Linearna algebra I

Determinanta

Determinanta

Teorem 3.2.21.(Laplaceov razvoj determinante)

Neka je $A = [a_{ij}] \in \mathcal{M}_n$, $n \geq 2$. Tada je

$$\det A = \sum_{j=1}^{n} a_{ij} A_{ij}, \quad \forall i = 1, 2, \ldots, n,$$

i

$$\det A = \sum_{i=1}^{n} a_{ij} A_{ij}, \quad \forall j = 1, 2, \ldots, n,$$

pri čemu za sve i, j, vrijedi

$$A_{ij} = (-1)^{i+j} \Delta_{ij},$$

a Δ_{ij} je determinanta matrice $n-1$. reda koja nastaje tako da iz matrice A uklonimo $i-$ti redak i $j-$ti stupac.

Determinanta

Propozicija 3.2.22.

Neka je $A = [a_{ij}] \in \mathcal{M}_n$. Tada vrijedi

$$\sum_{j=1}^{n} a_{ij} A_{kj} = 0, \quad \forall i \neq k,$$

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$$\sum_{i=1}^{n} a_{ij} A_{ik} = 0, \quad \forall j \neq k.$$

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Definicija 3.2.23.

Neka je $A \in \mathcal{M}_n$. Adjunkta matrice A je matrica $\widetilde{A} = [x_{ij}]$, pri čemu je

$$x_{ij} = A_{ji}, \quad \forall i, j = 1, 2, \ldots, n.$$

Korolar 3.2.24.

Za svaku kvadratnu matricu A vrijedi

$$A\widetilde{A} = \widetilde{A}A = (\det A)I.$$

Lema 3.2.25.
Neka su $X, Y \in \mathcal{M}_n$,

$$X = \left[\begin{array}{c|c} A & C \\ \hline 0 & B \end{array}\right], \qquad Y = \left[\begin{array}{c|c} A & 0 \\ \hline D & B \end{array}\right]$$

blok-matrice pri čemu je $n \geq 2$, $A \in \mathcal{M}_k$, $B \in \mathcal{M}_{n-k}$, $C \in \mathcal{M}_{k,n-k}$, $D \in \mathcal{M}_{n-k,k}$, $1 \leq k < n$. Tada je

$$\det X = \det A \cdot \det B \qquad \text{i} \qquad \det Y = \det A \cdot \det B.$$

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Teorem 3.2.26. (Binet-Cauchy)

Za sve $A, B \in \mathcal{M}_n$ vrijedi

$$\det(AB) = \det A \cdot \det B.$$

Determinanta

Teorem 3.2.27.

Matrica $A \in \mathcal{M}_n$ je regularna ako i samo ako je

$$\det A \neq 0.$$

U tom slučaju je inverzna matrica A^{-1} dana formulom

$$A^{-1} = \frac{1}{\det A}\widetilde{A}.$$

Još vrijedi

$$\det A^{-1} = \frac{1}{\det A}.$$