by the Mœglin-Tadić classification, if a twist by a character of the form ν^x, with $x\in\mathbb{R}$, of some irreducible unitarizable cuspidal representation $\rho\in R(GL)$ appears in the cuspidal support of a discrete series $\sigma\in R(G)$, then ρ is an F'/F-selfdual representation.

Let us recall the structural formula ([9] and [7, Section 15]).

Lemma 2.1. *Let $\rho\in R(GL)$ be an irreducible cuspidal representation and $k,l\in\mathbb{R}$ such that $k+l\in\mathbb{Z}_{\geqslant 0}$. Let $\sigma\in R(G)$ be an irreducible admissible representation. Write $\mu^*(\sigma)=\sum_{\pi,\sigma'}\pi\otimes\sigma'$. Then the following holds:*

$$\begin{aligned}\mu^*(\delta([\nu^{-k}\rho,\nu^l\rho])\rtimes\sigma) &= \sum_{i=-k-1}^{l}\sum_{j=i}^{l}\sum_{\pi,\sigma'}\delta([\nu^{-i}\check{\rho},\nu^k\check{\rho}])\times\delta([\nu^{j+1}\rho,\nu^l\rho])\times\pi\\ &\otimes\ \delta([\nu^{i+1}\rho,\nu^j\rho])\rtimes\sigma'.\end{aligned}$$

We omit $\delta([\nu^x\rho,\nu^y\rho])$ if $x>y$.

We use the subrepresentation version of the Langlands classification, and realize a non-tempered irreducible representation π of G_n as the unique irreducible (Langlands) subrepresentation of an induced representation of the form $\delta_1\times\delta_2\times\cdots\times\delta_k\rtimes\tau$, where τ is an irreducible tempered representation of some G_t, and $\delta_1,\delta_2,\dots,\delta_k\in R(GL)$ are irreducible essentially square-integrable representations such that $e(\delta_1)\leqslant e(\delta_2)\leqslant\cdots\leqslant e(\delta_k)<0$, where $e(\delta_i)$ is such that $\nu^{-e(\delta_i)}\delta_i$ is unitarizable. We write $\pi=L(\delta_1,\delta_2,\dots,\delta_k;\tau)$. We also use a similar classification for the general linear groups.

The following result ([3, Lemma 5.5]), whose proof carries directly to the unitary group case, is used several times in the paper.

Lemma 2.2. *Suppose that $\pi \in R(G_n)$ is an irreducible representation, λ an irreducible representation of the Levi subgroup M of G_n, and π is a subrepresentation of $Ind_M^{G_n}(\lambda)$. If $L > M$, then there is an irreducible subquotient ρ of $Ind_M^L(\lambda)$ such that π is a subrepresentation of $Ind_L^{G_n}(\rho)$.*

By the classification of discrete series ([5, 7]), which holds unconditionally due to [1], [6, Théorème 3.1.1] and [2, Theorem 7.8], a discrete series $\sigma \in G_n$ corresponds to an admissible triple which consists of the Jordan block, the partial cuspidal support, and the ϵ-function. For more details on these invariants we refer the reader to [7] and [8].

Through the paper we fix a discrete series σ, and we denote the corresponding admissible triple by $(\mathrm{Jord}, \sigma_{cusp}, \epsilon)$. For an irreducible F'/F-selfdual cuspidal representation ρ_1 of $GL(n_1, F')$ we write $\mathrm{Jord}_{\rho_1} = \{x : (x, \rho_1) \in \mathrm{Jord}\}$. If $\mathrm{Jord}_{\rho_1} \neq \emptyset$ and $x \in \mathrm{Jord}_{\rho_1}$, denote $x_- = \max\{y \in \mathrm{Jord}_{\rho_1} : y < x\}$, if it exists. Domain of the ϵ-function is a subset of $\mathrm{Jord} \cup (\mathrm{Jord} \times \mathrm{Jord})$, and to define the ϵ-function on the elements of $\mathrm{Jord} \times \mathrm{Jord}$, it is enough to define the ϵ-function on the elements of the form $((x_-, \rho_1), (x, \rho_1))$.

We fix an irreducible cuspidal representation $\rho \in R(GL)$, and determine the reducibility criterion for the induced representation of the form $\delta([\nu^x\rho, \nu^y\rho]) \rtimes \sigma$, $y - x \in \mathbb{Z}$ and $x + y \geqslant 0$, called the generalized principal series. We emphasize that in $R(G)$ holds $\delta([\nu^x\rho, \nu^y\rho]) \rtimes \sigma = \delta([\nu^{-y}\check{\rho}, \nu^{-x}\check{\rho}]) \rtimes \sigma$. Reducibility in the case $x = -y$ is an integral part of the discrete series classification, so we assume that $x + y > 0$. Also, it is rather well-know, and can be easily checked following the same lines as in the proof of Proposition 3.1, that $\delta([\nu^x\rho, \nu^y\rho]) \rtimes \sigma$ is irreducible if ρ is not F'/F-selfdual or if ρ is F'/F-selfdual but for α such that $\nu^\alpha\rho \rtimes \sigma_{cusp}$ reduces we have $x - \alpha \notin \mathbb{Z}$. Thus, through the paper we can also assume that ρ is F'/F-selfdual and $x - \alpha \in \mathbb{Z}$.

3 Non-positive case

Suppose that $0 \leqslant a \leqslant b$ and for α such that $\nu^\alpha\rho \rtimes \sigma_{cusp}$ reduces we have $a - \alpha \in \mathbb{Z}$. In this section we determine reducibility for $\delta([\nu^{-a}\rho, \nu^b\rho]) \rtimes \sigma$.

First we note that if $[2a + 1, 2b + 1] \cap \mathrm{Jord}_\rho(\sigma) = \emptyset$, by the classification of discrete series the induced representation $\delta([\nu^{-a}\rho, \nu^b\rho]) \rtimes \sigma$ contains two discrete series subrepresentations, so it reduces.

Proposition 3.1. *Suppose that* $\{2a+1, 2b+1\} \subseteq Jord_\rho(\sigma)$ *and for every* $x \in Jord_\rho(\sigma) \cap \langle 2a+1, 2b+1]$ *such that* x_- *is defined we have* $\epsilon((x_-, \rho), (x, \rho)) = -1$. *Then the induced representation* $\delta([\nu^{-a}\rho, \nu^b\rho]) \rtimes \sigma$ *is irreducible.*

Proof. Let us first show that there are no irreducible tempered subquotients. By the classification of discrete series, part of the cuspidal support of σ consisting of elements of the form $\nu^x\rho$, $x \in \mathbb{R}$, can be written as

$$(\bigcup_{i=1}^{k}[\nu^{-c_{2i-1}}\rho, \nu^{c_{2i}}\rho]) \bigcup (\bigcup_{j=1}^{l}[\nu^{\alpha-j+1}\rho, \nu^{d_j}\rho]),$$

where all k, l, c_i, d_j are non-negative, $c_i \neq d_j$ for all i, j, $c_i \neq c_j$ for $i \neq j$, $d_j > d_{j+1}$ for $j = 1, \dots, l-1$, $c_i > \alpha - l$ for all i, and $\alpha - l + 1 > 0$. Then we have $\mathrm{Jord}_\rho(\sigma) = \{2c_i + 1 : i = 1, \dots, 2k\} \cup \{2d_j + 1 : j = 1, \dots, l\} \cup \{2(\alpha - m) - 1 : m \in \mathbb{Z}, l \leqslant m, \alpha - m \geqslant 1\}$. From $2a+1, 2b+1 \in \mathrm{Jord}_\rho(\sigma)$ we obtain that $\delta([\nu^{-a}\rho, \nu^b\rho]) \rtimes \sigma$ does not have discrete series subquotients. Using similar cuspidal support considerations, we deduce that an irreducible tempered subquotient of $\delta([\nu^{-a}\rho, \nu^b\rho]) \rtimes \sigma$ is a subrepresentation of $\delta([\nu^{-b}\rho, \nu^b\rho]) \rtimes \pi$, for an irreducible representation π. This implies $\mu^*(\delta([\nu^{-a}\rho, \nu^b\rho]) \rtimes \sigma) \geqslant \delta([\nu^{-b}\rho, \nu^b\rho]) \otimes \pi$, and using the structural formula and the square-integrability of σ we deduce that $\mu^*(\sigma)$ contains an irreducible constituent of the form $\delta([\nu^{a+1}\rho, \nu^b\rho]) \otimes \pi'$. From [10, Proposition 7.2] follows $\epsilon(((2b+1)_-, \rho), (2b+1, \rho)) = 1$, a contradiction.

Now we determine the non-tempered irreducible subquotients. Every such irreducible subquotient is of the form $L(\delta([\nu^{x_1}\rho_1, \nu^{y_1}\rho_1]), \dots, \delta([\nu^{x_m}\rho_m, \nu^{y_m}\rho_m]); \tau)$ where $x_i + y_i < 0$ for $i = 1, \dots, m$, and $x_i + y_i \leqslant x_{i+1} + y_{i+1}$ for $i = 1, \dots, m-1$. Since $L(\delta([\nu^{x_1}\rho_1, \nu^{y_1}\rho_1]), \dots, \delta([\nu^{x_m}\rho_m, \nu^{y_m}\rho_m]); \tau)$ is a subrepresentation of

$$\delta([\nu^{x_1}\rho_1, \nu^{y_1}\rho_1]) \rtimes L(\delta([\nu^{x_2}\rho_2, \nu^{y_2}\rho_2]), \dots, \delta([\nu^{x_m}\rho_m, \nu^{y_m}\rho_m]); \tau),$$

it follows that $\mu^*(\delta([\nu^{-a}\rho, \nu^b\rho]) \rtimes \sigma)$ contains

$$\delta([\nu^{x_1}\rho_1, \nu^{y_1}\rho_1]) \otimes L(\delta([\nu^{x_2}\rho_2, \nu^{y_2}\rho_2]), \dots, \delta([\nu^{x_m}\rho_m, \nu^{y_m}\rho_m]); \tau).$$

We directly obtain $\rho_1 \cong \rho$, $-b \leqslant x_1$ and $a \leqslant y_1$. If $a < y_1$, it follows that $\mu^*(\sigma)$ contains an irreducible constituent of the form $\delta([\nu^{a+1}\rho, \nu^x\rho]) \otimes \pi'$, for $x < b$, contradicting [10, Proposition 7.2]. Thus, $L(\delta([\nu^{x_2}\rho_2, \nu^{y_2}\rho_2]), \dots, \delta([\nu^{x_m}\rho_m, \nu^{y_m}\rho_m]); \tau)$ is contained in $\delta([\nu^{-x_1+1}\rho, \nu^b\rho]) \rtimes \sigma$. If $m \geqslant 2$, in the same way we conclude that $y_2 = x_1$, which leads to $x_2 + y_2 \leqslant x_1 + y_1$, a contradiction. Thus, $m = 1$ and $\delta([\nu^{-x_1+1}\rho, \nu^b\rho]) \rtimes \sigma$ contains an irreducible tempered subquotient. In the same way as in the first part of the proof we deduce that this is possible only if $x_1 = -b$. Thus, $L(\delta([\nu^{-b}\rho, \nu^a\rho]); \sigma)$ is a unique irreducible subquotient of $\delta([\nu^{-a}\rho, \nu^b\rho]) \rtimes \sigma$, and it is well-known that it appears with multiplicity one. Thus, $\delta([\nu^{-a}\rho, \nu^b\rho]) \rtimes \sigma$ is irreducible. □

Lemma 3.2. *Suppose that* $[2a+1, 2b+1] \cap Jord_\rho(\sigma) \neq \emptyset$, *but* $\{2a+1, 2b+1\} \nsubseteq Jord_\rho(\sigma)$. *Then the induced representation* $\delta([\nu^{-a}\rho, \nu^b\rho]) \rtimes \sigma$ *reduces.*

Proof. Note that $L(\delta([\nu^{-b}\rho, \nu^a\rho]); \sigma)$ is contained in $\delta([\nu^{-a}\rho, \nu^b\rho]) \rtimes \sigma$. Let us first assume that $2a+1 \notin \mathrm{Jord}_\rho(\sigma)$. Let $x_m = \min([2a+1, 2b+1] \cap \mathrm{Jord}_\rho(\sigma))$. It is easy to see, using [10, Theorem 8.2], that there is a discrete series σ_1 such that σ is a unique irreducible subrepresentation of $\delta([\nu^{a+1}\rho, \nu^{\frac{x_m-1}{2}}\rho]) \rtimes \sigma_1$. Furthermore, σ is a unique irreducible subquotient of $\delta([\nu^{a+1}\rho, \nu^{\frac{x_m-1}{2}}\rho]) \rtimes \sigma_1$ which contains an irreducible constituent of the form $\delta([\nu^{a+1}\rho, \nu^{\frac{x_m-1}{2}}\rho]) \otimes \pi$ in the Jacquet module with respect to the appropriate parabolic subgroup. If $x_m \neq 2b+1$, note that $L(\delta([\nu^{-b}\rho, \nu^{\frac{x_m-1}{2}}\rho]); \sigma_1)$ is contained in

$$\delta([\nu^{-b}\rho, \nu^a\rho]) \times \delta([\nu^{a+1}\rho, \nu^{\frac{x_m-1}{2}}\rho]) \rtimes \sigma_1 = \delta([\nu^{-a}\rho, \nu^b\rho]) \times \delta([\nu^{a+1}\rho, \nu^{\frac{x_m-1}{2}}\rho]) \rtimes \sigma_1,$$

so there is an irreducible subquotient π_1 of $\delta([\nu^{a+1}\rho, \nu^{\frac{x_m-1}{2}}\rho]) \rtimes \sigma_1$ such that $L(\delta([\nu^{-b}\rho, \nu^{\frac{x_m-1}{2}}\rho]); \sigma_1)$ is contained in $\delta([\nu^{-a}\rho, \nu^b\rho]) \rtimes \pi_1$.

Obviously, $\mu^*(L(\delta([\nu^{-b}\rho, \nu^{\frac{x_m-1}{2}}\rho]); \sigma_1))$ contains an irreducible constituent of the form $\delta([\nu^{a+1}\rho, \nu^{\frac{x_m-1}{2}}\rho]) \otimes \pi$, so $\pi_1 \cong \sigma$. If $x_m = 2b+1$, in the same way we conclude that $\delta([\nu^{-a}\rho, \nu^b\rho]) \rtimes \sigma$ contains an irreducible tempered subrepresentation τ of $\delta([\nu^{-b}\rho, \nu^b\rho]) \rtimes \sigma_1$ such that $\mu^*(\tau)$ contains an irreducible constituent of the form $\delta([\nu^{a+1}\rho, \nu^b\rho]) \times \delta([\nu^{a+1}\rho, \nu^b\rho]) \otimes \pi$.

Let us now assume that $2b+1 \notin \mathrm{Jord}_\rho(\sigma)$, and let $x_M = \max([2a+1, 2b+1] \cap \mathrm{Jord}_\rho(\sigma))$. We denote by σ_2 a unique discrete series subrepresentation of $\delta([\nu^{\frac{x_M+1}{2}}\rho, \nu^b\rho]) \rtimes \sigma$. If $x_M > 2a+1$, then $L(\delta([\nu^{-\frac{x_M-1}{2}}\rho, \nu^a\rho]); \sigma_2)$ is contained in

$$\delta([\nu^{-a}\rho, \nu^{\frac{x_M-1}{2}}\rho]) \times \delta([\nu^{\frac{x_M+1}{2}}\rho, \nu^b\rho]) \rtimes \sigma,$$

so there is an irreducible subquotient π_2 of $\delta([\nu^{-a}\rho, \nu^{\frac{x_M-1}{2}}\rho]) \times \delta([\nu^{\frac{x_M+1}{2}}\rho, \nu^b\rho])$ such that $L(\delta([\nu^{-\frac{x_M-1}{2}}\rho, \nu^a\rho]); \sigma_2)$ is contained in $\pi_2 \rtimes \sigma$, and in the same way as in the previously considered case we deduce that $\pi_2 \cong \delta([\nu^{-a}\rho, \nu^b\rho])$. If $x_M = 2a+1$, following the same lines we obtain that $\delta([\nu^{-a}\rho, \nu^b\rho]) \rtimes \sigma$ contains an irreducible tempered subrepresentation τ of $\delta([\nu^{-a}\rho, \nu^a\rho]) \rtimes \sigma_2$ such that $\mu^*(\tau)$ contains an irreducible constituent of the form $\delta([\nu^{a+1}\rho, \nu^b\rho]) \otimes \pi$. $\square$

Lemma 3.3. *Suppose that $x \in Jord_\rho(\sigma)$, x_- is defined and we have $\epsilon((x_-, \rho), (x, \rho)) = 1$. Then the induced representation $\delta([\nu^{\frac{x_-+1}{2}}\rho, \nu^{\frac{x-1}{2}}\rho]) \rtimes \sigma$ has a unique irreducible tempered subrepresentation.*

Proof. From $\epsilon((x_-, \rho), (x, \rho)) = 1$ follows that there is a discrete series σ_1 such that σ is a subrepresentation of $\delta([\nu^{-\frac{x_--1}{2}}\rho, \nu^{\frac{x-1}{2}}\rho]) \rtimes \sigma_1$. By the classification of discrete series, $\delta([\nu^{-\frac{x_--1}{2}}\rho, \nu^{\frac{x-1}{2}}\rho]) \rtimes \sigma_1$ has two mutually non-isomorphic discrete series subrepresentations, and we denote by σ' a discrete series subrepresentation non-isomorphic to σ. The induced representation $\delta([\nu^{-\frac{x-1}{2}}\rho, \nu^{\frac{x-1}{2}}\rho]) \rtimes \sigma_1$ is a direct sum of two mutually non-isomorphic irreducible tempered representations τ_1 and τ_2. In a similar way as in the proof of Proposition 3.1 we deduce that $\delta([\nu^{\frac{x_-+1}{2}}\rho, \nu^{\frac{x-1}{2}}\rho]) \rtimes \sigma_1$ is irreducible, so for $i = 1, 2$ we have

$$\begin{aligned}\tau_i &\hookrightarrow \delta([\nu^{-\frac{x-1}{2}}\rho, \nu^{\frac{x-1}{2}}\rho]) \rtimes \sigma_1 \hookrightarrow \delta([\nu^{-\frac{x_--1}{2}}\rho, \nu^{\frac{x-1}{2}}\rho]) \times \delta([\nu^{-\frac{x-1}{2}}\rho, \nu^{-\frac{x_-+1}{2}}\rho]) \rtimes \sigma_1 \\ &\cong \delta([\nu^{-\frac{x_--1}{2}}\rho, \nu^{\frac{x-1}{2}}\rho]) \times \delta([\nu^{\frac{x_-+1}{2}}\rho, \nu^{\frac{x-1}{2}}\rho]) \rtimes \sigma_1 \\ &\cong \delta([\nu^{\frac{x_-+1}{2}}\rho, \nu^{\frac{x-1}{2}}\rho]) \times \delta([\nu^{-\frac{x_--1}{2}}\rho, \nu^{\frac{x-1}{2}}\rho]) \rtimes \sigma_1.\end{aligned}$$

Consequently, for $i = 1, 2$ there is an irreducible subquotient π_1 of $\delta([\nu^{-\frac{x_--1}{2}}\rho, \nu^{\frac{x-1}{2}}\rho]) \rtimes \sigma_1$ such that τ_i is a subrepresentation of $\delta([\nu^{\frac{x_-+1}{2}}\rho, \nu^{\frac{x-1}{2}}\rho]) \rtimes \pi_i$. Frobenius reciprocity implies that $\mu^*(\tau_i)$ contains an irreducible constituent of the form $\delta([\nu^{\frac{x_-+1}{2}}\rho, \nu^{\frac{x-1}{2}}\rho]) \times \delta([\nu^{\frac{x_-+1}{2}}\rho, \nu^{\frac{x-1}{2}}\rho]) \otimes \pi$, for $i = 1, 2$. It follows that $\mu^*(\pi_i)$ contains an irreducible constituent of the form $\delta([\nu^{\frac{x_-+1}{2}}\rho, \nu^{\frac{x-1}{2}}\rho]) \otimes \pi$, for $i = 1, 2$. From the classification of discrete series we conclude that $\mu^*(\delta([\nu^{-\frac{x_--1}{2}}\rho, \nu^{\frac{x-1}{2}}\rho]) \rtimes \sigma_1)$ does not contain an irreducible constituent of the form $\delta([\nu^{\frac{x_-+1}{2}}\rho, \nu^{\frac{x-1}{2}}\rho]) \times \delta([\nu^{\frac{x_-+1}{2}}\rho, \nu^{\frac{x-1}{2}}\rho]) \otimes \pi$, and it contains exactly two irreducible constituents of the form $\delta([\nu^{\frac{x_-+1}{2}}\rho, \nu^{\frac{x-1}{2}}\rho]) \otimes \pi$, each of them appearing with multiplicity one. Furthermore, both $\mu^*(\sigma)$ and $\mu^*(\sigma')$ contain a unique irreducible constituent of the form $\delta([\nu^{\frac{x_-+1}{2}}\rho, \nu^{\frac{x-1}{2}}\rho]) \otimes \pi$. Thus, $\{\pi_1, \pi_2\} = \{\sigma, \sigma'\}$ and there is a unique $i \in \{1, 2\}$ such that τ_i is a unique irreducible tempered subrepresentation of $\delta([\nu^{\frac{x_-+1}{2}}\rho, \nu^{\frac{x-1}{2}}\rho]) \rtimes \sigma$. $\square$

Lemma 3.4. *Suppose that $\{2a + 1, 2b + 1\} \subseteq Jord_\rho(\sigma)$ and there is an $x \in Jord_\rho(\sigma) \cap \langle 2a + 1, 2b + 1]$ such that x_- is defined and $\epsilon((x_-, \rho), (x, \rho)) = 1$. Then the induced representation $\delta([\nu^{-a}\rho, \nu^b\rho]) \rtimes \sigma$ reduces.*

Proof. Let us denote by y the minimal element of $\mathrm{Jord}_\rho(\sigma) \cap \langle 2a+1, 2b+1]$ such that y_- is defined and $\epsilon((y_-, \rho), (y, \rho)) = 1$. If $y_- = 2a+1$, by the classification of discrete series there is an irreducible tempered representation τ such that σ is a subrepresentation of $\delta([\nu^{a+1}\rho, \nu^{\frac{y-1}{2}}\rho]) \rtimes \tau$, and it can be seen in the same way as in the proof of Lemma 3.2 that $\delta([\nu^{-a}\rho, \nu^{b}\rho]) \rtimes \sigma$ contains $L(\delta([\nu^{-b}\rho, \nu^{\frac{y-1}{2}}\rho]); \tau)$ if $y < 2b+1$, or an irreducible tempered subrepresentation τ' of $\delta([\nu^{-b}\rho, \nu^{b}\rho]) \rtimes \tau$ such that $\mu^*(\tau')$ contains an irreducible constituent of the form $\delta([\nu^{a+1}\rho, \nu^{b}\rho]) \times \delta([\nu^{a+1}\rho, \nu^{b}\rho]) \otimes \pi$, if $y = 2b+1$. Let us assume that $y_- > 2a+1$. Then there is a discrete series σ_1 such that σ is a subrepresentation of $\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{\frac{y-1}{2}}\rho]) \rtimes \sigma_1$. Thus,

$$\sigma \hookrightarrow \delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{\frac{y-1}{2}}\rho]) \rtimes \sigma_1 \hookrightarrow \delta([\nu^{a+1}\rho, \nu^{\frac{y-1}{2}}\rho]) \times \delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{a}\rho]) \rtimes \sigma_1,$$

so there is an irreducible subquotient π_1 of $\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{a}\rho]) \rtimes \sigma_1$ such that σ is a subrepresentation of $\delta([\nu^{a+1}\rho, \nu^{\frac{y-1}{2}}\rho]) \rtimes \pi_1$. We claim that since $y_- \notin \mathrm{Jord}_\rho(\sigma_1)$, the induced representation $\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{a}\rho]) \rtimes \sigma_1$ reduces.

If $(y_-)_- = 2a+1$, it follows from the proof of Lemma 3.2 that $\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{a}\rho]) \rtimes \sigma_1$ contains an irreducible tempered subquotient, and in the same way as in the proof of Proposition 3.1 we obtain that $L(\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{a}\rho]); \sigma_1)$ is its unique irreducible non-tempered subquotient.

If $(y_-)_- > 2a+1$, in the same way we conclude that $\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{a}\rho]) \rtimes \sigma_1$ contains exactly two mutually non-isomorphic non-tempered subquotients. Also, it does not contain an irreducible tempered subquotient, since such a subquotient would be a subrepresentation of an induced representation of the form $\delta([\nu^{-a}\rho, \nu^{a}\rho]) \rtimes \sigma_{ds}$, for a discrete series σ_{ds}, and the structural formula implies that σ_{ds} is contained in $\delta([\nu^{a+1}\rho, \nu^{\frac{y_--1}{2}}\rho]) \rtimes \sigma_1$. This gives $\mathrm{Jord}_\rho(\sigma_{ds}) = \mathrm{Jord}_\rho(\sigma_1) \cup \{y_-\} \setminus \{2a+1\}$, so $\mu^*(\sigma_{ds})$ contains an irreducible constituent of the form $\delta([\nu^{a+1}\rho, \nu^{\frac{z-1}{2}}\rho]) \otimes \pi$, for $z \in \mathrm{Jord}_\rho(\sigma)$ such that $z_- = 2a+1$, but $\mu^*(\delta([\nu^{a+1}\rho, \nu^{\frac{y_--1}{2}}\rho]) \rtimes \sigma_1)$ does not contain an irreducible constituent of such a form, since $\epsilon((2a+1, \rho), (z, \rho)) = -1$ by the minimality of y.

It can be seen, using an easy Jacquet module calculation, that $\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{a}\rho]) \rtimes \sigma_1$ is a length two representation, so an irreducible subquotient non-isomorphic to $L(\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{a}\rho]); \sigma_1)$ is a subrepresentation of $\delta([\nu^{-a}\rho, \nu^{\frac{y_--1}{2}}\rho]) \rtimes \sigma_1$, and we denote it by π'.

If $\pi_1 \cong \pi'$, we have

$$\begin{aligned}
\sigma &\hookrightarrow \delta([\nu^{a+1}\rho, \nu^{\frac{y-1}{2}}\rho]) \times \delta([\nu^{-a}\rho, \nu^{\frac{y_--1}{2}}\rho]) \rtimes \sigma_1 \\
&\hookrightarrow \delta([\nu^{a+1}\rho, \nu^{\frac{y-1}{2}}\rho]) \times \delta([\nu^{a+1}\rho, \nu^{\frac{y_--1}{2}}\rho]) \times \delta([\nu^{-a}\rho, \nu^{a}\rho]) \rtimes \sigma_1 \\
&\cong \delta([\nu^{a+1}\rho, \nu^{\frac{y_--1}{2}}\rho]) \times \delta([\nu^{a+1}\rho, \nu^{\frac{y-1}{2}}\rho]) \times \delta([\nu^{-a}\rho, \nu^{a}\rho]) \rtimes \sigma_1,
\end{aligned}$$

and $\epsilon(((y_-)_-, \rho), (y_-, \rho)) = 1$, contradicting the minimality of y. It follows that $\pi_1 \cong L(\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{a}\rho]); \sigma_1)$.

Since the induced representation $\delta([\nu^{-a}\rho, \nu^{a}\rho]) \rtimes \sigma_1$ is irreducible, using the structural formula and definition of σ_1 we conclude that

$$\delta([\nu^{a+1}\rho, \nu^{\frac{y_--1}{2}}\rho]) \otimes \delta([\nu^{-a}\rho, \nu^{a}\rho]) \rtimes \sigma_1$$

is a unique irreducible constituent of the form $\delta([\nu^{a+1}\rho, \nu^{\frac{y_--1}{2}}\rho]) \otimes \pi$ appearing in $\mu^*(\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{a}\rho]) \rtimes \sigma_1)$, and it obviously appears in $\mu^*(\pi')$, so it does not appear in $\mu^*(L(\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{a}\rho]); \sigma_1))$. It can now be easily seen, using the structural formula, that $\delta([\nu^{a+1}\rho, \nu^{\frac{y-1}{2}}\rho]) \otimes \delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{a}\rho]) \otimes \sigma_1$ is a unique irreducible constituent of the form $\delta([\nu^{a+1}\rho, \nu^{\frac{y-1}{2}}\rho]) \otimes \delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{a}\rho]) \otimes \pi$ appearing in the Jacquet module of $\delta([\nu^{a+1}\rho, \nu^{\frac{y-1}{2}}\rho]) \rtimes L(\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{a}\rho]); \sigma_1)$ with respect to the appropriate parabolic subgroup, and appears with multiplicity one. Also, it has to appear in the Jacquet module of σ with respect to the appropriate parabolic subgroup.

Suppose that $y < 2b + 1$. We have

$$\begin{gathered}
L(\delta([\nu^{-b}\rho, \nu^{\frac{y-1}{2}}\rho]), \delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{a}\rho]); \sigma_1) \leqslant \\
\delta([\nu^{-b}\rho, \nu^{\frac{y-1}{2}}\rho]) \rtimes L(\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{a}\rho]); \sigma_1) \leqslant \\
\delta([\nu^{-a}\rho, \nu^{b}\rho]) \times \delta([\nu^{a+1}\rho, \nu^{\frac{y-1}{2}}\rho]) \rtimes L(\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{a}\rho]); \sigma_1),
\end{gathered}$$

so there is an irreducible subquotient π_2 of $\delta([\nu^{a+1}\rho, \nu^{\frac{y-1}{2}}\rho]) \rtimes L(\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{a}\rho]); \sigma_1)$ such that $L(\delta([\nu^{-b}\rho, \nu^{\frac{y-1}{2}}\rho]), \delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{a}\rho]); \sigma_1)$ is contained in $\delta([\nu^{-a}\rho, \nu^{b}\rho]) \rtimes \pi_2$. Since the Jacquet module of $L(\delta([\nu^{-b}\rho, \nu^{\frac{y-1}{2}}\rho]), \delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{a}\rho]); \sigma_1)$ with respect to the appropriate parabolic subgroup contains

$$\delta([\nu^{-b}\rho, \nu^{\frac{y-1}{2}}\rho]) \otimes \delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{a}\rho]) \otimes \sigma_1,$$

it follows from the structural formula that the Jacquet module of π_2 with respect to the appropriate parabolic subgroup contains $\delta([\nu^{a+1}\rho, \nu^{\frac{y-1}{2}}\rho]) \otimes \delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{a}\rho]) \otimes \sigma_1$, so $\pi_2 \cong \sigma$.

It remains to discuss the case $y = 2b+1$. Let us denote by τ a unique irreducible tempered subrepresentation of $\delta([\nu^{\frac{y_-+1}{2}}\rho, \nu^b\rho]) \rtimes \sigma_1$, given by Lemma 3.3. Note that we have

$$\begin{aligned} L(\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^a\rho]); \tau) &\hookrightarrow \delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^a\rho]) \times \delta([\nu^{-b}\rho, \nu^b\rho]) \rtimes \sigma_1 \\ &\cong \delta([\nu^{-b}\rho, \nu^b\rho]) \times \delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^a\rho]) \rtimes \sigma_1, \end{aligned}$$

and, by Lemma 2.2, there is an irreducible subquotient π_3 of the representation $\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^a\rho]) \rtimes \sigma_1$ such that $L(\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^a\rho]); \tau)$ is a subrepresentation of $\delta([\nu^{-b}\rho, \nu^b\rho]) \rtimes \pi_3$. Since $\mu^*(L(\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^a\rho]); \tau))$ contains an irreducible constituent of the form $\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^a\rho]) \otimes \pi$, it follows that $\pi_3 \cong L(\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^a\rho]); \sigma_1)$. Thus,

$$\begin{aligned} L(\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^a\rho]); \tau) &\hookrightarrow \delta([\nu^{-b}\rho, \nu^b\rho]) \rtimes L(\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^a\rho]); \sigma_1) \\ &\leqslant \delta([\nu^{-a}\rho, \nu^b\rho]) \times \delta([\nu^{a+1}\rho, \nu^b\rho]) \rtimes L(\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^a\rho]); \sigma_1), \end{aligned}$$

and in the same way as in the previously considered case we obtain

$$L(\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^a\rho]); \tau) \leqslant \delta([\nu^{-a}\rho, \nu^b\rho]) \rtimes \sigma.$$

□

Results obtained in this section lead to

Theorem 3.5. *Suppose that $0 \leqslant a \leqslant b$ and for α such that $\nu^\alpha\rho \rtimes \sigma_{cusp}$ reduces we have $a - \alpha \in \mathbb{Z}$. The induced representation $\delta([\nu^{-a}\rho, \nu^b\rho]) \rtimes \sigma$ is irreducible if and only if $\{2a+1, 2b+1\} \subseteq Jord_\rho(\sigma)$ and for every $x \in Jord_\rho(\sigma) \cap \langle 2a+1, 2b+1]$ such that x_- is defined we have $\epsilon((x_-, \rho), (x, \rho)) = -1$.*

4 Case $\frac{1}{2}$

Suppose that $\frac{1}{2} \leqslant b$, $b - \frac{1}{2}$ is a non-negative integer, and for α such that $\nu^\alpha\rho \rtimes \sigma_{cusp}$ reduces we have $\alpha - \frac{1}{2} \in \mathbb{Z}$. In this section we determine reducibility for $\delta([\nu^{\frac{1}{2}}\rho, \nu^b\rho]) \rtimes \sigma$.

Proposition 4.1. *Suppose that $2b+1 \in Jord_\rho(\sigma)$, $\epsilon(\min(Jord_\rho(\sigma)), \rho) = -1$, and for $x \in Jord_\rho(\sigma) \cap \langle \min(Jord_\rho(\sigma)), 2b+1]$ such that x_- is defined we have $\epsilon((x_-, \rho), (x, \rho)) = -1$. Then the induced representation $\delta([\nu^{\frac{1}{2}}\rho, \nu^b\rho]) \rtimes \sigma$ is irreducible.*

Proof. Note that $\mu^*(\sigma)$ does not contain an irreducible constituent of the form $\delta([\nu^x\rho, \nu^y\rho]) \otimes \pi$ for $x \leqslant \frac{1}{2}$ and $y \in \mathrm{Jord}_\rho(\sigma) \cap [\min(\mathrm{Jord}_\rho(\sigma)), 2b+1]$. Now the proof follows same lines as in the one of Proposition 3.1. □

The following result is used several times and might be of independent interest.

Lemma 4.2. *Let $\sigma' \in G_{n'}$ denote a discrete series and let us denote the corresponding admissible triple by $(Jord', \sigma'_{cusp}, \epsilon')$. We denote the domain of ϵ' by D. Suppose that for $\rho' \in R(GL)$ we have $Jord'_{\rho'} \neq \emptyset$, $2c+1 = \min(Jord'_{\rho'})$ is even, and $\epsilon'(2c+1, \rho') = 1$. Let $D' = D \backslash (\{(2c+1, \rho')\} \cup \{((2c+1, \rho'), (x, \rho')), ((x, \rho'), (2c+1, \rho')) : x \in Jord'_\rho, x \neq 2c+1\})$. Then σ' is a subrepresentation of $\delta([\nu^{\frac{1}{2}}\rho, \nu^c\rho]) \rtimes \sigma''$, for a discrete series σ'' corresponding to the admissible triple $(Jord' \backslash \{(2c+1, \rho')\}, \sigma'_{cusp}, \epsilon'')$, where $\epsilon'' : D' \to \{\pm 1\}$ is defined by:*

- *$\epsilon''(x, \rho_1) = \epsilon'(x, \rho_1)$ for $(x, \rho_1) \in D'$ such that $(x_-, \rho_1) \neq (2c+1, \rho')$,*
- *$\epsilon''((x_-, \rho_1), (x, \rho_1)) = \epsilon'((x_-, \rho_1), (x, \rho_1))$ for $((x_-, \rho_1), (x, \rho_1)) \in D'$,*
- *If $Jord'_{\rho'} \neq \{2c+1\}$ and $x_{\min} \in Jord'_{\rho'}$ is such that $(x_{\min})_- = 2c+1$, then $\epsilon''(x_{\min}, \rho') = \epsilon'((2c+1, \rho'), (x_{\min}, \rho'))$.*

Proof. It can be easily verified that $(\mathrm{Jord}' \backslash \{(2c+1, \rho')\}, \sigma'_{cusp}, \epsilon'')$ is an admissible triple. From $\epsilon'(2c+1, \rho') = 1$ follows that there is an irreducible representation π_1 such that σ' is a subrepresentation of $\delta([\nu^{\frac{1}{2}}\rho', \nu^c\rho']) \rtimes \pi_1$. Suppose that π_1 is not a discrete series representation. Then there is an embedding $\pi_1 \hookrightarrow \delta([\nu^{y_1}\rho_1, \nu^{y_2}\rho_1]) \rtimes \pi_2$, where $y_1 + y_2 \leqslant 0$, which leads to $\sigma' \hookrightarrow \delta([\nu^{\frac{1}{2}}\rho', \nu^c\rho']) \times \delta([\nu^{y_1}\rho_1, \nu^{y_2}\rho_1]) \rtimes \pi_2$. If $\delta([\nu^{\frac{1}{2}}\rho', \nu^c\rho']) \times \delta([\nu^{y_1}\rho_1, \nu^{y_2}\rho_1])$ is irreducible or $(y_2, \rho_1) = (-\frac{1}{2}, \rho')$, this immediately contradicts the square-integrability of σ'. If $\rho_1 \cong \rho'$ and $\frac{1}{2} \leqslant y_2 < c$, we have an embedding $\sigma' \hookrightarrow \nu^{y_2}\rho' \times \delta([\nu^{\frac{1}{2}}\rho', \nu^c\rho']) \times \delta([\nu^{y_1}\rho', \nu^{y_2-1}\rho']) \rtimes \pi_2$, which implies $2y_2 + 1 \in \mathrm{Jord}_{\rho'}(\sigma')$, a contradiction.

Thus, π_1 is a discrete series representation and we denote the corresponding admissible triple by $(\mathrm{Jord}(\pi_1), \sigma''_{cusp}, \epsilon_{\pi_1})$. Obviously, $\mathrm{Jord}(\pi_1) = \mathrm{Jord}(\sigma'')$ and $\sigma''_{cusp} \cong \sigma'_{cusp}$. To prove that $\pi_1 \cong \sigma''$ we only consider the case $\mathrm{Jord}_{\rho'} \neq \{2c+1\}$ and show $\epsilon_{\pi_1}(x_{\min}, \rho') = \epsilon''(x_{\min}, \rho')$, other properties can be obtained from the embedding $\sigma' \hookrightarrow \delta([\nu^{\frac{1}{2}}\rho', \nu^c\rho']) \rtimes \pi_1$ and definition of σ'', in the same way as in [10, Section 8].

If $\epsilon'((2c+1,\rho'),(x_{\min},\rho'))=1$, $\mu^*(\sigma')$ contains an irreducible constituent of the form $\delta([\nu^{-c}\rho',\nu^{\frac{x_{\min}-1}{2}}\rho'])\otimes\pi$. Using the structural formula we obtain that $\mu^*(\pi_1)$ contains an irreducible constituent of the form $\delta([\nu^{\frac{1}{2}}\rho',\nu^{\frac{x_{\min}-1}{2}}\rho'])\otimes\pi$, and from [10, Proposition 7.4] we get $\epsilon_{\pi_1}(x_{\min},\rho')=1$.

On the other hand, if $\epsilon_{\pi_1}(x_{\min},\rho')=1$ then π_1 is a subrepresentation of an induced representation of the form $\delta([\nu^{\frac{1}{2}}\rho',\nu^{\frac{x_{\min}-1}{2}}\rho'])\rtimes\pi_2$, and σ' is a subrepresentation of $\delta([\nu^{\frac{1}{2}}\rho',\nu^{\frac{x_{\min}-1}{2}}\rho'])\times\delta([\nu^{\frac{1}{2}}\rho',\nu^{c}\rho'])\rtimes\pi_2$. Now [10, Proposition 7.1] implies $\epsilon'((2c+1,\rho'),(x_{\min},\rho'))=1$. Consequently, $\epsilon'((2c+1,\rho'),(x_{\min},\rho'))=-1$ implies $\epsilon_{\pi_1}(x_{\min},\rho')=-1$ and we have $\epsilon_{\pi_1}(x_{\min},\rho')=\epsilon'(x_{\min},\rho')$. Thus, $\pi_1\cong\sigma''$. □

Lemma 4.3. *Suppose that* $2b+1\notin Jord_\rho(\sigma)$. *Then the induced representation* $\delta([\nu^{\frac{1}{2}}\rho,\nu^{b}\rho])\rtimes\sigma$ *reduces.*

Proof. If $\mathrm{Jord}_\rho(\sigma)\neq\emptyset$ and $\min(\mathrm{Jord}_\rho(\sigma))<2b+1$, the lemma follows in the same way as Lemma 3.2. In other cases, the previous lemma can be used to obtain a discrete series subrepresentation of $\delta([\nu^{\frac{1}{2}}\rho,\nu^{b}\rho])\rtimes\sigma$, so $\delta([\nu^{\frac{1}{2}}\rho,\nu^{b}\rho])\rtimes\sigma$ reduces. □

Lemma 4.4. *Suppose that* $\epsilon(\min(Jord_\rho(\sigma)),\rho)=1$. *Then the induced representation* $\delta([\nu^{\frac{1}{2}}\rho,\nu^{b}\rho])\rtimes\sigma$ *reduces.*

Proof. By the previous lemma, it is enough to consider the case $2b+1\in\mathrm{Jord}_\rho(\sigma)$. Again we denote $\min(\mathrm{Jord}_\rho(\sigma))$ by $x_{\min}$. By Lemma 4.2, there is a discrete series σ' such that σ is a subrepresentation of $\delta([\nu^{\frac{1}{2}}\rho,\nu^{x_{\min}}\rho])\rtimes\sigma'$. Also, it follows from the structural formula that σ is a unique irreducible subquotient of $\delta([\nu^{\frac{1}{2}}\rho,\nu^{x_{\min}}\rho])\rtimes\sigma'$ which contains an irreducible constituent of the form $\delta([\nu^{\frac{1}{2}}\rho,\nu^{x_{\min}}\rho])\otimes\pi$ in the Jacquet module with respect to the appropriate parabolic subgroup.

If $x_{\min}<b$, in the same way as in the proof of Lemma 3.2 we obtain that $L(\delta([\nu^{-b}\rho,\nu^{\frac{x_{\min}-1}{2}}\rho]);\sigma')$ is a subquotient of $\delta([\nu^{\frac{1}{2}}\rho,\nu^{b}\rho])\rtimes\sigma$.

Suppose that $x_{\min}=b$, and denote by τ an irreducible tempered subrepresentation of $\delta([\nu^{-b}\rho,\nu^{b}\rho])\rtimes\sigma'$ such that $\mu^*(\tau)$ contains $\delta([\nu^{\frac{1}{2}}\rho,\nu^{b}\rho])\times\delta([\nu^{\frac{1}{2}}\rho,\nu^{b}\rho])\otimes\sigma'$. Since τ is contained in $\delta([\nu^{\frac{1}{2}}\rho,\nu^{b}\rho])\times\delta([\nu^{-b}\rho,\nu^{-\frac{1}{2}}\rho])\rtimes\sigma'$, there is an irreducible subquotient π_1 of $\delta([\nu^{-b}\rho,\nu^{-\frac{1}{2}}\rho])\rtimes\sigma'$ such that τ is contained in $\delta([\nu^{\frac{1}{2}}\rho,\nu^{b}\rho])\rtimes\pi_1$. By the definition of τ, $\mu^*(\pi_1)$ has to contain $\delta([\nu^{\frac{1}{2}}\rho,\nu^{x_{\min}}\rho])\otimes\sigma'$, so $\pi_1\cong\sigma$. □

Lemma 4.5. *Suppose that $2b+1 \in Jord_\rho(\sigma)$ and $\epsilon(\min(Jord_\rho(\sigma)), \rho) = -1$. If there is an $x \in Jord_\rho(\sigma) \cap \langle \min(Jord_\rho(\sigma)), 2b+1]$ such that x_- is defined and $\epsilon((x_-, \rho), (x, \rho)) = 1$, then the induced representation $\delta([\nu^{\frac{1}{2}}\rho, \nu^b\rho]) \rtimes \sigma$ reduces.*

Proof. Let us denote by y the minimal element of $\mathrm{Jord}_\rho(\sigma) \cap \langle \min(\mathrm{Jord}_\rho(\sigma)), 2b+1]$ such that y_- is defined and $\epsilon((y_-, \rho), (y, \rho)) = 1$. Then there is a discrete series σ_1 such that σ is a subrepresentation of $\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{\frac{y-1}{2}}\rho]) \rtimes \sigma_1$, and an irreducible subquotient π_1 of $\delta([\nu^{\frac{1}{2}}\rho, \nu^{\frac{y_--1}{2}}\rho]) \rtimes \sigma_1$ such that σ is a subrepresentation of $\delta([\nu^{\frac{1}{2}}\rho, \nu^{\frac{y-1}{2}}\rho]) \rtimes \pi_1$.

If π_1 is tempered, since $\mu^*(\delta([\nu^{\frac{1}{2}}\rho, \nu^{\frac{y-1}{2}}\rho]) \rtimes \pi_1)$ contains $\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{\frac{y-1}{2}}\rho]) \otimes \sigma_1$, it follows that $\mu^*(\pi_1)$ has to contain an irreducible constituent of the form $\delta([\nu^{\frac{1}{2}}\rho, \nu^{\frac{y_--1}{2}}\rho]) \otimes \pi$. If $y_- = \min(\mathrm{Jord}_\rho(\sigma))$, it follows that π_1 is a subrepresentation of an induced representation of the form $\delta([\nu^{\frac{1}{2}}\rho, \nu^{\frac{y_--1}{2}}\rho]) \rtimes \pi$. Otherwise, it follows that π_1 is a subrepresentation of an induced representation of the form $\delta([\nu^{-\frac{(y_-)_--1}{2}}\rho, \nu^{\frac{y_--1}{2}}\rho]) \rtimes \pi$ and, consequently, there is an irreducible subquotient π' of $\delta([\nu^{-\frac{(y_-)_--1}{2}}\rho, \nu^{\frac{(y_-)_--1}{2}}\rho]) \rtimes \pi$ such that π_1 is a subrepresentation of $\delta([\nu^{\frac{(y_-)_-+1}{2}}\rho, \nu^{\frac{y_--1}{2}}\rho]) \rtimes \pi'$. In any case, using the embedding $\sigma \hookrightarrow \delta([\nu^{\frac{1}{2}}\rho, \nu^{\frac{y-1}{2}}\rho]) \rtimes \pi_1$ we get a contradiction with the description of the ϵ-function of σ. Consequently, π_1 is non-tempered.

If $y_- = \min(\mathrm{Jord}_\rho(\sigma))$, in the same way as in the proof of Proposition 3.1 we obtain that $L(\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{-\frac{1}{2}}\rho]); \sigma_1)$ is a unique irreducible non-tempered subquotient of $\delta([\nu^{\frac{1}{2}}\rho, \nu^{\frac{y_--1}{2}}\rho]) \rtimes \sigma_1$. If $y_- > \min(\mathrm{Jord}_\rho(\sigma))$, in the same way as in the previous section we conclude that an irreducible non-tempered subquotient of $\delta([\nu^{\frac{1}{2}}\rho, \nu^{\frac{y_--1}{2}}\rho]) \rtimes \sigma_1$ is either isomorphic to $L(\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{-\frac{1}{2}}\rho]); \sigma_1)$ or to $L(\delta([\nu^{-\frac{(y_-)_--1}{2}}\rho, \nu^{-\frac{1}{2}}\rho]); \sigma_2)$, where σ_2 is a unique discrete series subrepresentation of $\delta([\nu^{\frac{(y_-)_-+1}{2}}\rho, \nu^{\frac{y_--1}{2}}\rho]) \rtimes \sigma_1$. If

$$\pi_1 \cong L(\delta([\nu^{-\frac{(y_-)_--1}{2}}\rho, \nu^{-\frac{1}{2}}\rho]); \sigma_2),$$

we get that σ is a subrepresentation of an induced representation of the form $\delta([\nu^{\frac{(y_-)_-+1}{2}}\rho, \nu^{\frac{y_--1}{2}}\rho]) \rtimes \pi$, which contradicts the minimality of y.

Thus, $\pi_1 \cong L(\delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{-\frac{1}{2}}\rho]); \sigma_1)$ and the rest of the proof follows in the same way as in the proof of Lemma 3.4. $\square$

Let us now summarize the results of this section.

Theorem 4.6. *Suppose that* $\frac{1}{2} \leqslant b$*,* $b - \frac{1}{2}$ *is a non-negative integer, and for* α *such that* $\nu^\alpha \rho \rtimes \sigma_{cusp}$ *reduces we have* $\alpha - \frac{1}{2} \in \mathbb{Z}$*. The induced representation* $\delta([\nu^{\frac{1}{2}}\rho, \nu^b\rho]) \rtimes \sigma$ *is irreducible if and only if* $2b+1 \in Jord_\rho(\sigma)$*,* $\epsilon(\min(Jord_\rho(\sigma)), \rho) = -1$*, and for every* $x \in Jord_\rho(\sigma) \cap [\min(Jord_\rho(\sigma)), 2b+1]$ *such that* x_- *is defined we have* $\epsilon((x_-, \rho), (x, \rho)) = -1$*.*

5 The remaining case

In this section we determine reducibility for $\delta([\nu^a\rho, \nu^b\rho]) \rtimes \sigma$, where $1 \leqslant a \leqslant b$ and for α such that $\nu^\alpha \rho \rtimes \sigma_{cusp}$ reduces we have $a - \alpha \in \mathbb{Z}$.

The following result can be proved in the same way as Proposition 3.1.

Proposition 5.1. *Suppose that one of the following holds:*

1. $[2a-1, 2b+1] \cap Jord_\rho(\sigma) = \varnothing$*,*
2. $2b+1 \in Jord_\rho(\sigma)$ *and for every* $x \in [2a+1, 2b+1] \cap Jord_\rho(\sigma)$ *such that* x_- *is defined and* $x_- \geqslant 2a-1$ *we have* $\epsilon((x_-, \rho), (x, \rho)) = -1$*.*

Then the induced representation $\delta([\nu^a\rho, \nu^b\rho]) \rtimes \sigma$ *is irreducible.*

Lemma 5.2. *Suppose that* $2b+1 \notin Jord_\rho(\sigma)$ *and* $[2a-1, 2b+1\rangle \cap Jord_\rho(\sigma) \neq \varnothing$*. Then the induced representation* $\delta([\nu^a\rho, \nu^b\rho]) \rtimes \sigma$ *reduces.*

Proof. Let us write $x = \max([2a-1, 2b+1\rangle \cap \mathrm{Jord}_\rho(\sigma))$. Following the same lines as in the proof of Lemma 3.2, we deduce that $\delta([\nu^a\rho, \nu^b\rho]) \rtimes \sigma$ contains $L(\delta([\nu^{-\frac{x-1}{2}}\rho, \nu^{-a}\rho]); \sigma_1)$, where σ_1 is a unique discrete series subrepresentation of $\delta([\nu^{\frac{x+1}{2}}\rho, \nu^b\rho]) \rtimes \sigma$. □

Lemma 5.3. *Suppose that* $2b+1 \in Jord_\rho(\sigma)$ *and there is an* $x \in [2a+1, 2b+1] \cap Jord_\rho(\sigma)$ *such that* x_- *is defined,* $x_- \geqslant 2a-1$*, and we have* $\epsilon((x_-, \rho), (x, \rho)) = 1$*. Then the induced representation* $\delta([\nu^a\rho, \nu^b\rho]) \rtimes \sigma$ *reduces.*

Proof. If $\epsilon(((2b+1)_-, \rho), (2b+1, \rho)) = 1$, in the same way as in the proof of Lemma 3.2 we obtain that $\delta([\nu^a\rho, \nu^b\rho]) \rtimes \sigma$ contains $L(\delta([\nu^{\frac{(2b+1)_- -1}{2}}\rho, \nu^{-a}\rho]); \tau)$, where τ is an irreducible tempered subrepresentation of $\delta([\nu^{\frac{(2b+1)_- +1}{2}}\rho, \nu^b\rho]) \rtimes \sigma$ given by Lemma 3.3.

Let us now assume that $\epsilon(((2b+1)_{-},\rho),(2b+1,\rho)) = -1$, and let $y = \max\{x \in [2a+1, 2b+1] \cap \mathrm{Jord}_{\rho}(\sigma) : x_{-} \geqslant 2a-1, \epsilon((x_{-},\rho),(x,\rho)) = 1\}$. Then there is a discrete series σ_1 such that σ is a subrepresentation of $\delta([\nu^{-\frac{y_{-}-1}{2}}\rho, \nu^{\frac{y-1}{2}}\rho]) \rtimes \sigma_1$. Let us now discuss the case $y_{-} > 2a-1$.

We show that $\delta([\nu^{a}\rho, \nu^{b}\rho]) \rtimes \sigma$ contains

$$L(\delta([\nu^{-b}\rho, \nu^{\frac{y-1}{2}}\rho]), \delta([\nu^{-\frac{y_{-}-1}{2}}\rho, \nu^{-a}\rho]); \sigma_1).$$

Since $\delta([\nu^{-b}\rho, \nu^{\frac{y-1}{2}}\rho]) \times \delta([\nu^{-\frac{y_{-}-1}{2}}\rho, \nu^{-a}\rho])$ is irreducible, it follows at once that $L(\delta([\nu^{-b}\rho, \nu^{\frac{y-1}{2}}\rho]), \delta([\nu^{-\frac{y_{-}-1}{2}}\rho, \nu^{-a}\rho]); \sigma_1)$ is a subquotient of

$$\delta([\nu^{a}\rho, \nu^{\frac{y_{-}-1}{2}}\rho]) \rtimes L(\delta([\nu^{-b}\rho, \nu^{\frac{y-1}{2}}\rho]); \sigma_1),$$

and the Jacquet module of $L(\delta([\nu^{-b}\rho, \nu^{\frac{y-1}{2}}\rho]), \delta([\nu^{-\frac{y_{-}-1}{2}}\rho, \nu^{-a}\rho]); \sigma_1)$ with respect to the appropriate parabolic subgroup contains

$$\delta([\nu^{-\frac{y_{-}-1}{2}}\rho, \nu^{-a}\rho]) \otimes \delta([\nu^{-b}\rho, \nu^{\frac{y-1}{2}}\rho]) \otimes \sigma_1. \tag{1}$$

Note that $L(\delta([\nu^{-b}\rho, \nu^{\frac{y-1}{2}}\rho]); \sigma_1)$ is a subquotient of

$$\delta([\nu^{\frac{y_{-}+1}{2}}\rho, \nu^{b}\rho]) \times \delta([\nu^{-\frac{y_{-}-1}{2}}\rho, \nu^{\frac{y-1}{2}}\rho]) \rtimes \sigma_1. \tag{2}$$

Since $\mu^{*}(L(\delta([\nu^{-b}\rho, \nu^{\frac{y-1}{2}}\rho]); \sigma_1)) \geqslant \delta([\nu^{-b}\rho, \nu^{\frac{y-1}{2}}\rho]) \otimes \sigma_1$, using the structural formula and classification of discrete series we deduce that there is a discrete series subrepresentation σ_{ds} of $\delta([\nu^{-\frac{y_{-}-1}{2}}\rho, \nu^{\frac{y-1}{2}}\rho]) \rtimes \sigma_1$ such that $L(\delta([\nu^{-b}\rho, \nu^{\frac{y-1}{2}}\rho]); \sigma_1)$ is a subquotient of $\delta([\nu^{\frac{y_{-}+1}{2}}\rho, \nu^{b}\rho]) \rtimes \sigma_{ds}$. Also, it is easy to see that $\delta([\nu^{-b}\rho, \nu^{\frac{y-1}{2}}\rho]) \otimes \sigma_1$ appears with multiplicity one in $\mu^{*}(\delta([\nu^{\frac{y_{-}+1}{2}}\rho, \nu^{b}\rho]) \rtimes \sigma_{ds})$, so $L(\delta([\nu^{-b}\rho, \nu^{\frac{y-1}{2}}\rho]); \sigma_1)$ appears at most once in the composition series of $\delta([\nu^{\frac{y_{-}+1}{2}}\rho, \nu^{b}\rho]) \rtimes \sigma_{ds}$.

Let us prove that $L(\delta([\nu^{-b}\rho, \nu^{\frac{y-1}{2}}\rho]); \sigma_1)$ appears at least twice in the composition series of the induced representation (2).

In $R(G)$ we have

$$\begin{gathered}
\delta([\nu^{\frac{y_{-}+1}{2}}\rho, \nu^{b}\rho]) \times \delta([\nu^{-\frac{y_{-}-1}{2}}\rho, \nu^{\frac{y-1}{2}}\rho]) \rtimes \sigma_1 = \\
\delta([\nu^{-\frac{y_{-}-1}{2}}\rho, \nu^{b}\rho]) \times \delta([\nu^{\frac{y_{-}+1}{2}}\rho, \nu^{\frac{y-1}{2}}\rho]) \rtimes \sigma_1 + \\
L(\delta([\nu^{-\frac{y_{-}-1}{2}}\rho, \nu^{\frac{y-1}{2}}\rho]), \delta([\nu^{\frac{y_{-}+1}{2}}\rho, \nu^{b}\rho])) \rtimes \sigma_1.
\end{gathered}$$

From

$$L(\delta([\nu^{-b}\rho,\nu^{\frac{y-1}{2}}\rho]);\sigma_1) \leqslant \delta([\nu^{-b}\rho,\nu^{\frac{y_{-}-1}{2}}\rho]) \times \delta([\nu^{\frac{y_{-}+1}{2}}\rho,\nu^{\frac{y-1}{2}}\rho]) \rtimes \sigma_1,$$

we conclude that $L(\delta([\nu^{-b}\rho,\nu^{\frac{y-1}{2}}\rho]);\sigma_1)$ appears in $\delta([\nu^{-\frac{y_{-}-1}{2}}\rho,\nu^{b}\rho])\times\delta([\nu^{\frac{y_{-}+1}{2}}\rho,$ $\nu^{\frac{y-1}{2}}\rho]) \rtimes \sigma_1$.

On the other hand, by Proposition 5.1 we have

$$\begin{aligned} L(\delta([\nu^{-b}\rho,\nu^{\frac{y-1}{2}}\rho]);\sigma_1) &\hookrightarrow \delta([\nu^{-\frac{y_{-}-1}{2}}\rho,\nu^{\frac{y-1}{2}}\rho]) \times \delta([\nu^{-b}\rho,\nu^{-\frac{y_{-}+1}{2}}\rho]) \rtimes \sigma_1 \\ &\cong \delta([\nu^{-\frac{y_{-}-1}{2}}\rho,\nu^{\frac{y-1}{2}}\rho]) \times \delta([\nu^{\frac{y_{-}+1}{2}}\rho,\nu^{b}\rho]) \rtimes \sigma_1, \end{aligned}$$

so there is an irreducible representation

$$\pi_1 \in \{L(\delta([\nu^{-\frac{y_{-}-1}{2}}\rho,\nu^{\frac{y-1}{2}}\rho]),\delta([\nu^{\frac{y_{-}+1}{2}}\rho,\nu^{b}\rho])),\delta([\nu^{-\frac{y_{-}-1}{2}}\rho,\nu^{b}\rho])\times\delta([\nu^{\frac{y_{-}+1}{2}}\rho,\nu^{\frac{y-1}{2}}\rho])\}$$

such that $L(\delta([\nu^{-b}\rho,\nu^{\frac{y-1}{2}}\rho]);\sigma_1)$ is a subrepresentation of $\pi_1 \rtimes \sigma_1$. Suppose that $\pi_1 \cong \delta([\nu^{-\frac{y_{-}-1}{2}}\rho,\nu^{b}\rho]) \times \delta([\nu^{\frac{y_{-}+1}{2}}\rho,\nu^{\frac{y-1}{2}}\rho])$. Using Proposition 5.1 again, we have

$$\begin{aligned} L(\delta([\nu^{-b}\rho,\nu^{\frac{y-1}{2}}\rho]);\sigma_1) &\hookrightarrow \delta([\nu^{-\frac{y_{-}-1}{2}}\rho,\nu^{b}\rho]) \times \delta([\nu^{\frac{y_{-}+1}{2}}\rho,\nu^{\frac{y-1}{2}}\rho]) \rtimes \sigma_1 \\ &\cong \delta([\nu^{-\frac{y_{-}-1}{2}}\rho,\nu^{b}\rho]) \times \delta([\nu^{-\frac{y-1}{2}}\rho,\nu^{-\frac{y_{-}+1}{2}}\rho]) \rtimes \sigma_1, \end{aligned}$$

so there is an irreducible representation

$$\pi_2 \in \{L(\delta([\nu^{-\frac{y-1}{2}}\rho,\nu^{-\frac{y_{-}+1}{2}}\rho]),\delta([\nu^{-\frac{y_{-}-1}{2}}\rho,\nu^{b}\rho])),\delta([\nu^{-\frac{y-1}{2}}\rho,\nu^{b}\rho])\}$$

such that $L(\delta([\nu^{-b}\rho,\nu^{\frac{y-1}{2}}\rho]);\sigma_1)$ is a subrepresentation of $\pi_2 \rtimes \sigma_1$. If

$$\pi_2 \cong L(\delta([\nu^{-\frac{y-1}{2}}\rho,\nu^{-\frac{y_{-}+1}{2}}\rho]),\delta([\nu^{-\frac{y_{-}-1}{2}}\rho,\nu^{b}\rho])),$$

we get that $\mu^*(L(\delta([\nu^{-b}\rho,\nu^{\frac{y-1}{2}}\rho]);\sigma_1))$ contains an irreducible constituent of the form $\delta([\nu^{-\frac{y-1}{2}}\rho,\nu^{-\frac{y_{-}+1}{2}}\rho])\otimes\pi$, which is impossible since $L(\delta([\nu^{-b}\rho,\nu^{\frac{y-1}{2}}\rho]);\sigma_1)$ is a subquotient of $\delta([\nu^{-\frac{y-1}{2}}\rho,\nu^{b}\rho])\rtimes\sigma_1$. On the other hand, Theorem 3.5 implies that $\delta([\nu^{-\frac{y-1}{2}}\rho,\nu^{b}\rho])\rtimes\sigma_1$ reduces, so it can not contain $L(\delta([\nu^{-b}\rho,\nu^{\frac{y-1}{2}}\rho]);\sigma_1)$ as a subrepresentation. This gives $\pi_2 \not\cong \delta([\nu^{-\frac{y-1}{2}}\rho,\nu^{b}\rho])$.

Thus, $\pi_1 \cong L(\delta([\nu^{-\frac{y_{-}-1}{2}}\rho,\nu^{\frac{y-1}{2}}\rho]),\delta([\nu^{\frac{y_{-}+1}{2}}\rho,\nu^{b}\rho]))$, so $L(\delta([\nu^{-b}\rho,\nu^{\frac{y-1}{2}}\rho]);\sigma_1)$ also appears in $L(\delta([\nu^{-\frac{y_{-}-1}{2}}\rho,\nu^{\frac{y-1}{2}}\rho]),\delta([\nu^{\frac{y_{-}+1}{2}}\rho,\nu^{b}\rho])) \rtimes \sigma_1$.

Consequently, $L(\delta([\nu^{-b}\rho, \nu^{\frac{y-1}{2}}\rho]); \sigma_1)$ is a subquotient of $\delta([\nu^{\frac{y_-+1}{2}}\rho, \nu^b\rho]) \rtimes \sigma$, so $L(\delta([\nu^{-b}\rho, \nu^{\frac{y-1}{2}}\rho]), \delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{-a}\rho]); \sigma_1)$ is a subquotient of

$$\delta([\nu^a\rho, \nu^{\frac{y_--1}{2}}\rho]) \times \delta([\nu^{\frac{y_-+1}{2}}\rho, \nu^b\rho]) \rtimes \sigma,$$

and there is an irreducible representation

$$\pi_3 \in \{L(\delta([\nu^a\rho, \nu^{\frac{y_--1}{2}}\rho]), \delta([\nu^{\frac{y_-+1}{2}}\rho, \nu^b\rho])), \delta([\nu^a\rho, \nu^b\rho])\}$$

such that $L(\delta([\nu^{-b}\rho, \nu^{\frac{y-1}{2}}\rho]), \delta([\nu^{-\frac{y_--1}{2}}\rho, \nu^{-a}\rho]); \sigma_1)$ is a subquotient of $\pi_3 \rtimes \sigma$. Now (1) implies $\pi_3 \cong \delta([\nu^a\rho, \nu^b\rho])$.

If $y_- = 2a-1$, it can be proved in the same way that $\delta([\nu^a\rho, \nu^b\rho]) \rtimes \sigma$ contains $L(\delta([\nu^{-b}\rho, \nu^{\frac{y-1}{2}}\rho]); \sigma_1)$, details being left to the reader. $\square$

This leads to

Theorem 5.4. *Suppose that $1 \leqslant a \leqslant b$ and for α such that $\nu^\alpha\rho \rtimes \sigma_{cusp}$ reduces we have $a - \alpha \in \mathbb{Z}$. The induced representation $\delta([\nu^a\rho, \nu^b\rho]) \rtimes \sigma$ is irreducible if and only if one of the following holds:*

1. *$[2a-1, 2b+1] \cap Jord_\rho(\sigma) = \varnothing$,*
2. *$2b+1 \in Jord_\rho(\sigma)$ and for every $x \in [2a+1, 2b+1] \cap Jord_\rho(\sigma)$ such that x_- is defined and $x_- \geqslant 2a-1$ we have $\epsilon((x_-, \rho), (x, \rho)) = -1$.*

References

[1] J. Arthur, *The endoscopic classification of representations. Orthogonal and symplectic groups*, vol. 61 of American Mathematical Society Colloquium Publications, American Mathematical Society, Providence, RI, 2013.

[2] W. T. Gan and L. Lomelí, *Globalization of supercuspidal representations over function fields and applications*, J. Eur. Math. Soc., 20 (2018), pp. 2813–2858.

[3] C. Jantzen, *On supports of induced representations for symplectic and odd-orthogonal groups*, Amer. J. Math., 119 (1997), pp. 1213–1262.

[4] I. Matić, *Reducibility of representations induced from the Zelevinsky segment and discrete series*, Manuscripta Math., 164 (2021), pp. 349–374.

[5] C. Mœglin, *Sur la classification des séries discrètes des groupes classiques p-adiques: paramètres de Langlands et exhaustivité*, J. Eur. Math. Soc., 4 (2002), pp. 143–200.

[6] ——, *Paquets stables des séries discrètes accessibles par endoscopie tordue; leur paramètre de Langlands*, in Automorphic forms and related geometry: assessing the legacy of I. I. Piatetski-Shapiro, vol. 614 of Contemp. Math., Amer. Math. Soc., Providence, RI, 2014, pp. 295–336.

[7] C. Mœglin and M. Tadić, *Construction of discrete series for classical p-adic groups*, J. Amer. Math. Soc., 15 (2002), pp. 715–786.

[8] G. Muić, *Reducibility of generalized principal series*, Canad. J. Math., 57 (2005), pp. 616–647.

[9] M. Tadić, *Structure arising from induction and Jacquet modules of representations of classical p-adic groups*, J. Algebra, 177 (1995), pp. 1–33.

[10] ——, *On tempered and square integrable representations of classical p-adic groups*, Sci. China Math., 56 (2013), pp. 2273–2313.

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