Linearna algebra 1

Vježbe 11

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Zadatak 1.

Izračunajte sljedeće determinante n-tog reda:

a)

$$\begin{vmatrix} \alpha & \beta & 0 & \dots & 0 & 0 \\ 0 & \alpha & \beta & \dots & 0 & 0 \\ 0 & 0 & \alpha & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \alpha & \beta \\ \beta & 0 & 0 & \dots & 0 & \alpha \end{vmatrix}$$

b)

$$\begin{vmatrix} 2 & 4 & 6 & \dots & 2n \\ -2 & 0 & 6 & \dots & 2n \\ -2 & -4 & 0 & \dots & 2n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -2 & -4 & -6 & \dots & 0 \end{vmatrix}$$

c)

$$\begin{vmatrix} 0 & 1 & 1 & \dots & 1 \\ 1 & a_1 & 0 & \dots & 0 \\ 1 & 0 & a_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \dots & a_n \end{vmatrix}$$

d)

$$\begin{vmatrix} -1 & 2 & 2 & \dots & 2 \\ 2 & -1 & 2 & \dots & 2 \\ 2 & 2 & -1 & \dots & 2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 2 & 2 & 2 & \dots & -1 \end{vmatrix}$$

e)

$$\begin{vmatrix} x & a & a & \dots & a & a \\ -a & x & a & \dots & a & a \\ -a & -a & x & \dots & a & a \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -a & -a & -a & \dots & x & a \\ -a & -a & -a & \dots & -a & x \end{vmatrix}$$

f)

$$\begin{vmatrix} a_1 & a_2 & a_3 & \dots & a_{n-1} & a_n \\ -x & x & 0 & \dots & 0 & 0 \\ 0 & -x & x & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & x & 0 \\ 0 & 0 & 0 & \dots & -x & x \end{vmatrix}$$

g)

$$\begin{vmatrix} 0 & 0 & \dots & 0 & a_1 \\ 0 & 0 & \dots & a_2 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & a_{n-1} & \dots & 0 & 0 \\ a_n & 0 & \dots & 0 & 0 \end{vmatrix}$$

Zadatak 2.

Izračunajte sljedeće determinante koristeći rekurzivne formule:

a)

$$\begin{vmatrix} 1 & 1 & 1 & \dots & 1 & 1 & 1 \\ 1 & 1 & 0 & \dots & 0 & 0 & 0 \\ 0 & 1 & 1 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & 1 & 0 \\ 0 & 0 & 0 & \dots & 0 & 1 & 1 \end{vmatrix}$$

b)

$$\begin{vmatrix} 0 & 1 & 1 & \dots & 1 & 1 & 1 \\ -1 & 0 & 1 & \dots & 1 & 1 & 1 \\ -1 & -1 & 0 & \dots & 1 & 1 & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ -1 & -1 & -1 & \dots & -1 & 0 & 1 \\ -1 & -1 & -1 & \dots & -1 & -1 & 0 \end{vmatrix}$$

Pretpostavimo da determinante zadovoljavaju sljedeću rekurzijsku jednadžbu:

$$\Delta_n = p\Delta_{n-1} + q\Delta_{n-2}. \tag{1}$$

Za nju kažemo da je **linearna rekurzijska jednadžba drugog reda**. Rješavamo ju tako da najprije riješimo pripadnu **karakterističnu jednadžbu**

$$\lambda^2 = p\lambda + q$$

u kojoj razlikujemo sljedeće slučajeve:

i) $\lambda_1 \neq \lambda_2$; rješenje relacije (1) ima oblik

$$\Delta_n = C_1\lambda_1^n + C_2\lambda_2^n$$

ii) $\lambda_1 = \lambda_2$; rješenje relacije (1) ima oblik

$$\Delta_n = C_1\lambda_1^n + C_2 n\lambda_1^n$$

Konstante C_1 i C_2 određujemo iz poznatih vrijednosti determinante Δ_n za $n = 1$ i $n = 2$ (ili $n = 2$ i $n = 3$).

c)

$$\begin{vmatrix} 5 & 3 & 0 & \dots & 0 & 0 \\ 2 & 5 & 3 & \dots & 0 & 0 \\ 0 & 2 & 5 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 5 & 3 \\ 0 & 0 & 0 & \dots & 2 & 5 \end{vmatrix}$$

d)

$$\begin{vmatrix} 4 & 3 & 0 & \dots & 0 & 0 \\ 1 & 4 & 3 & \dots & 0 & 0 \\ 0 & 1 & 4 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 4 & 3 \\ 0 & 0 & 0 & \dots & 1 & 4 \end{vmatrix}$$

Zadatak 3.

Izračunajte Vandermondeovu determinantu

$$V(x_1, \ldots, x_n) = \begin{vmatrix} 1 & 1 & \ldots & 1 \\ x_1 & x_2 & \ldots & x_n \\ x_1^2 & x_2^2 & \ldots & x_n^2 \\ \vdots & \vdots & \ddots & \vdots \\ x_1^{n-1} & x_2^{n-1} & \ldots & x_n^{n-1} \end{vmatrix}$$

Zadatak 4.

a)

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 3 & 5 \\ 1 & 1 & 9 & 25 \\ 1 & 1 & 27 & 125 \end{vmatrix}$$

b)

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -1 \\ 0 & 1 & 9 & 1 \\ 0 & 1 & 27 & -1 \end{vmatrix}$$

Domaća zadaća

c) Izračunajte $\det A$ ako je $A = B^2$ pri čemu je B

$$B = \begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{bmatrix}.$$

Zadatak 5.

Odredite inverz sljedećih matrica koristeći adjunktu:

a)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & -1 \\ 1 & 3 & 4 \end{bmatrix}$$

b)

$$A = \begin{bmatrix} 3 & -4 & 5 \\ 2 & -3 & 1 \\ 3 & -5 & -1 \end{bmatrix}$$

c)

$$A = \begin{bmatrix} 2 & 7 & 3 \\ 3 & 9 & 4 \\ 1 & 5 & 3 \end{bmatrix}$$

Zadatak 6.

Riješite matrične jednadžbe:

a)

$$\begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} X = \begin{bmatrix} 3 & 5 \\ 5 & 9 \end{bmatrix}$$

b)

$$\begin{bmatrix} 1 & 3 \\ -3 & 1 \end{bmatrix} X \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 0 & 5 \end{bmatrix}$$