Rješenje 1

$$\left[\left(\frac{a}{0.01}\right)^{-0.01}\cdot\left(\frac{b}{0.02}\right)^{0.02}\right]^{100}=\left(\frac{a}{0.01}\right)^{-1}\cdot\left(\frac{b}{0.02}\right)^{2}=\frac{0.01}{a}\cdot\frac{b^2}{0.0004}=\frac{25b^2}{a}\quad\surd$$

Rješenje 2 $\frac{\sqrt{a}}{\sqrt{2}+\sqrt{3}}$, *gdje je* $a>0$

$$a)\qquad\frac{\sqrt{a}}{\sqrt{2}+\sqrt{3}}\cdot\frac{\sqrt{a}}{\sqrt{a}}=\frac{a}{\sqrt{2a}+\sqrt{3a}}\quad\surd$$

$$b)\qquad\frac{\sqrt{a}}{\sqrt{2}+\sqrt{3}}\cdot\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}=\frac{\sqrt{3a}-\sqrt{2a}}{3-2}=\sqrt{3a}-\sqrt{2a}\quad\surd$$

$$c)\qquad\frac{\sqrt{a}}{\sqrt{2}+\sqrt{3}}\cdot\frac{\sqrt{a}}{\sqrt{a}}=\frac{a}{\sqrt{2a}+\sqrt{3a}}\cdot\frac{-1}{-1}=\frac{-a}{-\sqrt{2a}-\sqrt{3a}}\quad\times$$

$$d)\qquad\frac{\sqrt{a}}{\sqrt{2}+\sqrt{3}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{2a}}{2+\sqrt{6}}\quad\surd$$

Rješenje 3

$$\left(\frac{2+3a}{2a-4}+\frac{3a-1}{a-2}\right)\cdot\left(\frac{15a-12}{3a^2b}-\frac{3}{ab}\right)\cdot\frac{a\sqrt{b^3}}{9}=$$

$$=\frac{2+3a+6a-2}{2(a-2)}\cdot\frac{15a-12-9a}{3a^2b}\cdot\frac{a\sqrt{b^3}}{9}=$$

$$=\frac{9a}{2(a-2)}\cdot\frac{6(a-2)}{3a^2b}\cdot\frac{a\sqrt{b^3}}{9}=\frac{b^{\frac{3}{2}}}{b}=\sqrt{b}\quad\surd$$

Rješenje 4

$$\left(\frac{1}{2}\right)^5\cdot\left(\left(\frac{2^{-1}}{3^{-2}}\right)^{-1}:\left(\frac{2}{3^{-1}}\right)^{-3}\right)=\frac{1}{2^5}\cdot\left(\left(\frac{\frac{1}{2}}{\frac{1}{9}}\right)^{-1}:\left(\frac{2}{\frac{1}{3}}\right)^{-3}\right)=$$

$$=\frac{1}{32}\cdot\left(\left(\frac{9}{2}\right)^{-1}:6^{-3}\right)=\frac{1}{32}\cdot\left(\frac{2}{9}:\frac{1}{6^3}\right)=\frac{1}{32}\cdot\frac{2}{9}\cdot6^3=\frac{3}{2}$$

Rješenje 5

$$\frac{x^3-y^3}{x+y-\frac{xy}{x+y}}-\frac{x^3+y^3}{x-y+\frac{xy}{x-y}}=$$

$$= \frac{x^3-y^3}{\frac{(x+y)^2-xy}{x+y}} - \frac{x^3+y^3}{\frac{(x-y)^2+xy}{x-y}} = \frac{x^3-y^3}{\frac{x^2+2xy+y^2-xy}{x+y}} - \frac{x^3+y^3}{\frac{x^2-2xy+y^2+xy}{x-y}} =$$

$$= \frac{(x^3-y^3)(x+y)}{x^2+xy+y^2} - \frac{(x^3+y^3)(x-y)}{x^2-xy+y^2} =$$

$$= \frac{(x-y)(x^2+xy+y^2)(x+y)}{x^2+xy+y^2} - \frac{(x+y)(x^2-xy+y^2)(x-y)}{x^2-xy+y^2} =$$

$$= (x-y)(x+y) - (x+y)(x-y) = 0 \qquad \surd$$

Rješenje 6

$$\frac{2\sqrt{6}}{\underbrace{\sqrt{2}+\sqrt{3}}+\sqrt{5}} \cdot \frac{\overbrace{(\sqrt{2}+\sqrt{3})}-\sqrt{5}}{(\sqrt{2}+\sqrt{3})-\sqrt{5}} = \frac{2\sqrt{6}(\sqrt{2}+\sqrt{3}-\sqrt{5})}{(\sqrt{2}+\sqrt{3})^2-5} =$$

$$= \frac{2\sqrt{6}(\sqrt{2}+\sqrt{3}-\sqrt{5})}{2\sqrt{6}} = \sqrt{2}+\sqrt{3}-\sqrt{5} \quad \surd$$

Rješenje 7

$$\frac{2^x(2^x-2\cdot 3^x)}{2^x+3^x} - \frac{2^{3x}+2^{2x}\cdot 3^x}{(2^x+3^x)^2} + \frac{3\cdot 2^x\cdot 3^x}{2^x+3^x} - \frac{2^{2x}\cdot 3^x-2^x\cdot 3^{2x}}{2^{2x}-3^{2x}} =$$

$$= \frac{2^x(2^x-2\cdot 3^x)}{2^x+3^x} + \frac{3\cdot 2^x\cdot 3^x}{2^x+3^x} - \frac{2^{3x}+2^{2x}\cdot 3^x}{(2^x+3^x)^2} - \frac{2^{2x}\cdot 3^x-2^x\cdot 3^{2x}}{2^{2x}-3^{2x}} =$$

$$= \frac{2^{2x}-2\cdot 2^x\cdot 3^x+3\cdot 2^x\cdot 3^x}{2^x+3^x} - \frac{2^{2x}\cdot(2^x+3^x)}{(2^x+3^x)^2} - \frac{2^x\cdot 3^x(2^x-3^x)}{(2^x-3^x)(2^x+3^x)} =$$

$$= \frac{2^{2x}+2^x\cdot 3^x}{2^x+3^x} - \frac{2^{2x}}{2^x+3^x} - \frac{2^x\cdot 3^x}{2^x+3^x} = \frac{2^{2x}+2^x\cdot 3^x-2^{2x}-2^x\cdot 3^x}{2^x+3^x} = 0 \quad \surd$$

Rješenje 8

$$\frac{1}{\sqrt{2\sqrt{2}}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2}{8} = \frac{1}{4} \quad \surd$$

Rješenje 9

$$\frac{4}{\sqrt[4]{64}} = \frac{4}{\sqrt[4]{2^6}} = \frac{2^2}{2^{\frac{3}{2}}} = 2^{2-\frac{3}{2}} = \sqrt{2} \quad \surd$$

Rješenje 10

$$\left(\frac{\sqrt{1+a}}{\sqrt{1+a}-\sqrt{1-a}}+\frac{1-a}{\sqrt{1-a^2}-1+a}\right)\left(\sqrt{\frac{1}{a^2}-1}-\frac{1}{a}\right)$$

$$\begin{aligned}
&= \left(\frac{\sqrt{1+a}}{\sqrt{1+a}-\sqrt{1-a}}+\frac{1-a}{\sqrt{1-a^2}-1+a}\right)\left(\sqrt{\frac{1}{a^2}-1}-\frac{1}{a}\right)\\
&= \left(\frac{\sqrt{1+a}}{\sqrt{1+a}-\sqrt{1-a}}\cdot\frac{\sqrt{1+a}+\sqrt{1-a}}{\sqrt{1+a}+\sqrt{1-a}}+\frac{1-a}{\sqrt{1-a^2}-(1-a)}\cdot\frac{\sqrt{1-a^2}+(1-a)}{\sqrt{1-a^2}+(1-a)}\right)\\
&\quad \left(\sqrt{\frac{1-a^2}{a^2}}-\frac{1}{a}\right)\\
&= \left(\frac{1+a+\sqrt{1-a^2}}{2a}+\frac{1-a+\sqrt{1-a^2}}{2a}\right)\left(\frac{\sqrt{1-a^2}}{a}-\frac{1}{a}\right)\\
&= \frac{2+2\sqrt{1-a^2}}{2a}\cdot\frac{\sqrt{1-a^2}-1}{a}\\
&= \frac{(1-a^2)-1}{a^2}\\
&= \frac{1-a^2-1}{a}\\
&= \frac{-a^2}{a^2}\\
&= -1
\end{aligned}$$

Rješenje 11

$$\left(\frac{1}{\sqrt{a}-\sqrt{a-b}}+\frac{1}{\sqrt{a}+\sqrt{a+b}}\right):\left(1+\sqrt{\frac{a+b}{a-b}}\right).$$

$$\begin{aligned}
&= \left(\frac{1}{\sqrt{a}-\sqrt{a-b}}\cdot\frac{\sqrt{a}+\sqrt{a-b}}{\sqrt{a}+\sqrt{a-b}}+\frac{1}{\sqrt{a}+\sqrt{a+b}}\cdot\frac{\sqrt{a}-\sqrt{a+b}}{\sqrt{a}-\sqrt{a+b}}\right):\left(\frac{\sqrt{a-b}+\sqrt{a+b}}{\sqrt{a-b}}\right)\\
&= \left(\frac{\sqrt{a}+\sqrt{a-b}}{b}+\frac{-\sqrt{a}+\sqrt{a+b}}{b}\right):\left(\frac{\sqrt{a-b}+\sqrt{a+b}}{\sqrt{a-b}}\right)\\
&= \frac{\sqrt{a+b}+\sqrt{a-b}}{b}\cdot\frac{\sqrt{a-b}}{\sqrt{a+b}+\sqrt{a-b}}\\
&= \frac{\sqrt{a-b}}{b}
\end{aligned}$$

Rješenje 12

$$\left(\frac{a^{-1}}{b^{-1}}-\frac{b^{-1}}{a^{-1}}\right)^{-1}:\left(\frac{1}{a^{-2}}-\frac{1}{b^{-2}}\right)^{-1}=\left(\left(\frac{a}{b}\right)^{-1}-\left(\frac{b}{a}\right)^{-1}\right)^{-1}:\left(a^2-b^2\right)^{-1}=$$

$$=\left(\frac{b}{a}-\frac{a}{b}\right)^{-1}\cdot\frac{1}{(a^2-b^2)^{-1}}=\left(\frac{b^2-a^2}{ab}\right)^{-1}\cdot(a^2-b^2)=$$

$$=\frac{-ab}{a^2-b^2}\cdot(a^2-b^2)=-ab \qquad \surd$$

Rješenje 13

$$3+\frac{2}{\sqrt{2}}-\frac{1}{1-\frac{1}{\sqrt{2}}}-\frac{1}{1-\frac{1}{1-\frac{1}{\sqrt{2}}}}=\frac{3\sqrt{2}+2}{\sqrt{2}}-\frac{1}{\frac{\sqrt{2}-1}{\sqrt{2}}}-\frac{1}{1-\frac{1}{\frac{\sqrt{2}-1}{\sqrt{2}}}}=$$

$$=\frac{3\sqrt{2}+2}{\sqrt{2}}-\frac{\sqrt{2}}{\sqrt{2}-1}-\frac{1}{1-\frac{\sqrt{2}}{\sqrt{2}-1}}=\frac{3\sqrt{2}+2}{\sqrt{2}}-\frac{\sqrt{2}}{\sqrt{2}-1}-\frac{1}{\frac{-1}{\sqrt{2}-1}}=$$

$$=\frac{3\sqrt{2}+2}{\sqrt{2}}-\frac{\sqrt{2}}{\sqrt{2}-1}+\frac{\sqrt{2}-1}{1}=\frac{3\sqrt{2}+2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}-\frac{\sqrt{2}}{\sqrt{2}-1}\cdot\frac{\sqrt{2}+1}{\sqrt{2}+1}+\sqrt{2}-1=$$

$$=\frac{6+2\sqrt{2}}{2}-\frac{2+\sqrt{2}}{1}+\sqrt{2}-1=3+\sqrt{2}-2-\sqrt{2}+\sqrt{2}-1=\sqrt{2} \quad \surd$$

Rješenje 14

$$(\sqrt{a}+\sqrt{b})^{-2}\cdot(a^{-1}+b^{-1})+\frac{2}{(\sqrt{a}+\sqrt{b})^3}\cdot(\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{b}})=$$

$$=\frac{1}{(\sqrt{a}+\sqrt{b})^2}\cdot\left(\frac{1}{a}+\frac{1}{b}\right)+\frac{2}{(\sqrt{a}+\sqrt{b})^3}\cdot\frac{\sqrt{a}+\sqrt{b}}{\sqrt{ab}}=$$

$$=\frac{1}{(\sqrt{a}+\sqrt{b})^2}\cdot\frac{b+a}{ab}+\frac{2}{(\sqrt{a}+\sqrt{b})^2}\cdot\frac{1}{\sqrt{ab}}=$$

$$\frac{1}{(\sqrt{a}+\sqrt{b})^2}\cdot\left(\frac{b+a}{ab}+\frac{2}{\sqrt{ab}}\right)=\frac{1}{(\sqrt{a}+\sqrt{b})^2}\cdot\left(\frac{a+2\sqrt{ab}+b}{ab}\right)=\frac{1}{ab} \quad \surd$$

Rješenje 15

$$\frac{9x^3-18x^2-x+2}{-3x^2+7x-2}$$

$$\begin{array}{rrrr} (9x^3 & -18x^2 & -x & +2 \quad) : (-3x^2+7x-2) = -3x-1 \\ 9x^3 & -21x^2 & +6x & \\ & 3x^2 & -7x & +2 \\ & 3x^2 & -7x & +2 \\ & & & 0 \end{array}$$

Sada gornji razlomak možemo napisati ovako:

$$\frac{(-3x^2+7x-2)(-3x-1)}{-3x^2+7x-2} = -3x-1$$

Rješenje 16

$$\frac{2a}{a^2-4x^2} + \frac{1}{2x^2+6x-ax-3a} \cdot \left(x + \frac{2x-6}{x-2} \right).$$

$$\begin{aligned} &= \frac{2a}{a^2-4x^2} + \frac{1}{x(2x-a)+3(2x-a)} \cdot \left(x + 3\frac{x-2}{x-2} \right) \\ &= \frac{2a}{(a-2x)(a+2x)} + \frac{1}{(2x-a)(x+3)} \cdot (x+3) \\ &= \frac{2a-(a+2x)}{(a-2x)(a+2x)} \\ &= \frac{-(a+2x)}{(a-2x)(a+2x)} \\ &= \frac{1}{2x-a} \end{aligned}$$

Rješenje 17

$$ab = cd$$

A

$$\frac{a}{d} = \frac{c}{b} \Rightarrow ab = cd$$

B

$$\frac{ab}{c} = \frac{cd}{b} \Rightarrow ab \cdot b = cd \cdot c \Rightarrow ab \cdot c = ab \cdot b$$

A to ne vrijedi ukoliko je $c \neq b$

C

$$\frac{a-c}{c} = \frac{d-b}{b} \Rightarrow \frac{a}{c} - 1 = \frac{d}{b} - 1 \Rightarrow \frac{a}{c} = \frac{d}{b} \Rightarrow ab = cd$$

D

$$\frac{cd}{a} + ae = \frac{cde}{b} + b \Rightarrow \frac{cd}{a} - b = e\left(\frac{cd}{b} - a\right) \Rightarrow \frac{cd-ab}{a} = e\frac{cd-ab}{b} \Rightarrow 0 = 0$$