Linearna algebra 1

Vježbe 14

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Zadatak 1.

Gaussovom metodom riješite sljedeće sustave:

a) $x + 2y + 3z = 5$
$2x - y - z = 1$
$x + 3y + 4z = 6$

b) $3x - y + 3z = 4$
$6x - 2y + 6z = 1$
$5x + 4y = 2$

c) Domaća zadaća
$x - 2y + z = 4$
$2x + 3y - z = 3$
$4x - y + z = 11$

d) $$\begin{aligned} x + 2y + 3z &= 3 \\ -2x + z &= -2 \\ x + 2y - z &= 3 \\ -x + 2y + 12z &= 1 \end{aligned}$$

e) $$\begin{aligned} x_1 - 2x_2 + x_3 &= 4 \\ 2x_1 + 3x_2 - x_3 &= 3 \\ 4x_1 - x_2 + x_3 &= 11 \end{aligned}$$

f) $$\begin{aligned} x_1 + 2x_2 - x_3 + x_4 &= -1 \\ 2x_1 + 5x_2 - x_3 + 2x_4 &= -2 \\ 3x_1 - x_2 - 2x_3 + x_4 &= 5 \\ x_1 - x_2 + 3x_3 - 5x_4 &= 6 \end{aligned}$$

g) $$\begin{aligned} 4x_1 - 2x_2 - 3x_3 - 2x_4 &= 1 \\ 2x_1 + 2x_2 + 3x_3 - 4x_4 &= 5 \\ 3x_1 + 2x_2 - 2x_3 - 5x_4 &= 1 \\ 2x_1 - 5x_2 - 3x_3 + 3x_4 &= -1 \end{aligned}$$

h) Domaća zadaća

$$\begin{aligned} 3x_1 + 2x_2 + 2x_3 + 2x_4 &= 2 \\ 2x_1 + 3x_2 + 2x_3 + 5x_4 &= 3 \\ 9x_1 + 2x_2 + 4x_3 - 5x_4 &= 1 \\ 2x_1 + 2x_2 + 3x_3 + 4x_4 &= 5 \end{aligned}$$

i) $$\begin{aligned} x_1 + x_2 - x_3 - 3x_4 + 4x_5 &= 2 \\ 3x_1 + x_2 - x_3 - x_4 &= 2 \\ 9x_1 + x_2 - 2x_3 - x_4 - 2x_5 &= 5 \\ x_1 - x_2 - x_4 + 2x_5 &= 1 \end{aligned}$$

Zadatak 2.

U ovisnosti o parametru $\lambda \in \mathrm{R}$ riješi sustave:

a) $\lambda x + 2y + z = 4$
$2x + y + 2z = 5$
$3x + 2y + 3z = 12$

b) $\lambda x + y + z = 1$
$x + \lambda y + z = \lambda$
$x + y + \lambda z = \lambda$

c) $$\begin{aligned} 2x_1 + 5x_2 + x_3 + 3x_4 &= 2 \\ 4x_1 + 6x_2 + 3x_3 + 5x_4 &= 4 \\ 4x_1 + 14x_2 + x_3 + 7x_4 &= 4 \\ 2x_1 - 3x_2 + 3x_3 + \lambda x_4 &= 7 \end{aligned}$$

d) $$\begin{aligned} 2x_1 + \lambda x_2 - 13x_3 &= -32 \\ x_1 - 2x_2 + x_3 &= \lambda \\ -2x_1 + 3x_2 + \lambda x_3 &= 8 \end{aligned}$$

Zadatak 3.

Riješite sustav:

a)

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix} X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

b) Domaća zadaća

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} X = 0$$

Zadatak 4.

Odredite LU dekompoziciju matrice $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 0 & 2 \\ 0 & -1 & 0 \end{bmatrix}$ i uz pomoć te dekompozicije riješite sustav

$$AX = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}.$$

Zadatak 5.

Odredite LU dekompoziciju matrice $A = \begin{bmatrix} 2 & 6 & -3 \\ -14 & -36 & 22 \\ 12 & 48 & -12 \end{bmatrix}$ i

uz pomoć te dekompozicije riješite sustav

$$AX = \begin{bmatrix} -3 \\ 28 \\ 0 \end{bmatrix}.$$

Neka je A kvadratna regularna matrica ($D = \det A \neq 0$).
i-ta komponenta rješenja sustava $Ax = b$ glasi

$$x_i = \frac{D_i}{D}$$

pri čemu je D_i determinanta matrice koju dobijemo tako da se u matrici A zamijeni i-ti stupac s vektorom b:

$$D_i = \begin{vmatrix} a_{11} & \dots & b_1 & \dots & a_{1n} \\ a_{21} & \dots & b_2 & \dots & a_{2n} \\ \vdots & & \vdots & & \vdots \\ a_{n1} & \dots & b_n & \dots & a_{nn} \end{vmatrix}$$

Zadatak 6.

Cramerovim pravilom riješi sustav:

a) $\begin{aligned} x + 2y + 3z &= 5 \\ 2x - y - z &= 1 \\ x + 3y + 4z &= 6 \end{aligned}$

b) $\begin{aligned} 3x + 2y - z &= 0 \\ 2x - y + 3z &= 0 \\ x + y - z &= 0 \end{aligned}$

c) $\begin{aligned} 2x - 3y + z - 2 &= 0 \\ x + 5y - 4z + 5 &= 0 \\ 4x + y - 3z + 4 &= 0 \end{aligned}$

d) $\begin{aligned} 5x_1 + 2x_2 + 5x_3 - 6x_4 &= 0 \\ 2x_1 + 3x_2 - 5x_4 &= 0 \\ x_1 + x_2 + x_3 + x_4 &= 4 \\ x_1 - x_2 + 2x_3 - x_4 &= 1 \end{aligned}$