



NUNC COGNOSCO EX PARTE



TRENT UNIVERSITY
LIBRARY



Digitized by the Internet Archive
in 2019 with funding from
Kahle/Austin Foundation

<https://archive.org/details/confluenthyperge0000slat>

CONFLUENT HYPERGEOMETRIC FUNCTIONS

BY

L. J. SLATER, D.LIT., PH.D.

*Formerly Bateson Research Fellow
Newnham College, Cambridge*



CAMBRIDGE
AT THE UNIVERSITY PRESS

1960

PUBLISHED BY
THE SYNDICS OF THE CAMBRIDGE UNIVERSITY PRESS

Bentley House, 200 Euston Road, London, N.W. 1
American Branch: 32 East 57th Street, New York 22, N.Y.

©

CAMBRIDGE UNIVERSITY PRESS

1960

*Printed in Great Britain at the University Press, Cambridge
(Brooke Crutchley, University Printer)*

ONULP

CONTENTS

PREFACE

page xi

CHAPTER I

DIFFERENTIAL EQUATIONS SATISFIED BY CONFLUENT HYPERGEOMETRIC FUNCTIONS

1.1	Introduction	1
1.1.1	Generalized hypergeometric functions	1
1.2	Two solutions of Kummer's equation	3
1.2.1	Two further solutions of Kummer's equation	4
1.3	The second form of solutions of Kummer's equation	5
1.4	Kummer's first theorem	6
1.5	The first logarithmic solutions when b is an integer	6
1.5.1	The second logarithmic solutions when b is an integer	8
1.6	Whittaker's normalized equation	9
1.7	An alternative solution for Whittaker's equation	10
1.7.1	The logarithmic solutions of Whittaker's equation when $2m$ is an integer	11
1.8	Kummer's second theorem	12
1.8.1	Bessel functions as special cases of confluent hypergeometric functions	12
1.9	Relations between Kummer's functions and Whittaker's functions	13

CHAPTER 2

DIFFERENTIAL PROPERTIES

2.1	The differentiation of Kummer's function	15
2.1.1	The derivatives of $U(a; b; x)$	16
2.1.2	The Wronskians of Kummer's equation	17
2.2	Recurrence relations for ${}_1F_1[a; b; x]$	19
2.2.1	Recurrence relations for $U(a; b; x)$	20
2.2.2	Continuation formulae for $U(a; b; x)$	21
2.3	Addition theorems for ${}_1F_1[a; b; x]$	21
2.3.1	Addition theorems for $U(a; b; x)$	22

006037

2.3.2	Multiplication theorems for ${}_1F_1[a; b; x]$	page 22
2.3.3	Multiplication theorems for $U(a; b; x)$	23
2.4	The derivatives of $M_{k,m}(x)$	23
2.4.1	The derivatives of $W_{k,m}(x)$	25
2.4.2	The Wronskians of Whittaker's equation	26
2.5	Recurrence relations for $M_{k,m}(x)$	26
2.5.1	Recurrence relations for $W_{k,m}(x)$	27
2.5.2	Continuation formulae for Whittaker's functions	28
2.6	Addition theorems for $M_{k,m}(x)$	29
2.6.1	Addition theorems for $W_{k,m}(x)$	29
2.6.2	Multiplication theorems for $M_{k,m}(x)$	30
2.6.3	Multiplication theorems for $W_{k,m}(x)$	30
2.7	Expansions in series of Bessel functions	30
2.7.1	An elementary proof of the ${}_4F_3[1]$ summation theorem	31
2.7.2	Expansion of Kummer's function in terms of $I_n(x)$	32
2.7.3	Some further expansions	32

CHAPTER 3

INTEGRAL PROPERTIES

3.1	Elementary integrals for Kummer's function	34
3.1.1	Barnes's integral for Kummer's function	35
3.1.2	Barnes and Euler type integrals for $U(a; b; x)$	37
3.1.3	Pochhammer's contour integrals for Kummer's function	38
3.1.4	The Pochhammer integrals for $U(a; b; x)$	41
3.2	Elementary indefinite integrals	42
3.2.1	The Laplace transforms of ${}_1F_1[a; b; x]$	43
3.2.2	The inverse Laplace transform	45
3.2.3	The Laplace transform of $U(a; b; x)$	46
3.3	Mellin transforms of ${}_1F_1[a; b; x]$	47
3.3.1	Mellin transforms of $U(a; b; x)$	49
3.4	The Hankel transforms	49

3.5	Elementary integrals for the Whittaker functions	<i>page</i> 50
3.5.1	Barnes type integrals for the Whittaker functions	51
3.5.2	Pochhammer contour integrals for the Whittaker functions	51
3.6	The Laplace transforms of the Whittaker functions	52
3.7	Integrals involving pairs of Kummer's functions	54
3.7.1	Integrals involving pairs of Whittaker functions	56
3.8	Some expansions in series	56

CHAPTER 4

ASYMPTOTIC EXPANSIONS

4.1	Introduction	58
4.1.1	The asymptotic expansions in x for Kummer's function	58
4.1.2	The asymptotic expansions in x for $U(a; b; x)$	60
4.1.3	The asymptotic expansions in x for Whittaker's functions	61
4.2	Converging factors for Kummer's functions	61
4.2.1	Converging factors for Whittaker's functions	65
4.3	Approximations when b is large	65
4.3.1	Approximations for Whittaker's functions when m is large	66
4.4	Bessel functions as limiting cases of Kummer functions	67
4.4.1	Approximations in terms of Bessel functions when a is large	68
4.4.2	Bessel functions as limiting cases of Whittaker functions	69
4.4.3	Approximations for Whittaker functions in terms of Bessel functions, when k is large	69
4.5	Approximations when a and x are real, $\frac{1}{4}x > \frac{1}{2}b - a$	71
4.5.1	Approximations when $\frac{1}{2}b - a \sim \frac{1}{4}x$	71
4.5.2	Approximations when $\frac{1}{2}b - a > \frac{1}{4}x$	72
4.5.3	Whittaker functions when k and x are large	73
4.6	Olver's theorems	76
4.6.1	Asymptotic expansions when a is large	79
4.6.2	Asymptotic expansions when k and x are large	81
4.6.3	Asymptotic expansions when $4k \neq x$	83
4.6.4	Asymptotic expansions when $4k \doteq x$	86

CHAPTER 5

RELATED DIFFERENTIAL EQUATIONS AND PARTICULAR
CASES OF THE FUNCTIONS

5.1	General transforms of Kummer's equation	<i>page</i> 89
5.2	Kummer's second theorem and the connection with Bessel functions	92
5.3	The Coulomb wave equation	93
5.4	Further forms of Whittaker's equation	93
5.4.1	Watson's fourth-order equation	94
5.5	The Laguerre polynomials	95
5.6	The incomplete gamma functions	96
5.7	Transformations of Kummer's equation when $m = 2$	98
5.7.1	The Poiseuille functions	100
5.8	The Schrödinger equation	100
5.9	Kamke's equation	101

CHAPTER 6

DESCRIPTIVE PROPERTIES

6.1	The distribution of the zeros	102
6.1.1	The curves of zeros	104
6.1.2	The zeros of $U(a; b; x)$	105
6.1.3	Approximations to the zeros	106
6.2	Expansions for the zeros	107
6.3	Nesting processes	110
6.4	Zeros in ' a '	110
6.5	The zeros in ' b '	112
6.5.1	The tabulation of zeros in x	112
6.6	The numerical evaluation of Kummer's function	113
6.7	Exponential and oscillatory regions	118
6.7.1	The Sonine–Polya theorem	119
6.7.2	Graphing Kummer's function	120

CONTENTS

ix

REFERENCES

page 121

APPENDIX I

Table of the smallest positive zeros of ${}_1F_1[a; b; x]$ over the range $a = -4.0(0.1) - 0.1$,
 $b = 0.1(0.1) 2.5$

125

APPENDIX II

Table of ${}_1F_1[a; b; x]$ over the range

$$a = -1.0(0.1) 1.0,$$

$$b = 0.1(0.1) 1.0,$$

$$x = 0.1(0.1) 10.0$$

131

APPENDIX III

Table of ${}_1F_1[a; b; 1]$ over the range

$$a = -11.0(0.2) 2.0,$$

$$b = -4.0(0.2) 1.0,$$

$$x = 1$$

233

SYMBOLIC INDEX OF DEFINITIONS

244

GENERAL INDEX

245

PREFACE

This work provides for the mathematical physicist a discussion of all the main properties of the confluent hypergeometric functions and the Whittaker functions. Outlines are given of the underlying functional analysis necessary in the development of the asymptotic expansions, and tables of the function ${}_1F_1[a; b; x]$ over those ranges most useful to the computer.

The author wishes to extend her thanks to the Principal and Fellows of Newnham College, Cambridge, for the award of the Bateson Research Fellowship whilst this book was being prepared, and to Dr J. C. P. Miller for his helpful criticism of most of the manuscript. The tables were computed by the author on the electronic calculator EDSAC I in the Cambridge University Mathematical Laboratory, and thanks are due also to Dr M. V. Wilkes and the staff of the laboratory for their courtesy in extending the use of the full facilities of the laboratory over the past six years.

L. J. S.

THE MATHEMATICAL LABORATORY

CAMBRIDGE

March 1959

DIFFERENTIAL EQUATIONS SATISFIED BY CONFLUENT HYPERGEOMETRIC FUNCTIONS

1.1 Introduction

There are four functions known as confluent hypergeometric functions, Kummer's function ${}_1F_1[a; b; x]$, its associated solution which we shall call $U(a; b; x)$ and the two Whittaker functions $M_{k,m}(x)$ and $W_{k,m}(x)$.

Physicists first meet these functions when they try to solve, by separation of the variables, equations of the type

$$\nabla^2 \Psi + \frac{8\pi^2 M}{h^2} \left(E + \frac{\mu}{r} \right) \Psi = 0, \quad \left(\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right), \quad (1.1.1)$$

where Ψ is a function of the spherical co-ordinates r , θ and ϕ . We find that

$$\Psi = R(r) \Theta(\theta) \Phi(\phi), \quad (1.1.2)$$

where $\Phi(\phi) = e^{mi\phi}$, $\Theta(\theta) = P_l^m(\cos \theta)$, a Legendre polynomial, and $R(r)$ is a function which satisfies a differential equation of the type $r^2 R'' + 2rR' + (ar^2 + br)R = c$.

$$(1.1.3)$$

As we shall see later the solutions of this equation are confluent hypergeometric functions, and these functions contain as special cases most of the commonly used functions of mathematical physics, including the Weber functions and Bessel functions. Before discussing their general properties and special cases, however, we shall derive the confluent hypergeometric functions in two ways from the more fundamental generalized hypergeometric functions.

1.1.1 Generalized hypergeometric functions

The generalized hypergeometric differential equation is

$$\left\{ x \frac{d}{dx} \left(x \frac{d}{dx} + b_1 - 1 \right) \left(x \frac{d}{dx} + b_2 - 1 \right) \dots \left(x \frac{d}{dx} + b_B - 1 \right) - x \left(x \frac{d}{dx} + a_1 \right) \left(x \frac{d}{dx} + a_2 \right) \dots \left(x \frac{d}{dx} + a_A \right) \right\} y = 0. \quad (1.1.4)$$

This is a linear equation of order $\max(A, B + 1)$, and its solution is in terms of the generalized hypergeometric function

$${}_A F_B[a_1, a_2, \dots, a_A; b_1, b_2, \dots, b_B; x] \equiv {}_A F_B[(a); (b); x] \equiv \sum_{n=0}^{\infty} \frac{(a_1)_n (a_2)_n \dots (a_A)_n x^n}{(b_1)_n (b_2)_n \dots (b_B)_n n!} \equiv \sum_{n=0}^{\infty} \frac{((a)_n) x^n}{((b)_n) n!}, \quad (1.1.5)$$

where $(a)_n \equiv a(a+1)(a+2) \dots (a+n-1)$, and $(a)_0 \equiv 1$, for all a_r , b_r and x real or complex provided that no b is a negative integer (Bailey, 1935, ch. II). Any function which can be expressed as an ascending power series in x of the above type is called a generalized hypergeometric function (from the Greek $\upsilon\pi\epsilon\rho$, [*hyper*] beyond), since this large class of functions arises from inserting coefficients in the simple geometric series

$$1 + x + x^2 + \dots + x^n + \dots$$

Here x is called the variable and a_r and b_r are called the parameters of the function.

If $A \leq B$, the series (I.I.5) is convergent for all finite values of x . In this case the differential equation (I.I.4) has a regular singularity at $x = 0$, and a solution in series of the form

$$u_0 + u_1x + u_2x^2 + \dots + u_nx^n + \dots,$$

but it also has an irregular singularity at $x = \infty$, and no simple solution in series of the form

$$u_0 + u_1/x + u_2/x^2 + \dots + u_n/x^n + \dots \quad (\text{Whittaker and Watson, 1946, ch. x}).$$

In particular if $A = B = 1$, the equation (I.I.4) reduces to

$$x \frac{d^2y}{dx^2} + (b-x) \frac{dy}{dx} - ay = 0. \quad (\text{I.I.6})$$

This equation is the confluent hypergeometric equation known as Kummer's equation, and any solution of equation (I.I.6) is a confluent hypergeometric function. The simplest solution is Kummer's function,

$${}_1F_1[a; b; x] \equiv 1 + \frac{a}{b}x + \frac{a(a+1)x^2}{b(b+1)2!} + \frac{a(a+1)(a+2)x^3}{b(b+1)(b+2)3!} + \dots \quad (\text{I.I.7})$$

This series is absolutely convergent for all values of a , b and x , real or complex, excluding $b = 0, -1, -2, \dots$. Its elementary properties were given by Kummer (1836), though a special case had been discussed by Lagrange as early as 1762.

There are several notations in use for the series (I.I.7). In this work it will be denoted by the symbol

$${}_1F_1[a; b; x] \equiv \sum_{n=0}^{\infty} \frac{(a)_n x^n}{(b)_n n!}. \quad (\text{I.I.8})$$

This is the standard hypergeometric notation, introduced by Pochhammer in 1870 and modified by Barnes (1908 *a, b*). Other notations for the series (I.I.7) are $M(a, b, x)$ used by Airey and Webb (1918), the symbol $\Phi(a; b; x)$ introduced by Humbert (1920), $u(a, b, x)$ used by Magnus and Oberhettinger (1948) and the original symbol used by Kummer, $F(\alpha, \beta, x)$. In this book any general solution of Kummer's equation will be denoted by

$$\mathfrak{F}[a; b; x].$$

There is a second approach to the confluent hypergeometric function, which exhibits the nature of its singularities more clearly and explains the use of the word 'confluent'. If in equation (I.I.4) we suppose that $A = B + 1$, the series still converges for $|x| < 1$, and the equation becomes a Fuchsian equation of the general type

$$x^{B(1-x)} \frac{d^{B+1}y}{dx^{B+1}} + \sum_{n=1}^B x^{n-1} (a_n x - b_n) \frac{d^n y}{dx^n} + a_0 y = 0. \quad (\text{I.I.9})$$

This equation has regular singularities at $x = 0, 1$ and ∞ . In particular if $B = 1$, equation (I.I.9) assumes the general form

$$x(1-x) \frac{d^2y}{dx^2} + \{c - (1+a+b)x\} \frac{dy}{dx} - aby = 0. \quad (\text{I.I.10})$$

This is Gauss's differential equation and one of its solutions is Gauss's function

$${}_2F_1[a, b; c; x]. \quad (\text{I.I.11})$$

This series is convergent when $|x| < 1$, when $x = 1$, provided that $\text{Re}(c-a-b) > 0$, and when $x = -1$, provided that $\text{Re}(c-a-b) > -1$.

In (1.1.10) and (1.1.11) let us replace x by x/b , and let $b \rightarrow \infty$. Then

$$(b)_n x^n / b^n \rightarrow x^n,$$

the equation (1.1.10) reduces to Kummer's equation (1.1.6) as before, and Gauss's function (1.1.11) reduces to Kummer's function (1.1.8). We see that the irregular singularity at ∞ of Kummer's equation is formed by the confluence of two regular singularities at b and ∞ .

1.2 Two solutions of Kummer's equation

Let us suppose that one solution of Kummer's equation is

$$y = a_0 x^c + a_1 x^{c+1} + a_2 x^{c+2} + \dots + a_n x^{c+n} + \dots \tag{1.2.1}$$

If we substitute this series and its first two derivatives in the differential equation, and then equate to zero the coefficients of powers of x , we find that

$$\begin{aligned} a_0 c(c+b-1) &= 0, \\ a_1(c+1)(c+b) &= a_0(c+a), \\ a_2(c+2)(c+b+1) &= a_1(c+a+1), \\ &\dots \end{aligned}$$

and in general
$$a_n(c+n)(c+b+n-1) = a_{n-1}(c+a+n-1). \tag{1.2.2}$$

The first of these recurrence relations is the indicial equation. Since $a_0 \neq 0$, it has two roots:

(i) $c = 0$, which gives one solution in terms of Kummer's series,

$$y = a_0 {}_1F_1[a; b; x], \tag{1.2.3}$$

provided that b is neither zero nor a negative integer; and

(ii) $c = 1 - b$, which leads to a second solution

$$y = a_0 x^{1-b} {}_1F_1[1+a-b; 2-b; x], \tag{1.2.4}$$

provided that $2 - b$ is not zero nor a negative integer, that is, provided that $b \neq 2, 3, 4, \dots$

There is thus no difficulty in defining either solution unless b is an integer. If b is an integer $\neq 1$, and a is also an integer, then one solution reduces to a finite polynomial in x , provided that $b \leq a \leq 0$, or that $b - 1 \geq a \geq 1$, since in both these cases the numerators of the terms of the function vanish before the denominators vanish.

This second solution to Kummer's equation can also be deduced from the second solution of Gauss's equation

$$x^{1-c} {}_2F_1[1+a-c, 1+b-c; 2-c; x]$$

by putting x/b for x and letting $b \rightarrow \infty$, as before.

The function ${}_1F_1[a; b; x]$ is an analytic function of x , that is to say, it is one-valued and differentiable for all values of x real or complex. It is also an analytic function of a , but it is not an analytic function of b , for, when b is zero or a negative integer, the function has simple poles at $b = 0, -1, -2, \dots$

However, since

$$\left. \begin{aligned} \lim_{b \rightarrow 1-m} \frac{1}{\Gamma(b+n)} &= 0, \quad \text{for } n = 0, 1, 2, \dots, m-1, \\ &= \frac{1}{(n-m)!}, \quad \text{for } n = m, m+1, m+2, \dots, \end{aligned} \right\} \tag{1.2.5}$$

we have

$$\lim_{b \rightarrow 1-n} \frac{{}_1F_1[a; b; x]}{\Gamma(b)} = \frac{(a)_n x^n}{n!} {}_1F_1[a+n; 1+n; x], \quad (1.2.6)$$

and so the modified solution

$$\frac{{}_1F_1[a; b; x]}{\Gamma(b)}$$

is an analytic function of b as well as of a and x , and is defined for all values of a , b and x real or complex. The special symbol

$$\Phi^*(a; b; x)$$

is used by Tricomi (1954) to indicate this modified form of the function.

The corresponding result for the solution y_2 is

$$\lim_{b \rightarrow 1+n} \frac{x^{1-b} {}_1F_1[1+a-b; 2-b; x]}{\Gamma(2-b)} = \frac{(1-a)_n (-1)^n}{n!} {}_1F_1[a; 1+n; x]. \quad (1.2.7)$$

Thus, apart from the special cases noted above, when both a and b are integers, there is only one solution of Kummer's equation in terms of functions of the type ${}_1F_1[a; b; x]$ when b is an integer.

1.2.1 Two further solutions of Kummer's equation

In Kummer's equation let $y = x^g e^{hx} Y$, and $x = fX$, where f , g and h are independent of x . Then the transformed equation is

$$X \frac{d^2 Y}{dX^2} + \{b + 2g - fX(1-2h)\} \frac{dY}{dX} + \left\{ \frac{g(g+b-1)}{X} - f(a-hb+g-2hg) + f^2 h(h-1) X \right\} Y = 0. \quad (1.2.8)$$

This equation reduces to one of the same form as Kummer's equation if $f(1-2h) = 1$, and

$$g = 0, \quad h = 0,$$

or

$$g = 0, \quad h = 1,$$

or

$$g = 1-b, \quad h = 0,$$

or

$$g = 1-b, \quad h = 1.$$

There are thus four ways of transforming Kummer's equation into an equation of the same form, including the identity transformation. The four transforms are

$$y = Y, \quad x = X, \quad (1.2.9)$$

$$y = e^x Y, \quad x = -X, \quad (1.2.10)$$

$$y = x^{1-b} Y, \quad x = X, \quad (1.2.11)$$

and

$$y = x^{1-b} e^x Y, \quad x = -X. \quad (1.2.12)$$

Hence we expect four possible forms of solution y_1, y_2, y_3, y_4 , but these need not all be linearly independent of one another. If y_m and y_n are any two linearly independent solutions, however, the complete solution of Kummer's equation is of the form

$$y = Ay_m + By_n, \quad (1.2.13)$$

where A and B are arbitrary constants which may depend on a and b but not on x .

Under the transforms (1.2.9) and (1.2.11) we find as before that

$$y_1 = {}_1F_1[a; b; x] \quad (1.2.14)$$

and

$$y_2 = x^{1-b} {}_1F_1[1+a-b; 2-b; x] \quad (1.2.15)$$

are two solutions of Kummer's equation.

Under the transforms (1.2.10) and (1.2.12) it follows that

$$y_3 = e^x {}_1F_1[b-a; b; -x] \quad (1.2.16)$$

and

$$y_4 = x^{1-b} e^x {}_1F_1[1-a; 2-b; -x] \quad (1.2.17)$$

are also solutions of Kummer's equation.

1.3 The second form of solutions of Kummer's equation

We define an alternative form of solution of Kummer's equation thus

$$y_5 = U(a; b; x) = \frac{\Gamma(1-b)}{\Gamma(1+a-b)} {}_1F_1[a; b; x] + \frac{\Gamma(b-1)}{\Gamma(a)} x^{1-b} {}_1F_1[1+a-b; 2-b; x]. \quad (1.3.1)$$

The fact that this is a solution can be verified by (1.2.13) with $y_m = y_1$, $y_n = y_2$. This solution is important because (§4.1.1) it remains finite as $x \rightarrow \infty$. There are many other notations for this type of solution. Tricomi (1954) uses $\Psi(a; b; x)$ with $\Phi(a; b; x)$ for his first type of solution, Meixner (1933) has

$$F_1(a, b, x) e^{-i\pi a} \frac{\Gamma(b-a)}{\Gamma(b)},$$

MacRobert (1942) has

$$\frac{x^{-a} E(a, 1+a-b; : x)}{\Gamma(a) \Gamma(1+a-b)},$$

and Magnus and Oberhettinger (1948) use

$$\overset{\infty}{V}(a, b, x).$$

Other notations are

$$G(a, b, x),$$

when $M(a; b; x)$ is the first type of solution, used by Tricomi, in his early papers, and

$$x^{-a} {}_2F_0[a, 1+a-b; ; -1/x],$$

which, as we shall see later, is in fact the asymptotic expansion of the function for large x .

Under the transformations (1.2.10), (1.2.11) and (1.2.12) we deduce three further forms of solution

$$y_6 = x^{1-b} U(1+a-b; 2-b; x), \quad (1.3.2)$$

$$y_7 = e^x U(b-a; b; -x) \quad (1.3.3)$$

and

$$y_8 = e^x x^{1-b} U(1-a; 2-b; -x). \quad (1.3.4)$$

The function $U(a; b; x)$ is a many-valued function of x , for complex values of a , b and x . We shall take as its principal branch that which lies in the x -plane cut along the negative real axis. This function is analytic for all values of a , b and x real or complex even when b is zero or a negative integer, for in these cases we can express the function in the form

$$U(a; b; x) = \frac{\pi}{\sin \pi b} \left\{ \frac{{}_1F_1[a; b; x]}{\Gamma(1+a-b) \Gamma(b)} - \frac{x^{1-b} {}_1F_1[1+a-b; 2-b; x]}{\Gamma(a) \Gamma(2-b)} \right\}, \quad (1.3.5)$$

which can be defined in the limit when $b \rightarrow \pm n$ or 0 . This expression follows from the definition of $U(a; b; x)$ in terms of ${}_1F_1[a; b; x]$ and the results (1.2.6) and (1.2.7).

1.4 Kummer's first theorem

The first theorem of Kummer states that

$$e^{-x} {}_1F_1[a; b; x] = {}_1F_1[b-a; b; -x], \quad (1.4.1)$$

for the coefficient of x^n on the left is

$$\sum_{r=0}^n \frac{(a)_r (-1)^{n-r}}{(b)_r r! (n-r)!} = \frac{(-1)^n}{n!} \sum_{r=0}^n \frac{(a)_r (-n)_r}{(b)_r r!} = \frac{(-1)^n (b-a)_n}{n! (b)_n},$$

which is the coefficient of x^n on the right of (1.4.1). The summation is by Vandermonde's theorem (Bailey, 1935, § 1.3).

A second proof, not using Vandermonde's theorem directly, is obtained by letting $b \rightarrow \infty$ in Euler's transform

$${}_2F_1[a, b; c; x/b] = (1-x/b)^{-b} {}_2F_1[c-a, b; c; x/(x-b)] \quad (\text{Bailey, 1935, § 2.4(1)}). \quad (1.4.2)$$

Yet a third proof follows from the fact that the four functions y_1, y_2, y_3 and y_4 are all solutions of Kummer's equation. But this equation can only have two linearly independent solutions. Hence any three of these four functions must be connected by a linear relation of the form

$$y_p = Ay_q + By_r, \quad (1.4.3)$$

where A and B are independent of x . We see that y_3 is of the order of x^0 at the origin, while y_4 is of the order of x^{1-b} . Hence these two solutions are linearly independent of each other. Similarly, we have already seen that y_1 and y_2 are linearly independent of each other. Hence in the relation

$$y_1 = Ay_2 + By_3,$$

we must have $A = 0$. Now $y_1(0) = y_3(0) = 1$ and $y_1'(0) = y_3'(0) = a/b - 1$.

But the differential equation cannot have more than one such solution. Hence $B = 1$, and

$$y_1 = y_3. \quad (1.4.4)$$

In a similar way it follows that

$$y_2 = y_4, \quad (1.4.5)$$

that is $x^{1-b} {}_1F_1[1+a-b; 2-b; x] = x^{1-b} e^x {}_1F_1[1-a; 2-b; -x]$. (1.4.6)

In terms of the second solution $U(a; b; x)$, Kummer's first theorem shows that

$$y_5 = y_6 \quad (1.4.7)$$

and

$$y_7 = \exp\{-\pi i \epsilon (1-b)\} y_8, \quad (1.4.8)$$

that is

$$U(a; b; x) = x^{1-b} U(1+a-b; 2-b; x) \quad (1.4.9)$$

and

$$e^x U(b-a; b; -x) = e^x x^{1-b} U(1-a; 2-b; -x) \exp\{-\pi i \epsilon (1-b)\}, \quad (1.4.10)$$

where $\epsilon = 1$, if $\arg x > 0$, and $\epsilon = -1$ if $\arg x \leq 0$. We shall adopt this convention for ϵ throughout this work, and we write $e^{-\pi i \epsilon} x$ for $-x$ in order to diminish the possibility of error when dealing with a many-valued function.

1.5 The first logarithmic solutions when b is an integer

When b is an integer, the definition as expansions in series of the solutions of Kummer's equation no longer holds, and we need to consider precisely what forms y_1, y_2, y_5 and y_7 will assume when b is

an integer or zero. Returning to the methods of Frobenius, used in § 1.2, we consider first the case when $b = 1$. Kummer's equation then becomes

$$xy'' + (1-x)y' - ay = 0. \quad (1.5.1)$$

The indicial equation is $c^2 a_0 = 0$, and the recurrence relation is

$$(c+p)^2 a_p = (c+p+a-1) a_{p-1}. \quad (1.5.2)$$

The general form of the solution is

$$y(c) = a_0 x^c \left\{ 1 + \frac{(c+a)x}{(c+1)(c+1)} + \dots \right\}. \quad (1.5.3)$$

When $c = 0$ this gives the first solution y_1 , and the second solution y_2 is only a constant multiple of y_1 , instead of an independent solution, so that we have to seek another form of the solution corresponding to y_2 .

The function $y(c)$ satisfies the equation

$$xy'' + (1-x)y' - ay = a_0 c^2 x^c, \quad (1.5.4)$$

and $\partial y(c)/\partial c \equiv Y$ satisfies the equation

$$xY'' + (1-x)Y' - aY = a_0(2cx^c + c^2 x^c \log x). \quad (1.5.5)$$

This function also vanishes when $c = 0$. Hence

$$Y_2 = \left. \frac{\partial y(c)}{\partial c} \right|_{c=0} \quad (1.5.6)$$

will provide the second solution we require. Now

$$\frac{\partial y(c)}{\partial c} = a_0 \frac{\partial}{\partial c} \{u_c + u_{c+1} + \dots + u_{c+p} + \dots\}, \quad (1.5.7)$$

where

$$u_{c+p} = \frac{(c+a)_p x^{c+p}}{(c+1)_p^2} \quad (1.5.8)$$

and so,

$$\begin{aligned} \log u_{c+p} &= \log(c+a) + \log(c+a+1) + \dots + \log(c+a+p-1) \\ &\quad - 2 \log(c+1) - 2 \log(c+2) - \dots - 2 \log(c+p) + (c+p) \log x. \end{aligned} \quad (1.5.9)$$

Hence if we differentiate the logarithm, we find

$$\frac{\partial u_{c+p}}{\partial c} = u_{c+p} \left(\frac{1}{c+a} + \frac{1}{c+a+1} + \dots + \frac{1}{c+a+p-1} - \frac{2}{c+1} - \frac{2}{c+2} - \dots - \frac{2}{c+p} + \log x \right). \quad (1.5.10)$$

Thus, when $c = 0$, the second solution is

$$Y_2 = a_0 \left\{ y_1 \log x + \frac{ax}{1!1!} \left(\frac{1}{a} - \frac{2}{1} \right) + \dots + \frac{(a)_r x^r}{r!r!} \left(\frac{1}{a} + \dots + \frac{1}{a+r-1} - \frac{2}{1} - \dots - \frac{2}{r} \right) + \dots \right\}. \quad (1.5.11)$$

Next let us suppose that the roots of the indicial equation differ by an integer. There are two cases $n > 0$ and $n < 0$. In the first case, b is a negative integer $1-n$, and $a \neq 0, -1, -2, \dots, 1-n$ (see § 1.2). The indicial equation is

$$a_0 c(c-n) = 0, \quad (1.5.12)$$

and the recurrence relation is

$$(c+p)(c+p-n) a_p = (c+p+a-1) a_{p-1}, \quad (1.5.13)$$

whence we find that

$$y(c) = a_0 x^c \left\{ 1 + \frac{(c+a)x}{(c+1)(c+1-n)} + \frac{(c+a)_2 x^2}{(c+1)_2 (c+1-n)_2} + \dots \right\}. \quad (1.5.14)$$

The root $c = 0$ does not lead to a solution, since in this case the factor $(c + 1 - n)_p$ in the denominator vanishes at the $(n - 1)$ th term and the numerator cannot vanish. The root $c = n$ gives one solution y_2 .

We require to build up a function of c^2 in the indicial equation so that we can form the partial derivative in c . So let us try the function $cy(c)$ which satisfies the differential equation

$$xy'' + (1 - n - x)y' - ay = a_0c^2(c - n)x^c. \quad (1.5.15)$$

We find that the derivative with respect to c satisfies the equation

$$xy'' + (1 - n - x)y' - ay = 2a_0c(c - n)x^c + a_0c^2x^c + a_0c^2(c - n)x^c \log x, \quad (1.5.16)$$

and the expression on the right-hand side of this equation also vanishes when $c = 0$. Hence

$$\left\{ \frac{\partial(cy(c))}{\partial c} \right\}_{c=0} \equiv Y_1 \quad (1.5.17)$$

is our second solution. In fact

$$Y_1 = \sum_{r=0}^{n-1} \frac{(a)_r x^r}{(1-n)_r r!} + \frac{(a)_{n-1} x^{n-1}}{(1-n)_{n-1} (n-1)!} \sum_{r=1}^{\infty} \frac{(a+n-1)_r x^r}{(n)_r r!} \\ \times \left\{ \log x + \sum_{s=1}^{n-1+r} \frac{1}{a+s-1} - \sum_{s=1}^r \frac{1}{n-1+s} - \sum_{s=1}^{r-1} \frac{1}{s} \right\}. \quad (1.5.18)$$

For the case, $b = 1 + n$, $a \neq 1, 2, \dots, n+1$, the corresponding equations are

$$a_0c(c+n) = 0, \quad (1.5.19)$$

and

$$(c+p)(c+p+n)a_p = (c+p+a-1)a_{p-1}. \quad (1.5.20)$$

The roots of the indicial equation are

$$c = 0,$$

which leads to the solution y_1 , and

$$c = -n,$$

which does not lead to any solution. Again we take the second solution as

$$Y_2 = \left\{ \frac{\partial((c+n)y(c))}{\partial c} \right\}_{c=-n} \quad (1.5.21)$$

and find that

$$Y_2 = \sum_{r=0}^{n-1} \frac{(a-n)_r x^{r-n}}{(1-n)_r r!} + \sum_{r=0}^{\infty} \frac{(a-n)_{n+r} x^{r-1}}{(1-n)_{n-1} (1)_{n+r} r!} \\ \times \left(\log x + \sum_{s=1}^{n+r-1} \frac{1}{a+s-n-1} - \sum_{s=1}^r \frac{1}{s+n-1} - \sum_{s=1}^{r-1} \frac{1}{s} \right). \quad (1.5.22)$$

The second form of solution was stated incorrectly for a number of years (Airey and Webb, 1918), the correct forms being given by Archibald (1938).

1.5.1 The second logarithmic solutions when b is an integer

The corresponding results for the solutions y_5 and y_7 follow from the definition of these functions in terms of y_1 and y_2 . Thus if $c = -n$, $b = 1 + n$,

and

$$Y_5 = U(a; 1 + n; x), \quad (1.5.23)$$

then

$$Y_5 = \frac{(-1)^{n-1}}{n! \Gamma(a-n)} \sum_{r=0}^{\infty} \frac{(a)_r x^r}{(1+n)_r r!} \{ \log x + \Psi'(a+r) - \Psi'(1+r) - \Psi'(1+n+r) \} \\ + \frac{(n-1)!}{\Gamma(a)} \sum_{r=0}^{n-1} \frac{(a-n)_r x^{r-n}}{(1-n)_r r!}. \quad (1.5.24)$$

This last sum is zero for $n = 0$, and was omitted altogether by Webb and Airey. Here

$$\Psi'(a) = \frac{d\Gamma(a)}{da} / \Gamma(a) = \lim_{n \rightarrow \infty} \left(\log n - \frac{1}{a} - \frac{1}{a+1} - \dots - \frac{1}{a+n} \right), \quad (1.5.25)$$

(Jeffreys and Jeffreys, 1950, §23.051 and Miller, 1957). With this definition of $U(a; 1+n; x)$, the other solution is

$$Y_7 = e^x U(1+n-a; 1+n; -x), \quad (1.5.26)$$

and, if $c = 0$, so that $b = 1-n$, the two solutions are

$$Y_5 = U(a; 1-n; x) = x^n U(a+n; 1+n; x) \quad (1.5.27)$$

and
$$Y_7 = e^x U(1-a-n; 1-n; -x) = e^x (-x)^n U(1-a; 1+n; -x), \quad (1.5.28)$$

using (1.4.9) and (1.4.10).

1.6 Whittaker's normalized equation

In Kummer's equation, let us put

$$y = \exp\left\{\frac{1}{2} \int \left(1 - \frac{b}{x}\right) dx\right\} z = x^{-\frac{1}{2}b} e^{\frac{1}{2}x} z, \quad (1.6.1)$$

so that the coefficient of dy/dx becomes zero. The equation then assumes the normalized form

$$\frac{d^2 z}{dx^2} + \left(-\frac{1}{4} + \frac{k}{x} + \frac{\frac{1}{4} - m^2}{x^2}\right) z = 0, \quad (1.6.2)$$

where $m = \frac{1}{2}b - \frac{1}{2}$ and $k = \frac{1}{2}b - a$. The inverse transform from Whittaker's equation to Kummer's equation is

$$z = x^{\frac{1}{2}b} e^{-\frac{1}{2}x} y, \quad (1.6.3)$$

where $a = \frac{1}{2} + m - k$ and $b = 1 + 2m$.

This equation was studied first by Whittaker (1904) and one of its solutions is Whittaker's function

$$z_1 = M_{k,m}(x) = x^{\frac{1}{2}+m} e^{-\frac{1}{2}x} {}_1F_1\left[\frac{1}{2} + m - k; 1 + 2m; x\right]. \quad (1.6.4)$$

Several notations are in use for this function. Erdélyi (1953) defines the function with μ and κ in place of the present m and k . Buchholz (1953) has $\frac{1}{2}\mu$ for m , and he has also used the symbol $m^{(\rho)}(x)$, (1943), where

$$\sqrt{(2x/\pi)} m_k^{(2\rho)}(x) = M_{k,\rho}(x).$$

Any general solution of Whittaker's equation is called a Whittaker function, and in this work we shall denote such a general solution by the symbol

$$\mathfrak{W}_{k,m}(x).$$

We see from the above transformation that any Whittaker function can be expressed in terms of a confluent hypergeometric function, so that in general there exist relations of the form

$$\mathfrak{F}[a; b; x] = e^{\frac{1}{2}x} x^{-\frac{1}{2}b} \mathfrak{W}_{\frac{1}{2}b-a, \frac{1}{2}b-\frac{1}{2}}(x) \quad (1.6.5)$$

between the two types of function.

The function $M_{k,m}(x)$ is analytic everywhere except at the points $2m = -1, -3, -5, \dots$, where it has simple poles. At these points, however, the function

$$\frac{M_{k,m}(x)}{\Gamma(1+2m)} \quad (1.6.6)$$

is analytic. Two alternative notations for this analytic function are

$$\mathfrak{M}_{k,m}(x) \quad (\text{Buchholz, 1953}),$$

and

$$x^{\frac{1}{2}-m}N_{k,m}(x) \quad (\text{Erdélyi, 1936}).$$

For x complex, $M_{k,m}(x)$ is a many-valued function of x . We shall take as its principal branch that which lies in the x -plane, cut along the negative real axis.

The singularities of Whittaker's equation are a regular singularity at $x = 0$ and an irregular singularity at $x = \infty$, so that, as for Kummer's equation, we may assume a solution near the origin of the form

$$a_0x^c + a_1x^{c+1} + a_2x^{c+2} + \dots + a_nx^{c+n} + \dots, \quad (1.6.7)$$

($a_0 \neq 0$) and apply Frobenius's process. The indicial equation is now

$$a_0(c^2 - c + \frac{1}{4} - m^2) = 0, \quad (1.6.8)$$

and the recurrence relation between the coefficients is

$$(c+n+2)(c+n+1)a_{n+2} - \frac{1}{4}a_n + ka_{n+1} + (\frac{1}{4} - m^2)a_{n+2} = 0. \quad (1.6.9)$$

Provided that $2m \neq -1, -3, -5, \dots$, the root $c = \frac{1}{2} + m$ of the indicial equation leads to the solution

$$M_{k,m}(x) = z_1, \quad (1.6.10)$$

and, provided that $2m \neq 1, 3, 5, \dots$, the root $c = \frac{1}{2} - m$ leads to a second solution

$$M_{k,-m}(x) = z_2. \quad (1.6.11)$$

We can also arrive at these two solutions, z_1 and z_2 , by the transform (1.6.3) by rewriting the solutions y_1 and y_2 of Kummer's equation in terms of the Whittaker functions $M_{k,m}(x)$. Similarly, if we express the solutions y_3 and y_4 in terms of Whittaker functions, we see that

$$z_3 = M_{-k,m}(-x) \quad (1.6.12)$$

and

$$z_4 = M_{-k,-m}(-x) \quad (1.6.13)$$

are also solutions of Whittaker's equation. We can verify that these are solutions by substituting the functions and their derivatives directly in Whittaker's equation, or alternatively, we can arrive at the same four solutions by considering under what conditions the general equation (1.2.8) will reduce to an equation of Whittaker's type.

1.7 An alternative solution for Whittaker's equation

We define the second Whittaker function as

$$W_{k,m}(x) = \frac{\pi}{\sin 2m\pi} \left\{ \frac{-M_{k,m}(x)}{\Gamma(\frac{1}{2} - m - k)\Gamma(1 + 2m)} + \frac{M_{k,-m}(x)}{\Gamma(\frac{1}{2} + m - k)\Gamma(1 - 2m)} \right\} \quad (1.7.1)$$

for all values real or complex of k , m and x . It is a many-valued function of x , so again we shall take as its principal branch that which lies in the x -plane cut along the negative real axis. That this function is in fact a solution of Whittaker's equation can be verified by substituting the function and its second derivative in Whittaker's equation.

Another notation for this function is

$$\sqrt{\frac{2x}{\pi}} w_k^{(2\rho)}(x)$$

used by Buchholz (1943).

If we rewrite the four functions y_5, y_6, y_7 , and y_8 in terms of Whittaker functions and use the transform (1.6.3) we arrive at the four further solutions of Whittaker's equation,

$$z_5 = W_{k,m}(x), \quad (1.7.2)$$

$$z_6 = W_{k,-m}(x), \quad (1.7.3)$$

$$z_7 = W_{-k,m}(-x), \quad (1.7.4)$$

and
$$z_8 = W_{-k,-m}(-x). \quad (1.7.5)$$

Again this form of solution is chosen because of its simple behaviour when $x \rightarrow \infty$.

Kummer's first theorem shows that the eight solutions of Whittaker's equation are connected by the following relations:

$$z_1 = \exp\{\epsilon i\pi(\frac{1}{2} + m)\} z_3, \quad (1.7.6)$$

that is
$$M_{k,m}(x) = \exp\{\epsilon i\pi(\frac{1}{2} + m)\} M_{-k,m}(-x), \quad (1.7.7)$$

$$z_2 = \exp\{\epsilon i\pi(\frac{1}{2} - m)\} z_4, \quad (1.7.8)$$

that is
$$M_{k,-m}(x) = \exp\{\epsilon i\pi(\frac{1}{2} - m)\} M_{-k,-m}(-x), \quad (1.7.9)$$

$$z_5 = z_6, \quad (1.7.10)$$

that is
$$W_{k,m}(x) = W_{k,-m}(x), \quad (1.7.11)$$

and
$$z_7 = z_8, \quad (1.7.12)$$

that is
$$W_{-k,m}(-x) = W_{-k,-m}(-x). \quad (1.7.13)$$

1.7.1 The logarithmic solutions of Whittaker's equation when $2m$ is an integer

If $1 + 2m$ is an integer and $\frac{1}{2} + m - k$ is also an integer, one solution of Whittaker's equation reduces to a multiple of a finite polynomial in x , provided that $1 + 2m \leq \frac{1}{2} + m - k \leq 0$, or that $2m \geq \frac{1}{2} + m - k \geq 1$, since in both these cases the numerators of the series vanish before the denominators vanish. But if $\frac{1}{2} + m - k$ is not such an integer, the two solutions are, when $m = \frac{1}{2}n - \frac{1}{2}$ and n is a positive integer,

$$(y(c))_{c=0} = M_{k, \frac{1}{2}n - \frac{1}{2}}(x), \quad (1.7.14)$$

and
$$\left\{ \frac{\partial c y(c)}{\partial c} \right\}_{c=0} = x^{\frac{1}{2}n} e^{-\frac{1}{2}x} \sum_{r=0}^{n-2} x^{1-n+r} \frac{(1 - \frac{1}{2}n - k)_r}{(2-n)_r r!} + x^{\frac{1}{2}n} e^{-\frac{1}{2}x} \sum_{r=0}^{\infty} \frac{(1 - \frac{1}{2}n - k)_{n+r-1} x^r}{(2-n)_{n-2} (1)_{n+r-1} r!}$$

$$\times \left(\log x + \sum_{s=1}^{n+r-2} \frac{1}{1+s-k-\frac{1}{2}n} - \sum_{s=1}^r \frac{1}{s+r-1} - \sum_{s=1}^{r-1} \frac{1}{s} \right). \quad (1.7.15)$$

By the symmetry of the solutions in m , the same two solutions hold when $m = -\frac{1}{2}n - \frac{1}{2}$.

For the Whittaker functions of the second kind, the two solutions are, when $m = \frac{1}{2}n - \frac{1}{2}$,

$$z_5 = W_{k,m}(x) \quad (1.7.16)$$

$$= (-1)^{2m} \frac{\partial}{\partial m} \left\{ \frac{M_{k,-m}(x)}{\Gamma(\frac{1}{2} + m - k) \Gamma(1 - 2m)} - \frac{M_{k,m}(x)}{\Gamma(\frac{1}{2} - m - k) \Gamma(1 + 2m)} \right\} \quad (1.7.17)$$

$$= \frac{(-1)^{2m+1} M_{k,m}(x) \log x}{\Gamma(\frac{1}{2} - m - k) \Gamma(1 + 2m)} + \frac{(-1)^{2m+1} x^{\frac{1}{2}+m} e^{-\frac{1}{2}x}}{\Gamma(\frac{1}{2} - m - k)}$$

$$\times \left[\sum_{r=0}^{\infty} x^r \frac{(\frac{1}{2} + m - k)_r}{(1)_{2m+r} r!} \{ \Psi(\frac{1}{2} + m - k + r) - \Psi(1 + r) - \Psi(1 + 2m + r) \} \right.$$

$$\left. - \sum_{r=1}^{2m} \frac{(r-1)! x^{-r}}{(1)_{2m-r} (\frac{1}{2} - m + k)_r} \right], \quad (1.7.18)$$

and
$$z_7 = W_{-k,m}(-x). \quad (1.7.19)$$

Again, by symmetry, the same solutions hold when $m = -\frac{1}{2}n - \frac{1}{2}$.

1.8 Kummer's second theorem

In the standard notation for Bessel functions (§ 3.7, Watson, 1948), we have, as the definition of the Bessel function $I_a(x)$,

$${}_0F_1[; a + \frac{1}{2}; x^2/16] = (\frac{1}{4}x)^{\frac{1}{2}-a} \Gamma(a + \frac{1}{2}) I_{a-\frac{1}{2}}(\frac{1}{2}x). \quad (1.8.1)$$

We now show that
$${}_1F_1[a; 2a; x] = e^{\frac{1}{2}x} {}_0F_1[; a + \frac{1}{2}; x^2/16] \quad (1.8.2)$$

that is
$${}_1F_1[a; 2a; x] = e^{\frac{1}{2}x} (\frac{1}{4}x)^{\frac{1}{2}-a} \Gamma(a + \frac{1}{2}) I_{a-\frac{1}{2}}(\frac{1}{2}x), \quad (1.8.3)$$

for $2a$ not zero nor a negative integer.

Let
$$e^{-\frac{1}{2}x} {}_1F_1[a; 2a; x] = \sum_{t=0}^{\infty} u_t x^t.$$

Then, expanding in powers of x , we find that

$$\begin{aligned} u_t &= \sum_{s=0}^t \frac{(a)_{t-s} (-1)^s}{(2a)_{t-s} (t-s)! s! 2^s} \\ &= \frac{(a)_t}{(2a)_t t!} {}_2F_1[1-2a-t, -t; 1-a-t; \frac{1}{2}] \\ &= \frac{(a)_t}{(2a)_t t!} {}_2F_1[\frac{1}{2}-a-\frac{1}{2}t, -\frac{1}{2}t; 1-a-t; 1], \end{aligned}$$

since, by Kummer's transform (Bailey, 1935, § 2.4)

$${}_2F_1[2a, 2b; a+b+\frac{1}{2}; \frac{1}{2}] = {}_2F_1[a, b; a+b+\frac{1}{2}; 1]. \quad (1.8.4)$$

Hence

$$u_t = \frac{(a)_t (\frac{1}{2} - \frac{1}{2}t)_{\frac{1}{2}t}}{(2a)_t (1-a-t)_{\frac{1}{2}t} t!},$$

summing by Vandermonde's theorem for t even, and

$$u_t = 0,$$

for t odd. So

$$u_{2n} = \frac{(a)_{2n} (\frac{1}{2} - n)_n}{(2a)_{2n} (1-a-2n)_n 2n!} = \frac{1}{(a + \frac{1}{2})_n n! 16^n},$$

and we have
$$e^{-\frac{1}{2}x} {}_1F_1[a; 2a; x] = \sum_{n=0}^{\infty} u_{2n} x^{2n} = {}_0F_1[; a + \frac{1}{2}; x^2/16],$$

which is the required result.

1.8.1 Bessel functions as special cases of confluent hypergeometric functions

If x is purely imaginary the theorem becomes

$$\begin{aligned} {}_1F_1[a; 2a; 2ix] &= e^{ix} {}_0F_1[; a + \frac{1}{2}; -x^2/4] \\ &= e^{ix} (\frac{1}{2}x)^{\frac{1}{2}-a} \Gamma(a + \frac{1}{2}) J_{a-\frac{1}{2}}(x). \end{aligned} \quad (1.8.5)$$

In terms of the function $U(a; b; x)$, Kummer's second theorem becomes

$$U(a; 2a; x) = \frac{\Gamma(1-2a)\Gamma(a+\frac{1}{2})}{\Gamma(1-a)} e^{\frac{1}{2}x} \left(\frac{1}{4}x\right)^{\frac{1}{2}-a} I_{a-\frac{1}{2}}\left(\frac{1}{2}x\right) + \frac{\Gamma(2a-1)\Gamma(3/2-a)}{\Gamma(a)} e^{\frac{1}{2}x} \left(\frac{1}{4}x\right)^{a-\frac{1}{2}} I_{\frac{1}{2}-a}\left(\frac{1}{2}x\right), \quad (1.8.6)$$

that is, in the standard notation for Bessel functions,

$$U(a; 2a; 2x) = \frac{(2x)^{\frac{1}{2}-a}}{\sqrt{\pi}} e^x K_{a-\frac{1}{2}}(x). \quad (1.8.7)$$

For a purely imaginary variable this gives

$$U(a; 2a; -2ix) = \frac{1}{2}\sqrt{\pi} \exp(\pi ia - ix) H_{a-\frac{1}{2}}^{(1)}(x) (2x)^{\frac{1}{2}-a}, \quad (1.8.8)$$

and

$$U(a; 2a; 2ix) = \frac{1}{2}\sqrt{\pi} \exp(ix - \pi ia) H_{a-\frac{1}{2}}^{(2)}(x) (2x)^{\frac{1}{2}-a}, \quad (1.8.9)$$

whence it follows that

$$Y_{a-\frac{1}{2}}(x) = \frac{(2x)^{a-\frac{1}{2}}}{\sqrt{\pi}} \{ \exp(\pi ia + \frac{1}{2}\pi i - ix) U(a; 2a; 2ix) + \exp(ix - \pi ia - \frac{1}{2}\pi i) U(a; 2a; -2ix) \}. \quad (1.8.10)$$

In terms of Whittaker's functions, Kummer's second theorem gives the following results,

$$M_{0,m}(2x) = \Gamma(1+m) 2^{2m+\frac{1}{2}} x^{\frac{1}{2}} I_m(x), \quad (1.8.11)$$

$$W_{0,m}(x) = \sqrt{\frac{x}{\pi}} K_m\left(\frac{1}{2}x\right), \quad (1.8.12)$$

$$M_{0,m}(\pm 2ix) = \Gamma(1+m) \exp\left\{\pm \frac{1}{2}\pi i\left(\frac{1}{2}+m\right)\right\} 2^{2m+\frac{1}{2}} x^{\frac{1}{2}} J_m(x), \quad (1.8.13)$$

$$W_{0,m}(2ix) = \sqrt{\left(\frac{1}{2}\pi x\right)} \exp\left\{-\frac{1}{4}i\pi(2m+1)\right\} H_m^{(2)}(x), \quad (1.8.14)$$

and

$$W_{0,m}(-2ix) = \sqrt{\left(\frac{1}{2}\pi x\right)} \exp\left\{\frac{1}{4}i\pi(2m+1)\right\} H_m^{(1)}(x). \quad (1.8.15)$$

1.9 Relations between Kummer's functions and Whittaker's functions

For convenience we repeat here the definitions of the three other confluent hypergeometric functions in terms of Kummer's function. From these definitions (1.9.4), (1.9.7) and (1.9.12), the remaining nine relations are deduced.

$${}_1F_1[a; b; x] = e^{\epsilon i\pi a} \frac{\Gamma(b) U(a; b; x)}{\Gamma(b-a)} + \exp\{\epsilon i\pi(a-b)\} e^x \frac{\Gamma(b) U(b-a; b; -x)}{\Gamma(a)}, \quad (1.9.1)$$

$$= e^{\frac{1}{2}x} x^{-\frac{1}{2}b} M_{\frac{1}{2}b-a, \frac{1}{2}b-\frac{1}{2}}(x), \quad (1.9.2)$$

$$= e^{\frac{1}{2}x} x^{-\frac{1}{2}b} \left[\exp\left\{-\epsilon i\pi\left(\frac{1}{2}b-a\right)\right\} \frac{\Gamma(b) W_{a-\frac{1}{2}b, \frac{1}{2}b-\frac{1}{2}}(-x)}{\Gamma(a)} + e^{\epsilon i\pi a} \frac{\Gamma(b) W_{\frac{1}{2}b-a, \frac{1}{2}b-\frac{1}{2}}(x)}{\Gamma(b-a)} \right], \quad (1.9.3)$$

$$U(a; b; x) = \frac{\Gamma(1-b) {}_1F_1[a; b; x]}{\Gamma(1+a-b)} + \frac{\Gamma(b-1) x^{1-b} {}_1F_1[1+a-b; 2-b; x]}{\Gamma(a)}, \quad (1.9.4)$$

$$= e^{\frac{1}{2}x} x^{-\frac{1}{2}b} \left\{ \frac{\Gamma(1-b) M_{\frac{1}{2}b-a, \frac{1}{2}b-\frac{1}{2}}(x)}{\Gamma(1+a-b)} + \frac{\Gamma(b-1) M_{\frac{1}{2}b-a, \frac{1}{2}-\frac{1}{2}b}(x)}{\Gamma(a)} \right\}, \quad (1.9.5)$$

$$= e^{\frac{1}{2}x} x^{-\frac{1}{2}b} W_{\frac{1}{2}b-a, \frac{1}{2}b-\frac{1}{2}}(x), \quad (1.9.6)$$

$$M_{k,m}(x) = e^{-\frac{1}{2}x} x^{\frac{1}{2}+m} {}_1F_1\left[\frac{1}{2}+m-k; 1+2m; x\right], \quad (1.9.7)$$

$$\begin{aligned} &= \Gamma(1+2m) e^{-\frac{1}{2}x} x^{\frac{1}{2}+m} \left[\exp\{ci\pi(\frac{1}{2}+m-k)\} \frac{U(\frac{1}{2}+m-k; 1+2m; x)}{\Gamma(\frac{1}{2}+m+k)} \right. \\ &\quad \left. + e^x \exp\{-\epsilon\pi i(\frac{1}{2}+m+k)\} \frac{U(\frac{1}{2}+m+k; 1+2m; -x)}{\Gamma(\frac{1}{2}+m-k)} \right], \end{aligned} \quad (1.9.8)$$

$$\begin{aligned} &= \Gamma(1+2m) \left[\exp\{ci\pi(\frac{1}{2}+m-k)\} \frac{W_{k,m}(x)}{\Gamma(\frac{1}{2}+m+k)} \right. \\ &\quad \left. + \exp\{-\epsilon i\pi k\} \frac{W_{-k,m}(-x)}{\Gamma(\frac{1}{2}+m-k)} \right], \end{aligned} \quad (1.9.9)$$

$$\begin{aligned} W_{k,m}(x) &= \Gamma(-2m) \frac{{}_1F_1[\frac{1}{2}+m-k; 1+2m; x]}{\Gamma(\frac{1}{2}-m-k)} e^{-\frac{1}{2}x} x^{m+\frac{1}{2}} \\ &\quad + \Gamma(2m) \frac{{}_1F_1[\frac{1}{2}-m-k; 1-2m; x]}{\Gamma(\frac{1}{2}+m-k)} e^{-\frac{1}{2}x} x^{\frac{1}{2}-m}, \end{aligned} \quad (1.9.10)$$

$$= e^{-\frac{1}{2}x} x^{m+\frac{1}{2}} U(\frac{1}{2}+m-k; 1+2m; x), \quad (1.9.11)$$

$$= \frac{\Gamma(-2m) M_{k,m}(x)}{\Gamma(\frac{1}{2}-m-k)} + \frac{\Gamma(2m) M_{k,-m}(x)}{\Gamma(\frac{1}{2}+m-k)}, \quad (1.9.12)$$

where

$$\begin{aligned} \epsilon &= \text{sign}(\text{Im } x) = +1, & \text{if } \arg x > 0, \\ &= -1, & \text{if } \arg x \leq 0. \end{aligned}$$

DIFFERENTIAL PROPERTIES

2.1 The differentiation of Kummer's function

Since the series ${}_1F_1[a; b; x]$ is absolutely convergent we can differentiate it term by term. We find that

$$\begin{aligned} \frac{d}{dx} \{ {}_1F_1[a; b; x] \} &= \sum_{n=1}^{\infty} \frac{a(a+1)_{n-1} n x^{n-1}}{b(b+1)_{n-1} n(n-1)!} \\ &= \frac{a}{b} {}_1F_1[a+1; b+1; x], \end{aligned} \quad (2.1.1)$$

and in general
$$\frac{d^n}{dx^n} \{ {}_1F_1[a; b; x] \} = \frac{(a)_n}{(b)_n} {}_1F_1[a+n; b+n; x], \quad (2.1.2)$$

for $n = 1, 2, 3, \dots$. In the same way we have

$$\frac{d}{dx} \{ x^a {}_1F_1[a; b; x] \} = a x^{a-1} {}_1F_1[a+1; b; x], \quad (2.1.3)$$

and in general
$$\frac{d^n}{dx^n} \{ x^{a+n-1} {}_1F_1[a; b; x] \} = (a)_n x^{a-1} {}_1F_1[a+n; b; x], \quad (2.1.4)$$

also
$$\frac{d}{dx} \{ x^{b-1} {}_1F_1[a; b; x] \} = (b-1) x^{b-2} {}_1F_1[a; b-1; x], \quad (2.1.5)$$

and in general
$$\frac{d^n}{dx^n} \{ x^{b-1} {}_1F_1[a; b; x] \} = (-1)^n (1-b)_n x^{b-1-n} {}_1F_1[a; b-n; x]. \quad (2.1.6)$$

If we apply Kummer's first theorem to (2.1.2), (2.1.4) and (2.1.6), we find three further results.

$$\frac{d^n}{dx^n} \{ e^{-x} {}_1F_1[a; b; x] \} = \frac{(-1)^n (b-a)_n}{(b)_n} {}_1F_1[b-a+n; b+n; -x], \quad (2.1.7)$$

$$= \frac{(-1)^n (b-a)_n}{(b)_n} e^{-x} {}_1F_1[a; b+n; x], \quad (2.1.8)$$

$$\frac{d^n}{dx^n} \{ e^{-x} x^{b-a+n-1} {}_1F_1[a; b; x] \} = (b-a)_n x^{b-a-1} {}_1F_1[b-a+n; b; -x], \quad (2.1.9)$$

$$= (b-a)_n e^{-x} x^{b-a-1} {}_1F_1[a-n; b; x], \quad (2.1.10)$$

$$\frac{d^n}{dx^n} \{ e^{-x} x^{b-1} {}_1F_1[a; b; x] \} = (-1)^n (1-b)_n x^{b-1-n} {}_1F_1[b-a; b-n; -x], \quad (2.1.11)$$

$$= (-1)^n (1-b)_n e^{-x} x^{b-n-1} {}_1F_1[a-n; b-n; x]. \quad (2.1.12)$$

In particular, when $n = 1$, we have

$${}_1F_1[a+1; b; x] = \frac{1}{a} x^{1-a} \frac{d}{dx} \{ x^a {}_1F_1[a; b; x] \}, \quad (2.1.13)$$

$$= {}_1F_1[a; b; x] + \frac{x}{a} {}_1F_1'[a; b; x], \quad (2.1.14)$$

$${}_1F_1[a-1; b; x] = \frac{1}{b-a} e^x x^{1+a-b} \frac{d}{dx} \{ e^{-x} x^{b-a} {}_1F_1[a; b; x] \}, \quad (2.1.15)$$

$$= \left(1 - \frac{x}{b-a} \right) {}_1F_1[a; b; x] + \frac{x}{b-a} {}_1F_1'[a; b; x], \quad (2.1.16)$$

$${}_1F_1[a; b+1; x] = \frac{-b}{b-a} e^x \frac{d}{dx} \{e^{-x} {}_1F_1[a; b; x]\}, \quad (2.1.17)$$

$$= \frac{b}{b-a} {}_1F_1[a; b; x] - \frac{b}{b-a} {}_1F_1'[a; b; x], \quad (2.1.18)$$

$${}_1F_1[a; b-1; x] = \frac{1}{b-1} x^{2-b} \frac{d}{dx} \{x^{b-1} {}_1F_1[a; b; x]\}, \quad (2.1.19)$$

$$= {}_1F_1[a; b; x] + \frac{x}{b-1} {}_1F_1'[a; b; x], \quad (2.1.20)$$

$${}_1F_1[a+1; b+1; x] = \frac{b}{a} {}_1F_1'[a; b; x], \quad (2.1.21)$$

$${}_1F_1[a-1; b-1; x] = \frac{e^x x^{2-b}}{b-1} \frac{d}{dx} \{e^{-x} x^{b-1} {}_1F_1[a; b; x]\}, \quad (2.1.22)$$

$$= \frac{b-1-x}{b-1} {}_1F_1[a; b; x] + \frac{x}{b-1} {}_1F_1'[a; b; x]. \quad (2.1.23)$$

2.1.1 The derivatives of $U(a; b; x)$

If we differentiate $U(a; b; x)$ term by term using its definition (1.3.1), we find that

$$\frac{d}{dx} \{U(a; b; x)\} = -aU(a+1; b+1; x), \quad (2.1.24)$$

and, in general,
$$\frac{d^n}{dx^n} \{U(a; b; x)\} = (-1)^n (a)_n U(a+n; b+n; x). \quad (2.1.25)$$

Also
$$\frac{d}{dx} \{x^a U(a; b; x)\} = a(1+a-b) x^{a-1} U(a+1; b; x), \quad (2.1.26)$$

with, in general,
$$\frac{d^n}{dx^n} \{x^{a+n-1} U(a; b; x)\} = (a)_n (1+a-b)_n x^{a-1} U(a+n; b; x), \quad (2.1.27)$$

and
$$\frac{d}{dx} \{x^{b-1} U(a; b; x)\} = (b-a-1) x^{b-2} U(a; b-1; x), \quad (2.1.28)$$

with, in general,
$$\frac{d^n}{dx^n} \{x^{b-1} U(a; b; x)\} = (-1)^n (1+a-b)_n x^{b-n-1} U(a; b-n; x). \quad (2.1.29)$$

Again, if we apply Kummer's first theorem to (2.1.25), (2.1.27) and (2.1.29) we find that

$$\frac{d^n}{dx^n} \{e^{-x} U(a; b; x)\} = (-1)^n e^{-x} U(a; b+n; x), \quad (2.1.30)$$

$$\frac{d^n}{dx^n} \{e^{-x} x^{b-a+n-1} U(a; b; x)\} = (-1)^n e^{-x} x^{b-a-1} U(a-n; b; x), \quad (2.1.31)$$

and
$$\frac{d^n}{dx^n} \{e^{-x} x^{b-1} U(a; b; x)\} = (-1)^n e^{-x} x^{b-n-1} (1-a)_n U(a-n; b-n; x). \quad (2.1.32)$$

In particular, when $n = 1$, we have

$$U(a+1; b+1; x) = \frac{-1}{a} U'(a; b; x), \quad (2.1.33)$$

$$U(a+1; b; x) = \frac{x^{1-a}}{a(1+a-b)} \frac{d}{dx} \{x^a U(a; b; x)\}, \quad (2.1.34)$$

$$= \frac{1}{1+a-b} U(a; b; x) + \frac{x}{a(1+a-b)} U'(a; b; x), \quad (2.1.35)$$

$$U(a; b-1; x) = \frac{-x^{2-b}}{1+a-b} \frac{d}{dx} \{x^{b-1} U(a; b; x)\}, \quad (2.1.36)$$

$$= \frac{1-b}{1+a-b} U(a; b; x) - \frac{x}{1+a-b} U'(a; b; x), \quad (2.1.37)$$

$$U(a; b+1; x) = -e^x \frac{d}{dx} \{e^{-x} U(a; b; x)\}, \quad (2.1.38)$$

$$= U(a; b; x) - U'(a; b; x), \quad (2.1.39)$$

$$U(a-1; b; x) = -e^x x^{1+a-b} \frac{d}{dx} \{e^{-x} x^{b-a} U(a; b; x)\}, \quad (2.1.40)$$

$$= (x+a-b) U(a; b; x) - x U'(a; b; x), \quad (2.1.41)$$

$$U(a-1; b-1; x) = -e^x x^{2-b} \frac{d}{dx} \{e^{-x} x^{b-1} U(a; b; x)\}, \quad (2.1.42)$$

$$= (1+x-b) U(a; b; x) - x U'(a; b; x). \quad (2.1.43)$$

2.1.2. The Wronskians of Kummer's equation

The Wronskian of a differential equation is defined as

$$K_{m,n} \equiv y_m \frac{d}{dx} y_n - y_n \frac{d}{dx} y_m, \quad (2.1.44)$$

where y_m and y_n are any two solutions of the differential equation. If this Wronskian is not zero then the two solutions of the equation, y_m and y_n are linearly independent and provide a complete solution of the equation. But if $K_{m,n} = 0$, then the two solutions y_m and y_n are not independent, and one solution is a multiple of the other. It is easy to verify from the definition that

$$K_{m,n} = -K_{n,m} \quad (2.1.45)$$

and

$$K_{m,n} y_p + K_{n,p} y_m + K_{p,m} y_n = 0. \quad (2.1.46)$$

We have already seen by Kummer's first theorem that four of the eight solutions of Kummer's equation are dependent on the other four solutions, so that we have

$$K_{1,3} = K_{2,4} = K_{5,6} = K_{7,8} = 0. \quad (2.1.47)$$

By considering the definitions in series of the remaining four functions y_1 , y_2 , y_5 and y_7 and their derivatives, we find that

$$K_{1,5} = -\frac{\Gamma(b)}{\Gamma(a)} e^x x^{-b}, \quad (2.1.48)$$

$$K_{1,7} = \frac{\Gamma(b)}{\Gamma(b-a)} e^{\epsilon i \pi b} e^x x^{-b}, \quad (2.1.49)$$

$$K_{1,2} = (1-b) e^x x^{-b}, \quad (2.1.50)$$

$$K_{2,5} = -\frac{\Gamma(2-b)}{\Gamma(1+a-b)} e^x x^{-b}, \quad (2.1.51)$$

$$K_{2,7} = -\frac{\Gamma(2-b)}{\Gamma(1-a)} e^x x^{-b}, \quad (2.1.52)$$

and

$$K_{5,7} = \exp\{\epsilon i \pi (b-a)\} e^x x^{-b}, \quad (2.1.53)$$

where $\epsilon = 1$, if $\arg x > 0$ and $\epsilon = -1$ if $\arg x \leq 0$. If we introduce constants $1/\Gamma(b)$ and $1/\Gamma(2-b)$ into the two solutions y_1 and y_2 , as we have already seen, these two solutions can be defined for all values of b including integers or zero. However, the only Wronskian which never vanishes for any values of a , b or x , except $x = 0$, is $K_{5,7}$. Hence y_5 and y_7 form a fundamental system of solutions under all circumstances. If a , b and $b-a$ are not integers nor zero, none of the above six Wronskians vanish, and so any two of the solutions y_1 , y_2 , y_5 and y_7 can be taken to form a complete system of solutions. If a is zero or a negative integer, $K_{1,5} = 0$, and y_5 is a multiple of y_1 . Similarly, if a is a positive integer, $K_{2,7} = 0$, and y_7 is a multiple of y_2 .

From (2.1.46) and their Wronskians we deduce the following relations between these four solutions of Kummer's equation,

$$y_1 = \frac{\Gamma(b)}{\Gamma(b-a)} \exp\{\epsilon i \pi a\} y_5 + \frac{\Gamma(b)}{\Gamma(a)} \exp\{\epsilon i \pi (a-b)\} y_7, \quad (2.1.54)$$

$$y_2 = -\frac{\Gamma(2-b)}{\Gamma(1-a)} \exp\{\epsilon i \pi (a-b)\} y_5 + \frac{\Gamma(2-b)}{\Gamma(1+a-b)} \exp\{\epsilon i \pi (a-b)\} y_7, \quad (2.1.55)$$

$$y_5 = \frac{\Gamma(1-b)}{\Gamma(1+a-b)} y_1 + \frac{\Gamma(b-1)}{\Gamma(a)} y_2 \quad (2.1.56)$$

and

$$y_7 = \frac{\Gamma(1-b)}{\Gamma(1-a)} y_1 - \frac{\Gamma(b-1)}{\Gamma(b-a)} \exp\{\epsilon i \pi b\} y_2. \quad (2.1.57)$$

Eight further relations between these four solutions can be deduced immediately. These are

$$y_1 = \frac{-\Gamma(b)\Gamma(1-a)}{\Gamma(b-a)\Gamma(2-b)} e^{\epsilon b \pi i} y_2 + \frac{\Gamma(1-a)}{\Gamma(1-b)} y_7, \quad (2.1.58)$$

$$y_1 = \frac{\Gamma(b)\Gamma(1+a-b)}{\Gamma(a)\Gamma(2-b)} y_2 + \frac{\Gamma(1+a-b)}{\Gamma(1-b)} y_5, \quad (2.1.59)$$

$$y_2 = \frac{\Gamma(a)\Gamma(2-b)}{\Gamma(b)\Gamma(1+a-b)} y_1 + \frac{\Gamma(a)}{\Gamma(b-1)} y_5, \quad (2.1.60)$$

$$y_2 = \frac{-\Gamma(2-b)\Gamma(b-a)}{\Gamma(b)\Gamma(1-a)} e^{-\epsilon b \pi i} y_1 - \frac{\Gamma(b-a)}{\Gamma(b-1)} e^{-\epsilon b \pi i} y_7, \quad (2.1.61)$$

$$y_5 = \frac{\Gamma(b-a)}{\Gamma(b)} e^{-\epsilon a \pi i} y_1 - \frac{\Gamma(b-a)}{\Gamma(a)} e^{-\epsilon b \pi i} y_7, \tag{2.1.62}$$

$$y_5 = -\frac{\Gamma(1-a)}{\Gamma(2-b)} \exp\{\epsilon(b-a)\pi i\} y_2 - \frac{\Gamma(1-a)}{\Gamma(1+a-b)} y_7, \tag{2.1.63}$$

$$y_7 = \frac{\Gamma(a)}{\Gamma(b)} \exp\{\epsilon(b-a)\pi i\} y_1 - \frac{\Gamma(a)}{\Gamma(b-a)} e^{\epsilon b \pi i} y_5, \tag{2.1.64}$$

$$y_7 = \frac{\Gamma(1+a-b)}{\Gamma(2-b)} \exp\{\epsilon(b-a)\pi i\} y_2 + \frac{\Gamma(1+a-b)}{\Gamma(1-a)} y_5. \tag{2.1.65}$$

A similar set of twelve results exists connecting any four solutions.

2.2 Recurrence relations for ${}_1F_1[a; b; x]$

For any confluent hypergeometric function with parameters a and b the four functions with parameters $a-1; b, a+1; b, a; b-1$, and $a; b+1$ are called the contiguous functions. It follows, from the differential equation satisfied by a confluent hypergeometric function and its derivatives, that a homogeneous linear relation exists between any such function and any two of its contiguous functions. These recurrence relations are of particular value in building up a table of a confluent hypergeometric function for a fixed value of x , when a and b are variable.

The confluent hypergeometric functions with parameters $a \pm m; b \pm n$ for $m, n, = 0, 1, 2, \dots$, are called associated functions. Again it follows from the differential equation, that a homogeneous linear relation exists between any three associated functions, so that any confluent hypergeometric function can be expressed in terms of any two of its associated functions.

The function ${}_1F_1[a; b; x]$ satisfies the following recurrence relations:

$$(b-a) {}_1F_1[a-1; b; x] + (2a-b+x) {}_1F_1[a; b; x] - a {}_1F_1[a+1; b; x] = 0, \tag{2.2.1}$$

$$b(b-1) {}_1F_1[a; b-1; x] - b(b-1+x) {}_1F_1[a; b; x] + (b-a)x {}_1F_1[a; b+1; x] = 0, \tag{2.2.2}$$

$$(1+a-b) {}_1F_1[a; b; x] - a {}_1F_1[a+1; b; x] + (b-1) {}_1F_1[a; b-1; x] = 0, \tag{2.2.3}$$

$$b {}_1F_1[a; b; x] - b {}_1F_1[a-1; b; x] - x {}_1F_1[a; b+1; x] = 0, \tag{2.2.4}$$

$$b(a+x) {}_1F_1[a; b; x] - (b-a)x {}_1F_1[a; b+1; x] - ab {}_1F_1[a+1; b; x] = 0, \tag{2.2.5}$$

$$(a-1+x) {}_1F_1[a; b; x] + (b-a) {}_1F_1[a-1; b; x] - (b-1) {}_1F_1[a; b-1; x] = 0. \tag{2.2.6}$$

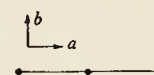


Fig. 2.1



Fig. 2.2



Fig. 2.3



Fig. 2.4



Fig. 2.5



Fig. 2.6

These six results can be deduced directly from (2.1.13–23) by eliminating ${}_1F_1'[a; b; x]$ from each of the results. They can also be verified by expanding the confluent hypergeometric functions in ascending powers of x . In every case the coefficient of x^n is zero for all values of n . From any two of the six results the remaining four recurrence relations can be deduced.

As an example of a relation between associated functions, we have from (2.2.3) and (2.2.4) above,

$$b(1-b-x){}_1F_1[a; b; x] + b(b-1){}_1F_1[a-1; b-1; x] - ax{}_1F_1[a+1; b+1; x] = 0. \quad (2.2.7)$$

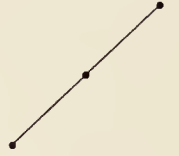


Fig. 2.7

Many more relations between associated functions have been worked out by A. J. Thompson.

2.2.1 Recurrence relations for $U(a; b; x)$

If we eliminate the derivative $U'(a; b; x)$ in the results (2.1.33–43), or if we compare the coefficients of like powers of x in the expansions as series, we can prove the following recurrence relations for the solution $U(a; b; x)$,

$$U(a-1; b; x) - (2a-b+x)U(a; b; x) + a(1+a-b)U(a+1; b; x) = 0, \quad (2.2.8)$$

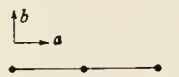


Fig. 2.8

$$(b-a-1)U(a; b-1; x) - (x+b-1)U(a; b; x) + xU(a; b+1; x) = 0, \quad (2.2.9)$$



Fig. 2.9

$$U(a; b; x) - aU(a+1; b; x) - U(a; b-1; x) = 0, \quad (2.2.10)$$



Fig. 2.10

$$(b-a)U(a; b; x) + U(a-1; b; x) - xU(a; b+1; x) = 0, \quad (2.2.11)$$



Fig. 2.11

$$(a+x)U(a; b; x) - xU(a; b+1; x) + a(b-a-1)U(a+1; b; x) = 0, \quad (2.2.12)$$



Fig. 2.12

$$(a+x-1)U(a; b; x) - U(a-1; b; x) + (1+a-b)U(a; b-1; x) = 0. \quad (2.2.13)$$



Fig. 2.13

2.2.2 Continuation formulae for $U(a; b; x)$

When x is a complex number, the function ${}_1F_1[a; b; x]$ is a single-valued function of x for all values of $\arg x$, and so no continuation formulae are necessary. But $U(a; b; x)$ is a many-valued function of x with its principal value given by $-\pi < \arg x \leq \pi$, so that, when $\arg x$ lies outside this range of values, some continuation formulae are needed.

Now $x^a = \exp\{a(\log|x| + i \arg x)\}$, (2.2.14)

and we take as the principal branch of the many-valued function x^a , that which makes $\arg x = 0$ when $x > 0$, given by $-\pi < \arg x \leq \pi$. Then we have, from the definition of $U(a; b; x)$, {(1.3.5)},

$$U(a; b; x e^{\pm \pi i}) = \frac{\pi}{\sin \pi b} \left\{ \frac{{}_1F_1[a; b; -x]}{\Gamma(\Gamma + a - b) \Gamma(b)} - \frac{\exp\{\pm \pi i(\Gamma - b)\} x^{1-b}}{\Gamma(a) \Gamma(2-b)} {}_1F_1[\Gamma + a - b; 2 - b; -x] \right\}, \quad (2.2.15)$$

$$= \frac{\pi}{\sin \pi b} e^{-x} \left[\frac{{}_1F_1[b - a; b; x]}{\Gamma(\Gamma + a - b) \Gamma(b)} - \frac{\exp\{\pm \pi i(\Gamma - b)\} x^{1-b}}{\Gamma(a) \Gamma(2-b)} {}_1F_1[\Gamma - a; 2 - b; x] \right], \quad (2.2.16)$$

by Kummer's theorem. These two functions $U(a; b; x e^{\pi i})$ and $U(a; b; x e^{-\pi i})$ have a position in the theory of Kummer's functions similar to that of the functions $H_v^{(1)}(x)$ and $H_v^{(2)}(x)$ in the theory of Bessel functions. Here we have defined $x e^{\pi i} = \lim_{\eta \rightarrow 0} (x + i\eta)$ and $x e^{-\pi i} = \lim_{\eta \rightarrow 0} (x - i\eta)$ both for $\eta > 0$, so that (2.2.16) shows the difference in the behaviour of $U(a; b; x)$ as x approaches the negative real axis from the positive or negative sides, that is with $\text{Im } x > 0 \rightarrow 0$ or $\text{Im } x < 0 \rightarrow 0$.

In all these formulae either the upper or lower signs should be read throughout.

From (2.2.16) it follows that

$${}_1F_1[b - a; b; x] = \frac{e^x \Gamma(\Gamma + a - b) \Gamma(b)}{2\pi i} \{e^{\pi i b} U(a; b; x e^{\pi i}) - e^{-\pi i b} U(a; b; x e^{-\pi i})\}, \quad (2.2.17)$$

and $U(b - a; b; x) = \frac{e^x \Gamma(a) \Gamma(\Gamma + a - b)}{2\pi i} \{e^{\pi i a} U(a; b; x e^{\pi i}) - e^{-\pi i a} U(a; b; x e^{-\pi i})\}. \quad (2.2.18)$

From the definition of $U(a; b; x)$ we have in general

$$U(a; b; x e^{2\pi i n}) = \frac{\pi}{\sin \pi b} \left[\frac{{}_1F_1[a; b; x]}{\Gamma(\Gamma + a - b) \Gamma(b)} - \frac{\exp\{2\pi i n(\Gamma - b)\} x^{1-b}}{\Gamma(a) \Gamma(2-b)} {}_1F_1[\Gamma + a - b; 2 - b; x] \right], \quad (2.2.19)$$

so that when we express ${}_1F_1[\Gamma + a - b; 2 - b; x]$ in terms of $U(a; b; x)$ and ${}_1F_1[a; b; x]$ we find that

$$U(a; b; x e^{2\pi i n}) = \frac{(\Gamma - e^{-2\pi i b n}) \Gamma(\Gamma - b)}{\Gamma(\Gamma + a - b)} {}_1F_1[a; b; x] + e^{-2\pi i b n} U(a; b; x). \quad (2.2.20)$$

Similarly, we find that

$$U(b - a; b; x \exp\{(2n \pm 1)\pi i\}) = \frac{\pi e^{-x}}{\sin(\pi b)} \left[\frac{{}_1F_1[a; b; x]}{\Gamma(b) \Gamma(\Gamma - a)} - \frac{\exp\{-\pi i(b - 1)(2n \pm 1)\} x^{1-b}}{\Gamma(b - a) \Gamma(2 - b)} {}_1F_1[\Gamma - a; 2 - b; x] \right], \quad (2.2.21)$$

and

$$U(b - a; b; x \exp\{(2n \pm 1)\pi i\}) = \frac{\Gamma(\Gamma - b) [\Gamma - \exp\{\pi i(\Gamma - b)(2n \pm 1)\}]}{\Gamma(\Gamma - a)} {}_1F_1[b - a; b; -x] + \exp\{\pi i(\Gamma - b)(2n \pm 1)\} U(b - a; b; x e^{\pm \pi i}). \quad (2.2.22)$$

2.3 Addition theorems for ${}_1F_1[a; b; x]$

Taylor's theorem states that, if $f(x)$ is an analytic function convergent for $|x| < \rho$, then

$$f(x + y) = \sum_{n=0}^{\infty} f^{(n)}(x) \frac{y^n}{n!} \quad \text{for } |y| < \rho \quad (2.3.1)$$

(Whittaker and Watson, 1946, § 5.4). From the expressions for the derivatives of Kummer's function (2.1.1-12) we deduce these addition theorems:

$${}_1F_1[a; b; x+y] = \sum_{n=0}^{\infty} \frac{(a)_n y^n}{(b)_n n!} {}_1F_1[a+n; b+n; x], \quad (2.3.2)$$

$$= \left(\frac{x+y}{x}\right)^{1-b} \sum_{n=0}^{\infty} \frac{(1-b)_n (-y)^n}{n! x^n} {}_1F_1[a; b-n; x], \quad (2.3.3)$$

$$= \left(\frac{x}{x+y}\right)^a \sum_{n=0}^{\infty} \frac{(a)_n y^n}{n! (x+y)^n} {}_1F_1[a+n; b; x], \quad (2.3.4)$$

$$= e^y \sum_{n=0}^{\infty} \frac{(b-a)_n (-y)^n}{(b)_n n!} {}_1F_1[a; b+n; x], \quad (2.3.5)$$

$$= e^y \left(\frac{x}{x+y}\right)^{b-a} \sum_{n=0}^{\infty} \frac{(b-a)_n y^n (x+y)^n}{n!} {}_1F_1[a-n; b; x], \quad (2.3.6)$$

$$= e^y \left(\frac{x+y}{x}\right)^{1-b} \sum_{n=0}^{\infty} \frac{(-y)^n (1-b)_n}{n! x^n} {}_1F_1[a-n; b-n; x]. \quad (2.3.7)$$

2.3.1 Addition theorems for $U(a; b; x)$

From the expressions for the derivatives of $U(a; b; x)$ (2.1.24-32), we deduce these addition theorems:

$$U(a; b; x+y) = \sum_{n=0}^{\infty} \frac{(a)_n}{n!} (-1)^n y^n U(a+n; b+n; x), \quad (2.3.8)$$

$$= \left(\frac{x}{x+y}\right)^a \sum_{n=0}^{\infty} \frac{(a)_n (1+a-b)_n}{n!} \left(\frac{y}{x+y}\right)^n U(a+n; b; x), \quad (2.3.9)$$

$$= \left(\frac{x}{x+y}\right)^{b-1} \sum_{n=0}^{\infty} \frac{(1+a-b)_n (-y)^n}{n! x^n} U(a; b-n; x), \quad (2.3.10)$$

$$= e^y \sum_{n=0}^{\infty} \frac{(-1)^n y^n}{n!} U(a; b+n; x), \quad (2.3.11)$$

$$= e^y \left(\frac{x}{x+y}\right)^{b-a} \sum_{n=0}^{\infty} \frac{(-1)^n y^n}{n! (x+y)^n} U(a-n; b; x), \quad (2.3.12)$$

$$= e^y \left(\frac{x}{x+y}\right)^{b-1} \sum_{n=0}^{\infty} \frac{(-1)^n y^n}{x^n n!} U(a-n; b-n; x). \quad (2.3.13)$$

2.3.2 Multiplication theorems for ${}_1F_1[a; b; x]$

If we replace y by $(y-1)x$ in (2.3.1), we see that Taylor's theorem can also be stated in the form

$$f(xy) = \sum_{n=0}^{\infty} \frac{(y-1)^n x^n}{n!} \frac{d^n}{dx^n} \{f(x)\} \quad (2.3.14)$$

for $|y-1| < \text{Rl } x/|x|$. When the appropriate expansions for the derivatives are inserted, the theorem in this form gives the following multiplication theorems for Kummer's function:

$${}_1F_1[a; b; xy] = \sum_{n=0}^{\infty} \frac{(a)_n x^n (y-1)^n}{(b)_n n!} {}_1F_1[a+n; b+n; x], \quad (2.3.15)$$

$$= y^{1-b} \sum_{n=0}^{\infty} \frac{(1-b)_n (1-y)^n}{n!} {}_1F_1[a; b-n; x], \quad (2.3.16)$$

$$= y^{-a} \sum_{n=0}^{\infty} \frac{(a)_n (y-1)^n}{n! y^n} {}_1F_1[a+n; b; x], \quad (2.3.17)$$

$$= e^{(y-1)x} \sum_{n=0}^{\infty} \frac{(b-a)_n (1-y)^n x^n}{(b)_n n!} {}_1F_1[a; b+n; x], \quad (2.3.18)$$

$$= e^{(y-1)x} y^{1-b} \sum_{n=0}^{\infty} \frac{(1-b)_n (1-y)^n}{n!} {}_1F_1[a-n; b-n; x], \quad (2.3.19)$$

$$= e^{(y-1)x} y^{a-b} \sum_{n=0}^{\infty} \frac{(b-a)_n (y-1)^n}{n! y^n} {}_1F_1[a-n; b; x]. \quad (2.3.20)$$

2.3.3 Multiplication theorems for $U(a; b; x)$

Next let us substitute the derivatives of $U(a; b; x)$ in (2.3.14). Then we have

$$U(a; b; xy) = \sum_{n=0}^{\infty} \frac{(a)_n (1-y)^n x^n}{n!} U(a+n; b+n; x), \quad (2.3.21)$$

$$= y^{-a} \sum_{n=0}^{\infty} \frac{(a)_n (1+a-b)_n (y-1)^n}{n! y^n} U(a+n; b; x), \quad (2.3.22)$$

$$= y^{1-b} \sum_{n=0}^{\infty} \frac{(1+a-b)_n (1-y)^n}{n!} U(a; b-n; x), \quad (2.3.23)$$

$$= e^{(y-1)x} \sum_{n=0}^{\infty} \frac{(1-y)^n x^n}{n!} U(a; b+n; x), \quad (2.3.24)$$

$$= e^{(y-1)x} y^{a-b} \sum_{n=0}^{\infty} \frac{(1-y)^n}{n! y^n} U(a-n; b; x), \quad (2.3.25)$$

$$= e^{(y-1)x} y^{1-b} \sum_{n=0}^{\infty} \frac{(1-y)^n}{n!} U(a-n; b-n; x). \quad (2.3.26)$$

2.4 The derivatives of $M_{k,m}(x)$

From the definition of $M_{k,m}(x)$ in terms of Kummer's function (1.6.4) and the results of §2.1, we have immediately

$$\frac{d^n}{dx^n} \{e^{\frac{1}{2}x} x^{m-\frac{1}{2}} M_{k,m}(x)\} = (-1)^n (-2m)_n x^{m-\frac{1}{2}n-\frac{1}{2}} e^{\frac{1}{2}x} M_{k-\frac{1}{2}n, m-\frac{1}{2}n}(x), \quad (2.4.1)$$

$$\frac{d^n}{dx^n} \{e^{\frac{1}{2}x} x^{-m-\frac{1}{2}} M_{k,m}(x)\} = \frac{(\frac{1}{2}+m-k)_n}{(1+2m)_n} e^{\frac{1}{2}x} x^{-\frac{1}{2}-m-\frac{1}{2}n} M_{k-\frac{1}{2}n, m+\frac{1}{2}n}(x), \quad (2.4.2)$$

$$\frac{d^n}{dx^n} \{e^{\frac{1}{2}x} x^{n-k-1} M_{k,m}(x)\} = \left(\frac{1}{2} + m - k\right)_n e^{\frac{1}{2}x} x^{-k-1} M_{k-n,m}(x), \quad (2.4.3)$$

$$\frac{d^n}{dx^n} \{e^{-\frac{1}{2}x} x^{m-\frac{1}{2}} M_{k,m}(x)\} = (-1)^n (-2m)_n e^{-\frac{1}{2}x} x^{m-\frac{1}{2}-n} M_{k+\frac{1}{2}n, m-\frac{1}{2}n}(x), \quad (2.4.4)$$

$$\frac{d^n}{dx^n} \{e^{-\frac{1}{2}x} x^{-m-\frac{1}{2}} M_{k,m}(x)\} = \frac{(-1)^n \left(\frac{1}{2} + m + k\right)_n}{(1 + 2m)_n} e^{-\frac{1}{2}x} x^{-\frac{1}{2}-m-\frac{1}{2}n} M_{k+\frac{1}{2}n, m+\frac{1}{2}n}(x), \quad (2.4.5)$$

$$\frac{d^n}{dx^n} \{e^{-\frac{1}{2}x} x^{k+n-1} M_{k,m}(x)\} = \left(\frac{1}{2} + m + k\right)_n e^{-\frac{1}{2}x} x^{k-1} M_{k+n,m}(x). \quad (2.4.6)$$

The results (2.4.4–6) can also be deduced by applying Kummer's first theorem to (2.4.1–3) above.

In particular when $n = 1$, we have

$$\begin{aligned} M_{k-\frac{1}{2}, m-\frac{1}{2}}(x) &= \frac{e^{-\frac{1}{2}x} x^{1-m}}{2m} \frac{d}{dx} \{e^{\frac{1}{2}x} x^{m-\frac{1}{2}} M_{k,m}(x)\}, \\ &= \frac{x + 2m - 1}{4m\sqrt{x}} M_{k,m}(x) + \frac{\sqrt{x}}{2m} M'_{k,m}(x), \end{aligned} \quad (2.4.7)$$

$$\begin{aligned} M_{k-\frac{1}{2}, m+\frac{1}{2}}(x) &= \frac{1 + 2m}{\frac{1}{2} + m - k} e^{-\frac{1}{2}x} x^{m+1} \frac{d}{dx} \{e^{\frac{1}{2}x} x^{-m-\frac{1}{2}} M_{k,m}(x)\}, \\ &= \frac{(1 + 2m) \left(\frac{1}{2}x - m - \frac{1}{2}\right)}{\sqrt{x} \left(\frac{1}{2} + m - k\right)} M_{k,m}(x) + \frac{(1 + 2m)\sqrt{x}}{\frac{1}{2} + m - k} M'_{k,m}(x), \end{aligned} \quad (2.4.8)$$

$$\begin{aligned} M_{k+\frac{1}{2}, m-\frac{1}{2}}(x) &= \frac{e^{\frac{1}{2}x} x^{1-m}}{2m} \frac{d}{dx} \{e^{-\frac{1}{2}x} x^{m-\frac{1}{2}} M_{k,m}(x)\}, \\ &= \frac{2m - 1 - x}{4m\sqrt{x}} M_{k,m}(x) + \frac{\sqrt{x}}{2m} M'_{k,m}(x), \end{aligned} \quad (2.4.9)$$

$$\begin{aligned} M_{k+\frac{1}{2}, m+\frac{1}{2}}(x) &= -\frac{1 + 2m}{\frac{1}{2} + m + k} e^{\frac{1}{2}x} x^{m+1} \frac{d}{dx} \{e^{-\frac{1}{2}x} x^{-m-\frac{1}{2}} M_{k,m}(x)\}, \\ &= \frac{1 + 2m}{\frac{1}{2} + m + k} \left\{ \frac{1 + 2m + x}{2\sqrt{x}} M_{k,m}(x) - \sqrt{x} M'_{k,m}(x) \right\}, \end{aligned} \quad (2.4.10)$$

$$\begin{aligned} M_{k+1, m}(x) &= \frac{2e^{\frac{1}{2}x} x^{1-k}}{1 + 2m + 2k} \frac{d}{dx} \{e^{-\frac{1}{2}x} x^k M_{k,m}(x)\}, \\ &= \frac{k - \frac{1}{2}x}{\frac{1}{2} + m + k} M_{k,m}(x) + \frac{x}{\frac{1}{2} + m + k} M'_{k,m}(x), \end{aligned} \quad (2.4.11)$$

$$\begin{aligned} M_{k-1, m}(x) &= \frac{2e^{-\frac{1}{2}x} x^{k+1}}{1 + 2m - 2k} \frac{d}{dx} \{e^{\frac{1}{2}x} x^{-k} M_{k,m}(x)\}, \\ &= \frac{\frac{1}{2}x - k}{\frac{1}{2} + m - k} M_{k,m}(x) + \frac{x}{\frac{1}{2} + m - k} M'_{k,m}(x). \end{aligned} \quad (2.4.12)$$

2.4.1 The derivatives of $W_{k,m}(x)$

From the definition of $W_{k,m}(x)$ in terms of $U(a; b; x)$ (1.9.11), and the derivatives of $U(a; b; x)$, we deduce immediately that

$$\frac{d^n}{dx^n} \{e^{\frac{1}{2}x} x^{-\frac{1}{2}-m} W_{k,m}(x)\} = (-1)^n (\frac{1}{2} + m - k)_n e^{\frac{1}{2}x} x^{-\frac{1}{2}-m-\frac{1}{2}n} W_{k-\frac{1}{2}n, m+\frac{1}{2}n}(x), \quad (2.4.13)$$

$$\frac{d^n}{dx^n} \{e^{\frac{1}{2}x} x^{m-\frac{1}{2}} W_{k,m}(x)\} = (-1)^n (\frac{1}{2} - m - k)_n e^{\frac{1}{2}x} x^{m-\frac{1}{2}-\frac{1}{2}n} W_{k-\frac{1}{2}n, m-\frac{1}{2}n}(x), \quad (2.4.14)$$

and
$$\frac{d^n}{dx^n} \{e^{\frac{1}{2}x} x^{n-k-1} W_{k,m}(x)\} = (\frac{1}{2} + m - k)_n (\frac{1}{2} - m - k)_n x^{-k-1} e^{\frac{1}{2}x} W_{k-n, m}(x). \quad (2.4.15)$$

Now let us apply Kummer's first theorem to these three results. We find that

$$\frac{d^n}{dx^n} \{e^{-\frac{1}{2}x} x^{-\frac{1}{2}-m} W_{k,m}(x)\} = (-1)^n e^{-\frac{1}{2}x} x^{-\frac{1}{2}-m-\frac{1}{2}n} W_{k+\frac{1}{2}n, m+\frac{1}{2}n}(x), \quad (2.4.16)$$

$$\frac{d^n}{dx^n} \{e^{-\frac{1}{2}x} x^{m-\frac{1}{2}} W_{k,m}(x)\} = (-1)^n e^{-\frac{1}{2}x} x^{m-\frac{1}{2}-\frac{1}{2}n} W_{k+\frac{1}{2}n, m-\frac{1}{2}n}(x), \quad (2.4.17)$$

and
$$\frac{d^n}{dx^n} \{e^{-\frac{1}{2}x} x^{k+n-1} W_{k,m}(x)\} = (-1)^n e^{-\frac{1}{2}x} x^{k-1} W_{k+n, m}(x). \quad (2.4.18)$$

In particular when $n = 1$, we have

$$\begin{aligned} W_{k-\frac{1}{2}, m+\frac{1}{2}}(x) &= \frac{x^{m+1}}{k-m-\frac{1}{2}} e^{-\frac{1}{2}x} \frac{d}{dx} \{e^{\frac{1}{2}x} x^{-\frac{1}{2}-m} W_{k,m}(x)\}, \\ &= \frac{1+2m-x}{(1-2m-2k)\sqrt{x}} W_{k,m}(x) + \frac{\sqrt{x}}{k-m-\frac{1}{2}} W'_{k,m}(x), \end{aligned} \quad (2.4.19)$$

$$\begin{aligned} W_{k-\frac{1}{2}, m-\frac{1}{2}}(x) &= \frac{e^{-\frac{1}{2}x}}{k+m-\frac{1}{2}} x^{1-m} \frac{d}{dx} \{e^{\frac{1}{2}x} x^{m-\frac{1}{2}} W_{k,m}(x)\}, \\ &= \frac{1-2m-x}{(1-2m-2k)\sqrt{x}} W_{k,m}(x) + \frac{\sqrt{x}}{k-\frac{1}{2}+m} W'_{k,m}(x), \end{aligned} \quad (2.4.20)$$

$$\begin{aligned} W_{k-1, m}(x) &= \frac{e^{-\frac{1}{2}x} x^{k+1}}{(\frac{1}{2}+m-k)(\frac{1}{2}-m-k)} \frac{d}{dx} \{e^{\frac{1}{2}x} x^{-k} W_{k,m}(x)\}, \\ &= \frac{\frac{1}{2}x-k}{(\frac{1}{2}+m-k)(\frac{1}{2}-m-k)} W_{k,m}(x) + \frac{x}{(\frac{1}{2}+m-k)(\frac{1}{2}-m-k)} W'_{k,m}(x), \end{aligned} \quad (2.4.21)$$

$$\begin{aligned} W_{k+\frac{1}{2}, m+\frac{1}{2}}(x) &= -e^{\frac{1}{2}x} x^{1+m} \frac{d}{dx} \{e^{-\frac{1}{2}x} x^{-\frac{1}{2}-m} W_{k,m}(x)\}, \\ &= \frac{1+2m+x}{2\sqrt{x}} W_{k,m}(x) - \sqrt{x} W'_{k,m}(x), \end{aligned} \quad (2.4.22)$$

$$\begin{aligned} W_{k+\frac{1}{2}, m-\frac{1}{2}}(x) &= -e^{\frac{1}{2}x} x^{1-m} \frac{d}{dx} \{e^{-\frac{1}{2}x} x^{m-\frac{1}{2}} W_{k,m}(x)\}, \\ &= \frac{1-2m+x}{2\sqrt{x}} W_{k,m}(x) - \sqrt{x} W'_{k,m}(x), \end{aligned} \quad (2.4.23)$$

$$\begin{aligned} W_{k+1, m}(x) &= -e^{\frac{1}{2}x} x^{1-k} \frac{d}{dx} \{e^{-\frac{1}{2}x} x^k W_{k,m}(x)\}, \\ &= (\frac{1}{2}x-k) W_{k,m}(x) - x W'_{k,m}(x). \end{aligned} \quad (2.4.24)$$

2.4.2 The Wronskians of Whittaker's equation

Again we have eight possible forms of solution for Whittaker's equation, but, by Kummer's first theorem, we have seen that only four of these solutions are independent, so that we have

$$K_{1,3} = K_{2,4} = K_{5,6} = K_{7,8} = 0, \quad (2.4.25)$$

and for the four solutions z_1, z_2, z_5 and z_7 we find that

$$K_{1,2} = -2m, \quad (2.4.26)$$

$$K_{1,5} = -\frac{\Gamma(1+2m)}{\Gamma(\frac{1}{2}+m-k)}, \quad (2.4.27)$$

$$K_{1,7} = \exp\{e\pi i(\frac{1}{2}-m)\} \frac{\Gamma(1+2m)}{\Gamma(\frac{1}{2}+m+k)}, \quad (2.4.28)$$

$$K_{2,5} = -\frac{\Gamma(1-2m)}{\Gamma(\frac{1}{2}-m-k)}, \quad (2.4.29)$$

$$K_{2,7} = -\exp\{e\pi i(\frac{1}{2}+m)\} \frac{\Gamma(1-2m)}{\Gamma(\frac{1}{2}-m+k)}, \quad (2.4.30)$$

and

$$K_{5,7} = e^{-e\pi k}. \quad (2.4.31)$$

Again, $K_{5,7}$ is the only Wronskian which never vanishes for any values of k and m , so that z_5 and z_7 are the only pair of solutions which will always provide a complete solution to Whittaker's equation. If $\pm 2m$ and $2k \pm 2m$ are not integers nor zero, any two of the solutions z_1, z_2, z_5 or z_7 can be taken to form a complete system of solutions. If $2m - 2k$ is zero or a negative integer $K_{1,5} = 0$, and z_5 is a multiple of z_1 . Similarly, if $-2m + 2k$ is zero or a negative integer, $K_{2,7} = 0$ and z_7 is a multiple of z_2 .

From the result $K_{m,n}y_p + K_{n,p}y_m + K_{p,m}y_n = 0$, and the Wronskians we deduce the following relations between the four solutions of Whittaker's equation,

$$z_1 = \Gamma(1+2m) \left[\frac{\exp\{e\pi i(k-m+\frac{1}{2})\}}{\Gamma(\frac{1}{2}+m+k)} z_5 + \frac{\exp(e\pi k)}{\Gamma(\frac{1}{2}+m-k)} z_7 \right], \quad (2.4.32)$$

$$z_2 = \Gamma(1-2m) \left[\frac{\exp\{e\pi i(m+k-\frac{1}{2})\}}{\Gamma(\frac{1}{2}-m+k)} z_5 + \frac{\exp(e\pi k)}{\Gamma(\frac{1}{2}-m-k)} z_7 \right], \quad (2.4.33)$$

$$z_5 = \frac{\pi}{\sin \pi 2m} \left[-\frac{z_1}{\Gamma(\frac{1}{2}-m-k)\Gamma(1+2m)} + \frac{z_2}{\Gamma(\frac{1}{2}+m-k)\Gamma(1-2m)} \right], \quad (2.4.34)$$

and

$$z_7 = \frac{\pi}{\sin \pi 2m} \left[-\frac{\exp\{e\pi i(\frac{1}{2}+m)\}}{\Gamma(\frac{1}{2}-m+k)\Gamma(1+2m)} z_1 + \frac{\exp\{e\pi i(\frac{1}{2}-m)\}}{\Gamma(\frac{1}{2}+m+k)\Gamma(1-2m)} z_2 \right]. \quad (2.4.35)$$

There are eight further relations connecting these four solutions, and twelve relations connecting any four solutions.

2.5 Recurrence relations for $M_{k,m}(x)$

If we eliminate $M_{k,m}(x)$ from the results (2.4.7-12) we can deduce immediately the following recurrence relations for $M_{k,m}(x)$:

$$\frac{2m}{\sqrt{x}} M_{k,m}(x) - 2m M_{k-\frac{1}{2},m-\frac{1}{2}}(x) + \frac{1+2m-2k}{2(1+2m)} M_{k-\frac{1}{2},m+\frac{1}{2}}(x) = 0, \quad (2.5.1)$$



Fig. 2.14

$$2mM_{k-\frac{1}{2},m-\frac{1}{2}}(x) - \sqrt{x}M_{k,m}(x) - 2mM_{k+\frac{1}{2},m-\frac{1}{2}}(x) = 0, \quad (2.5.2)$$



Fig. 2.15

$$\frac{1+2m-2k}{2(1+2m)}M_{k-\frac{1}{2},m+\frac{1}{2}}(x) + \frac{2m-x}{\sqrt{x}}M_{k,m}(x) - 2mM_{k+\frac{1}{2},m-\frac{1}{2}}(x) = 0, \quad (2.5.3)$$



Fig. 2.16

$$2mM_{k-\frac{1}{2},m-\frac{1}{2}}(x) + \frac{2m+x}{\sqrt{x}}M_{k,m}(x) - \frac{1+2m+2k}{2(1+2m)}M_{k+\frac{1}{2},m+\frac{1}{2}}(x) = 0, \quad (2.5.4)$$



Fig. 2.17

$$\frac{1+2m-2k}{2\sqrt{x}(1+2m)}M_{k-\frac{1}{2},m+\frac{1}{2}}(x) - M_{k,m}(x) + \frac{1+2m+2k}{2\sqrt{x}(1+2m)}M_{k+\frac{1}{2},m+\frac{1}{2}}(x) = 0, \quad (2.5.5)$$



Fig. 2.18

$$2mM_{k+\frac{1}{2},m-\frac{1}{2}}(x) - \frac{2m}{\sqrt{x}}M_{k,m}(x) + \frac{1+2m+2k}{2(1+2m)}M_{k+\frac{1}{2},m+\frac{1}{2}}(x) = 0, \quad (2.5.6)$$



Fig. 2.19

and

$$(1+2m+2k)M_{k+1,m}(x) - (1+2m-2k)M_{k-1,m}(x) + 2(x-2k)M_{k,m}(x) = 0. \quad (2.5.7)$$

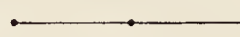


Fig. 2.20

2.5.1 Recurrence relations for $W_{k,m}(x)$

From the results (2.4.19–24) we have the following recurrence relations for $W_{k,m}(x)$,

$$W_{k+\frac{1}{2},m}(x) - \sqrt{x}W_{k,m-\frac{1}{2}}(x) - (m-k)W_{k-\frac{1}{2},m}(x) = 0, \quad (2.5.8)$$



Fig. 2.21

$$W_{k+\frac{1}{2},m}(x) - \sqrt{x}W_{k,m+\frac{1}{2}}(x) + (m+k)W_{k-\frac{1}{2},m}(x) = 0, \quad (2.5.9)$$



Fig. 2.22

$$2mW_{k,m}(x) - \sqrt{x}W_{k+\frac{1}{2},m+\frac{1}{2}}(x) - \sqrt{x}W_{k+\frac{1}{2},m-\frac{1}{2}}(x) = 0, \quad (2.5.10)$$



Fig. 2.23

$$(2k-x)W_{k,m}(x) + W_{k+1,m}(x) - (m-k+\frac{1}{2})(m+k-\frac{1}{2})W_{k-1,m}(x) = 0, \quad (2.5.11)$$



Fig. 2.24

$$(k-m-\frac{1}{2})W_{k-\frac{1}{2},m+\frac{1}{2}}(x) + \frac{2m}{\sqrt{x}}W_{k,m}(x) + (\frac{1}{2}-m-k)W_{k-\frac{1}{2},m-\frac{1}{2}}(x) = 0, \quad (2.5.12)$$



Fig. 2.25

$$(k+m-\frac{1}{2})W_{k-\frac{1}{2},m-\frac{1}{2}}(x) - \frac{2m+x}{\sqrt{x}}W_{k,m}(x) + W_{k+\frac{1}{2},m+\frac{1}{2}}(x) = 0, \quad (2.5.13)$$

and

$$(k-m-\frac{1}{2})W_{k-\frac{1}{2},m+\frac{1}{2}}(x) + \frac{2m-x}{\sqrt{x}}W_{k,m}(x) + W_{k+\frac{1}{2},m-\frac{1}{2}}(x) = 0. \quad (2.5.14)$$

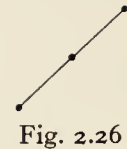


Fig. 2.26



Fig. 2.27

2.5.2 Continuation formulae for Whittaker's functions

By Kummer's theorem we have immediately

$$M_{-k,m}(x e^{\pm\pi i}) = \exp\{\pm\pi i(\frac{1}{2}+m)\} M_{k,m}(x), \quad (2.5.15)$$

and again in all the formulae of this section either the upper or the lower signs should be read throughout.

From the definition of $W_{k,m}(x)$ in terms of $M_{k,m}(x)$ we have

$$W_{-k,m}(x e^{\pm\pi i}) = \frac{\pi}{\sin(2m\pi)} \left\{ -\frac{M_{-k,m}(x e^{\pm\pi i})}{\Gamma(\frac{1}{2}-m+k)\Gamma(1+2m)} + \frac{M_{-k,-m}(x e^{\pm\pi i})}{\Gamma(\frac{1}{2}+m+k)\Gamma(1-2m)} \right\}, \quad (2.5.16)$$

and so, by (2.5.15) above,

$$W_{-k,m}(x e^{\pm\pi i}) = \frac{\pi}{\sin(2m\pi)} \left[-\frac{\exp\{\pm\pi i(\frac{1}{2}+m)\} M_{k,m}(x)}{\Gamma(\frac{1}{2}-m+k)\Gamma(1+2m)} + \frac{\exp\{\pm\pi i(\frac{1}{2}-m)\} M_{k,-m}(x)}{\Gamma(\frac{1}{2}+m+k)\Gamma(1-2m)} \right]. \quad (2.5.17)$$

This equation defines the two functions $W_{-k,m}(x e^{\pi i})$ and $W_{-k,m}(x e^{-\pi i})$ which occupy a position in the theory of Whittaker functions similar to that of the functions $H_v^{(1)}(x)$ and $H_v^{(2)}(x)$ in the theory of Bessel functions. The two results (2.5.16) and (2.5.17) are only alternative statements of the relation between solutions of Whittaker's equation, which has already been stated in (2.4.35). Another pair of continuation formulae are given by (1.9.9) and (1.9.12), with $x e^{\pm\pi i}$ written for $-x$.

From (2.5.17) above, if we write $-k$ for k and eliminate the function $M_{-k,-m}(x)$ from the results, we deduce that

$$M_{-k,m}(x) = \frac{\Gamma(\frac{1}{2}-m-k)\Gamma(1+2m)}{2\pi i} [\exp\{\pi i(\frac{1}{2}+m)\} W_{k,m}(x e^{\pi i}) - \exp\{-\pi i(\frac{1}{2}+m)\} W_{k,m}(x e^{-\pi i})] \quad (2.5.18)$$

and

$$W_{-k,m}(x) = \frac{\Gamma(\frac{1}{2}+m-k)\Gamma(\frac{1}{2}-m-k)}{2\pi i} \{e^{-\pi i k} W_{k,m}(x e^{\pi i}) - e^{\pi i k} W_{k,m}(x e^{-\pi i})\}. \quad (2.5.19)$$

In general we can re-apply (2.5.15) and obtain

$$M_{k,m}(x e^{2n\pi i}) = \exp\{\pi i(1+2m)n\} M_{k,m}(x), \quad (2.5.20)$$

and

$$M_{k,m}(x \exp\{(2n \pm 1)\pi i\}) = \exp\{\pi i(2n \pm 1)(\frac{1}{2}+m)\} M_{k,m}(x). \quad (2.5.21)$$

From the definition of $W_{k,m}(x)$ in terms of $M_{k,m}(x)$ and $M_{k,-m}(x)$, using (2.5.20), we deduce that

$$W_{k,m}(x e^{2n\pi i}) = \frac{\pi}{\sin(2m\pi)} \left\{ -\frac{\exp\{\pi i(\frac{1}{2}+m)2n\} M_{k,m}(x)}{\Gamma(\frac{1}{2}-m-k)\Gamma(1+2m)} + \frac{\exp\{\pi i(\frac{1}{2}-m)2n\} M_{k,-m}(x)}{\Gamma(\frac{1}{2}+m-k)\Gamma(1-2m)} \right\}. \quad (2.5.22)$$

Again if we use (1.9.12) to express $M_{k,-m}(x)$ in terms of $M_{k,m}(x)$ and $W_{k,m}(x)$ we deduce that

$$W_{k,m}(x e^{2n\pi i}) = \frac{(-1)^{n+1} 2\pi i \sin(2mn\pi)}{\Gamma(\frac{1}{2}-m-k)\sin(2m\pi)} \frac{M_{k,m}(x)}{\Gamma(1+2m)} + (-1)^n e^{-2\pi i mn} W_{k,m}(x). \quad (2.5.23)$$

Similarly,

$$W_{-k,m}(x \exp\{(2n \pm 1)\pi i\}) = \frac{\pi}{\sin(2m\pi)} \left[-\frac{\exp\{\pi i(\frac{1}{2} + m)(2n \pm 1)\}}{\Gamma(\frac{1}{2} - m + k)\Gamma(1 + 2m)} M_{k,m}(x) + \frac{\exp\{\pi i(\frac{1}{2} - m)(2n \pm 1)\}}{\Gamma(\frac{1}{2} + m + k)\Gamma(1 - 2m)} M_{k,-m}(x) \right], \quad (2.5.24)$$

and

$$W_{-k,m}(x \exp\{(2n \pm 1)\pi i\}) = \frac{(-1)^{n+1} 2\pi i \sin(2mn\pi)}{\Gamma(\frac{1}{2} - m + k) \sin(2m\pi) \Gamma(1 + 2m)} M_{-k,m}(x e^{\pm\pi i}) + (-1)^n e^{-2\pi imn} W_{-k,m}(x e^{\pm\pi i}). \quad (2.5.25)$$

2.6 Addition theorems for $M_{k,m}(x)$

From (2.4.1-6) and (2.3.1) we deduce these addition theorems for $M_{k,m}(x)$,

$$M_{k,m}(x+y) = e^{-\frac{1}{2}y} \left(\frac{x}{x+y}\right)^{m-\frac{1}{2}} \sum_{n=0}^{\infty} \frac{(-2m)_n (-1)^n y^n}{n! x^{\frac{1}{2}n}} M_{k-\frac{1}{2}n, m-\frac{1}{2}n}(x), \quad (2.6.1)$$

$$= e^{-\frac{1}{2}y} \left(\frac{x+y}{x}\right)^{\frac{1}{2}+m} \sum_{n=0}^{\infty} \frac{(\frac{1}{2} + m - k)_n y^n}{n! x^{\frac{1}{2}n} (1 + 2m)_n} M_{k-\frac{1}{2}n, m+\frac{1}{2}n}(x), \quad (2.6.2)$$

$$= e^{-\frac{1}{2}y} \left(\frac{x+y}{x}\right)^k \sum_{n=0}^{\infty} \frac{(\frac{1}{2} + m - k)_n y^n}{n!} M_{k-n, m}(x), \quad (2.6.3)$$

$$= e^{\frac{1}{2}y} \left(\frac{x+y}{x}\right)^{-m+\frac{1}{2}} \sum_{n=0}^{\infty} \frac{(-2m)_n (-1)^n y^n}{n! x^{\frac{1}{2}n}} M_{k+\frac{1}{2}n, m-\frac{1}{2}n}(x), \quad (2.6.4)$$

$$= e^{\frac{1}{2}y} \left(\frac{x+y}{x}\right)^{m+\frac{1}{2}} \sum_{n=0}^{\infty} \frac{(\frac{1}{2} + m + k)_n (-1)^n y^n}{n! x^{\frac{1}{2}n} (1 + 2m)_n} M_{k+\frac{1}{2}n, m+\frac{1}{2}n}(x), \quad (2.6.5)$$

$$= e^{\frac{1}{2}y} \left(\frac{x+y}{x}\right)^{-k} \sum_{n=0}^{\infty} \frac{(\frac{1}{2} + m + k)_n y^n}{n!} M_{k+n, m}(x). \quad (2.6.6)$$

2.6.1 Addition theorems for $W_{k,m}(x)$

From (2.4.13-18) and (2.3.1) we deduce these addition theorems for $W_{k,m}(x)$,

$$W_{k,m}(x+y) = e^{-\frac{1}{2}y} \left(\frac{x+y}{x}\right)^{m+\frac{1}{2}} \sum_{n=0}^{\infty} \frac{(\frac{1}{2} + m - k)_n (-1)^n y^n}{n! x^{\frac{1}{2}n}} W_{k-\frac{1}{2}n, m+\frac{1}{2}n}(x), \quad (2.6.7)$$

$$= e^{-\frac{1}{2}y} \left(\frac{x+y}{x}\right)^{\frac{1}{2}-m} \sum_{n=0}^{\infty} \frac{(\frac{1}{2} - m - k)_n (-1)^n y^n}{n! x^{\frac{1}{2}n}} W_{k-\frac{1}{2}n, m-\frac{1}{2}n}(x), \quad (2.6.8)$$

$$= e^{-\frac{1}{2}y} \left(\frac{x+y}{x}\right)^k \sum_{n=0}^{\infty} \frac{(\frac{1}{2} + m - k)_n (\frac{1}{2} - m - k)_n}{n!} \left(\frac{y}{x+y}\right)^n W_{k-n, m}(x), \quad (2.6.9)$$

$$= e^{\frac{1}{2}y} \left(\frac{x+y}{x}\right)^{\frac{1}{2}+m} \sum_{n=0}^{\infty} \frac{(-1)^n y^n}{n! x^{\frac{1}{2}n}} W_{k+\frac{1}{2}n, m+\frac{1}{2}n}(x), \quad (2.6.10)$$

$$= e^{\frac{1}{2}y} \left(\frac{x+y}{x}\right)^{\frac{1}{2}-m} \sum_{n=0}^{\infty} \frac{(-1)^n y^n}{n! x^{\frac{1}{2}n}} W_{k+\frac{1}{2}n, m-\frac{1}{2}n}(x), \quad (2.6.11)$$

$$= e^{\frac{1}{2}y} \left(\frac{x}{x+y}\right)^k \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left(\frac{y}{x+y}\right)^n W_{k+n, m}(x). \quad (2.6.12)$$

2.6.2 Multiplication theorems for $M_{k,m}(x)$

Next let us substitute the derivatives of $M_{k,m}(x)$ in (2.3.14). We have then

$$M_{k,m}(xy) = e^{\frac{1}{2}x(1-y)} y^{\frac{1}{2}-m} \sum_{n=0}^{\infty} \frac{(y-1)^n (-1)^n (-2m)_n}{n! x^{-\frac{1}{2}n}} M_{k-\frac{1}{2}n, m-\frac{1}{2}n}(x), \quad (2.6.13)$$

$$= e^{\frac{1}{2}x(1-y)} y^{\frac{1}{2}+m} \sum_{n=0}^{\infty} \frac{(\frac{1}{2}+m-k)_n (y-1)^n x^{\frac{1}{2}n}}{n! (1+2m)_n} M_{k-\frac{1}{2}n, m+\frac{1}{2}n}(x), \quad (2.6.14)$$

$$= e^{\frac{1}{2}x(1-y)} y^k \sum_{n=0}^{\infty} \frac{(y-1)^n}{n!} x^n (\frac{1}{2}+m-k)_n M_{k-n, m}(x), \quad (2.6.15)$$

$$= e^{\frac{1}{2}x(y-1)} y^{\frac{1}{2}-m} \sum_{n=0}^{\infty} \frac{(y-1)^n x^{\frac{1}{2}n}}{n!} (-1)^n (-2m)_n M_{k+\frac{1}{2}n, m-\frac{1}{2}n}(x), \quad (2.6.16)$$

$$= e^{\frac{1}{2}x(y-1)} y^{\frac{1}{2}+m} \sum_{n=0}^{\infty} \frac{(y-1)^n}{n!} x^{\frac{1}{2}n} (-1)^n \frac{(\frac{1}{2}+m+k)_n}{(1+2m)_n} M_{k+\frac{1}{2}n, m+\frac{1}{2}n}(x), \quad (2.6.17)$$

$$= e^{\frac{1}{2}x(y-1)} y^{-k} \sum_{n=0}^{\infty} \frac{(y-1)^n}{n! y^n} (\frac{1}{2}+m+k)_n M_{k+n, m}(x). \quad (2.6.18)$$

2.6.3 Multiplication theorems for $W_{k,m}(x)$

Finally let us substitute the derivatives of $W_{k,m}(x)$ in (2.3.14). Then we find that

$$W_{k,m}(xy) = e^{\frac{1}{2}x(1-y)} y^{\frac{1}{2}+m} \sum_{n=0}^{\infty} \frac{(1-y)^n}{n!} x^{\frac{1}{2}n} (\frac{1}{2}+m-k)_n W_{k-\frac{1}{2}n, m+\frac{1}{2}n}(x), \quad (2.6.19)$$

$$= e^{\frac{1}{2}x(1-y)} y^{\frac{1}{2}-m} \sum_{n=0}^{\infty} \frac{(1-y)^n}{n!} x^{\frac{1}{2}n} (\frac{1}{2}-m-k)_n W_{k-\frac{1}{2}n, m-\frac{1}{2}n}(x), \quad (2.6.20)$$

$$= e^{\frac{1}{2}x(1-y)} y^k \sum_{n=0}^{\infty} \frac{(y-1)^n}{n!} x^n (\frac{1}{2}+m-k)_n (\frac{1}{2}-m-k)_n W_{k-n, m}(x), \quad (2.6.21)$$

$$= e^{\frac{1}{2}x(y-1)} y^{\frac{1}{2}+m} \sum_{n=0}^{\infty} \frac{(1-y)^n}{n!} x^{\frac{1}{2}n} W_{k+\frac{1}{2}n, m+\frac{1}{2}n}(x), \quad (2.6.22)$$

$$= e^{\frac{1}{2}x(y-1)} y^{\frac{1}{2}-m} \sum_{n=0}^{\infty} \frac{(1-y)^n}{n!} x^{\frac{1}{2}n} W_{k+\frac{1}{2}n, m-\frac{1}{2}n}(x), \quad (2.6.23)$$

$$= e^{\frac{1}{2}x(y-1)} y^k \sum_{n=0}^{\infty} \frac{(y-1)^n}{n!} y^{-n} (-1)^n W_{k+n, m}(x). \quad (2.6.24)$$

2.7 Expansions in series of Bessel functions

We have seen that

$${}_1F_1[n + \frac{1}{2}; 2n + 1; x] = 2^{2n} n! e^{\frac{1}{2}x} x^{-n} I_n(\frac{1}{2}x). \quad (2.7.1)$$

But

$${}_1F_1'[a; b; x] = a {}_1F_1[a + 1; b + 1; x]/b.$$

Hence

$${}_1F_1[n + 3/2; 2n + 2; x] = 2^{2n} n! e^{\frac{1}{2}x} x^{-n} \{(1-n) I_n(\frac{1}{2}x) + I_n'(\frac{1}{2}x)\}. \quad (2.7.2)$$

Also

$$n I_n(\frac{1}{2}x)/x = \frac{1}{4} \{I_{n-1}(\frac{1}{2}x) - I_{n+1}(\frac{1}{2}x)\},$$

and

$$I_n'(\frac{1}{2}x) = \frac{1}{2} \{I_{n-1}(\frac{1}{2}x) + I_{n+1}(\frac{1}{2}x)\}.$$

Hence, substituting for $nI_n(\frac{1}{2}x)/x$ and $I'_n(\frac{1}{2}x)$, we have, after some reduction,

$${}_1F_1[n+3/2; 2n+2; x] = 2^{2n}n! e^{\frac{1}{2}x} x^{-n} \{I_n(\frac{1}{2}x) + I_{n+1}(\frac{1}{2}x)\}. \quad (2.7.3)$$

Similarly, on repeating this process, we find that

$${}_1F_1[n+5/2; 2n+3; x] = 2^{2n}n! e^{\frac{1}{2}x} x^{-n} \left\{ I_n(\frac{1}{2}x) + \frac{4n+4}{2n+3} I_{n+1}(\frac{1}{2}x) + \frac{2n+1}{2n+3} I_{n+2}(\frac{1}{2}x) \right\}. \quad (2.7.4)$$

The existence of relations of the types (2.7.1), (2.7.3) and (2.7.4) suggests the existence of general expansions of the form

$$\mathfrak{F}[n+t-\frac{1}{2}; 2n+t; x] = 2^{2n}n! e^{\frac{1}{2}x} x^{-n} \sum_{r=0}^{\infty} A_{n+r}(t) \mathfrak{C}_{n+r}(\frac{1}{2}x), \quad (2.7.5)$$

where $\mathfrak{F}[a; b; x]$ is any confluent hypergeometric function and where $\mathfrak{C}_n(\frac{1}{2}x)$ is any Bessel function. A group of results of this type will now be discussed. They are fairly elementary and they all depend on the following summation theorem:

$${}_4F_3 \left[\begin{matrix} a, 1+\frac{1}{2}a, & b, & -n; \\ & \frac{1}{2}a, 1+a-b, 1+a+n; & 1 \end{matrix} \right] = \frac{(1+a)_n (\frac{1}{2} + \frac{1}{2}a - b)_n}{(\frac{1}{2} + \frac{1}{2}a)_n (1+a-b)_n}. \quad (2.7.6)$$

Further notes on similar theorems will be found in Bailey (1935, ch. IV) and in Slater (1954c).

2.7.1 An elementary proof of the ${}_4F_3[1]$ summation theorem

Since $\Gamma(1+a+n)/\Gamma(1+a) = (1+a)_n$, the theorem can be written in the form

$${}_4F_3 \left[\begin{matrix} a, 1+\frac{1}{2}a, & b, & c; \\ & \frac{1}{2}a, 1+a-b, 1+a-c; & 1 \end{matrix} \right] = \frac{\Gamma(\frac{1}{2} + \frac{1}{2}a) \Gamma(1+a-b) \Gamma(1+a-c) \Gamma(\frac{1}{2} + \frac{1}{2}a - b - c)}{\Gamma(1+a) \Gamma(\frac{1}{2} + \frac{1}{2}a - b) \Gamma(\frac{1}{2} + \frac{1}{2}a - c) \Gamma(1+a-b-c)}, \quad (2.7.7)$$

where b or c is a negative integer.

This is symmetrical in b and c . Suppose that the result is true when c has the values

$$0, -1, -2, \dots, -(n-1).$$

Then by its symmetry it also is true when b has these n values, and c has any value. In particular when $c = -n$, we can write (2.7.1) in the form

$$(\frac{1}{2} + \frac{1}{2}a)_n (1+a-b)_n {}_4F_3 \left[\begin{matrix} a, 1+\frac{1}{2}a, & b, & -n; \\ & \frac{1}{2}a, 1+a-b, 1+a+n; & 1 \end{matrix} \right] = (1+a)_n (\frac{1}{2} + \frac{1}{2}a - b)_n. \quad (2.7.8)$$

This is an equation between two polynomials each of degree n in b . Let $b = a+n$. This is a pole of the last term of the series and the truth of (2.7.8) for $c = -n, b = a+n$ follows easily. Hence we have shown that (2.7.8) is an equation of degree n in b which is satisfied for the $n+1$ values of b ,

$$b = 0, -1, -2, \dots, -(n-1), \quad \text{and} \quad b = a+n.$$

Hence (2.7.8) is true for all values of b when $c = -n$, if it is true for all values of b when $c = -(n-1)$. But it is certainly true when $c = 0$, and the proof is completed by induction. Such a series, in which the sum of each pair of numerator and denominator parameters is constant, is said to be a 'well-poised' series.

2.7.2 Expansion of Kummer's function in terms of $I_n(x)$

We now prove that

$${}_1F_1\left[n+t-\frac{1}{2}; 2n+t; x\right] = 2^{2n}n! e^{\frac{1}{2}x} x^{-n} \sum_{r=0}^{\infty} A_{n+r}(t) I_{n+r}\left(\frac{1}{2}x\right), \quad (2.7.9)$$

where

$$A_{n+r}(t) = \frac{(n+r)(2n)_r(-1)^r(1-t)_r}{nr!(2n+t)_r}$$

for all n and t , real or complex (Slater, 1954c). Let

$$S = 2^n n! e^{\frac{1}{2}x} x^{-n} \sum_{r=0}^{\infty} A_{n+r}(t) I_{n+r}\left(\frac{1}{2}x\right).$$

Then

$$\begin{aligned} S &= \sum_{r=0}^{\infty} \sum_{s=0}^{\infty} \frac{(2n)_r(1-t)_r(-1)^r(n+r+\frac{1}{2})_s x^{r+s}}{r!(n)_r(2n+t)_r 2^{2r} s! (2n+2r+1)_s} \\ &= \sum_{r=0}^{\infty} \frac{(2n)_r(1-t)_r(-1)^r x^r}{r!(n)_r(2n+t)_r} \sum_{m=r}^{\infty} \frac{(n+r+\frac{1}{2})_{m-r} x^{m-r}}{(m-r)!(2n+2r+1)_{m-r}} \\ &= \sum_{m=0}^{\infty} \frac{(n+\frac{1}{2})_m x^m}{m!(2n+1)_m} \sum_{r=0}^m \frac{(2n)_r(n+1)_r(1-t)_r(-m)_r}{r!(n)_r(2n+t)_r(2n+m+1)_r} \\ &= \sum_{m=0}^{\infty} \frac{(n+\frac{1}{2})_m x^m}{m!(2n+1)_m} {}_4F_3 \left[\begin{matrix} 2n, n+1, 1-t, -m; \\ n, 2n+t, 2n+m+1; 1 \end{matrix} \right]. \end{aligned}$$

This inner series is a ${}_4F_3[1]$ series with the special form of the second parameters, well-poised in $2n+1$, and so it has the sum

$$\frac{(2n+1)_m (n+t-\frac{1}{2})_m}{(n+\frac{1}{2})_m (2n+t)_m}.$$

Hence

$$S = \sum_{m=0}^{\infty} \frac{(n+t-\frac{1}{2})_m x^m}{m!(2n+1)_m} = {}_1F_1\left[n+t-\frac{1}{2}; 2n+t; x\right].$$

The proof of this result has been given in some detail as it forms an example of the important technique of inversion of the order of summation in order to reduce a doubly infinite sum to an infinite sum and a finite sum.

2.7.3 Some further expansions

We can rewrite this result in the form

$${}_1F_1[a; b; x] = e^{\frac{1}{2}x} \sum_{n=0}^{\infty} \frac{(2b-2a-1)_n (b-2a)_n (-\frac{1}{4}x)^n}{n! (b-a-\frac{1}{2})_n (b)_n} {}_0F_1\left[; b-a+\frac{1}{2}+n; (\frac{1}{4}x)^2\right], \quad (2.7.10)$$

where

$${}_0F_1\left[; a; (\frac{1}{2}x)^2\right] = \Gamma(a) (\frac{1}{2}x)^{1-a} I_{a-1}(x). \quad (2.7.11)$$

But

$$I_a(x) = e^{-\frac{1}{2}\pi i a} J_a(x e^{\frac{1}{2}\pi i}) \quad \text{for } -\pi < \arg x \leq \frac{1}{2}\pi, \quad (2.7.12)$$

and

$$I_a(x) = e^{\frac{3}{2}\pi i a} J_a(x e^{-\frac{3}{2}\pi i}) \quad \text{for } \frac{1}{2}\pi < \arg x \leq \pi \quad (2.7.13)$$

(Watson, 1948, §§3-7). These two expressions lead to the same expansion in $J_a(\frac{1}{2}x)$ for both ranges of $\arg x$. Hence our result becomes finally

$${}_1F_1[a; b; x] = e^{\frac{1}{2}x} \Gamma(b-a-\frac{1}{2}) (\frac{1}{4}x)^{a-b+\frac{1}{2}} \sum_{n=0}^{\infty} \frac{(2b-2a-1)_n (b-2a)_n (-1)^n}{n! (b)_n} I_{b-a-\frac{1}{2}+n}\left(\frac{1}{2}x\right) \quad \text{for all } \arg x, \quad (2.7.14)$$

and ${}_1F_1[a; b; -ix] = e^{-\frac{1}{2}xi} \Gamma(b - a - \frac{1}{2}) (\frac{1}{4}x)^{a + \frac{1}{2} - b}$
 $\times \sum_{n=0}^{\infty} \frac{(2b - 2a - 1)_n (b - 2a)_n i^n}{n! (b)_n} J_{b - a - \frac{1}{2} + n}(\frac{1}{2}x)$ for all $\arg x$. (2.7.15)

In terms of $M_{k,m}(x)$ these results become

$$M_{k,m}(x) = 2^{2m+2k} x^{\frac{1}{2}-k} \Gamma(m+k) \sum_{n=0}^{\infty} \frac{(2m+2k)_n (2k)_n (-1)^n}{n! (1+2m)_n} I_{m+k+n}(\frac{1}{2}x)$$
 for all $\arg x$, (2.7.16)

and $M_{k,m}(-ix) = 2^{2m+2k} x^{\frac{1}{2}-k} \Gamma(m+k) \exp\{-\frac{1}{2}\pi i(\frac{1}{2}+m)\}$
 $\times \sum_{n=0}^{\infty} \frac{(2m+2k)_n (2k)_n i^n}{n! (1+2m)_n} J_{m+k+n}(\frac{1}{2}x)$ for all $\arg x$. (2.7.17)

INTEGRAL PROPERTIES

3.1 Elementary integrals for Kummer's function

Let us consider the integral

$$\begin{aligned} I &= \int_0^1 e^{xt} t^{a-1} (1-t)^{b-a-1} dt \\ &= \int_0^1 \sum_{n=0}^{\infty} \frac{x^n}{n!} t^{a+n-1} (1-t)^{b-a-1} dt \\ &= \sum_{n=0}^{\infty} \frac{x^n}{n!} \int_0^1 t^{a+n-1} (1-t)^{b-a-1} dt. \end{aligned} \quad (3.1.1)$$

This last integral is an Euler beta function. Hence it can be evaluated in terms of gamma functions (Watson, 1946, § 12.4), and we find that

$$I = \sum_{n=0}^{\infty} \frac{x^n \Gamma(a+n) \Gamma(b-a)}{n! \Gamma(b+n)} = \frac{\Gamma(b-a) \Gamma(a)}{\Gamma(b)} {}_1F_1[a; b; x] \quad \text{for } \operatorname{Re} b > \operatorname{Re} a > 0,$$

that is
$${}_1F_1[a; b; x] = \frac{\Gamma(b)}{\Gamma(b-a) \Gamma(a)} \int_0^1 e^{xt} t^{a-1} (1-t)^{b-a-1} dt \quad \text{for } \operatorname{Re} b > \operatorname{Re} a > 0. \quad (3.1.2)$$

This elementary integral for Kummer's function can be transformed in several ways. First let us put $tx = v$, then

$${}_1F_1[a; b; x] = \frac{\Gamma(b)}{\Gamma(b-a) \Gamma(a)} x^{1-b} \int_0^x e^v v^{a-1} (x-v)^{b-a-1} dv. \quad (3.1.3)$$

If we put $w = 1-t$, then

$${}_1F_1[a; b; x] = \frac{\Gamma(b) e^x}{\Gamma(b-a) \Gamma(a)} \int_0^1 e^{-wx} w^{b-a-1} (1-w)^{a-1} dw. \quad (3.1.4)$$

If further we put $s = 2w - 1 = 1 - 2t$, then

$${}_1F_1[a; b; x] = \frac{2^{1-b} \Gamma(b) e^{\frac{1}{2}x}}{\Gamma(b-a) \Gamma(a)} \int_{-1}^{+1} e^{-\frac{1}{2}xs} (1+s)^{b-2} \left(\frac{1-s}{1+s} \right)^{a-1} ds, \quad (3.1.5)$$

and if we put $s' = 1 - 2w = 2t - 1$, then

$${}_1F_1[a; b; x] = \frac{2^{1-b} \Gamma(b) e^{\frac{1}{2}x}}{\Gamma(b-a) \Gamma(a)} \int_{-1}^{+1} e^{\frac{1}{2}xs'} (1-s')^{b-2} \left(\frac{1+s'}{1-s'} \right)^{a-1} ds'. \quad (3.1.6)$$

If $\cos \theta = s = 1 - 2t$,

$${}_1F_1[a; b; x] = \frac{2^{1-b} \Gamma(b) e^{\frac{1}{2}x}}{\Gamma(b-a) \Gamma(a)} \int_0^{\pi} \exp\left(-\frac{1}{2}x \cos \theta\right) \sin \theta^{b-1} \cot\left(\frac{1}{2}\theta\right)^{b-2a} d\theta \quad (3.1.7)$$

and if $\cos \phi = \cos(\pi - \theta) = 2t - 1$,

$${}_1F_1[a; b; x] = \frac{2^{1-b} \Gamma(b) e^{\frac{1}{2}x}}{\Gamma(b-a) \Gamma(a)} \int_0^{\pi} \exp\left(\frac{1}{2}x \cos \phi\right) \sin \phi^{b-1} \tan\left(\frac{1}{2}\phi\right)^{b-2a} d\phi. \quad (3.1.8)$$

If $u = c + (d - c)t$, we get

$${}_1F_1[a; b; x] = \frac{\Gamma(b) \exp\{-cx/(d-c)\}}{\Gamma(b-a)\Gamma(a)} (d-c)^{1-b} \int_c^d \exp\{xu/(d-c)\} (u-c)^{a-1} (d-u)^{b-a-1} du, \quad (3.1.9)$$

all under the conditions $\text{Re } b > \text{Re } a > 0$.

3.1.1 Barnes's integral for Kummer's function

Let us consider the integral

$$I_R = \frac{1}{2\pi i} \int_{ABCD} \frac{\Gamma(-s)\Gamma(a+s)}{\Gamma(b+s)} z^s ds \quad (3.1.10)$$

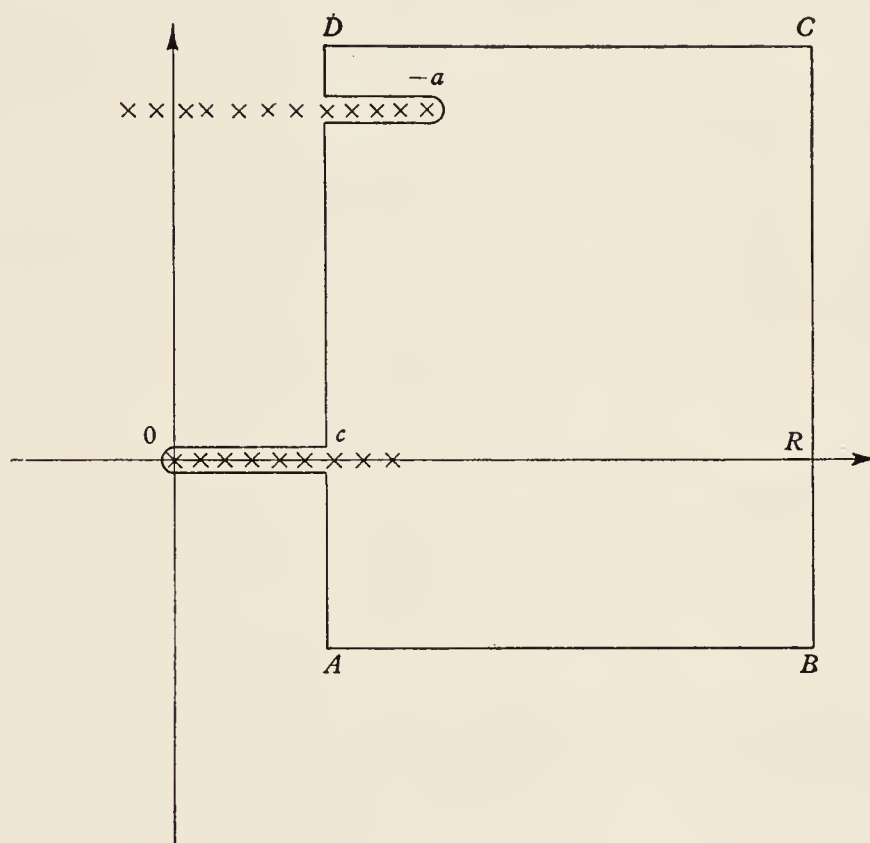


Fig. 3.1

taken round the rectangular contour

$$A(c - iN), \quad B(R - iN), \quad C(R + iM), \quad D(c + iM)$$

in the s -plane. The integrand has an increasing sequence of poles at the points

$$s = 0, 1, 2, \dots,$$

and a decreasing sequence of poles at the points

$$s = -a, -a - 1, -a - 2, \dots$$

It is assumed that the M and N are large enough for both sequences of poles to fall between the lines $y = M$ and $y = -N$, and that the contour $ABCD$ is indented so that the first R poles of the sequence $s = 0, 1, 2, \dots$ fall within $ABCD$ and all the poles of the sequence $s = -a, -a - 1, -a - 2, \dots$ fall

outside $ABCD$, otherwise we should require the unnecessary restriction $-\operatorname{Re} a < \operatorname{Re} s = c < 0$. Now

$$I_R = \int_{AB} + \int_{BC} + \int_{CD} + \int_{DA},$$

and on AB , $s = x - iN$; on BC , $s = R + t$; on CD , $s = x + iM$ and on DA , $s = c + iy$. Hence

$$I_R = J_1 + J_2 + J_3 - I_{M,N},$$

where

$$I_{M,N} = \int_{c-iN}^{c+iM} \frac{\Gamma(-s)\Gamma(a+s)}{\Gamma(b+s)} z^s ds,$$

$$J_1 = \int_c^R \frac{\Gamma(-x+iN)\Gamma(a+x-iN)}{\Gamma(b+x-iN)} z^{x-iN} dx,$$

$$J_2 = \int_{c-iN}^{c+iM} \frac{\Gamma(-R-t)\Gamma(a+R+t)}{\Gamma(b+R+t)} z^{R+t} dt,$$

and

$$J_3 = - \int_c^R \frac{\Gamma(-x-iM)\Gamma(a+x+iM)}{\Gamma(b+x+iM)} z^{x+iM} dx.$$

Now from a consideration of the residues of the integrand at those of its poles which fall within $ABCD$, that is at $s = 0, 1, 2, \dots, [R]$, we have

$$I_R = \Sigma_R \equiv \sum_{n=0}^{[R]} \frac{\Gamma(a+n)}{\Gamma(b+n)} z^n \frac{(-1)^{n-1}}{n!},$$

since the residue of $\Gamma(-s)$ at $s = n$ is $\frac{(-1)^{n-1}}{n!}$.

We now let M and $N \rightarrow \infty$ separately, using the results

$$|z^s| = |z|^{\operatorname{Re} s} e^{-\operatorname{Im} s \cdot \arg z} \quad (3.1.11)$$

and

$$|\Gamma(x+iy)| \sim \sqrt{(2\pi)} |x+iy|^{x-\frac{1}{2}} e^{-\frac{1}{2}\pi|y|} \quad (3.1.12)$$

(Watson, 1946, § 12.33), for $|x|$ finite and $|y|$ large, and we find that

$$I_{M,N} \rightarrow I \equiv \int_{c-i\infty}^{c+i\infty} \frac{\Gamma(-s)\Gamma(a+s)}{\Gamma(b+s)} z^s ds,$$

and

$$|J_1| \leq C \cdot N^{\operatorname{Re}(a-b)-\frac{1}{2}} \exp\{N(\arg z - \frac{1}{2}\pi)\} \int_c^R N^{-x} |z|^x dx.$$

Hence $J_1 \sim |z|^R N^{-R} e^{-N}$, provided that $|\arg z| < \frac{1}{2}\pi$, so that $J_1 \rightarrow 0$ as $N \rightarrow \infty$. Similarly, $J_3 \rightarrow 0$ as $M \rightarrow \infty$. Also

$$J_2 \rightarrow J_R \equiv \int_{c-i\infty}^{c+i\infty} \frac{\Gamma(-R-t)\Gamma(a+R+t)}{\Gamma(b+R+t)} z^{R+t} dt, \quad (3.1.13)$$

as M and $N \rightarrow \infty$. Hence

$$\Sigma_R = I_R = J_R - I. \quad (3.1.14)$$

Next we let $R \rightarrow \infty$. Then

$$\Sigma_R \rightarrow \Sigma_\infty \equiv - \sum_{n=0}^{\infty} \frac{\Gamma(a+n) z^n (-1)^n}{\Gamma(b+n) n!} = - \frac{\Gamma(a)}{\Gamma(b)} {}_1F_1[a; b; -z].$$

Also

$$|J_R| \leq C \cdot R^{\operatorname{Re}(a-b)-c-R} |z|^R \left| \int_{c-i\infty}^{c+i\infty} \exp\{t(\arg z - \frac{1}{2}\pi)\} dt \right|,$$

that is $J_R \sim |z|^R R^{-R}$. Hence $J_R \rightarrow 0$ as $R \rightarrow \infty$, and we have

$$\Sigma_\infty = -I,$$

that is, changing the sign of z throughout,

$$\frac{\Gamma(a)}{\Gamma(b)} {}_1F_1[a; b; z] = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{\Gamma(-s) \Gamma(a+s)}{\Gamma(b+s)} (-z)^s ds, \quad (3.1.15)$$

provided that $|\arg(-z)| < \frac{1}{2}\pi$, and $b \neq 0, -1, -2, \dots$. This is Barnes's integral for Kummer's function (Barnes, 1908 *a, b*). It holds for all finite values of c provided that the contour of integration can always be deformed so as to separate the two sequences of poles.

3.1.2 Barnes and Euler type integrals for $U(a; b; x)$

Next let us consider the integral

$$I_R = \frac{1}{2\pi i} \int_{ABCD} \Gamma(-s) \Gamma(a+s) \Gamma(1+a-b+s) z^{-s} ds, \quad (3.1.16)$$

taken round a similar contour. On evaluating the residues of the two sequences of poles

$$s = -a-n \quad \text{and} \quad s = b-a-1-n, \quad n = 0, 1, 2, \dots,$$

we find that as $R \rightarrow \infty$

$$\begin{aligned} I_R &\rightarrow z^a \sum_{n=0}^{\infty} \Gamma(a+n) \Gamma(1-b-n) z^n \frac{(-1)^n}{n!} + z^a \sum_{n=0}^{\infty} \Gamma(b-1-n) \Gamma(1+a-b+n) z^{1-b+n} \frac{(-1)^n}{n!}, \\ &= z^a \Gamma(a) \Gamma(1-b) {}_1F_1[a; b; z] + \Gamma(b-1) \Gamma(1+a-b) z^{1+a-b} {}_1F_1[1+a-b; 2-b; z], \\ &= z^a \Gamma(a) \Gamma(1+a-b) U(a; b; z). \end{aligned}$$

But on the remainder of the contour the integrand $\rightarrow 0$ as $R \rightarrow \infty$, and so

$$I_R \rightarrow I = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \Gamma(-s) \Gamma(a+s) \Gamma(1+a-b+s) z^{-s} ds, \quad (3.1.17)$$

provided that $|\arg z| < \frac{3}{2}\pi$. Hence we have the Mellin-Barnes integral for $U(a; b; z)$,

$$\Gamma(a) \Gamma(1+a-b) z^a U(a; b; z) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \Gamma(-s) \Gamma(a+s) \Gamma(1+a-b+s) z^{-s} ds, \quad \text{for } |\arg z| < \frac{3}{2}\pi, \quad (3.1.18)$$

$a \neq 0, 1, 2, \dots$ and $b-a \neq 0, 1, 2, \dots$

From this we can deduce the corresponding Euler integral for $U(a; b; x)$. We have

$$\begin{aligned} I &= \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \Gamma(-s) \Gamma(a+s) \Gamma(1+a-b+s) x^{-s-a} ds \\ &= \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \Gamma(-s) \Gamma(1+a-b+s) \left\{ \int_0^\infty e^{-xt} t^{a+s-1} dt \right\} ds \\ &= \int_0^\infty e^{-xt} t^{a-1} \left\{ \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \Gamma(-s) \Gamma(1+a-b+s) t^s ds \right\} dt \\ &= \int_0^\infty e^{-xt} t^{a-1} (1+t)^{b-a-1} \Gamma(1+a-b) dt, \end{aligned}$$

since $\text{Re } a > 0$, and so if $\text{Re } x > 0$, this interchange in the order of summation is justified. Hence

$$U(a; b; x) = \frac{1}{\Gamma(a)} \int_0^\infty e^{-xt} t^{a-1} (1+t)^{b-a-1} dt \quad (3.1.19)$$

for $\text{Re } a > 0$, $\text{Re } x > 0$.

This integral can also be transformed in various ways. If $tx = v$,

$$U(a; b; x) = \frac{x^{1-b}}{\Gamma(a)} \int_0^\infty e^{-v} v^{a-1} (x+v)^{b-a-1} dv. \quad (3.1.20)$$

If $w = 1+t$,
$$U(a; b; x) = \frac{e^x}{\Gamma(a)} \int_1^\infty e^{-xw} (w-1)^{a-1} w^{b-a-1} dw. \quad (3.1.21)$$

If $s = 1+2t$,
$$U(a; b; x) = \frac{2^{1-b} e^{\frac{1}{2}x}}{\Gamma(a)} \int_1^\infty e^{-\frac{1}{2}xs} (s-1)^{a-1} (s+1)^{b-a-1} ds, \quad (3.1.22)$$

and if $\cosh \theta = s$,
$$U(a; b; x) = \frac{2^{1-b} e^{\frac{1}{2}x}}{\Gamma(a)} \int_\pi^\infty e^{-\frac{1}{2}x \cosh \theta} \sinh \theta^{b-1} \coth \left(\frac{1}{2}\theta\right)^{b-2a} d\theta. \quad (3.1.23)$$

If $u = d - (d-c)/w$,

$$U(a; b; x) = \frac{(d-c)^{b-a} e^x}{\Gamma(a)} \int_c^d \exp\{-x(d-c)/(d-u)\} (u-c)^{a-1} (d-u)^{-b} du. \quad (3.1.24)$$

3.1.3 Pochhammer's contour integrals for Kummer's function

The elementary Euler type of integral (3.1.2) has the restrictions $0 < \text{Re } a < \text{Re } b$. If required, these restrictions can be relaxed or removed entirely provided that the integral is defined as a contour integral round a suitable contour. Let us consider first the integral round the contour OGG' , Fig. 3.2.

$$I_1 \equiv \int_0^{(1+)} e^{xt} t^{a-1} (1-t)^{b-a-1} dt. \quad (3.1.25)$$

This contour can be deformed into the real axis from 0 to $A(1-\epsilon)$, the circle C centre 1, radius ϵ , and the real axis again $A'(1-\epsilon)$ to 0. Then

$$\begin{aligned} I_1 &= [1 - \exp\{2(b-a)\pi i\}] \int_0^{1-\epsilon} e^{xt} t^{a-1} (1-t)^{b-a-1} dt + \int_C e^{xt} t^{a-1} (1-t)^{b-a-1} dt, \\ &= [1 - \exp\{2(b-a)\pi i\}] I - [1 - \exp\{2(b-a)\pi i\}] \int_{1-\epsilon}^1 e^{xt} t^{a-1} (1-t)^{b-a-1} dt \\ &\quad + \int_c e^{xt} t^{a-1} (1-t)^{b-a-1} dt, \end{aligned}$$

where

$$I \equiv \int_0^1 e^{xt} t^{a-1} (1-t)^{b-a-1} dt.$$

But
$$\lim_{\epsilon \rightarrow 0} \int_c e^{xt} t^{a-1} (1-t)^{b-a-1} dt = \lim_{\epsilon \rightarrow 0} \int_{1-\epsilon}^1 e^{xt} t^{a-1} (1-t)^{b-a-1} dt = 0, \quad \text{for } \text{Re } a > 0.$$

Hence
$$I_1 = [1 - \exp\{2(b-a)\pi i\}] I, \quad (3.1.26)$$

if $b-a$ is not a positive integer, so that

$${}_1F_1[a; b; x] = \frac{\Gamma(b)}{\Gamma(a) \Gamma(b-a) [1 - \exp\{2(b-a)\pi i\}]} I_1,$$

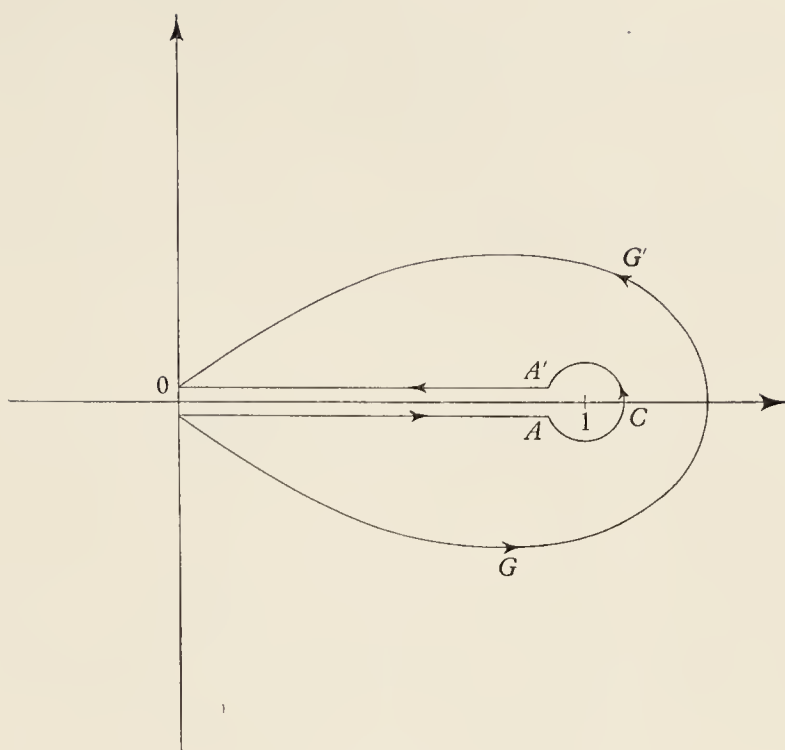


Fig. 3.2

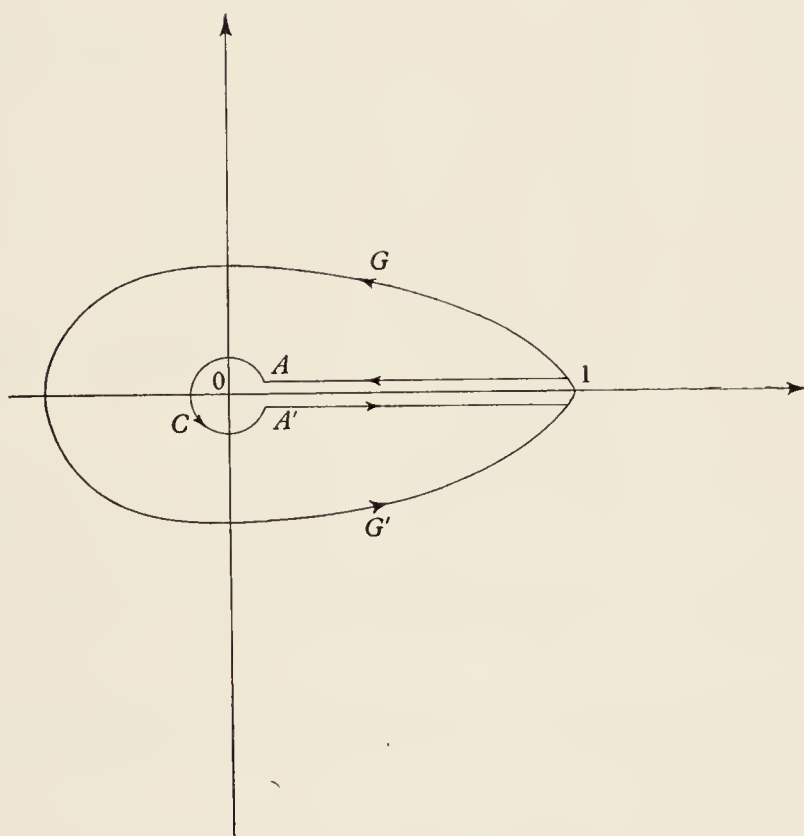


Fig. 3.3

that is, since

$$\frac{(1-t)^{b-a-1}}{\Gamma(b-a)[1-\exp\{2(b-a)\pi i\}]} = \frac{(t-1)^{b-a-1}\Gamma(1+a-b)}{2\pi i},$$

$${}_1F_1[a; b; x] = \frac{\Gamma(1+a-b)\Gamma(b)}{\Gamma(a)2\pi i} \int_0^{(1+)} e^{xt} t^{a-1}(t-1)^{b-a-1} dt. \quad (3.1.27)$$

But this expression is analytic for all values of b . Hence this result holds under the conditions that $\text{Re } a > 0$, and $b-a$ is not a positive integer.

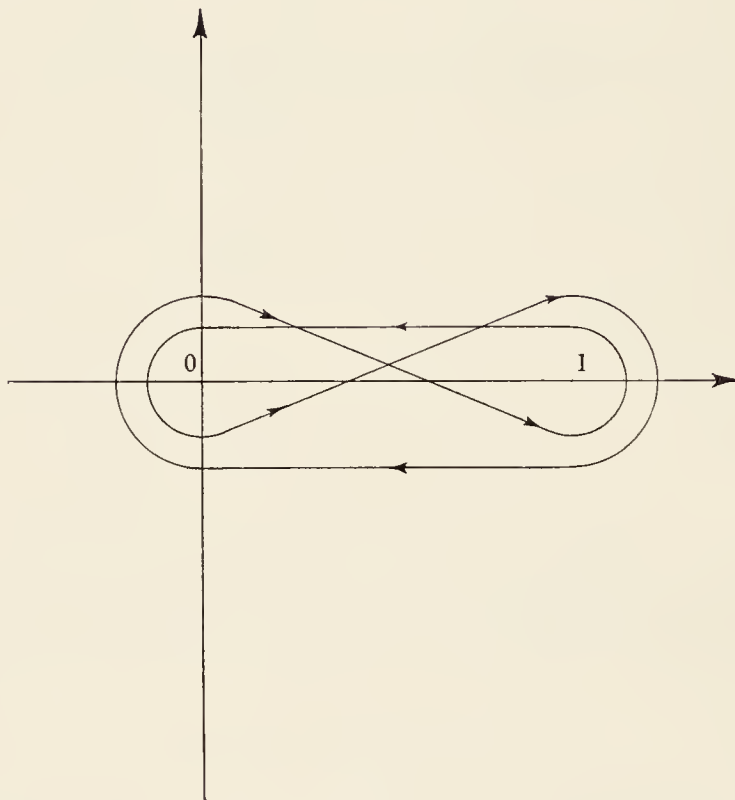


Fig. 3.4

In a similar way we can remove the condition $\text{Re } a > 0$, by considering this integral round the contour IGG' in Fig. 3.3

$$I_2 \equiv \int_1^{(0+)} e^{xt} t^{a-1}(1-t)^{b-a-1} dt, \quad (3.1.28)$$

where IGG' is a contour which deforms into the real axis ($0+\epsilon, 1$) and the circle centre 0 radius ϵ . The result in this case is

$${}_1F_1[a; b; x] = \frac{e^{-\pi i} \Gamma(1-a) \Gamma(b)}{2\pi i \Gamma(b-a)} \int_1^{(0+)} e^{xt} t^{a-1}(1-t)^{b-a-1} dt, \quad (3.1.29)$$

for $\text{Re } b > \text{Re } a$, $a \neq 1, 2, 3, \dots$

We can finally remove both conditions at once by taking the integral round Pochhammer's contour (Watson, 1946, § 12.43) (Fig. 3.4).

Then we find that

$${}_1F_1[a; b; x] = \frac{-e^{-\pi i b} \Gamma(1-a) \Gamma(b) \Gamma(1+a-b)}{4\pi^2} \int^{(1+, 0+, 1-, 0-)} e^{xt} t^{a-1}(1-t)^{b-a-1} dt, \quad (3.1.30)$$

where a and $b-a$ are not positive integers.

The contour of integration has been deformed into the real axis ($0 + \epsilon, 1 - \epsilon$) traversed four times, twice in each direction, and the two circles radius ϵ , centres 0 and 1 traversed twice, once in each direction.

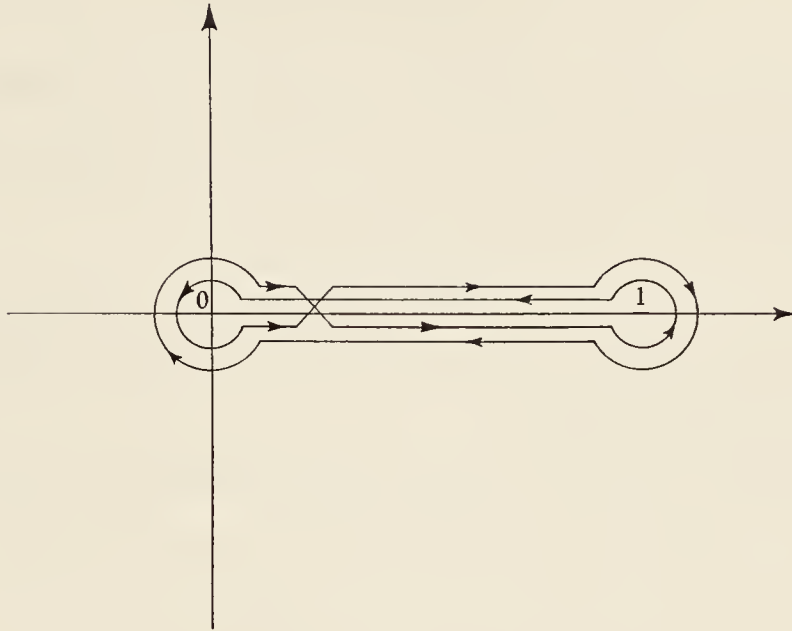


Fig. 3.5

3.1.4 *The Pochhammer integrals for $U(a; b; x)$*

We have already seen that

$$U(a; b; x) = \int_0^\infty e^{-xt} t^{a-1} (1+t)^{b-a-1} dt,$$

where $\text{Re } a > 0$, and $\text{Re } x > 0$. If we wish to remove the unduly restrictive condition on x , we can

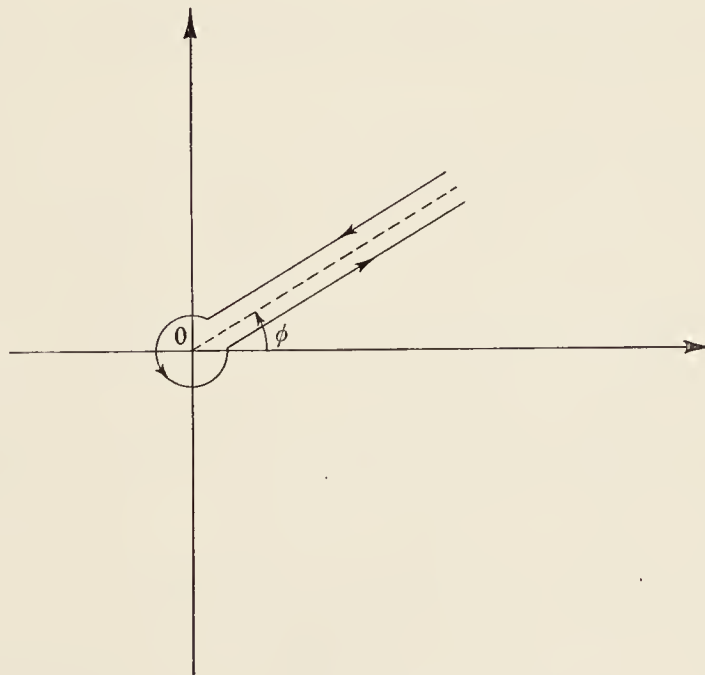


Fig. 3.6

extend this to $\text{Re}(xt) > 0$, where t is along a ray through the origin making an angle ϕ with the positive x -axis. This gives the line integral

$$U(a; b; x) = \frac{1}{\Gamma(a)} \int_0^{\infty e^{i\phi}} e^{-xt} t^{a-1} (1+t)^{b-a-1} dt, \quad (3.1.31)$$

for $-\frac{1}{2}\pi < \arg x + \phi < \frac{1}{2}\pi$ and $\text{Re} a > 0$. Here t^{a-1} and $(1+t)^{b-a-1}$ are assumed to have their principal values.

If we extend the integration round the contour we can relax the condition on a and we find the contour integral (Fig. 3.6)

$$U(a; b; x) = e^{-a\pi i} \frac{\Gamma(1-a)}{2\pi i} \int_{\infty e^{i\phi}}^{(0+)} e^{-xt} t^{a-1} (1+t)^{b-a-1} dt,$$

where $|\phi| < \pi$ for $|\arg x| < \frac{1}{2}\pi$ and $a \neq 1, 2, 3, \dots$. Here $\arg t = \arg(t+1) = \phi$ at the start of the loop.

3.2 Elementary indefinite integrals

The following integrals are deduced directly from the corresponding formulae for the derivatives in §§ 2.1 and 2.1.1:

$$\int {}_1F_1[a; b; x] dx = \frac{b-1}{a-1} {}_1F_1[a-1; b-1; x] + C \quad (a \neq 1), \quad (3.2.1)$$

$$\int x^{b-1} {}_1F_1[a; b; x] dx = \frac{x^b}{b} {}_1F_1[a; 1+b; x] + C, \quad (3.2.2)$$

$$\int x^{a-2} {}_1F_1[a; b; x] dx = \frac{x^{a-1}}{a-1} {}_1F_1[a-1; b; x] + C \quad (a \neq 1), \quad (3.2.3)$$

$$\int e^{-x} {}_1F_1[a; b; x] dx = \frac{e^{-x}(b-1)}{1+a-b} {}_1F_1[a; b-1; x] + C \quad (b-a \neq 1), \quad (3.2.4)$$

$$\int e^{-x} x^{b-1} {}_1F_1[a; b; x] dx = \frac{e^{-x} x^b}{b} {}_1F_1[1+a; 1+b; x] + C, \quad (3.2.5)$$

and
$$\int e^{-x} x^{b-a-2} {}_1F_1[a; b; x] dx = \frac{e^{-x} x^{b-a-1}}{b-a-1} {}_1F_1[1+a; b; x] + C \quad (b-a \neq 1), \quad (3.2.6)$$

where C is the constant of integration.

The corresponding integrals for $U(a; b; x)$ are

$$\int U(a; b; x) dx = \frac{1}{1-a} U(a-1; b-1; x) + C \quad (a \neq 1), \quad (3.2.7)$$

$$\int x^{b-1} U(a; b; x) dx = \frac{x^b}{b-a} U(a; b+1; x) + C \quad (a \neq b), \quad (3.2.8)$$

$$\int x^{a-2} U(a; b; x) dx = \frac{x^{a-1}}{(a-1)(a-b)} U(a-1; b; x) + C \quad (a \neq 1 \text{ and } a \neq b), \quad (3.2.9)$$

$$\int e^{-x} U(a; b; x) dx = -e^{-x} U(a; b-1; x) + C, \quad (3.2.10)$$

$$\int e^{-x} x^{b-1} U(a; b; x) dx = -e^{-x} x^b U(1+a; 1+b; x) + C \quad (3.2.11)$$

and
$$\int e^{-x} x^{b-a-2} U(a; b; x) dx = -e^{-x} x^{b-a-1} U(1+a; b; x) + C. \quad (3.2.12)$$

3.2.1 The Laplace transforms of ${}_1F_1[a; b; x]$

We shall write

$$g(s) \equiv \int_0^{\infty} e^{-st} f(t) dt = \mathfrak{L}_s\{f(t)\}, \quad (3.2.13)$$

and we define the Laplace transform of $f(t)$ as the integral $g(s)$, whenever this integral exists in the Lebesgue sense. See Erdélyi (1954) for references, and Titchmarsh (1937) or Widder (1941) for the background and theory of these transforms. Since

$$\int_0^{\infty} e^{-st} t^{\nu} dt = \Gamma(\nu+1) s^{-\nu-1}, \quad (3.2.14)$$

for $\text{Rl } \nu > -1$, $\text{Rl } s > 0$, that is $\mathfrak{L}_s(t^{\nu}) = \Gamma(\nu+1) s^{-\nu-1}$,

we find that the Laplace transform of the generalized hypergeometric series is, interchanging the order of summation and integration,

$$\int_0^{\infty} e^{-st} t^{\nu-1} {}_A F_B[(a); (b); kt] dt = \Gamma(\nu) s^{-\nu} {}_{A+1} F_B[(a), \nu; (b); k/s], \quad (3.2.15)$$

for $A < B$, $\text{Rl } \nu > 0$ and $\text{Rl } s > 0$, or $A = B$, $\text{Rl } \nu > 0$, $\text{Rl } s > \text{Rl } k$.

In particular for $A = B = 1$, we find that

$$\int_0^{\infty} e^{-st} t^{b-1} {}_1 F_1[a; c; kt] dt = \Gamma(b) s^{-b} {}_2 F_1[a, b; c; k/s], \quad (3.2.16)$$

for $\text{Rl } b > 0$, $\text{Rl } s > 0$ and $\text{Rl } s > \text{Rl } k$, $|s| > |k|$. If $k = \frac{1}{2}s$ and either $c = \frac{1}{2}(a+b+1)$ or $b = 1-a$, the ${}_2 F_1[\frac{1}{2}]$ series are summable (Bailey, 1935, §2.4), and we find that

$$\int_0^{\infty} e^{-st} t^{b-1} {}_1 F_1[a; \frac{1}{2}(a+b+1); \frac{1}{2}st] dt = s^{-b} \frac{\Gamma(\frac{1}{2}) \Gamma(b) \Gamma(\frac{1}{2} + \frac{1}{2}a + \frac{1}{2}b)}{\Gamma(\frac{1}{2} + \frac{1}{2}a) \Gamma(\frac{1}{2} + \frac{1}{2}b)}, \quad (3.2.17)$$

for $\text{Rl } b > 0$ and $\text{Rl } s > 0$, and that

$$\int_0^{\infty} e^{-st} t^{-a} {}_1 F_1[a; c; \frac{1}{2}st] dt = s^{a-1} \frac{\Gamma(1-a) \Gamma(\frac{1}{2}c) \Gamma(\frac{1}{2}c + \frac{1}{2})}{\Gamma(\frac{1}{2}c + \frac{1}{2}a) \Gamma(\frac{1}{2} + \frac{1}{2}c - \frac{1}{2}a)}, \quad (3.2.18)$$

for $\text{Rl}(1-a) > 0$ and $\text{Rl } s > 0$.

Alternatively, we can transform the ${}_2 F_1[x]$ series and we find that

$$\int_0^{\infty} e^{-st} t^{b-1} {}_1 F_1[a; c; kt] dt = \Gamma(b) (s-k)^{-b} {}_2 F_1[c-a, b; c; k/(k-s)] \quad (3.2.19)$$

for $|s-k| > |k|$ by Euler's transform (Bailey, 1935, §2.4. (1)). Further, when $b = c$, in (3.2.16),

$$\begin{aligned} \int_0^{\infty} e^{-st} t^{b-1} {}_1 F_1[a; b; kt] dt &= \Gamma(b) s^{-b} {}_1 F_0[a; ; k/s] \\ &= \Gamma(b) s^{-b} (1-k/s)^{-a}, \end{aligned} \quad (3.2.20)$$

for $\text{Rl } b > 0$, $\text{Rl } s > \text{Rl } k$, $|s| > |k|$, if we carry out the summation by the binomial theorem.

From this result we have

$$\begin{aligned} \int_0^{\infty} e^{-st} t^{b-1} {}_1 F_1[a; b; kt] dt &\times \int_0^{\infty} e^{-st} t^{b'-1} {}_1 F_1[a'; b'; kt] dt \\ &= \frac{\Gamma(b) \Gamma(b')}{\Gamma(b+b')} \int_0^{\infty} e^{-st} t^{b+b'-1} {}_1 F_1[a+a'; b+b'; kt] dt, \end{aligned} \quad (3.2.21)$$

where $\text{Rl } b > 0$, $\text{Rl } b' > 0$, and $\text{Rl } s > 1$. Also, if $A = 0$, $B = 1$, in (3.2.15),

$$\int_0^\infty e^{-st} t^{a-1} {}_0F_1[; b; kt] dt = \Gamma(a) s^{-a} {}_1F_1[a; b; k/s], \quad (3.2.22)$$

for $\text{Rl } a > 0$, $\text{Rl } s > 0$. But ${}_0F_1[; b; -x] = \Gamma(b) x^{\frac{1}{2}-b} J_{b-1}(2x^{\frac{1}{2}})$,

a Bessel function. Hence, if we write $k = -x$,

$${}_1F_1[a; b; -x/s] = \frac{\Gamma(b)}{\Gamma(a)} x^{\frac{1}{2}-b} s^a \int_0^\infty e^{-st} t^{a-\frac{1}{2}-b} J_{b-1}(2x^{\frac{1}{2}} t^{\frac{1}{2}}) dt, \quad (3.2.23)$$

for $\text{Rl } a > 0$ and $\text{Rl } s > 0$. In particular, when $a = b$ this result becomes

$$\int_0^\infty e^{-st} t^{a-1} {}_0F_1[; a; kt] dt = \Gamma(a) s^{-a} e^{k/s}. \quad (3.2.24)$$

If we apply Kummer's first theorem to this result (3.2.23) we get

$${}_1F_1[a; b; x/s] = \frac{\Gamma(b)}{\Gamma(b-a)} e^{x/s} x^{\frac{1}{2}-b} s^{b-a} \int_0^\infty e^{-st} t^{\frac{1}{2}b-\frac{1}{2}-a} J_{b-1}(2x^{\frac{1}{2}} t^{\frac{1}{2}}) dt, \quad (3.2.25)$$

and if we put τ for st , z for x/s , this becomes

$${}_1F_1[a; b; z] = \frac{\Gamma(b)}{\Gamma(b-a)} e^z z^{\frac{1}{2}-b} \int_0^\infty e^{-\tau} \tau^{\frac{1}{2}b-\frac{1}{2}-a} J_{b-1}(2z^{\frac{1}{2}} \tau^{\frac{1}{2}}) d\tau. \quad (3.2.26)$$

If in (3.2.23) we put $-x$ for x we get

$${}_1F_1[a; b; x/s] = \frac{\Gamma(b)}{\Gamma(a)} x^{\frac{1}{2}-b} s^a \int_0^\infty e^{-st} t^{a-\frac{1}{2}-b} I_{b-1}(2x^{\frac{1}{2}} t^{\frac{1}{2}}) dt. \quad (3.2.27)$$

The general product theorem for Laplace transforms is: if the integrals $g_1(t)$ and $g_2(t)$ are absolutely convergent then

$$g_1(t)g_2(t) = \int_0^\infty \left\{ \int_0^t f_1(\tau) f_2(t-\tau) d\tau \right\} e^{-st} dt. \quad (3.2.28)$$

This theorem can be applied to each pair of the above results to produce further results. Thus

$$\begin{aligned} & \int_0^\infty e^{-st} \left\{ \int_0^t \tau^{\mu-1} {}_A F_B[(a); (b); k\tau] (t-\tau)^{\nu-1} {}_{A'} F_{B'}[(a'); (b'); k'(t-\tau)] d\tau \right\} dt \\ & = \Gamma(\mu) \Gamma(\nu) s^{-\mu-\nu} {}_{A+1} F_B[(a), \mu; (b); k/s] {}_{A'+1} F_{B'}[(a'), \nu; (b'); k'/s], \end{aligned} \quad (3.2.29)$$

for $A < B$, $A' < B'$, $\text{Rl } \mu > 0$, $\text{Rl } \nu > 0$, $\text{Rl } s > 0$, or for $A = B$, $A' = B'$, $\text{Rl } \mu > 0$, $\text{Rl } \nu > 0$, $\text{Rl } s > \text{Rl } k$. If $A = B = 1 = A' = B'$, we have

$$\begin{aligned} & \int_0^\infty e^{-st} \left\{ \int_0^t \tau^{\mu-1} {}_1F_1[a; b; k\tau] (t-\tau)^{\nu-1} {}_1F_1[a'; b'; k'(t-\tau)] d\tau \right\} dt \\ & = \Gamma(\mu) \Gamma(\nu) s^{-\mu-\nu} {}_2F_1[a, \mu; b; k/s] {}_2F_1[a', \nu; b'; k'/s], \end{aligned} \quad (3.2.30)$$

for $\text{Rl } b > 0$, $\text{Rl } b' > 0$, $\text{Rl } s > 0$, $\text{Rl } s > \text{Rl } k$, and $|s| > |k|$. In particular if $\mu = b$, $\nu = b'$,

$$\begin{aligned} & \int_0^\infty e^{-st} \left\{ \int_0^t \tau^{b-1} {}_1F_1[a; b; k\tau] (t-\tau)^{b'-1} {}_1F_1[a'; b'; k(t-\tau)] d\tau \right\} dt \\ & = \frac{\Gamma(b) \Gamma(b')}{\Gamma(b+b')} \int_0^\infty e^{-st} t^{b+b'-1} {}_1F_1[a+a'; b+b'; kt] dt, \end{aligned} \quad (3.2.31)$$

by (3.2.21). But if

$$\mathfrak{L}\{g_1(s)\} = \mathfrak{L}\{g_2(s)\}$$

then

$$g_1(s) = g_2(s).$$

Hence we have the integral addition formula for Kummer's function

$$\int_0^t \tau^{b-1} {}_1F_1[a; b; k\tau] (t-\tau)^{b'-1} {}_1F_1[a'; b'; k(t-\tau)] d\tau = \frac{\Gamma(b)\Gamma(b')}{\Gamma(b+b')} t^{b+b'-1} {}_1F_1[a+a'; b+b'; kt], \quad (3.2.32)$$

for $\text{Rl } b > 0$, and $\text{Rl } b' > 0$, and if $a' = 0$, and $b+b' = c$,

$$\int_0^t \tau^{b-1} (t-\tau)^{c-b-1} {}_1F_1[a; b; k\tau] d\tau = \frac{\Gamma(b)\Gamma(c-b)}{\Gamma(c)} t^{c-1} {}_1F_1[a; c; kt], \quad (3.2.33)$$

for $\text{Rl } c > \text{Rl } b > 0$. In particular when $t = 1$, and $b = 2a$,

$${}_1F_1[a; c; x] = \frac{\Gamma(c)\sqrt{\pi}}{\Gamma(a)\Gamma(c-2a)} x^{\frac{1}{2}-a} \int_0^1 e^{\frac{1}{2}x\tau} \tau^{a-\frac{1}{2}} (1-\tau)^{c-2a-1} I_{a-\frac{1}{2}}(\frac{1}{2}x\tau) d\tau \quad (3.2.34)$$

for $\text{Rl } c > 2\text{Rl } a > 0$, since ${}_1F_1[a; 2a; x] = \Gamma(a + \frac{1}{2}) (\frac{1}{4}x)^{\frac{1}{2}-a} e^{\frac{1}{2}x} I_{a-\frac{1}{2}}(\frac{1}{2}x)$. (3.2.35)

It should be noted that each Laplace transform can be extended in definition to become a contour integral round a single or a double loop contour of the Pochhammer type. Each such extension will result in a relaxation of the conditions for convergence.

3.2.2 The inverse Laplace transform

We write

$$F(t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{ts} g(s) ds = \mathfrak{L}_x^{-1}\{g(s)\}, \quad (3.2.36)$$

and call $F(t)$ the inverse Laplace transform of $g(s)$. This inversion is always valid if $\mathfrak{L}\{F(t)\}$ converges absolutely on $\text{Rl } s = c$ and $F(x)$ is of bounded variation in some neighbourhood of t .

We deduce immediately that, since

$$\frac{t^{\nu-1}}{\Gamma(\nu)} = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{st} s^{-\nu} ds, \quad (3.2.37)$$

for $\text{Rl } \nu > 0$, $\text{Rl } s > 0$; the inverse Laplace transform for the generalized hypergeometric series is

$$\frac{t^{\nu-1}}{\Gamma(\nu)} {}_A F_{B+1}[(a); (b), \nu; kt] = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{st} s^{-\nu} {}_A F_B[(a); (b); k/s] ds, \quad (3.2.38)$$

for $A < B+1$, $\text{Rl } s > 0$, $\text{Rl } \nu > 0$; or for $A = B+1$ and $\text{Rl } s > \text{Rl } k$, $\text{Rl } \nu > 0$. In particular for $A = B = 1$, we have

$$\frac{t^{\nu-1}}{\Gamma(\nu)} {}_1F_2[a; b, \nu; kt] = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{st} s^{-\nu} {}_1F_1[a; b; k/s] ds, \quad (3.2.39)$$

for $\text{Rl } s > \text{Rl } k$, $\text{Rl } \nu > 0$; and if $a = \nu$,

$$\frac{t^{a-1}}{\Gamma(a)} {}_0F_1[; b; kt] = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{st} s^{-a} {}_1F_1[a; b; k/s] ds, \quad (3.2.40)$$

for $\text{Rl } s > \text{Rl } k$ and $\text{Rl } a > 0$, or, expressed as a Bessel function,

$$\frac{t^{a-\frac{1}{2}b-\frac{1}{2}}}{\Gamma(a)} k^{\frac{1}{2}-\frac{1}{2}b} \Gamma(b) I_{b-1}(2k^{\frac{1}{2}}t^{\frac{1}{2}}) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{st} s^{-a} {}_1F_1[a; b; k/s] ds, \quad (3.2.41)$$

for $\text{Rl } s > \text{Rl } k$, $\text{Rl } a > 0$. If $A = 1$, and $B = 0$, we have

$$\begin{aligned} \frac{t^{\nu-1}}{\Gamma(\nu)} {}_1F_1[a; \nu; kt] &= \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{st} s^{-\nu} {}_1F_0[a; ; k/s] ds, \\ &= \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{st} s^{-\nu} (1-k/s)^{-a} ds, \end{aligned} \quad (3.2.42)$$

for $\text{Rl } \nu > 0$ and $\text{Rl } s > \text{Rl } k$. Again if $A = 2$, $B = 1$ and $a_2 = \nu$, in (3.2.38),

$${}_1F_1[a; b; kt] = \frac{1}{2\pi i} \Gamma(\nu) t^{1-\nu} \int_{c-i\infty}^{c+i\infty} e^{st} s^{-\nu} {}_2F_1[a, \nu; b; k/s] ds, \quad (3.2.43)$$

for $\text{Rl } \nu > 0$ and $\text{Rl } s > \text{Rl } k$. In particular if $\nu = 1, 2, \dots, n+1$, a positive integer, the integrand is a one-valued function of s , and the path of integration may be replaced by a closed contour, for example by a circle centre O , radius $|s| > 1$, and then

$${}_1F_1[a; b; x] = \frac{n!}{2\pi i} x^{-n} \int_c e^{sx} s^{-n-1} {}_2F_1[a, 1+n; b; 1/s] ds, \quad (3.2.44)$$

for n a positive integer.

3.2.3 The Laplace transform of $U(a; b; x)$

From the definition of $U(a; b; x)$ in terms of two Kummer functions we can deduce from (3.2.22) that

$$U(a; b; x/s) = \frac{2x^{\frac{1}{2}-\frac{1}{2}b} s^a}{\Gamma(a)\Gamma(1+a-b)} \int_0^\infty e^{-st} t^{a-\frac{1}{2}-\frac{1}{2}b} K_{b-1}(2x^{\frac{1}{2}} t^{\frac{1}{2}}) dt, \quad (3.2.45)$$

for $\text{Rl } a > 0$, $\text{Rl}(a-b) > -1$, and $\text{Rl } s > 0$; where

$$K_{b-1}(2\sqrt{x}) = \frac{\pi}{2 \sin \pi b} \left\{ \frac{x^{\frac{1}{2}b-\frac{1}{2}}}{\Gamma(b)} {}_0F_1[; b; x] - \frac{x^{\frac{1}{2}-\frac{1}{2}b}}{\Gamma(2-b)} {}_0F_1[; 2-b; x] \right\}, \quad (3.2.46)$$

$$\text{and inversely } 2t^{a-\frac{1}{2}-\frac{1}{2}b} K_{b-1}(2x^{\frac{1}{2}} t^{\frac{1}{2}}) = \frac{\Gamma(a)\Gamma(1+a-b)}{2\pi i x^{\frac{1}{2}-\frac{1}{2}b}} \int_{c-i\infty}^{c+i\infty} e^{ts} s^{-a} U(a; b; x/s) ds, \quad (3.2.47)$$

for $\text{Rl } a > 0$, $\text{Rl } s > 0$.

The elementary integral (3.1.19) can be written

$$\mathfrak{L}\{t^{a-1}(1+t)^{b-a-1}\} = U(a; b; s) \quad (3.2.48)$$

for $\text{Rl } a > 0$, and $\text{Rl } s > 0$; and this gives the inverse

$$t^{a-1}(1+t)^{b-a-1} = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{ts} U(a; b; s) ds, \quad (3.2.49)$$

for $\text{Rl } a > 0$ and $\text{Rl } s > 0$.

From (3.2.16), since

$$\begin{aligned} {}_2F_1[a, b; c; x] &= \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} {}_2F_1[a, b; 1+a+b-c; 1-x] \\ &\quad + (1-x)^{c-a-b} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} {}_2F_1[c-a, c-b; 1+c-a-b; 1-x] \end{aligned} \quad (3.2.50)$$

(Bailey, 1935, § 1.4.1), we find that

$$\int_0^\infty e^{-st} t^{b-1} U(a; c; t) dt = \frac{\Gamma(b)\Gamma(1+b-c)}{\Gamma(1+a+b-c)} {}_2F_1[b, 1+b-c; 1+a+b-c; 1-s], \quad (3.2.51)$$

for $\text{Rl } b > 0$, $\text{Rl } c < \text{Rl } b + 1$, $|1-s| < 1$,

$$= \frac{\Gamma(b) \Gamma(1+b-c)}{\Gamma(1+a+b-c)} s^{-b} {}_2F_1[a, b; 1+a+b-c; 1-s] \quad \text{for } \text{Rl } s > \frac{1}{2}. \quad (3.2.52)$$

On inversion these give

$$U(a; d; t) = \frac{\Gamma(b) \Gamma(1+b-d)}{\Gamma(1+a+b-d)} \frac{t^{1-b}}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{st} {}_2F_1[b, 1+b-d; 1+a+b-d; 1-s] ds, \quad (3.2.53)$$

$$= \frac{\Gamma(b) \Gamma(1+b-d)}{\Gamma(1+a+b-d)} \frac{t^{1-b}}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{st} s^{-b} {}_2F_1[a, b; 1+a+b-d; 1-s] ds, \quad (3.2.54)$$

for $\text{Rl } b > 0$, $\text{Rl } d < 1 + \text{Rl } b$, $\text{Rl } s > \frac{1}{2}$.

An integral connected with these is Meijer's integral

$$U(a; d; x) = \frac{x^{a-b}}{\Gamma(b)} \int_0^\infty e^{-xt} t^{b-1} {}_2F_1[a, 1+a-d; b; -t] dt, \quad (3.2.55)$$

for $\text{Rl } b > 0$, $\text{Rl } x > 0$ (Meijer, 1936*b*); and the inverse of this integral is

$${}_2F_1[a, 1+a-d; b; -t] = \frac{\Gamma(b) t^{1-b}}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{st} s^{b-a} U(a; d; s) ds, \quad (3.2.56)$$

under the same conditions. We can deduce (3.2.55) from the Barnes integral for $U(a; d; x)$ (3.1.18), which is written as

$$\Gamma(a) \Gamma(1+a-d) U(a; d; x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{\Gamma(-s) \Gamma(a+s) \Gamma(1+a-d+s)}{\Gamma(s+b)} x^{-s-a} \Gamma(s+b) ds. \quad (3.2.57)$$

But
$$x^{s+b} \int_0^\infty e^{-st} t^{s+b-1} dt = \Gamma(s+b).$$

Hence interchanging the order of the two integrations we find that

$$\Gamma(a) \Gamma(1+a-d) U(a; d; x) = \frac{x^{b-a}}{2\pi i} \int_0^\infty e^{-xt} t^{b-1} \left\{ \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{\Gamma(-s) \Gamma(a+s) \Gamma(1+a-d+s)}{\Gamma(s+b)} t^s ds \right\} dt \quad (3.2.58)$$

and this inner integral is the Barnes integral for the Gauss function

$${}_2F_1[a, 1+a-d; b; -t],$$

(Bailey, 1935, § 1.6).

3.3 Mellin transforms of ${}_1F_1[a; b; x]$

We define the Mellin transform as

$$g(s) = \int_0^\infty f(x) x^{s-1} dx \equiv \mathfrak{M}_s\{f(x)\} \quad (3.3.1)$$

and the inverse Mellin transform as

$$f(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} g(s) x^{-s} ds \equiv \mathfrak{M}'_x\{g(s)\}. \quad (3.3.2)$$

These transforms hold under the condition that $g(s)$ exists in the Lebesgue sense over the range $0, \infty$.

For the generalized hypergeometric function we find that

$$\Gamma(s) k^{-s} {}_A F_B[(a), s; (b); y/k] = \int_0^\infty x^{s-1} e^{-kx} {}_A F_B[(a); (b); xy] dx, \quad (3.3.3)$$

by expanding the series under the integral and using (3.2.14), for $\text{Rl } k > 0$, and $\text{Rl } s > 0$ and either $A < B$ or $A = B$ and $|y| < 1$. Inversely we have

$$e^{-kx} {}_A F_B[(a); (b); xy] = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} k^{-s} x^{-s} \Gamma(s) {}_A F_B[(a), s; (b); y/k] ds, \quad (3.3.4)$$

under the same conditions (see Slater, 1955*d* for more general results). These two results should be compared with the corresponding results (3.2.15) and (3.2.38) for the Laplace transform. The first integral is substantially the same as (3.2.15) but its inverse is new. In particular if $A = 0$, $B = 1$,

$$\Gamma(s) {}_1 F_1[s; b; y] = \int_0^\infty e^{-x} x^{s-1} {}_0 F_1[; b; xy] dx, \quad (3.3.5)$$

for $\text{Rl } s > 0$, and

$$e^{-x} {}_0 F_1[; b; xy] = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} x^{-s} \Gamma(s) {}_1 F_1[s; b; y] ds, \quad (3.3.6)$$

for $\text{Rl } s > 0$; that is

$$\Gamma(s) {}_1 F_1[s; b; y] = \Gamma(b) \int_0^\infty x^{s-1} (xy)^{\frac{1}{2}-\frac{1}{2}b} e^{-x} I_{b-1}(2x^{\frac{1}{2}}y^{\frac{1}{2}}) dx, \quad (3.3.7)$$

for $\text{Rl } s > 0$, and

$$e^{-x} I_{b-1}(2x^{\frac{1}{2}}y^{\frac{1}{2}}) (xy)^{\frac{1}{2}b-\frac{1}{2}} \Gamma(b) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} x^{-s} \Gamma(s) {}_1 F_1[s; b; y] ds, \quad (3.3.8)$$

for $\text{Rl } s > 0$. From (3.3.7) we deduce

$$\Gamma(s + \frac{1}{2}) {}_1 F_1[-s; \frac{1}{2}; y] = \int_0^\infty x^{s-\frac{1}{2}} e^{-x} \cos(2x^{\frac{1}{2}}y^{\frac{1}{2}}) dx, \quad (3.3.9)$$

for $\text{Rl } s > \frac{1}{2}$; and

$$\sqrt{(2y)} \Gamma(s + \frac{1}{2}) {}_1 F_1[1-s; \frac{3}{2}; y] = \int_0^\infty x^{s-1} e^{-x} \sin(2x^{\frac{1}{2}}y^{\frac{1}{2}}) dx, \quad (3.3.10)$$

for $\text{Rl } s > -\frac{1}{2}$.

If $A = B = 1$, in (3.3.3),

$${}_2 F_1[b, s; d; k/a] a^{-s} \Gamma(s) = \int_0^\infty e^{-ax} x^{s-1} {}_1 F_1[b; d; kx] dx, \quad (3.3.11)$$

for $\text{Rl } a > 0$, $\text{Rl } a > \text{Rl } k$, and $\text{Rl } s > 0$; and inversely

$$e^{-ax} {}_1 F_1[b; d; kx] = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} x^{-s} a^{-s} \Gamma(s) {}_2 F_1[b, s; d; k/a] ds, \quad (3.3.12)$$

under the same conditions.

From the Barnes integral

$${}_1 F_1[a; b; x] = \frac{\Gamma(b)}{\Gamma(a)} \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{\Gamma(-s) \Gamma(a+s) (-x)^s}{\Gamma(b+s)} ds, \quad (3.3.13)$$

we get by inversion

$$\frac{\Gamma(b) \Gamma(s) \Gamma(a-s)}{\Gamma(a) \Gamma(b-s)} = \int_0^\infty t^{s-1} {}_1 F_1[a; b; -t] dt, \quad (3.3.14)$$

for $0 < \text{Rl } s < \text{Rl } a$.

3.3.1 The Mellin transforms of $U(a; b; x)$

From (3.3.5) and (3.2.46) we deduce

$$U(s; b; y) = \frac{2}{\Gamma(s) \Gamma(\mathbf{1} - b + s)} \int_0^\infty e^{-x} x^{s-1} (xy)^{\frac{1}{2} - \frac{1}{2}b} K_{b-1}(2x^{\frac{1}{2}} y^{\frac{1}{2}}) dx, \quad (3.3.15)$$

for $\text{Rl } s > 0$, $\text{Rl}(s-b) > -1$, and inversely

$$K_{b-1}(2x^{\frac{1}{2}} y^{\frac{1}{2}}) = \frac{e^x (xy)^{\frac{1}{2}b - \frac{1}{2}}}{4\pi i} \int_{c-i\infty}^{c+i\infty} x^{-s} \Gamma(s) \Gamma(\mathbf{1} + s - b) U(s; b; y) ds, \quad (3.3.16)$$

under the same conditions. From (3.3.11) using (3.2.50) we deduce

$$a^{-s} \Gamma(s) {}_2F_1[b, s; \mathbf{1} + b + s - d; \mathbf{1} - k/a] \Gamma(\mathbf{1} + s - d) = \Gamma(\mathbf{1} + b + s - d) \int_0^\infty e^{-ax} x^{s-1} U(b; d; kx) dx, \quad (3.3.17)$$

for $\text{Rl } s > 0$, $\mathbf{1} + \text{Rl } s > \text{Rl } d$, and inversely

$$U(b; d; kx) = \frac{e^{ax}}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{\Gamma(\mathbf{1} + s - d) a^{-s} x^{-s} \Gamma(s)}{\Gamma(\mathbf{1} + b + s - d)} {}_2F_1[b, s; \mathbf{1} + b + s - d; \mathbf{1} - k/a] ds, \quad (3.3.18)$$

under the same conditions. From the Barnes integral

$$U(a; b; x) = \frac{\mathbf{1}}{\Gamma(a) \Gamma(\mathbf{1} + a - b)} \frac{\mathbf{1}}{2\pi i} \int_{c-i\infty}^{c+i\infty} \Gamma(-s) \Gamma(a+s) \Gamma(\mathbf{1} - b + s + a) x^{-s-a} ds, \quad (3.3.19)$$

we find that

$$\frac{\Gamma(s) \Gamma(a-s) \Gamma(\mathbf{1} - b + a - s)}{\Gamma(a) \Gamma(\mathbf{1} + a - b)} = \int_0^\infty x^{a-s-1} U(a; b; x) dx, \quad (3.3.20)$$

for $\text{Rl } s > 0$, $\text{Rl } a > 0$, $\mathbf{1} + \text{Rl}(a-s) > \text{Rl } b$.

3.4 The Hankel transforms

We define the Hankel transform of order ν as

$$g(y, \nu) = \mathfrak{H}_\nu\{f(x); y\} = \int_0^\infty f(x) J_\nu(xy) \sqrt{(xy)} dx, \quad (3.4.1)$$

where y is a positive real variable. The transform is its own inverse and it reduces to the Fourier sine and cosine transforms respectively when $\nu = \pm \frac{1}{2}$.

Let us consider

$$I = \int_0^\infty x^{2b-\nu-\frac{3}{2}} {}_1F_1[a; b; -kx^2] J_\nu(xy) \sqrt{(xy)} dx. \quad (3.4.2)$$

If we apply Kummer's first theorem to the ${}_1F_1$ function, interchange the order of summation and integration and then integrate by the Laplace integral, we find that

$$I = \frac{\Gamma(b)}{\Gamma(a-b+\nu+\mathbf{1})} 2^{2b-2a-\nu-1} k^{-a} y^{2a-2b+\nu+\frac{1}{2}} {}_1F_1[a; \mathbf{1} + a - b + \nu; -y^2/4k], \quad (3.4.3)$$

for $\text{Rl } k > 0$, $y > 0$, $0 < \text{Rl } b < \frac{3}{4} + \text{Rl}(a + \frac{1}{2}\nu)$; whence, if $k = \frac{1}{2}$ and $b = \frac{1}{2} + \frac{1}{2}a + \frac{1}{2}\nu$,

$$\int_0^\infty x^{a-\frac{1}{2}} {}_1F_1[a; \frac{1}{2} + \frac{1}{2}a + \frac{1}{2}\nu; -\frac{1}{2}x^2] J_\nu(xy) \sqrt{(xy)} dx = y^{a-\frac{1}{2}} {}_1F_1[a; \frac{1}{2} + \frac{1}{2}a + \frac{1}{2}\nu; -\frac{1}{2}y^2], \quad (3.4.4)$$

for $\text{Rl } a > -\frac{1}{2}$ and $\text{Rl}(a + \nu) > -1$; and if $k = \frac{1}{2}$ and $b = 1 + \nu - a$ then

$$\begin{aligned} \int_0^\infty x^{\nu+\frac{1}{2}-2a} {}_1F_1[a; 1+\nu-a; -\frac{1}{2}x^2] J_\nu(xy) \sqrt{(xy)} dx \\ = \sqrt{\pi} 2^{-2a+\nu+\frac{1}{2}} \frac{\Gamma(1+\nu-a)}{\Gamma(a)} y^{2a-\nu-\frac{1}{2}} e^{-\frac{1}{4}y^2} I_{a-\frac{1}{2}}(\frac{1}{4}y^2), \end{aligned} \quad (3.4.5)$$

for $\text{Rl}(a + \frac{1}{2}\nu + \frac{3}{4}) > \text{Rl}(1 + \nu - a) > 0$.

The inverse of this integral is

$$\begin{aligned} {}_1F_1[a; 1+\nu-a; -\frac{1}{2}x^2] = \sqrt{\pi} \frac{\Gamma(1+\nu-a)}{\Gamma(a)} 2^{\frac{1}{2}+\nu-2a} x^{2a-\nu-\frac{1}{2}} \\ \times \int_0^\infty e^{-\frac{1}{4}y^2} y^{2a-\nu-\frac{1}{2}} I_{a-\frac{1}{2}}(\frac{1}{4}y^2) J_\nu(xy) \sqrt{(xy)} dy, \end{aligned} \quad (3.4.6)$$

under the same conditions. Also

$$\frac{\Gamma(1+a)\sqrt{\pi}}{\Gamma(2a-\nu)} x^{\nu+\frac{1}{2}} {}_1F_1[2a-\nu; 1+a; -\frac{1}{2}x^2] = 2^{\nu-a+\frac{1}{2}} \int_0^\infty y^{2a-\nu-\frac{1}{2}} e^{-\frac{1}{4}y^2} K_{a-\nu-\frac{1}{2}}(\frac{1}{4}y^2) J_\nu(xy) \sqrt{(xy)} dy, \quad (3.4.7)$$

and inversely,

$$\frac{\Gamma(1+a)\sqrt{\pi}}{\Gamma(2a-\nu)} \int_0^\infty x^{\nu+\frac{1}{2}} {}_1F_1[2a-\nu; 1+a; -\frac{1}{2}x^2] J_\nu(xy) \sqrt{(xy)} dx = 2^{\nu-a+\frac{1}{2}} y^{2a-\nu-\frac{1}{2}} e^{-\frac{1}{4}y^2} K_{a-\nu-\frac{1}{2}}(\frac{1}{4}y^2), \quad (3.4.8)$$

for $\text{Rl } \nu > -1$, $\text{Rl}(4a - 3\nu) > \frac{1}{2}$.

The Hankel transforms can be deduced directly from the Laplace transforms (Tricomi, 1954) and they are also special cases of the Mellin transforms (Slater, 1955 *d*, § 5.6).

3.5 Elementary integrals for the Whittaker functions

From the elementary integrals for ${}_1F_1[a; b; x]$ and the definition of $M_{k,m}(x)$ we deduce the following elementary integrals for $M_{k,m}(x)$:

$$\frac{\Gamma(\frac{1}{2}+m-k)\Gamma(\frac{1}{2}+m+k)}{\Gamma(1+2m)} M_{k,m}(x) = x^{\frac{1}{2}+m} e^{-\frac{1}{2}x} \int_0^1 e^{xu} u^{m-k-\frac{1}{2}} (1-u)^{m+k-\frac{1}{2}} du, \quad (3.5.1)$$

$$= x^{\frac{1}{2}-m} e^{-\frac{1}{2}x} \int_0^x e^v v^{m-k-\frac{1}{2}} (x-v)^{m+k-\frac{1}{2}} dv, \quad (3.5.2)$$

$$= x^{\frac{1}{2}+m} e^{\frac{1}{2}x} \int_0^1 e^{-vx} v^{m+k-\frac{1}{2}} (1-v)^{m-k-\frac{1}{2}} dv, \quad (3.5.3)$$

$$= x^{m+\frac{1}{2}} 2^{-2m} \int_{-1}^{+1} (1+u)^{m-k-\frac{1}{2}} (1-u)^{m+k-\frac{1}{2}} e^{\frac{1}{2}xu} du, \quad (3.5.4)$$

$$= x^{m-\frac{1}{2}} 2^{-2m} \int_{-1}^{+1} e^{-\frac{1}{2}sx} (1-s)^{m-k-\frac{1}{2}} (1+s)^{m+k-\frac{1}{2}} ds, \quad (3.5.5)$$

$$= x^{m+\frac{1}{2}} 2^{-2m} \int_0^\pi e^{-\frac{1}{2}x \cos \theta} \sin^{2m} \theta \cotan^{2k}(\frac{1}{2}\theta) d\theta, \quad (3.5.6)$$

$$= x^{m+\frac{1}{2}} 2^{-2m} \int_0^\pi e^{\frac{1}{2}x \cos \phi} \sin^{2m} \phi \tan^{2k}(\frac{1}{2}\phi) d\phi, \quad (3.5.7)$$

$$\begin{aligned} = x^{m+\frac{1}{2}} (d-c)^{-2m} e^{-xc/(d-c)} e^{-\frac{1}{2}x} \int_c^d e^{xu/(d-c)} (d-u)^{m+k-\frac{1}{2}} \\ \times (u-c)^{m-k-\frac{1}{2}} du, \end{aligned} \quad (3.5.8)$$

all under the conditions $\text{Rl}(\frac{1}{2} + m \pm k) > 0$, $|\arg x| < \pi$.

The corresponding integrals for $W_{k,m}(x)$ are

$$\Gamma(\tfrac{1}{2} + m - k) W_{k,m}(x) = x^{\frac{1}{2}-m} e^{-\frac{1}{2}x} \int_0^{\infty} e^{-u} u^{m-\frac{1}{2}-k} (x+u)^{m-\frac{1}{2}+k} du, \quad (3.5.9)$$

$$= x^{\frac{1}{2}+m} e^{-\frac{1}{2}x} \int_0^{\infty} e^{-xt} t^{m-k-\frac{1}{2}} (1+t)^{m+k-\frac{1}{2}} dt, \quad (3.5.10)$$

$$= x^{\frac{1}{2}+m} e^{\frac{1}{2}x} \int_1^{\infty} e^{-xw} (w-1)^{m-k-\frac{1}{2}} w^{m+k-\frac{1}{2}} dw, \quad (3.5.11)$$

$$= x^{\frac{1}{2}+m} 2^{-2m} \int_1^{\infty} e^{-\frac{1}{2}xs} (s-1)^{m-k-\frac{1}{2}} (s+1)^{m+k-\frac{1}{2}} ds, \quad (3.5.12)$$

$$= x^{\frac{1}{2}+m} 2^{-2m} \int_{\pi}^{\infty} e^{-\frac{1}{2}x \cosh \theta} \sinh^{2m} \theta \coth^{2k} (\tfrac{1}{2}\theta) d\theta, \quad (3.5.13)$$

$$= x^{\frac{1}{2}+m} e^{\frac{1}{2}x} (d-c)^{\frac{1}{2}+m+k} \int_c^d \exp\left\{\frac{x(d-c)}{d-u}\right\} (u-c)^{m-k-\frac{1}{2}} (d-u)^{-1-2m} du, \quad (3.5.14)$$

all under the conditions $\text{Re}(\tfrac{1}{2} + m \pm k) > 0$, $|\arg x| < \pi$.

3.5.1 Barnes type integrals for the Whittaker functions

The Barnes integral for $M_{k,m}(x)$ is

$$M_{k,m}(x) = \frac{x^{m+\frac{1}{2}} e^{-\frac{1}{2}x} \Gamma(1+2m)}{\Gamma(\tfrac{1}{2} + m - k) 2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{\Gamma(-s) \Gamma(\tfrac{1}{2} + m - k + s)}{\Gamma(1+2m+s)} (-x)^s ds, \quad (3.5.15)$$

for $|\arg x| < \frac{1}{2}\pi$, and $1+2m \neq 0, -1, -2, \dots$. This follows from the definition of $M_{k,m}(x)$ in terms of ${}_1F_1[a; b; x]$ and the corresponding integral (3.1.15) for ${}_1F_1[a; b; x]$, or it can be proved directly by considering the integral round a rectangular contour as for Kummer's function.

The Barnes integral for $W_{k,m}(x)$ is

$$\Gamma(\tfrac{1}{2} - m - k) \Gamma(\tfrac{1}{2} + m - k) W_{k,m}(x) = \frac{x^k e^{-\frac{1}{2}x}}{2\pi i} \int_{c-i\infty}^{c+i\infty} \Gamma(-s) \Gamma(\tfrac{1}{2} + s - m - k) \Gamma(\tfrac{1}{2} + s + m - k) x^{-s} ds, \quad (3.5.16)$$

for $|\arg x| < \frac{3}{2}\pi$, and $\frac{1}{2} + k \pm m \neq 0, 1, 2, \dots$. This also follows from the expression of $W_{k,m}(x)$ in terms of $U(a; b; x)$ and the corresponding integral for $U(a; b; x)$ (3.1.18), or directly, as for $M_{k,m}(x)$, by contour integration.

3.5.2 Pochhammer contour integrals for the Whittaker functions

Again we can relax the conditions of validity by extending the elementary integrals for Whittaker's functions to contour integrals of Pochhammer's type. Thus for $M_{k,m}(x)$ we have

$$M_{k,m}(x) = \Gamma(\tfrac{1}{2} - m + k) \Gamma(\tfrac{1}{2} - m - k) \Gamma(1+2m) x^{\frac{1}{2}+m} 2^{-2m} \exp\left\{-\pi i(m - \tfrac{1}{2} \pm k)\right\} \\ \times \frac{1}{4\pi^2} \int_{\pm 1}^{(+1+, -1+, +1-, -1-)} e^{\mp \frac{1}{2}xs} (s^2 - 1)^{m-\frac{1}{2}} \left(\frac{s+1}{s-1}\right)^{\pm k} ds, \quad (3.5.17)$$

for $\arg(s+1) = 0$, $\arg(s-1) = -\pi$ initially, where the integration is round a contour similar to Fig. 3.4, about the points -1 and $+1$, and the upper or lower signs should be taken throughout the equation.

For $W_{k,m}(x)$ we have

$$W_{k,m}(x) = \frac{-1}{2\pi i} \Gamma(k + \frac{1}{2} - m) e^{-\frac{1}{2}x} x^k \int_{\infty}^{(0+)} (-t)^{m-k-\frac{1}{2}} (1+t/x)^{m+k-\frac{1}{2}} e^{-t} dt, \quad (3.5.18)$$

for $|\arg(-t)| \leq \pi$, $k - \frac{1}{2} - m \neq -1, -2, \dots$. Here the integration is round the contour of Fig. 3.6. This integral is substantially equivalent to Whittaker's original definition of the function (Whittaker, 1904). Both integrals have various transformations similar to those of the elementary integrals (see Buchholz, 1953, §§ 5.(1)-(7)).

3.6 The Laplace transforms of the Whittaker functions

From the corresponding integrals for Kummer's functions we can deduce the Laplace integrals for the Whittaker functions, and the inverse Laplace transforms. Thus we have from (3.2.16)

$$\int_0^{\infty} e^{-st} t^{\nu-1} M_{k,m}(bt) dt = b^{\frac{1}{2}+m} \Gamma(\nu + \frac{1}{2} + m) (s + \frac{1}{2}b)^{-\nu-\frac{1}{2}-m} {}_2F_1[\frac{1}{2} + m - k, \frac{1}{2} + m + \nu; 1 + 2m; b/(s + \frac{1}{2}b)], \quad (3.6.1)$$

for $\text{Rl } s > \text{Rl } \frac{1}{2}b$ and $\text{Rl}(\nu + \frac{1}{2} + m) > 0$. In particular when $s = \frac{1}{2}b$ the ${}_2F_1[1]$ series is summable by Gauss's theorem (Bailey, 1935, § 1.3) and the integral reduces to

$$\int_0^{\infty} e^{-\frac{1}{2}bt} t^{\nu-1} M_{k,m}(bt) dt = \frac{\Gamma(k-\nu) \Gamma(\frac{1}{2} + m + \nu) \Gamma(1 + 2m)}{\Gamma(\frac{1}{2} + m + k) \Gamma(\frac{1}{2} + m - \nu)} b^{-\nu}, \quad (3.6.2)$$

for $\text{Rl}(\nu + \frac{1}{2} + m) > 0$ and $\text{Rl}(k - \nu) > 0$. If $\nu = m + \frac{1}{2}$ the ${}_2F_1$ series reduces to a binomial series and

$$\int_0^{\infty} e^{-st} t^{m-\frac{1}{2}} M_{k,m}(bt) dt = b^{m+\frac{1}{2}} \Gamma(1 + 2m) (s - \frac{1}{2}b)^{k-m-\frac{1}{2}} (s + \frac{1}{2}b)^{-k-m-\frac{1}{2}}, \quad (3.6.3)$$

for $\text{Rl } m > -\frac{1}{2}$ and $\text{Rl } s > |\text{Rl } \frac{1}{2}b|$.

For the integrals in terms of Bessel function we have from (3.2.27)

$$\int_0^{\infty} e^{-st} t^{-k-\frac{1}{2}} I_{2m}(2x^{\frac{1}{2}}t^{\frac{1}{2}}) dt = \frac{\Gamma(\frac{1}{2} + m - k) s^k}{\Gamma(1 + 2m) \sqrt{x}} e^{x/2s} M_{k,m}(x/s), \quad (3.6.4)$$

for $\text{Rl}(k - m) > \frac{1}{2}$; and from (3.2.26)

$$\int_0^{\infty} e^{-t} t^{k-\frac{1}{2}} J_{2m}(2x^{\frac{1}{2}}t^{\frac{1}{2}}) dt = \frac{e^{-\frac{1}{2}x} \Gamma(\frac{1}{2} + m + k)}{\sqrt{x} \Gamma(1 + 2m)} M_{k,m}(x). \quad (3.6.5)$$

The inverse Laplace transform gives, from (3.2.39),

$$x^{\nu-1} b^{\frac{1}{2}+m} {}_1F_2[\frac{1}{2} + m - k; 1 + 2m, \nu; bx] = \frac{\Gamma(\nu)}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{sx+b/2s} s^{\frac{1}{2}+m-\nu} M_{k,m}(b/s) ds, \quad (3.6.6)$$

for $\text{Rl } s > \text{Rl } b$, and $\text{Rl } \nu > 0$. If $\frac{1}{2} + m - k = \nu$,

$$x^{m-k-\frac{1}{2}} {}_0F_1[; 1 + 2m; bx] = \frac{\Gamma(\frac{1}{2} + m - k)}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{sx+b/2s} s^k M_{k,m}(b/s) ds, \quad (3.6.7)$$

for $\text{Rl } s > \text{Rl } b$, and $\text{Rl}(\frac{1}{2} + m - k) > 0$. This integral can also be expressed as a Bessel function,

$$\frac{x^{-k-\frac{1}{2}} \Gamma(1 + 2m)}{\Gamma(\frac{1}{2} + m - k)} I_{2m}(2b^{\frac{1}{2}}x^{\frac{1}{2}}) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{sx+b/2s} s^k M_{k,m}(b/s) ds, \quad (3.6.8)$$

under the same conditions.

From (3.2.42), we deduce

$$\frac{x^{2m} e^{\frac{1}{2}bx}}{\Gamma(\mathbf{I} + 2m)} (bx)^{-\frac{1}{2}-m} M_{k,m}(bx) = \frac{\mathbf{I}}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{sx} s^{-1-2m} (\mathbf{I} - b/s)^{k-\frac{1}{2}-m} ds, \quad (3.6.9)$$

for $\text{Rl } 2m > -\mathbf{I}$ and $\text{Rl } s > \text{Rl } k$; and from (3.2.43), we deduce

$$e^{\frac{1}{2}bx} (bx)^{-\frac{1}{2}-m} M_{k,m}(bx) = \frac{\Gamma(\nu) x^{1-\nu}}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{st} s^{-\nu} {}_2F_1\left[\frac{1}{2} + m - k, \nu; \mathbf{I} + 2m; b/s\right] ds. \quad (3.6.10)$$

For the Whittaker function $W_{k,m}(x)$ the Laplace transform is, from (3.2.51),

$$\int_0^\infty e^{-st} t^{\nu-1} W_{k,m}(t) dt = \frac{\Gamma(\nu + \frac{1}{2} + m) \Gamma(\nu + \frac{1}{2} - m)}{\Gamma(\mathbf{I} + \nu - k)} (s + \frac{1}{2})^{-\nu-\frac{1}{2}-m} \\ \times {}_2F_1\left[\frac{1}{2} + m - k, \frac{1}{2} + m + \nu; \mathbf{I} + \nu - k; (s - \frac{1}{2})/(s + \frac{1}{2})\right], \quad (3.6.11)$$

$$= \frac{\Gamma(\nu + \frac{1}{2} + m) \Gamma(\nu + \frac{1}{2} - m)}{\Gamma(\mathbf{I} + \nu - k)} {}_2F_1\left[\frac{1}{2} - m + \nu, \frac{1}{2} + m + \nu; \nu - k + \mathbf{I}; \frac{1}{2} - s\right], \quad (3.6.12)$$

both for $\text{Rl}(s + \frac{1}{2}) > 0$ and $\text{Rl}(\nu + \frac{1}{2} \pm m) > 0$. These ${}_2F_1$ series can be transformed in several ways, and in some cases the series may become summable. Thus if $s = \frac{3}{2}$ and $\nu = k + 2m$,

$$\int_0^\infty e^{-\frac{3}{2}t} t^{k+2m-1} W_{k,m}(t) dt = \frac{\Gamma(k + m + \frac{1}{2}) \Gamma(\frac{1}{2}k + \frac{5}{4} + \frac{3}{2}m)}{(k + \frac{1}{2} + 3m) \Gamma(-\frac{1}{2}k + \frac{3}{4} + \frac{1}{2}m)}, \quad (3.6.13)$$

$$\text{if } s = \frac{1}{2}, \quad \int_0^\infty e^{-\frac{1}{2}t} t^{\nu-1} W_{k,m}(t) dt = \frac{\Gamma(\nu + \frac{1}{2} - m) \Gamma(\nu + \frac{1}{2} + m)}{\Gamma(\nu + \mathbf{I} - k)}, \quad (3.6.14)$$

$$\text{and if } s = -\frac{1}{2}, \quad \int_0^\infty e^{\frac{1}{2}t} t^{\nu-1} W_{k,m}(t) dt = \frac{\Gamma(-k - \nu) \Gamma(\frac{1}{2} - m - \nu) \Gamma(\frac{1}{2} + m - \nu)}{\Gamma(\frac{1}{2} - m - k) \Gamma(\frac{1}{2} + m - k)}. \quad (3.6.15)$$

The inverse transform has the forms

$$\frac{\mathbf{I}}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{e^{sx}}{(s + \frac{1}{2})^{\nu+m+\frac{1}{2}}} {}_2F_1\left[\frac{1}{2} + m - k, \frac{1}{2} + m + \nu; \mathbf{I} + \nu - k; (s - \frac{1}{2})/(s + \frac{1}{2})\right] ds \\ = \frac{\Gamma(\mathbf{I} + \nu - k) x^{\nu-1}}{\Gamma(\frac{1}{2} + \nu + m) \Gamma(\frac{1}{2} + \nu - m)} W_{k,m}(x), \quad (3.6.16)$$

$$\text{and } \frac{\mathbf{I}}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{sx} {}_2F_1\left[\frac{1}{2} - m + \nu, \frac{1}{2} + m + \nu; \mathbf{I} + \nu - k; \frac{1}{2} - s\right] ds \\ = \frac{\Gamma(\mathbf{I} + \nu - k) t^{\nu-1}}{\Gamma(\frac{1}{2} + \nu + m) \Gamma(\frac{1}{2} + \nu - m)} W_{k,m}(x), \quad (3.6.17)$$

under the conditions $\text{Rl}(\nu + \frac{1}{2} \pm m) > 0$, $\text{Rl } s > -\frac{1}{2}$.

The elementary integral (3.5.10) has the inverse

$$t^{m-k-\frac{1}{2}} (\mathbf{I} + t)^{m+k-\frac{1}{2}} = \frac{\Gamma(\frac{1}{2} + m - k)}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{t(s+\frac{1}{2})} s^{-\frac{1}{2}-m} W_{k,m}(s) ds, \quad (3.6.18)$$

for $\text{Rl}(\frac{1}{2} + m - k) > 0$, and there are corresponding inverses for the other integrals of §(3.5).

The process used to deduce (3.2.55) also leads to

$$W_{k,m}(x) = \frac{x^{k+b} e^{-\frac{1}{2}x}}{\Gamma(b)} \int_0^\infty e^{-xt} t^{b-1} {}_2F_1\left[\frac{1}{2} + m - k, \frac{1}{2} - m - k; b; -t\right] dt, \quad (3.6.19)$$

for $\text{Rl } b > 0$ and $|\arg x| < \frac{1}{2}\pi$, and the inverse of this integral is

$${}_2F_1\left[\frac{1}{2} + m - k, \frac{1}{2} - m - k; b; -t\right] = \frac{\Gamma(b) t^{1-b}}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{(t+\frac{1}{2})s} s^{b-\frac{1}{2}-m+k} W_{k,m}(s) ds, \quad (3.6.20)$$

for $\text{Rl } s > 0$.

Finally from (3.2.45) we find that

$$W_{m,\nu}(x/s) = \frac{2\sqrt{x} s^{-m} e^{-\frac{1}{2}x/s}}{\Gamma(\frac{1}{2} + \nu - m) \Gamma(\frac{1}{2} - \nu - m)} \int_0^\infty e^{-st} t^{-m-\frac{1}{2}} K_{2\nu}(2x^{\frac{1}{2}} t^{\frac{1}{2}}) dt, \quad (3.6.21)$$

for $\text{Rl } \nu > \text{Rl } m - \frac{1}{2}$ and $\text{Rl } s > 0$; and its inverse is

$$K_{2\nu}(2x^{\frac{1}{2}} t^{\frac{1}{2}}) = \frac{\Gamma(\frac{1}{2} - \nu - m) \Gamma(\frac{1}{2} + \nu - m)}{\sqrt{x} 4\pi i} t^{\frac{1}{2}+m} \int_{c-i\infty}^{c+i\infty} e^{st} s^m e^{\frac{1}{2}x/s} W_{m,\nu}(x/s) ds, \quad (3.6.22)$$

for $\text{Rl } s > 0$.

3.7 Integrals involving pairs of Kummer's functions

There are large numbers of integrals involving pairs of confluent hypergeometric functions throughout the literature. Lists of the main results are to be found in Erdélyi (1954) and those for the Whittaker functions in Buchholz (1953). Here space forbids any attempt at completeness so we shall only consider some elementary integrals which do not seem to have been stated before, as examples of the main methods used to deduce such results. We start from the general integral

$$I \equiv \int_0^\infty e^{-st} t^{d-1} {}_A F_B[(a); (b); kt] {}_{A'} F_{B'}[(a'); (b'); k't] dt, \quad (3.7.1)$$

where $\text{Rl } d > 0$ and $\text{Rl } s > 0$. If we interchange the order of the double summation and the integration, we find, by (3.2.14), Laplace's integral, that

$$I = s^{-d} \Gamma(d) \sum_{m=0}^\infty \frac{((a)_m) k^m (d)_m}{((b)_m) m! s^m} {}_{A'+1} F_{B'}[(a'), d+m; (b'); k'/s]. \quad (3.7.2)$$

This inner series is not in general summable, but if $s = k'$ and $A' = B' = 1$, it becomes

$${}_2F_1[a', d+m; b'; 1],$$

which is summable by Gauss's theorem, and we deduce the result

$$\begin{aligned} & \int_0^\infty e^{-k't} t^{d-1} {}_A F_B[(a); (b); kt] {}_1 F_1[a'; b'; k't] dt \\ &= k'^{-d} \frac{\Gamma(d) \Gamma(b') \Gamma(b' - a' - d)}{\Gamma(b' - a') \Gamma(b' - d)} {}_{A+2} F_{B+1}[(a), d, 1+d-b'; (b), 1+d+a'-b'; k/k'], \end{aligned} \quad (3.7.3)$$

for $\text{Rl } d > 0$ and $\text{Rl } k' > 0$, $A \leq B$, and $|k'| > |k|$.

If $A = B = 1$ this result reduces to

$$\begin{aligned} & \int_0^\infty e^{-k't} t^{d-1} {}_1 F_1[a; b; kt] {}_1 F_1[a'; b'; k't] dt \\ &= k'^{-d} \frac{\Gamma(d) \Gamma(b') \Gamma(b' - a' - d)}{\Gamma(b' - a') \Gamma(b' - d)} {}_3 F_2[a, d, 1+d-b'; b, 1+d+a'-b'; k/k'], \end{aligned} \quad (3.7.4)$$

for $\text{Re } d > 0$ and $\text{Re } k' > 0$, $|k'| > |k|$, or $k = k'$ and $\text{Re}(\mathbf{I} + a' + b - a - d) > 0$. Further if $b = d$,

$$\begin{aligned} & \int_0^\infty e^{-k't} t^{b-1} {}_1F_1[a; b; kt] {}_1F_1[a'; b'; k't] dt \\ &= k'^{-b} \frac{\Gamma(b) \Gamma(b') \Gamma(b' - a' - b)}{\Gamma(b' - a') \Gamma(b' - b)} {}_2F_1[a, \mathbf{I} + b - b'; \mathbf{I} + a' + b - b'; k/k'], \end{aligned} \quad (3.7.5)$$

for $\text{Re } b > 0$ and $\text{Re } k' > 0$, $|k'| > |k|$ or $k = k'$ and $\text{Re}(\mathbf{I} + a' - a) > 0$. If $k = k'$, this ${}_2F_1[\mathbf{I}]$ series is summable by Gauss's theorem and we get

$$\begin{aligned} & \int_0^\infty e^{-kt} t^{b-1} {}_1F_1[a; b; kt] {}_1F_1[a'; b'; kt] dt \\ &= k^{-b} \frac{\Gamma(b) \Gamma(b') \Gamma(b' - a' - b) \Gamma(\mathbf{I} + a' + b - b') \Gamma(a' - a)}{\Gamma(b' - a') \Gamma(b' - b) \Gamma(\mathbf{I} + a' - a + b - b') \Gamma(a')}, \end{aligned} \quad (3.7.6)$$

for $\text{Re } b > 0$ and $\text{Re } k > 0$, $\text{Re}(\mathbf{I} + a' - a) > 0$.

We can proceed in exactly the same way starting from the integral

$$\int_0^\infty v^{x-1} (\mathbf{I} + v)^{-y} dv = \frac{\Gamma(x) \Gamma(y-x)}{\Gamma(y)}, \quad (3.7.7)$$

for $\text{Re } x > 0$, $\text{Re } y > 0$, instead of from the gamma function integral. In this case we have

$$I \equiv \int_0^\infty v^{x-1} (\mathbf{I} + v)^{-y} {}_A F_B[(a); (b); kv] {}_{A'} F_{B'}[(a'); (b'); k'v] dv, \quad (3.7.8)$$

and we find, on inverting the order of summation and integration, that

$$I = \frac{\Gamma(x) \Gamma(y-x)}{\Gamma(y)} \sum_{m=0}^{\infty} \frac{((a)_m) (-\mathbf{I})^m k^m (x)_m}{((b)_m) m! (\mathbf{I} + x - y)_m} {}_{A'+1} F_{B'+1}[(a'), x+m; (b'), \mathbf{I} + x - y + m; -k']. \quad (3.7.9)$$

First we let $k' = -\mathbf{I}$, $A' = \mathbf{I}$ and $B' = 0$. Then we can sum the inner series by Gauss's theorem and we get

$$\begin{aligned} & \int_0^\infty v^{x-1} (\mathbf{I} + v)^{-y} {}_A F_B[(a); (b); -kv] (\mathbf{I} + v)^{-a'} dv \\ &= \frac{\Gamma(y-x) \Gamma(x) \Gamma(\mathbf{I} + x - y) \Gamma(\mathbf{I} - y - a')}{\Gamma(y) \Gamma(\mathbf{I} + x - y - a') \Gamma(\mathbf{I} - y)} {}_{A+1} F_{B+1}[(a), x; (b), \mathbf{I} + x - y - a'; k], \end{aligned} \quad (3.7.10)$$

for $\text{Re } x > 0$ and $\text{Re } y > 0$, $A \leq B + \mathbf{I}$. In particular if $A = B = \mathbf{I}$,

$$\begin{aligned} & \int_0^\infty v^{x-1} (\mathbf{I} + v)^{-y-a'} {}_1F_1[a; b; -kv] dv \\ &= \frac{\sin \pi y \Gamma(x) \Gamma(\mathbf{I} - y - a')}{\sin \pi(y-x) \Gamma(\mathbf{I} + x - y - a')} {}_2F_2[a, x; b, \mathbf{I} + x - y - a'; k], \end{aligned} \quad (3.7.11)$$

if $A = \mathbf{I}$ and $B = 0$ we have an elementary integral for the Gauss function and if $A = B = 0$ we have an Euler integral for Kummer's function.

Secondly, if $\mathbf{I} + x - y = x$, that is if $y = \mathbf{I}$, the inner sum is independent of m and the two sums are separable so that we get

$$\begin{aligned} & \int_0^\infty v^{x-1} (\mathbf{I} + v)^{-1} {}_A F_B[(a); (b); kv] {}_{A'} F_{B'}[(a'); (b'); k'v] dv \\ &= \Gamma(x) \Gamma(\mathbf{I} - x) {}_A F_B[(a); (b); -k] {}_{A'} F_{B'}[(a'); (b'); -k'], \end{aligned} \quad (3.7.12)$$

for $\text{Re } x > 0$, $A \leq B$ and $A' \leq B'$.

In particular, when $A = B = A' = B' = 1$,

$$\int_0^\infty v^{x-1}(1+v)^{-1} {}_1F_1[a; b; kv] {}_1F_1[a'; b'; k'v] dv = \frac{\pi}{\sin \pi x} {}_1F_1[a; b; -k] {}_1F_1[a'; b'; -k'], \quad (3.7.13)$$

for $\text{Re } x > 0$.

3.7.1 Integrals involving pairs of Whittaker functions

In terms of Whittaker functions, (3.7.4) gives

$$\begin{aligned} \int_0^\infty e^{\frac{1}{2}(b-b')t} t^{d-2-m-m'} M_{k,m}(bt) M_{k',m'}(b't) dt &= b^{\frac{1}{2}+m} b'^{\frac{1}{2}+m'-d} \frac{\Gamma(d) \Gamma(1+2m') \Gamma(\frac{1}{2}+m'+k'-d)}{\Gamma(\frac{1}{2}+m'+k') \Gamma(1+2m'-d)} \\ &\times {}_3F_2[\frac{1}{2}+m-k, d-2m, d; 1+2m, \frac{1}{2}-m'-k'+d; b/b'], \end{aligned} \quad (3.7.14)$$

for $\text{Re } d > 0$, $\text{Re } b' > 0$, $|b'| < |b|$ or $b' = b$ and $\text{Re}(2+m'-k'+m+k-d) > 0$. If $d = 1+2m$

$$\begin{aligned} \int_0^\infty e^{\frac{1}{2}(b-b')t} t^{m-m'-1} M_{k,m}(bt) M_{k',m'}(b't) dt &= b^{\frac{1}{2}+m} b'^{m'-\frac{1}{2}-2m} \frac{\Gamma(1+2m) \Gamma(1+2m') \Gamma(m'+k'-2m-\frac{1}{2})}{\Gamma(\frac{1}{2}+m'+k') \Gamma(2m'-2m)} \\ &\times {}_2F_1[\frac{1}{2}+m-k, 1+2m-2m'; \frac{3}{2}+2m-m'-k'; b/b'], \end{aligned} \quad (3.7.15)$$

for $\text{Re}(1+2m) > 0$, $\text{Re } b' > 0$, $|b'| < |b|$ or $b = b'$ and $\text{Re}(1+m'-k'-m+k) > 0$. If further $b = b'$, we can sum the ${}_2F_1[1]$ series and

$$\begin{aligned} \int_0^\infty t^{m-m'-1} M_{k,m}(bt) M_{k',m'}(bt) dt \\ = b^{m'-m} \frac{\Gamma(1+2m) \Gamma(1+2m') \Gamma(m'+k'-2m-\frac{1}{2}) \Gamma(\frac{3}{2}+2m-k'-m')}{\Gamma(\frac{1}{2}+m'+k') \Gamma(2m'-2m) \Gamma(1+m+k-m'-k') \Gamma(\frac{1}{2}+m'-k')}, \end{aligned} \quad (3.7.16)$$

for $\text{Re}(1+2m) > 0$, $\text{Re } b > 0$ and $\text{Re}(1+m'-k'-m+k) > 0$. From (3.7.13) we find that

$$\int_0^\infty v^{x-2-m-m'} (1+v)^{-1} e^{\frac{1}{2}(b+b')v} M_{k,m}(bv) M_{k',m'}(b'v) dv = \frac{\pi}{\sin \pi x} e^{-\frac{1}{2}(b+b')} M_{-k,m}(b) M_{-k',m'}(b'), \quad (3.7.17)$$

for $\text{Re } x > 0$.

3.8 Some expansions in series

Let $A_n(k, \frac{1}{2}b)$ be defined by the equation

$$e^{2kx} (1-x)^{k-\frac{1}{2}b} (1+x)^{-k-\frac{1}{2}b} = \sum_{n=0}^{\infty} A_n(k, \frac{1}{2}b) x^n, \quad (3.8.1)$$

where $k \equiv \frac{1}{2}b - a$, so that $A_n(k, \frac{1}{2}b)$ satisfies the recurrence relation

$$(n+1) A_{n+1}(k, \frac{1}{2}b) = (n+b-1) A_{n-1}(k, \frac{1}{2}b) - 2k A_{n-2}(k, \frac{1}{2}b), \quad (3.8.2)$$

for $n = 2, 3, 4, \dots$, where

$$A_0(k, \frac{1}{2}b) = 1, \quad A_1(k, \frac{1}{2}b) = 0 \quad \text{and} \quad A_2(k, \frac{1}{2}b) = \frac{1}{2}b.$$

Now let us apply the Laplace transform to both sides of (3.8.1). We find using (3.2.20) and (3.2.24) that

$$\begin{aligned} \int_0^\infty e^{-(s+\frac{1}{2})t} x^{-b-1} {}_1F_1[a; b; t] dt &= \Gamma(b) (s+\frac{1}{2})^{a-b} (s-\frac{1}{2})^{-a}, \\ &= \Gamma(b) \sum_{n=0}^{\infty} \frac{A_n(\frac{1}{2}b-a, \frac{1}{2}b)}{2^n} s^{-b-n} e^{(a-\frac{1}{2}b)s}, \\ &= \Gamma(b) \sum_{n=0}^{\infty} \frac{A_n(\frac{1}{2}b-a, \frac{1}{2}b)}{2^n} \int_0^\infty e^{-st} \frac{t^{b+n-1}}{\Gamma(b+n)} {}_0F_1[; b+n; (a-\frac{1}{2}b)t] dt, \end{aligned}$$

and so, putting $\frac{1}{2s} = x$, we have

$${}_1F_1[a; b; x] = \Gamma(b) e^{\frac{1}{2}x} (kx)^{\frac{1}{2}-\frac{1}{2}b} \sum_{n=0}^{\infty} A_n(k, \frac{1}{2}b) \left(\frac{x}{4k}\right)^{\frac{1}{2}n} J_{b+n-1}(2k^{\frac{1}{2}}x^{\frac{1}{2}}), \quad (3.8.3)$$

where $k \equiv \frac{1}{2}b - a$,

$$A_3(k, \frac{1}{2}b) = -\frac{2}{3}k, \quad A_4(k, \frac{1}{2}b) = \frac{1}{4}b(1 + \frac{1}{2}b), \quad A_5(k, \frac{1}{2}b) = -\frac{2k}{15} \left(\frac{5b}{2} + 3\right),$$

$$A_6(k, \frac{1}{2}b) = \frac{1}{12}b(1 + \frac{1}{2}b) \left(2 + \frac{1}{2}b\right) + \frac{2}{9}k^2.$$

Again, if $B_n(a, b, h)$ is defined by the equation

$$e^{-ax} \{1 + (h-1)x\}^{-a} (1+hx)^{a-b} = \sum_{n=0}^{\infty} B_n(a, b, h) x^n, \quad (3.8.4)$$

so that $(n+1)B_{n+1}(a, b, h) = \{(1-2h)n - hb\} B_n(a, b, h)$

$$+ \{(1-2h)a - h(h-1)(b+n-1)\} B_{n-1}(a, b, h) - h(h-1)a B_{n-2}(a, b, h), \quad (3.8.5)$$

for $n = 2, 3, 4, \dots$, where

$$B_0(a, b, h) = 1, \quad B_1(a, b, h) = -bh, \quad B_2(a, b, h) = -\frac{1}{2}(2h-1)a + \frac{1}{2}b(b+1)h^2,$$

then, if we apply the Laplace transform to both sides of (3.8.4), we find by (3.1.5) and (3.2.25) that

$${}_1F_1[a; b; x] = \Gamma(b) e^{hx} \sum_{n=0}^{\infty} B_n(a, b, h) x^n (-ax)^{\frac{1}{2}(1-b-n)} J_{b-1+n}(2ia^{\frac{1}{2}}x^{\frac{1}{2}}). \quad (3.8.6)$$

In particular if $h = 0$ or if $h = 1$ the coefficients $A_n(k, \frac{1}{2}b)$ and $B_n(a, b, h)$ reduce to polynomials. These results are due to Tricomi (1954, § 1.8).

ASYMPTOTIC EXPANSIONS

4.1 Introduction

The fact that this chapter is considerably longer than its neighbours seems to call for some explanation. The theory of the asymptotic expansions has been treated here in great detail because it seems to cause most difficulty to students of the subject. The theory is far from being complete, and it is in this region that important new developments are most likely to be made.

The first paragraphs deal with the expansions in x , of the Kummer and Whittaker functions, and then an outline is given of the method by which converging factors can be developed to improve the accuracy of these asymptotic approximations in x . After mentioning the elementary approximations which hold when b is large, or m in the case of the Whittaker functions, we study the case when a (or k) is large through a consideration of the various approximations in terms of Bessel functions, and then we approach the main theory when both one parameter and x are large together, through statements of the three forms assumed by the results in the case when x and the parameters are real. We then state three theorems of Olver which underlie these asymptotic approximations, and we show how these theorems may be applied to the confluent hypergeometric functions to deduce their corresponding asymptotic forms for complex variable and parameters.

4.1.1 *The asymptotic expansions in x for Kummer's function*

Let us consider the integral

$$I_L = \frac{1}{2\pi i} \int_{ADEF} \frac{\Gamma(-s)\Gamma(a+s)}{\Gamma(b+s)} x^s ds, \quad (4.1.1)$$

round the rectangular contour

$$A(c-iN), \quad D(c+iM), \quad E(c-L+iM), \quad F(c-L-iN)$$

in the s -plane. As before we have

$$I_L = \int_{AD} + \int_{DE} + \int_{EF} + \int_{FA} = I_{M,N} + J_4 + J_5 + J_6 \quad \text{say,}$$

where, as M and $N \rightarrow \infty$,

$$J_4 \rightarrow 0,$$

$$J_6 \rightarrow 0,$$

$$-J_5 \rightarrow J_L = \int_{c-i\infty}^{c+i\infty} \frac{\Gamma(L-s)\Gamma(a-L+s)}{2\pi i \Gamma(b-L+s)} x^{s-L} ds,$$

$$I_{M,N} \rightarrow I_1 = \int_{c-i\infty}^{c+i\infty} \frac{\Gamma(-s)\Gamma(a+s)}{2\pi i \Gamma(b+s)} x^s ds,$$

$$= \frac{\Gamma(a)}{\Gamma(b)} {}_1F_1[a; b; -x], \quad \text{by (3.1.15),}$$

and

$$I_L = - \sum_{n=0}^{[L]} \frac{\Gamma(a+n)}{\Gamma(b-a-n)} \frac{(-1)^{n-1}}{n!} x^{-a-n},$$

from the residues at the poles $s = -a, -a-1, \dots, -a-[L]$, within the contour, provided that $|\arg x| < \frac{1}{2}\pi$, and $a \neq 0, -1, -2, \dots$

Now
$$|J_L| < L^L |x|^{-L},$$

so that for L fixed and $|x|$ large, J_L is bounded, and

Hence we find that
$$J_L = O(|x|^{-a-[L]}).$$

$${}_1F_1[a; b; -x] = x^{-a} \frac{\Gamma(b)}{\Gamma(b-a)} \sum_{n=0}^{[L]} \frac{(a)_n (1+a-b)_n x^{-n}}{n!} + O(|x|^{-a-[L]-1}), \quad (4.1.2)$$

where $|\arg x| < \frac{1}{2}\pi$.

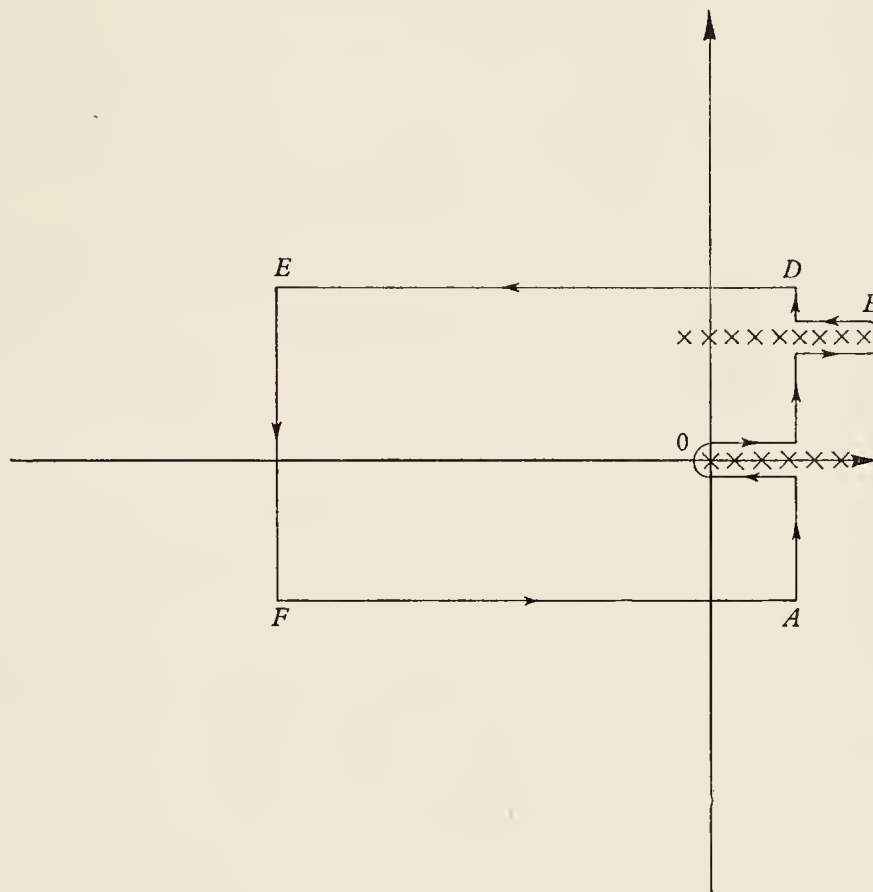


Fig. 4.1

In particular, for $|x|$ large, we have

$${}_1F_1[a; b; -x] = x^{-a} \frac{\Gamma(b)}{\Gamma(b-a)} \{1 + O(|x|^{-1})\}. \quad (4.1.3)$$

Again from the integral

$$I_2 = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \Gamma(a+s) \Gamma(-s) \Gamma(1-b-s) x^s ds, \quad (4.1.4)$$

we find, in the same way, that

$$\begin{aligned} & \Gamma(a) \Gamma(1+a-b) U(a; b; x) \\ & \equiv \Gamma(a) \Gamma(1-b) {}_1F_1[a; b; x] + x^{1-b} \Gamma(1+a-b) \Gamma(b-1) {}_1F_1[1+a-b; 2-b; x], \\ & = I_2, \\ & = x^{-a} \Gamma(a) \Gamma(1+a-b) \sum_{n=0}^{[L]} \frac{(a)_n (1+a-b)_n}{n!} (-x)^{-n} + O(|x|^{-a-[L]-1}), \end{aligned} \quad (4.1.5)$$

for $|\arg x| < \frac{3}{2}\pi$.

From the two results (4.1.2) and (4.1.5) we deduce that

$$\begin{aligned} {}_1F_1[a; b; x] &= \frac{\Gamma(b)}{\Gamma(b-a)} e^{i\pi a} x^{-a} \left\{ \sum_{n=0}^{R-1} \frac{(a)_n (1+a-b)_n}{n!} (-x)^{-n} + O(|x|^{-R}) \right\} \\ &\quad + \frac{\Gamma(b)}{\Gamma(a)} e^x x^{a-b} \left\{ \sum_{n=0}^{S-1} \frac{(b-a)_n (1-a)_n}{n!} x^{-n} + O(|x|^{-S}) \right\}, \end{aligned} \quad (4.1.6)$$

for $R, S = 0, 1, 2, \dots$, $\epsilon = 1$ if $0 < \arg x < \pi$, $\epsilon = -1$, if $-\pi < \arg x \leq 0$ as $x \rightarrow \infty$.

In particular, if $x \rightarrow \infty$, $\operatorname{Rl} x > 0$,

$${}_1F_1[a; b; x] = \frac{\Gamma(b)}{\Gamma(a)} e^x x^{a-b} \{1 + O(|x|^{-1})\}, \quad (4.1.7)$$

and if $x \rightarrow \infty$, $\operatorname{Rl} x < 0$,

$${}_1F_1[a; b; x] = \frac{\Gamma(b)}{\Gamma(b-a)} (-x)^{-a} \{1 + O(|x|^{-1})\}, \quad (4.1.8)$$

so that

$${}_1F_1[a; b; x] \sim \frac{\Gamma(b)}{\Gamma(b-a)} x^{-a} e^{i\pi a}, \quad (4.1.9)$$

where $\epsilon = 1$ for $\frac{1}{2}\pi < \arg x \leq \pi$, and $\epsilon = -1$, for $-\pi \leq \arg x < -\frac{1}{2}\pi$.

4.1.2 The asymptotic expansions in x for $U(a; b; x)$

From the integral (4.1.4) we have

$$U(a; b; x) \sim x^{-a} {}_2F_0[a, 1+a-b; \quad ; -1/x], \quad (4.1.10)$$

for

$$-\frac{3}{2}\pi < \arg x < \frac{3}{2}\pi, \quad \text{as } |x| \rightarrow \infty,$$

and in particular

$$U(a; b; x) = x^{-a} \{1 + O(|x|^{-1})\}, \quad (4.1.11)$$

as $\operatorname{Rl} x \rightarrow \infty$, so that

$$U(a; b; x) \sim x^{-a} \quad (4.1.12)$$

for $\operatorname{Rl} a > 0$, as $x \rightarrow \infty$.

From the definition of $U(a; b; x)$ in terms of Kummer's function, as $x \rightarrow 0$ we find that

$$U(a; b; x) = \frac{\Gamma(b-1)}{\Gamma(a)} x^{1-b} + O(|x|^{\operatorname{Rl} b - 2}), \quad \text{if } \operatorname{Rl} b \geq 2, b \neq 2, \quad (4.1.13)$$

$$= \frac{\Gamma(b-1)}{\Gamma(a)} x^{1-b} + O(|\log x|), \quad \text{if } b = 2, \quad (4.1.14)$$

$$= \frac{\Gamma(b-1)}{\Gamma(a)} x^{1-b} + O(1), \quad \text{if } 1 < \operatorname{Rl} b < 2, \quad (4.1.15)$$

$$= \frac{\Gamma(1-b)}{\Gamma(1+a-b)} + \frac{\Gamma(b-1)}{\Gamma(a)} x^{1-b} + O(|x|), \quad \text{if } \operatorname{Rl} b = 1, b \neq 1, \quad (4.1.16)$$

$$= \frac{-1}{\Gamma(a)} \{\log x + \psi(a) + 2C\} + O(|x \log x|), \quad \text{if } b = 1, \quad (4.1.17)$$

where

$$\psi(a) = \frac{\Gamma'(a)}{\Gamma(a)} \quad \text{and} \quad C = -\psi(1) = 0.577215665,$$

$$= \frac{\Gamma(1-b)}{\Gamma(1+a-b)} + O(|x|^{1-\operatorname{Rl} b}), \quad \text{if } 0 < \operatorname{Rl} b < 1, \quad (4.1.18)$$

$$= \frac{1}{\Gamma(1+a)} + O(|x \log x|), \quad \text{if } b = 0, \quad (4.1.19)$$

$$= \frac{\Gamma(1-b)}{\Gamma(1+a-b)} + O(|x|), \quad \text{if } \operatorname{Rl} b \leq 0, b \neq 0. \quad (4.1.20)$$

4.1.3 The asymptotic expansions in x for Whittaker's functions

From the expansions for ${}_1F_1[a; b; x]$, or directly from the integral

$$\frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{\Gamma(-s) \Gamma(\frac{1}{2} + m + k + s)}{\Gamma(1 + 2m + s)} x^s ds,$$

we deduce that

$$\begin{aligned} \frac{M_{k,m}(x)}{\Gamma(1 + 2m)} &\sim \frac{x^{-k} e^{\frac{1}{2}x}}{\Gamma(\frac{1}{2} + m - k)} {}_2F_0[\frac{1}{2} + m + k, \frac{1}{2} - m + k; \quad ; 1/x] \\ &+ \frac{x^k e^{-\frac{1}{2}x} e^{\epsilon\pi i(k-m-\frac{1}{2})}}{\Gamma(\frac{1}{2} + m + k)} {}_2F_0[\frac{1}{2} + m - k, \frac{1}{2} - m - k; \quad ; -1/x], \end{aligned} \quad (4.1.21)$$

as $|x| \rightarrow \infty$, where $\epsilon = 1$ for $-\frac{3}{2}\pi < \arg x < \frac{1}{2}\pi$, and $\epsilon = -1$ for $-\frac{1}{2}\pi < \arg x < \frac{3}{2}\pi$.

Similarly, from its definition in terms of $U(a; b; x)$ or directly from the integral

$$\frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \Gamma(\frac{1}{2} - m - k - s) \Gamma(\frac{1}{2} + m - k - s) \Gamma(s) x^s ds,$$

we deduce the asymptotic formulae for $W_{k,m}(x)$,

$$W_{k,m}(x) \sim x^k e^{-\frac{1}{2}x} {}_2F_0[\frac{1}{2} + m - k, \frac{1}{2} - m - k; \quad ; -1/x], \quad (4.1.22)$$

for $|\arg x| < \frac{3}{2}\pi$ as $|x| \rightarrow \infty$, and

$$W_{-k,m}(x e^{\epsilon\pi i}) \sim x^{-k} e^{-\epsilon\pi i k} e^{\frac{1}{2}x} {}_2F_0[\frac{1}{2} + m + k, \frac{1}{2} - m + k; \quad ; 1/x], \quad (4.1.23)$$

for $|\arg x + \epsilon\pi| < \frac{3}{2}\pi$ as $|x| \rightarrow \infty$, $\epsilon = \pm 1$.

Another method of deducing these results, from the Euler integrals for the functions is given in Whittaker and Watson, 1946, § 16.3.

4.2 Converging factors for Kummer's functions

We have shown that

$${}_1F_1[a; b; x] \sim K_1 S_1 + K_2 S_2,$$

where
$$K_1 = \frac{\Gamma(b)}{\Gamma(b-a)} e^{a\pi i} x^{-a}, \quad S_1 = \sum_{r=0}^{\infty} \frac{(-1)^r (a)_r (1+a-b)_r}{n!} x^{-r}$$

and
$$K_2 = \frac{\Gamma(b)}{\Gamma(a)} e^x x^{a-b}, \quad S_2 = \sum_{r=0}^{\infty} \frac{(1-a)_r (b-a)_r}{r!} x^{-r}.$$

The error in both series is less than the first term neglected, and so the accuracy is strictly limited. Thus for $a = 0.9$, $b = 0.1$, $x = 10$, the best we can say is

$$0.869 < S_1 < 0.871 \quad \text{and} \quad 0.99209723 < S_2 < 0.99209724.$$

When, however, converging factors have been developed for the function, the results can be improved immediately to

$$0.870089576 < S_1 < 0.870089583 \quad \text{and} \quad 0.9920972320 < S_2 < 0.9920972321.$$

The factors are developed by the method of Miller (1952) for converging factors of parabolic cylinder functions.

Let

$$K_1 S_1 = \sum_{s=0}^{r-1} (-1)^s v_s + (-1)^r C_r v_r. \quad (4.2.1)$$

Then $rxv_r = (a+r-1)(a+r-b)v_{r-1}$, (4.2.2)

and $(C_{r-1}-1)rx = -C_r(a+r-1)(a+r-b)$. (4.2.3)

Now let us make the approximation $r = 1 + b - 2a + x - k$, (4.2.4)

where k is small when the r th term is the smallest of the expansion, substitute for r in (4.2.3) and arrange the result in powers of x . We find that

$$x^2(C_r + C_{r-1} - 1) + x\{(C_r + C_{r-1} - 1)(1 + b - 2a - k) - kC_r\} + \{k^2 + (2a - b - 1)k + (a - b)(a - 1)\}C_r = 0. \quad (4.2.5)$$

We suppose that C_r has an expansion of the form

$$C_r = b_0 + b_1/x + b_2/x^2 + \dots$$

Now k becomes $k + 1$ when r becomes $r - 1$. Hence

$$C_{r-1} = b_0^* + b_1^*/x + b_2^*/x^2 + \dots,$$

where

$$b_r^* \equiv b_r^*(k) \equiv b_r(k+1).$$

Substituting for C_r and C_{r-1} in (4.2.5) and equating the coefficients of x to zero, we have the system of equations

$$\left. \begin{aligned} b_0^* + b_0 &= 1, \\ b_1^* + b_1 &= -K(b_0^* + b_0) + kb_0 + K, \\ b_2^* + b_2 &= -K(b_1^* + b_1) + kb_1 - K'b_0, \\ &\dots\dots\dots \\ b_n^* + b_n &= -K(b_{n-1}^* + b_{n-1}) + kb_{n-1} - K'b_{n-2}, \\ &\dots\dots\dots \end{aligned} \right\} \quad (4.2.6)$$

where $K = 1 + b - 2a - k$ and $K' = k^2 + (2a - b - 1)k + (a - b)(a - 1)$.

Again when $K_1 S_1$ is substituted for y in Kummer's equation we find that

$$x^2 \frac{d^2 C_r}{dx^2} + x(b - 2a - 2r - x) \frac{dC_r}{dx} + (1 + a - b + r)(a + r)C_r = rx(1 - C_r). \quad (4.2.7)$$

Let us express r in terms of k and x , and then, if $C_r' \equiv dC_r/dk$,

$$x^2(C_r'' - 3C_r' + 2C_r - 1) + x\{(2a - b - 2 + 2k)C_r' + (4 - 4a + 2b - 3k)C_r + 2a + k - 1 - b\} + (k + a - 2)(k + a - b - 1)C_r = 0. \quad (4.2.8)$$

Now we substitute for C_r , C_r' and C_r'' and equate to zero the coefficients of powers of x . Then

$$\left. \begin{aligned} b_0'' - 3b_0' + 2b_0 &= 1, \\ b_1'' - 3b_1' + 2b_1 &= (4a - 2b - 4 + 3k)b_0 + (2 + b - 2a - 2k)b_0' + 1 + b - 2a - k, \\ b_2'' - 3b_2' + 2b_2 &= (4a - 2b - 7 + 3k)b_1 + (b - 2a - 2k + 4)b_1' - (k + a - 2)(k + a - b - 1)b_0, \\ b_3'' - 3b_3' + 2b_3 &= (4a - 2b - 10 + 3k)b_2 + (b - 2a - 2k + 6)b_2' \\ &\quad - \{(k + a - 2)(k + a - b - 1) + b - 2a - 2k + 4\}b_1, \\ &\dots\dots\dots \\ b_n'' - 3b_n' + 2b_n &= (4a - 2b + 3k - 1 - 3n)b_{n-1} + (b - 2a - 2k + 2n)b_{n-1}' \\ &\quad - \{(k + a - 2)(k + a - b - 1) + (n - 2)(b - 2a - 2k + 1 + n)\}b_{n-2}, \\ &\dots\dots\dots \end{aligned} \right\} \quad (4.2.9)$$

When these equations are solved in succession we find, using (4.2.6) to check the results, that

$$\left. \begin{aligned} b_0 &= \frac{1}{2}, \\ b_1 &= \frac{1}{4}k - \frac{1}{8}, \\ b_2 &= \frac{1}{8}k^2 - \frac{3}{16}k + \frac{1}{32} - \frac{1}{4}\lambda, \\ b_3 &= \frac{1}{16}k^3 - \frac{3}{16}k^2 + \left(\frac{7}{64} - \frac{1}{2}\lambda\right)k + \frac{1}{128} + \frac{9}{16}\lambda - \frac{1}{4}\mu, \\ b_4 &= \frac{1}{32}k^4 - \frac{5}{32}k^3 + \left(\frac{25}{128} - \frac{1}{16}\lambda\right)k^2 + \left(\frac{7}{4}\lambda - \frac{3}{4}\mu - \frac{5}{256}\right)k \\ &\quad + \left(\frac{1}{8}\lambda^2 - \frac{1}{4}(2a-b)\mu + \frac{1}{16}\mu - \frac{5}{64}\lambda - \frac{1}{512}\right), \\ b_5 &= \frac{1}{64}k^5 - \frac{15}{128}k^4 + \left(\frac{65}{256} - \frac{1}{16}\lambda\right)k^3 + \left(\frac{89}{32}\lambda - \frac{23}{16}\mu - \frac{15}{128}\right)k^2 \\ &\quad + \left(\frac{15}{8}\mu - \frac{57}{16}\lambda^2 - \frac{49}{16}\lambda - \frac{83}{1024}\right)k + \left(\frac{41}{16}\lambda^2 - \frac{3}{4}\lambda\mu - \frac{19}{64}\mu + \frac{79}{256}\lambda + \frac{47}{2048}\right), \end{aligned} \right\} \quad (4.2.10)$$

where $\lambda = (a-b)(a-1)$ and $\lambda(2a-b) = \mu$.

If we put $\mu \equiv (a - \frac{1}{2})(a - \frac{3}{2})$ for $4\lambda = 4(a-b)(a-1)$, a for $2a-b$, k for $2k$ and x^2 for $2x$, we obtain Miller's result (1952), (4.3)).

For the series S_1 , we find that

r	k	C_r	$(-1)^r v_r$	$\sum_{s=0}^{r-1} (-1)^s v_s$	$\sum_{s=0}^{r-1} (-1)^s v_s + (-1)^r C_r v_r$
7	2.3	0.54791 84603	- 0.00216 47724	0.87127 57503	0.87008 96316
8	1.3	0.52020 24450	+ 188 11872	0.86911 09779	95761
9	0.3	0.49508 65909	- 182 30794	0.87099 21651	95829
10	-0.7	0.47223 61543	+ 194 92365	0.86916 90857	95612
11	-1.7	0.45136 88915	- 227 91891	0.87111 83222	95871

It will be noticed that five extra figures are given by this expansion, but that the accuracy is then limited. The series for C_r is probably also asymptotic, and the error here is certainly < 0.00000001 , which is of the order of $b_6 v_r / x^5$, the neglected term.

For the series $K_2 S_2$ the situation is more complicated. Let

$$K_2 S_2 = \sum_{s=0}^{r-1} u_s + C_r u_r. \quad (4.2.11)$$

Then $rxu_r = (r-a)(b+r-a-1)u_{r-1}$,

and so $(C_{r-1} - 1)rx = C_r(r-a)(r+b-a-1)$. (4.2.12)

We make the approximation $r = 1 - b + 2a - k + x$, and substitute for r in (4.2.12). Then

$$x^2(C_{r-1} - C_r - 1) + x\{(C_{r-1} - C_r - 1)(1 + 2a - b - k) + kC_r\} - C_r(1 + a - b - k)(a - k) = 0. \quad (4.2.13)$$

Again let $C_r = b_0 + b_1/x + \dots$ and $C_{r-1} = b_0^* + b_1^*/x + \dots$, where $b_r^*(k) \equiv b_r(k+1)$. Then from (4.2.13) equating the coefficients of x to zero we find that

$$\left. \begin{aligned} b_0^* - b_0 &= 1, \\ b_1^* - b_1 &= (k+b-2a-1)(b_0^* - b_0) - kb_0 - k - b + 2a + 1, \\ b_2^* - b_2 &= (k+b-2a-1)(b_1^* - b_1) - kb_1 + (k+b-a-1)(k-a)b_0, \\ &\dots\dots\dots \\ b_n^* - b_n &= (k+b-2a-1)(b_{n-1}^* - b_{n-1}) - kb_{n-1} + (k+b-a-1)(k-a)b_{n-2}, \\ &\dots\dots\dots \end{aligned} \right\} \quad (4.2.14)$$

Next we substitute for K_2S_2 in Kummer's equation and, on simplification, we have

$$x^2 \frac{d^2 C_r}{dx^2} + 2x \frac{dC_r}{dx} (x + a - b - r) + \{x^2 + 2x(a - b - r) + (a - b - r)(a - b - r - 1)\} C_r + (b - x) \left\{ x \frac{dC_r}{dx} + C_r(x + a - b - r) \right\} - axC_r = -rx. \quad (4.2.15)$$

We express r in terms of k and x and arrange in descending powers of x . Then

$$x^2(C_r'' - C_r' + 1) + x\{(2k - 2a - 2 + b)C_r' + (2 - k)C_r + 1 + 2a - b - k\} + C_r(k - a - 1)(k + b - a - 2) = 0. \quad (4.2.16)$$

On substituting the series for C_r , C_r' and C_r'' and equating the coefficients of powers of x to zero, we obtain

$$\left. \begin{aligned} b_0'' - b_1' &= -1, \\ b_1'' - b_1' &= (k - 2)b_0 + (2 + 2a - 2k - b)b_0' + k + b - 2a - 1, \\ b_2'' - b_2' &= (k - 3)b_1 + (4 + 2a - 2k - b)b_1' + (k - a - 1)(2 + a - b - k)b_0, \\ b_3'' - b_3' &= (k - 4)b_2 + (6 + 2a - 2k - b)b_2' \\ &\quad + \{2(k - a - 1) + b - 2 - (k - a - 1)(k - a + b - 2)\}b_1, \\ &\dots\dots\dots \\ b_{n+1}'' - b_{n+1}' &= (k - 2 - n)b_n + (2a - 2k + 2 + 2n - b)b_n' \\ &\quad + \{(n - 1)(b - n + 2k - 2a - 2) - (k - a - 1)(k - a - 2 + b)\}b_{n-1}, \\ &\dots\dots\dots \end{aligned} \right\} \quad (4.2.17)$$

In this case a constant arises in the integration of these equations and this has to be determined from substitution of the results in (4.2.14). After a considerable amount of algebra we find that

$$\left. \begin{aligned} b_0 &= k - \frac{1}{3}, \\ b_1 &= -\frac{1}{3}k^3 + \frac{2}{3}k^2 - \frac{1}{3}k + \lambda + \frac{4}{135}, \\ b_2 &= \frac{1}{15}k^5 - \frac{1}{3}k^4 + \frac{5}{9}k^3 - (\lambda + \frac{47}{135})k^2 + (\frac{4}{3}\lambda + \frac{8}{135})k - \mu - 2\lambda - \frac{8}{567}, \\ b_3 &= -\frac{1}{105}k^7 + \frac{4}{45}k^6 - \frac{14}{45}k^5 + (\frac{1}{3}\lambda + \frac{68}{135})k^4 - (\frac{5}{3}\lambda + \frac{149}{405})k^3 + (\mu + \frac{11}{3}\lambda + \frac{248}{2835})k^2 \\ &\quad + (-\lambda^2 - \frac{4}{3}\mu - \frac{364}{135}\lambda + \frac{8}{945})k + (\frac{7}{3}\mu - \frac{13}{3}\lambda^2 + \frac{643}{540}\lambda - \frac{16}{8505}), \\ b_4 &= \frac{1}{945}k^9 - \frac{1}{63}k^8 + \frac{2}{21}k^7 - (\frac{1}{15}\lambda + \frac{118}{405})k^6 + (\frac{2}{3}\lambda + \frac{193}{405})k^5 - (\frac{1}{3}\mu + \frac{26}{9}\lambda + \frac{221}{567})k^4 \\ &\quad + (\frac{2}{3}\lambda^2 + 2\mu + \frac{812}{135}\lambda + \frac{968}{8505})k^3 + (2\lambda^2 - 4\mu - \frac{1321}{180}\lambda + \frac{152}{8505})k^2 \\ &\quad + (2\lambda\mu - \lambda^2 + \frac{319}{135}\mu + \frac{2111}{567}\lambda - \frac{64}{8505})k + \epsilon_4, \end{aligned} \right\} \quad (4.2.18)$$

where $\lambda = a(b - a - 1)$ and $\mu = -\lambda(b - 2a)$.

If we put λ for -4λ , $c - 1$ for $2a - b$, h for k , and x^2 for $2x$ we obtain Miller's result (1952, 9.17). For the series S_2 we have

r	k	C_r	u_r	$\sum_{s=0}^{r-1} u_s$	$\sum_{s=0}^{r-1} u_s + C_r u_r$
10	2.7	2.06732 4	+ 0.05 00142 35	+ 0.99209 72031 26	+ 0.99209 72325 54
11	1.7	1.23865 5	120 25	173 61	2 56
12	0.7	0.24137 1	113 46	293 86	1 31
13	-0.3	-0.73550 1	118 28	407 32	0 32
14	-1.3	-1.51100 0	135 03	525 60	1 57

Here the constant term in b_3 makes the major contribution to its numerical value, and, since nothing is known about the size of ϵ_4 , C_r is only accurate to two decimals. Probably $\epsilon_4 \sim 80.0$ and so $b_4/x^4 \sim 0.008$, giving an error in the final result

$$< 0.00000000001.$$

Thus here the converging factors have given two extra figures compared with the ordinary asymptotic expansion.

4.2.1 Converging factors for Whittaker's functions

In the expressions for the b coefficients, (4.2.10) and (4.2.18), let us write h in place of k and then put $a = m - k + \frac{1}{2}$, $b = 1 + 2m$. Then

$$\lambda = -(m - k - \frac{1}{2})(m + k + \frac{1}{2}) \quad \text{and} \quad \mu = -2k\lambda,$$

and we have

$$W_{k,m}(x) \sim x^k e^{-\frac{1}{2}x} \left\{ \sum_{r=0}^{n-1} \frac{(m - k + \frac{1}{2})_r (-m - k + \frac{1}{2})_r}{r!} (-1)^r x^{-r} + \frac{(m - k + \frac{1}{2})_n (-m - k + \frac{1}{2})_n}{n!} (-1)^n x^{-n} \sum_{r=0}^{\infty} b_r x^{-r} \right\}, \quad (4.2.19)$$

where the coefficients b_r are defined by (4.2.10) with the above values of λ and μ .

Similarly,

$$W_{-k,m}(e^{\pm\pi i} x) \sim (e^{\pm\pi i} x)^{-k} e^{\frac{1}{2}x} \left\{ \sum_{r=0}^{n-1} \frac{(m + k + \frac{1}{2})_r (-m + k + \frac{1}{2})_r}{r!} x^{-r} + \frac{(m + k + \frac{1}{2})_n (-m + k + \frac{1}{2})_n}{n!} x^{-n} \sum_{r=0}^{\infty} b_r x^{-r} \right\}, \quad (4.2.20)$$

where the coefficients b_r are defined by (4.2.18) with $\lambda = (m + k - \frac{1}{2})(m - k + \frac{1}{2})$ and $\mu = -2k\lambda$.

The expansion for $M_{k,m}(x)$ follows immediately from (4.1.21).

4.3 Approximations when b is large

If b is large and a and x are bounded

$${}_1F_1[a; b; x] = \sum_{n=0}^{N-1} \frac{(a)_n x^n}{(b)_n n!} + O(|b|^{-N}), \quad (4.3.1)$$

$$= 1 + \frac{ax}{b} + O(|b|^{-2}), \quad (4.3.2)$$

$$= 1 + O(|b|^{-1}), \quad (4.3.3)$$

for a , b and x complex as $b \rightarrow \infty$.

If b is large and a is also large but $b - a$ and x are bounded then Kummer's transform gives

$${}_1F_1[a; b; x] = e^x \sum_{n=0}^{N-1} \frac{(b-a)_n (-x)^n}{(b)_n n!} + O(|b|^{-N}), \quad (4.3.4)$$

$$= e^x \left\{ 1 - \frac{(b-a)x}{b} + O(|b|^{-2}) \right\}, \quad (4.3.5)$$

$$= e^x \{ 1 + O(|b|^{-1}) \}, \quad (4.3.6)$$

as a and $b \rightarrow \infty$ in such a way that

$$\lim_{a, b \rightarrow \infty} \frac{(b-a)x}{b} = 0.$$

When b and x are both large, let $x = by$, then, since

$$\frac{1}{(b)_n} = \frac{1}{b^n} \left\{ 1 - \frac{n(n-1)}{2b} + O(|b|^{-2}) \right\},$$

we have
$${}_1F_1[a; b; by] = (1-y)^{-a} \left\{ 1 - \frac{a(a+1)}{2b} \left(\frac{y}{1-y} \right)^2 + O(|b|^{-2}) \right\}, \quad (4.3.7)$$

where a and y are complex and bounded but $b \rightarrow \infty$. If we apply Kummer's theorem to (4.3.7), we deduce

$${}_1F_1[a; b; by] = e^{-by} (1-y)^{a-b} \left\{ 1 - \frac{(b-a)(b-a+1)}{2b} \left(\frac{y}{1-y} \right)^2 + O(|b|^{-2}) \right\}, \quad (4.3.8)$$

where $b-a$ and y are bounded but a , x and $b \rightarrow \infty$.

For the function $U(a; b; x)$, since

$$\log \Gamma(x+a) = (x+a-\frac{1}{2}) \log x - x + \frac{1}{2} \log 2\pi + O(|x|^{-1}), \quad (4.3.9)$$

and
$$\frac{\Gamma(x+a)}{\Gamma(x+b)} = x^{a-b} \left\{ 1 + \frac{1}{2}(a-b)(a+b-1)x^{-1} + O(|x|^{-2}) \right\}, \quad (4.3.10)$$

we have
$$U(a; b; x) = e^{-\pi i a} b^{-a} \left\{ 1 + O(|b|^{-1}) \right\} + \sqrt{(2\pi)} x^{1-b} e^{x-b} b^{b-\frac{3}{2}} \left\{ 1 + O(|b|^{-1}) \right\}, \quad (4.3.11)$$

for $b \rightarrow \infty$, $|\arg(\pm b)| < \pi$, a and x bounded, and since

$$U(a; b; x) = x^{1-b} U(1+a-b; 2-b; x),$$

we have

$$U(a; b; x) = x^{1-b} \exp\{-\pi i(1+a-b)\} (2-b)^{b-a-1} \left\{ 1 + O(|b|^{-1}) \right\} + \sqrt{(2\pi)} e^{x+b-2} (2-b)^{\frac{1}{2}-b} \left\{ 1 + O(|b|^{-1}) \right\}, \quad (4.3.12)$$

for $b \rightarrow \infty$, $|\arg(\pm b)| < \pi$, $b-a$ and x bounded.

For a , b and x large, if $x = by$, and $b-a$ and y are bounded, we find that

$$U(a; b; by) = e^{-\pi i a} b^{-a} (1-y)^{-a} \left\{ 1 - \frac{a(a+1)}{2b} \left(\frac{y}{1-y} \right)^2 + O(|b|^{-2}) \right\} + \sqrt{(2\pi)} y^{1-b} e^{b(1-y)} b^{-\frac{1}{2}} (1-y)^{a-b} \left\{ 1 - \frac{(b-a)(1+b-a)}{2b} \left(\frac{y}{1-y} \right)^2 + O(|b|^{-2}) \right\}. \quad (4.3.13)$$

4.3.1 Approximations for Whittaker's functions when m is large

When $\text{Re } m > 0$, we have this integral for $M_{k,m}(x)$,

$$M_{k,m}(x) = \frac{(\frac{1}{4}x)^{\frac{1}{2}+m} \Gamma(1+2m)}{\Gamma(\frac{1}{2}+m-k) \Gamma(\frac{1}{2}+m+k)} \int_0^1 t^{m-\frac{1}{2}} \left\{ \left(\frac{1+\sqrt{(1-t)}}{1-\sqrt{(1-t)}} \right)^k e^{-\frac{1}{2}x/\sqrt{(1-t)}} + \left(\frac{1-\sqrt{(1-t)}}{1+\sqrt{(1-t)}} \right)^k e^{\frac{1}{2}x/\sqrt{(1-t)}} \right\} \frac{dt}{\sqrt{(1-t)}}, \quad (4.3.14)$$

for $\text{Re}(\frac{1}{2}+m \pm k) > 0$, from (3.5.4). Now

$$\left(\frac{1+v}{1-v} \right)^k = \sum_{r=0}^{\infty} \gamma_r v^r, \quad (4.3.15)$$

where $\gamma_0 = 1$, $\gamma_r = 2k {}_2F_1[1-r, 1-k; 2; 2]$, for $|v| < 1$, and so

$$\left(\frac{1+v}{1-v} \right)^k e^{-\frac{1}{2}vx} = \sum_{r=0}^{\infty} v^r \sum_{s=0}^r \frac{(-\frac{1}{2}x)^s}{s!} \gamma_{r-s} = \sum_{r=0}^{\infty} c_r v^r. \quad (4.3.16)$$

Hence we have this expansion for m large,

$$M_{k,m}(x) = \frac{\Gamma(\frac{1}{2} + m)^2}{\Gamma(\frac{1}{2} + m + k) \Gamma(\frac{1}{2} + m - k)} x^{\frac{1}{2} + m} \sum_{r=0}^{\infty} \frac{a_r r!}{(\mathbf{I} + m)_r}, \quad (4.3.17)$$

for $|\arg 2m| \leq \frac{1}{2}\pi$, where

$$\frac{2^{2r} r! r!}{2r!} a_r \equiv c_{2r} = \frac{\mathbf{I}}{2\pi i} \int^{(0+)} e^{-\frac{1}{2}xv} \left(\frac{\mathbf{I} + v}{\mathbf{I} - v}\right)^k v^{-2r-1} dv, \quad (4.3.18)$$

for $|v| < \mathbf{I}$, so that

$$\left. \begin{aligned} a_0 &= \mathbf{I}, \\ a_1 &= k^2(\mathbf{I} - x/4k)^2, \\ a_2 &= \frac{1}{2}k^2(\mathbf{I} - x/4k)^2 + \frac{1}{4}k^4(\mathbf{I} - x/4k)^4. \end{aligned} \right\} \quad (4.3.19)$$

In particular, as $m \rightarrow \infty$ in (4.3.17) we have

$$M_{k,m}(x) \sim x^{\frac{1}{2} + m} \{ \mathbf{I} + O(|\mathbf{I}/m|) \}, \quad \text{for } \text{Rl } m > 0. \quad (4.3.20)$$

Since

$$M_{k \pm m, a+m}(x) = x^{a+m+\frac{1}{2}} e^{\mp \frac{1}{2}x} {}_1F_1[a + \frac{1}{2} \mp k; 2a + \mathbf{I} + 2m; \pm x],$$

we have

$$M_{k \pm m, a+m}(x) \sim x^{a+m+\frac{1}{2}} e^{\mp \frac{1}{2}x} \{ \mathbf{I} + O(\mathbf{I}/| \mathbf{I} + 2m + 2a |) \}, \quad (4.3.21)$$

for $|m|$ large, $|\arg m| < \frac{1}{2}\pi$.

Corresponding approximations for $W_{k,m}(x)$ can be deduced from these results and (1.7.1).

4.4 Bessel functions as limiting cases of Kummer functions

Just as the confluent hypergeometric functions arise as limiting cases of the Gauss ${}_2F_1$ function, so the Bessel functions arise as limiting cases of the confluent hypergeometric functions. If we pass to the limit term by term we find

$$\lim_{a \rightarrow \infty} {}_1F_1[a; b; x/a] = \Gamma(b) x^{\frac{1}{2} - \frac{1}{2}b} I_{b-1}(2\sqrt{x}), \quad (4.4.1)$$

and

$$\lim_{a \rightarrow \infty} {}_1F_1[a; b; -x/a] = \Gamma(b) x^{\frac{1}{2} - \frac{1}{2}b} J_{b-1}(2\sqrt{x}). \quad (4.4.2)$$

Since

$$\lim_{a \rightarrow \infty} \{ \Gamma(\mathbf{I} + a - b) a^{b-1} / \Gamma(a) \} = \mathbf{I}, \quad (4.4.3)$$

it follows in the same way from the definition (1.3.1), of $U(a; b; x)$, that

$$\lim_{a \rightarrow \infty} \{ \Gamma(\mathbf{I} + a - b) U(a; b; x/a) \} = 2x^{\frac{1}{2} - \frac{1}{2}b} K_{b-1}(2\sqrt{x}), \quad (4.4.4)$$

and

$$\lim_{a \rightarrow \infty} \{ \Gamma(\mathbf{I} + a - b) U(a; b; -x/a) \} = \pi x^{\frac{1}{2} - \frac{1}{2}b} \operatorname{cosec}(\pi b) \{ J_{b-1}(2\sqrt{x}) + e^{i\pi b} J_{1-b}(2\sqrt{x}) \}, \quad (4.4.5)$$

$$= -i\pi e^{i\pi b} x^{\frac{1}{2} - \frac{1}{2}b} H_{b-1}^{(1)}(2\sqrt{x}), \quad \text{when } \text{Im } x > 0, \quad (4.4.6)$$

$$= i\pi e^{-i\pi b} x^{\frac{1}{2} - \frac{1}{2}b} H_{b-1}^{(2)}(2\sqrt{x}), \quad \text{when } \text{Im } x < 0. \quad (4.4.7)$$

If we apply Kummer's theorem to (4.4.1) and (4.4.2) above we find

$$\lim_{a \rightarrow \infty} {}_1F_1[b - a; b; x/a] = \Gamma(b) x^{\frac{1}{2} - \frac{1}{2}b} J_{b-1}(2\sqrt{x}), \quad (4.4.8)$$

and

$$\lim_{a \rightarrow \infty} {}_1F_1[b - a; b; -x/a] = \Gamma(b) x^{\frac{1}{2} - \frac{1}{2}b} I_{b-1}(2\sqrt{x}). \quad (4.4.9)$$

4.4.1 Approximations in terms of Bessel functions when a is large

We have already seen (§3.8) that

$${}_1F_1[a; b; x] = \Gamma(b) e^{\frac{1}{2}x} (kx)^{\frac{1}{2}-\frac{1}{2}b} \sum_{n=0}^{\infty} U_n, \quad (4.4.10)$$

where

$$k = \frac{1}{2}b - a, \quad U_n = u_{3n} + u_{3n+1} + u_{3n+2}$$

and

$$u_n = A_n(k, \frac{1}{2}b) \left(\frac{x}{4k}\right)^{\frac{1}{2}n} J_{b+n-1}(2k^{\frac{1}{2}}x^{\frac{1}{2}}).$$

Let $|x| = O(|k|^\rho)$ where $0 \leq \rho < 1$. Now we have

$$|J_{b+n-1}(2k^{\frac{1}{2}}x^{\frac{1}{2}})| \leq 1,$$

for b real and positive and $n = 1, 2, \dots$ (Watson, 1948, §13.42 (10)).

Hence

$$u_{3n} = O\{|k|^{n+\frac{1}{2}(\rho-1)3n}\},$$

$$u_{3n+1} = O\{|k|^{n+\frac{1}{2}(\rho-1)(3n+1)}\}$$

and

$$u_{3n+2} = O\{|k|^{n+\frac{1}{2}(\rho-1)(3n+2)}\}.$$

Hence if $\rho - 1 < 0$,

$$U_n = O\{|k|^{n+\frac{1}{2}(\rho-1)3n}\} = O\{|k|^{-\frac{1}{2}n(1-3\rho)}\}. \quad (4.4.11)$$

Thus, when $\rho < \frac{1}{3}$, $U_n \rightarrow 0$ as $k \rightarrow \infty$, and so

$$\left| \frac{U_{n+1}}{U_n} \right| = O(|k|^{-\frac{1}{2}(1-3\rho)}) \rightarrow 0, \quad \text{as } n \rightarrow \infty.$$

Hence the series $\sum_{n=0}^{\infty} U_n$ is an asymptotic representation of the function ${}_1F_1[a; b; x] e^{-\frac{1}{2}x} (kx)^{\frac{1}{2}b-\frac{1}{2}}/\Gamma(b)$, for large values of $|k|$.

Hence we have

$${}_1F_1[a; b; x] = \Gamma(b) e^{\frac{1}{2}x} (kx)^{\frac{1}{2}-\frac{1}{2}b} J_{b-1}(2x^{\frac{1}{2}}k^{\frac{1}{2}}) \{1 + O(|k|^{-\sigma})\}, \quad (4.4.12)$$

where $|x| = |k|^\rho$, $k = \frac{1}{2}b - a$, $\sigma = \min(1 - \rho, \frac{1}{2} - \frac{3}{2}\rho)$ and $0 \leq \rho < \frac{1}{3}$.

Again, since

$$J_\nu(\xi) = \sqrt{\frac{2}{\pi\xi}} \cos\left(\xi - \frac{1}{2}\pi\nu - \frac{1}{4}\pi\right) \{1 + O(|\xi|^{-1})\} \quad (4.4.13)$$

(Watson, 1948, §7.1), we have

$${}_1F_1[a; b; x] = \Gamma(b) e^{\frac{1}{2}x} (kx)^{\frac{1}{2}-\frac{1}{2}b} \pi^{-\frac{1}{2}} \cos\left\{2\sqrt{(kx)} - \frac{1}{2}b\pi + \frac{1}{4}\pi\right\} \{1 + O(|k|^{-\tau})\}, \quad (4.4.14)$$

where $\tau = \min(\frac{1}{2} + \frac{1}{2}\rho, \frac{1}{2} - \frac{3}{2}\rho) \leq \frac{1}{2}$, $|x| = O(|k|^\rho)$, $0 \leq \rho < \frac{1}{3}$, and $k = \frac{1}{2}b - a$. This is an extension of a result originally proved by Perron (1921). Tricomi (1954, §3.2) proves a general theorem for results of this type when k and x are real.

In particular, when $\rho = 0$, we have

$${}_1F_1[a; b; x] = \Gamma(b) e^{\frac{1}{2}x} (kx)^{\frac{1}{2}-\frac{1}{2}b} \pi^{-\frac{1}{2}} \cos\left\{2\sqrt{(kx)} - \frac{1}{2}b\pi + \frac{1}{4}\pi\right\} \{1 + O(|k|^{-\frac{1}{2}})\}, \quad (4.4.15)$$

as $a \rightarrow -\infty$ for b bounded and x real.

The corresponding results for $U(a; b; x)$ can be deduced in a similar way. They are

$$U(a; b; x) = \Gamma(k + \frac{1}{2}) e^{\frac{1}{2}x} x^{\frac{1}{2}-\frac{1}{2}b} [\cos\{(k - \frac{1}{2}b)\pi\} J_{b-1}(2k^{\frac{1}{2}}x^{\frac{1}{2}}) + \sin\{(k - \frac{1}{2}b)\pi\} Y_{b-1}(2k^{\frac{1}{2}}x^{\frac{1}{2}})] \{1 + O(|k|^{-\sigma'})\}, \quad (4.4.16)$$

where $\sigma' = \min(\mathbf{1} - \rho, \frac{1}{2} - \frac{3}{2}\rho, \mathbf{1})$, $k = \frac{1}{2}b - a$, so that

$$U(a; b; x) = \frac{\Gamma(k + \frac{1}{2})}{k^{\frac{1}{2}} \sqrt{\pi}} e^{\frac{1}{2}x} x^{\frac{1}{4} - \frac{1}{2}b} \cos\{2\sqrt{(kx)} - k\pi + \frac{1}{4}\pi\} \{1 + O(|k|^{-\tau})\}, \quad (4.4.17)$$

where $\tau = \min(\frac{1}{2} + \frac{1}{2}\rho, \frac{1}{2} - \frac{3}{2}\rho) \leq \frac{1}{2}$.

In particular if $a \rightarrow -\infty$, for b fixed, and x real

$$U(a; b; x) = \frac{\Gamma(k + \frac{1}{4})}{\sqrt{\pi}} e^{\frac{1}{2}x} x^{\frac{1}{4} - \frac{1}{2}b} \cos\{2\sqrt{(kx)} - k\pi + \frac{1}{4}\pi\} \{1 + O(|k|^{-\frac{1}{2}})\}, \quad (4.4.18)$$

since
$$\frac{\Gamma(k + \frac{1}{2})}{\Gamma(k + \frac{1}{4})} = k^{\frac{1}{2}} \{1 + O(|k|^{-1})\}. \quad (4.4.19)$$

It should be noted that all the results of this paragraph can be proved for complex values of k and x by methods similar to those of § 4.4.3.

4.4.2 Bessel functions as limiting cases of Whittaker functions

From § 4.4 we have immediately

$$\lim_{k \rightarrow \infty} \left(\frac{x}{k}\right)^{-\frac{1}{2}-m} M_{k,m}(x/k) = \Gamma(\mathbf{1} + 2m) x^{-m} I_{2m}(2\sqrt{x}) \quad (4.4.20)$$

and
$$\lim_{k \rightarrow \infty} \left(\frac{-x}{k}\right)^{-\frac{1}{2}-m} M_{k,m}(-x/k) = \Gamma(\mathbf{1} + 2m) x^{-m} J_{2m}(2\sqrt{x}). \quad (4.4.21)$$

Also
$$\lim_{k \rightarrow \infty} \{\Gamma(\frac{1}{2} - m - k) (x/k)^{-\frac{1}{2}-m} W_{k,m}(x/k)\} = 2x^{-m} K_{2m}(2\sqrt{x}) \quad (4.4.22)$$

and
$$\lim_{k \rightarrow \infty} \{\Gamma(\frac{1}{2} - m - k) (-x/k)^{-\frac{1}{2}-m} W_{k,m}(-x/k)\} = -i\pi e^{i\pi(1+2m)} x^{-m} H_{2m}^{(1)}(2\sqrt{x}) \quad \text{for } \text{Im } x > 0, \quad (4.4.23)$$

$$= i\pi e^{-i\pi(1+2m)} x^{-m} H_{2m}^{(2)}(2\sqrt{x}) \quad \text{for } \text{Im } x < 0. \quad (4.4.24)$$

4.4.3 Approximations for Whittaker functions in terms of Bessel functions, when k is large

When k is large, we can develop expansions in terms of Bessel functions similar to those already considered for Kummer's functions. We have the integral

$$M_{k,m}(x) = \Gamma(\mathbf{1} + 2m) 2^{2m} \frac{(x e^{-\pi i})^{\frac{1}{2}-m}}{2\pi i} \int^{(0-)} \frac{\exp\{2kt - x/(2 \tanh t)\} dt}{(i \sinh t)^{1-2m}}, \quad (4.4.25)$$

taken round a Pochhammer contour, encircling 0 in the negative direction, for $|\arg z| < \frac{1}{2}\pi$. Now

$$\exp\{\frac{1}{2}x(\coth t - t^{-1})\} (t \operatorname{cosech} t)^{1-2m} = \sum_{r=0}^{\infty} p_r^{(m)}(x) (-t/x)^r, \quad (4.4.26)$$

where
$$p_r^{(m)}(x) = \frac{(-x)^r}{2\pi i} \int^{(0+)} \exp\{-\frac{1}{2}x(\coth t - t^{-1})\} \frac{(\sinh t/t)^{2m-1}}{t^{r+1}} dt,$$

taken round a similar contour, encircling 0 in the positive direction.

Hence

$$M_{k,m}(x) = 2^{2m} (x e^{-2\pi i})^{\frac{1}{2}-m} \sum_{r=0}^{\infty} (-1)^r p_r^{(m)}(x) x^{m-\frac{1}{2}r} (4k)^{-m-\frac{1}{2}r} \\ \times \Gamma(1+2m) \frac{1}{2\pi i} \int^{(0-)} \exp\{\sqrt{(kx)}(v-1/v)\} v^{2m+r-1} dv, \quad (4.4.27)$$

for $|\arg \sqrt{(kx)}| < \frac{1}{2}\pi$, where $v = it$. But Watson (1948, § 6.2) gives this integral for the Bessel function

$$J_m(x) = \frac{e^{\pi i(1+2m)}}{2\pi i} \int^{(0-)} e^{\frac{1}{2}x(v-1/v)} v^{2m-1} dv, \quad (4.4.28)$$

for $|\arg z/v| < \frac{1}{2}\pi$. Hence we have, for all m, k and x , this expansion

$$M_{k,m}(x) = 2^{2m} x^{\frac{1}{2}+m} \Gamma(1+2m) \sum_{r=0}^{\infty} p_r^{(m)}(x) \frac{J_{2m+r}(2k^{\frac{1}{2}}x^{\frac{1}{2}})}{\{2\sqrt{(kx)}\}^{2m+r}}, \quad (4.4.29)$$

where

$$\left. \begin{aligned} p_0 &= 1, \\ p_1 &= \frac{1}{6}x^2, \\ p_2 &= \frac{1}{6}x^2\left(\frac{1}{12}x^2 + 2m - 1\right), \\ p_3 &= \frac{1}{36}x^4\left(\frac{1}{36}x^2 + 2m - \frac{7}{5}\right), \\ p_4 &= \frac{1}{36}x^4\left[\frac{1}{864}x^4 - \frac{1}{60}x^2\{4 + 5(1-2m)\} + \frac{1}{10}(1-2m)\{2 + 5(1-2m)\}\right]. \end{aligned} \right\} \quad (4.4.30)$$

Thus it follows from the known asymptotic approximations for $J_\nu(x)$, that

$$M_{k,m}(x) \sim \Gamma(1+2m) x^{\frac{1}{2}} \pi^{-\frac{1}{2}} k^{-\frac{1}{4}-m} \cos\{2\sqrt{(kx)} - \pi m - \frac{1}{4}\pi\} \{1 + O(|k|^{-\frac{1}{2}})\}, \quad (4.4.31)$$

as $k \rightarrow \infty$, for $\arg(kx) < 2\pi$ and for any m . Also

$$M_{-k,m}(x) \sim \Gamma(1+2m) x^{\frac{1}{2}} \pi^{-\frac{1}{2}} k^{-\frac{1}{4}-m} e^{\mp \pi i(m+\frac{1}{4})} \cos\{2\sqrt{(kx)} e^{\pm \frac{1}{2}\pi i} - \pi m - \frac{1}{4}\pi\} \\ \times \{1 + O(|k|^{-\frac{1}{2}})\}, \quad (4.4.32)$$

where the upper signs should be taken for $-3\pi < \arg(kx) < \pi$, and the lower signs for

$$-\pi < \arg(kx) < 3\pi.$$

The corresponding results for the function $W_{k,m}(x)$ are

$$W_{k,m}(x) \sim \sqrt{2} x^{\frac{1}{4}} k^{-\frac{1}{4}} k^k e^{-k} \cos\{2\sqrt{(kx)} - \pi k + \frac{1}{4}\pi\}, \quad (4.4.33)$$

for $|\arg k| < \pi$ and $|\arg kx| < 2\pi$, and

$$W_{k,m}(x) \sim 2^{-\frac{1}{2}} x^{\frac{1}{4}} k^{-\frac{1}{4}} k^k e^{-k} \exp[\mp i\{\pi k - \frac{1}{4}\pi - 2\sqrt{(kx)}\}], \quad (4.4.34)$$

where the upper sign holds for $\text{Im } k > 0$, and the lower sign for $\text{Im } k < 0$. Also

$$W_{-k,m}(x) \sim 2^{-\frac{1}{2}} x^{\frac{1}{4}} k^{-\frac{1}{4}} k^{-k} e^{k} e^{-2\sqrt{(kx)}}, \quad (4.4.35)$$

for $\text{Im } k > 0$ and $-\pi < \arg kx < 3\pi$, or $\text{Im } k < 0$ and $-3\pi < \arg kx < \pi$.

4.5 Approximations when a and x are real, $\frac{1}{4}x > \frac{1}{2}b - a$

We consider now the asymptotic approximations in the cases when x and $k = \frac{1}{2}b - a$ are real and $\rightarrow \infty$, together. There are three possibilities, $x > 4k > 1$, $x \sim 4k$, and $4k > x > 0$. No detailed proofs will be given as similar approximations will appear later as particular cases of asymptotic expansions in $1/k$.

First we shall consider the case in which $x > 4k > 1$, $k = \frac{1}{2}b - a$. We take $x \geq (1 + \eta)4k$, $\eta > 0$, and write

$$\sqrt{\frac{x}{4k}} = \cosh \theta. \quad (4.5.1)$$

Let
$$\phi(z) = \frac{1}{2} \log \left(\frac{1+z}{1-z} \right) - z \cosh^2 \theta,$$

and
$$\psi(z) = (1 - z^2)^{\frac{1}{2}b-1}.$$

Then we can develop a contour integral for ${}_1F_1[a; b; x]$ which can be written in the form

$$\frac{e^{-\frac{1}{2}x}}{\Gamma(b)} {}_1F_1[a; b; x] = \frac{-\Gamma(1-a)}{\Gamma(b-a)} \frac{e^{-a\pi i}}{2^b \pi i} \int_{-1}^{(+1)} e^{2k\phi(z)} \psi(z) dz, \quad (4.5.2)$$

taken round a contour starting from -1 and circling $+1$ in the positive direction.

Here the saddle point is $z_0 = \tanh \theta$, and, if N is a large positive integer, we find

$$\begin{aligned} \frac{e^{-\frac{1}{2}x}}{\Gamma(b)} {}_1F_1[a; b; x] &= -\frac{\Gamma(1-a)}{\Gamma(b-a)} \left\{ \frac{e^{-a\pi i}}{2^b \pi i} (1 - e^{2a\pi i}) \int_{-1}^{z_0} e^{2k\phi(z)} \psi(z) dz + O(k^{-N}) \right\}, \\ &= \frac{\Gamma(1-a)}{\Gamma(b-a)} \left\{ \frac{\sin a\pi}{2^{b-1} \pi} \int_{-1}^{z_0} e^{2k\phi(z)} \psi(z) dz + O(k^{-N}) \right\}, \\ &= \frac{\Gamma(1-a)}{\Gamma(b-a)} \left[\frac{\sin a\pi}{2^{b-1} \pi} e^{2k\phi(-z_0)} \psi(-z_0) \int_{-\infty}^{\infty} e^{k\phi''(-z_0)z} dz \{1 + O(k^{-1})\} + O(k^{-N}) \right], \\ &= \frac{\Gamma(1-a)}{\Gamma(b-a)} \left[\frac{\sin a\pi}{2^{b-1} \pi} \exp\{2k\phi(-z_0)\} \psi(-z_0) \sqrt{\left\{ \frac{\pi}{-k\phi''(-z_0)} \right\}} \{1 + O(k^{-1})\} + O(k^{-N}) \right]. \end{aligned} \quad (4.5.3)$$

But
$$\phi(-z_0) = \frac{1}{2}(\sinh 2\theta - \theta),$$

$$\psi(-z_0) = (\cosh \theta)^{2-b},$$

and
$$\phi''(-z_0) = -\cosh^2 \theta \sinh 2\theta.$$

Hence
$$\begin{aligned} {}_1F_1[a; b; x] &= \Gamma(b) \sin a\pi \exp\{2k(\frac{1}{2} \sinh 2\theta - \theta + \cosh^2 \theta)\} \\ &\quad \times \frac{(2k \cosh \theta)^{1-b}}{\sqrt{(\pi k \sinh 2\theta)}} \{1 + O(k^{-1})\}. \end{aligned} \quad (4.5.5)$$

4.5.1 Approximations when $\frac{1}{2}b - a \sim \frac{1}{4}x$

Secondly, we consider the case in which x is about the same size as $4k$, and x and $k \rightarrow \infty$ together. Let us take

$$x = 4k - 2 \left(\frac{2k}{3} \right)^{\frac{1}{3}} t,$$

where t is a variable over a limited range of values. Tricomi (1949a) has considered this case, using the Steckloff-Langer method based on the asymptotic approximations to the Airy integrals. Tricomi shows that an expansion must exist of the form

$$\begin{aligned} \frac{e^{-\frac{1}{2}x}}{\Gamma(b)} {}_1F_1[a; b; x] &= \gamma_1 [\text{Ai}(t') + 3^{-\frac{1}{3}} \{ \frac{1}{5} t'^2 \text{Ai}'(t') + \alpha(t - \beta) \text{Ai}(t') \\ &\quad + \alpha\beta 3^{-\frac{1}{2}} \text{Bi}(t') \} (2k)^{-\frac{2}{3}} + O(k^{-\frac{4}{3}})] \\ &\quad + \gamma_2 [\text{Bi}(t') + 3^{-\frac{1}{3}} \{ \frac{1}{5} t'^2 \text{Bi}'(t') + \alpha(t + \beta) \text{Bi}(t') - 3^{\frac{1}{2}} \alpha\beta \text{Ai}(t') \} (2k)^{-\frac{2}{3}} + O(k^{-\frac{4}{3}})], \end{aligned} \quad (4.5.6)$$

where

$$\alpha = \frac{5b-2}{10}, \quad \beta = \frac{1}{2} \frac{\Gamma(\frac{1}{3})}{\Gamma(\frac{2}{3})}, \quad t' = -3^{-\frac{1}{3}}t,$$

$$\gamma_1 = \left\{ \frac{3^{\frac{1}{2}}}{\Gamma(\frac{1}{3})} k_1 - \frac{2 \cdot 3^{\frac{1}{6}}}{\Gamma(\frac{2}{3})} (2k)^{\frac{1}{3}} k_2 \right\} \frac{\pi}{3^{\frac{1}{3}}},$$

$$\gamma_2 = \left\{ \frac{1}{\Gamma(\frac{1}{3})} k_1 + \frac{2 \cdot 3^{-\frac{1}{6}}}{\Gamma(\frac{2}{3})} (2k)^{\frac{1}{3}} k_2 \right\} \frac{\pi}{3^{\frac{1}{3}}},$$

and

$$k_1 = \left\{ \frac{e^{-\frac{1}{2}x}}{\Gamma(b)} {}_1F_1[a; b; x] \right\}_{x=4k},$$

$$k_2 = \left[\frac{d}{dx} \left\{ \frac{e^{-\frac{1}{2}x}}{\Gamma(b)} {}_1F_1[a; b; x] \right\} \right]_{x=4k}.$$

When $x = 4k$ exactly, it can be shown that

$$k_1 = \frac{\Gamma(\frac{1}{3}) \sin(\pi a + \frac{1}{6}\pi)}{\pi 3^{\frac{1}{6}} (2k)^{b-\frac{2}{3}}} \{1 + O(k^{-\frac{4}{3}})\},$$

and

$$k_2 = -\frac{3^{\frac{1}{2}}}{2\pi(2k)^{b-\frac{1}{3}}} \left\{ \frac{\Gamma(\frac{2}{3}) \cos(\pi a + \frac{1}{3}\pi)}{3^{\frac{1}{3}}} + \frac{(5b-2) \Gamma(\frac{1}{3}) \sin(\pi a + \frac{1}{6}\pi)}{10 \cdot 3^{\frac{2}{3}} (2k)^{\frac{2}{3}}} + O(k^{-\frac{4}{3}}) \right\},$$

so that

$$\gamma_1 = \frac{1}{3^{\frac{1}{3}} (2k)^{b-\frac{2}{3}}} \left\{ 3^{\frac{1}{3}} \cos a\pi + 2\alpha\beta \frac{\sin(a\pi + \frac{1}{6}\pi)}{(2k)^{\frac{2}{3}}} + O(k^{-\frac{4}{3}}) \right\}$$

and

$$\gamma_2 = \frac{1}{3^{\frac{1}{3}} (2k)^{b-\frac{2}{3}}} \left\{ 3^{\frac{1}{3}} \sin a\pi - \frac{2\alpha\beta \sin(\pi a + \frac{1}{6}\pi)}{3^{\frac{1}{3}} (2k)^{\frac{2}{3}}} + O(k^{-\frac{4}{3}}) \right\}.$$

When a is an integer, we see that γ_2 is of a higher order than γ_1 in $k^{-\frac{1}{3}}$ and we can take as our first approximation

$${}_1F_1[a; b; x] = \frac{e^{\frac{1}{2}x} \Gamma(b)}{(2k)^{b-\frac{2}{3}}} \{ \text{Ai}(t') \cos a\pi + \text{Bi}(t') \sin a\pi + O(k^{-\frac{2}{3}}) \}, \quad (4.5.7)$$

where $\frac{x}{4k} = 1 + (4k^2)^{-\frac{1}{3}}t'$.

This expansion should be compared with that deduced by Lauwerier {(1950), § 11} in the special case $b = 1$.

4.5.2 Approximations when $\frac{1}{2}b - a > \frac{1}{4}x$

Thirdly, if $0 < x < 4k$ the function ${}_1F_1[a; b; x]$ oscillates, for b positive and $-a$ and $x \rightarrow +\infty$ in such a way that $\eta k \leq x \leq (1-\eta)4k$ where η is a small positive number. So we let

$$\cos \theta = \sqrt{\frac{x}{4k}},$$

where

$$0 < \theta < \frac{1}{2}\pi.$$

Then the function has the integral representation

$${}_1F_1[a; b; x] = -\Gamma(b) \frac{e^{-a\pi i} \Gamma(1-a)}{2^b \pi i \Gamma(b-a)} e^{\frac{1}{2}x} \int_{-1}^{(+1)} e^{-\frac{1}{2}xz} \left(\frac{1+z}{1-z} \right)^k (1-z^2)^{\frac{1}{2}b-1} dz, \quad (4.5.8)$$

taken round a contour starting from -1 and circling $+1$ in the positive direction. Tricomi (1949*b*) shows that this integral can be expressed in the form

$${}_1F_1[a; b; x] = \frac{\Gamma(b)\Gamma(1-a)e^{\frac{1}{2}x}}{\pi 2^{b-1}\Gamma(b-a)} \operatorname{Im}(e^{a\pi i} H) + O(k^{-N}), \quad (4.5.9)$$

where $N > 0$ and H has the expansion, asymptotic in Poincaré's sense,

$$H \sim \frac{\exp(2k\theta i - ki \sin 2\theta + \frac{1}{4}i\pi)(\cos \theta)^{1-b}}{\sqrt{(\pi k \sin 2\theta)}} \sum_{p=0}^{\infty} \frac{i^{3p}}{(k \sin 2\theta)^p} \\ \times \sum_{m=0}^{\infty} \frac{1}{m!} \left(\frac{-2}{\sin 2\theta}\right)^m a_h^{(m)} C_m^{(1-\frac{1}{2}b)}(\sin \theta) \Gamma(p+m+\frac{1}{2}). \quad (4.5.10)$$

In this expansion $C_n^{(b)}(x) = {}_2F_1[-n, 2b-1+n; b; x]$, an ultraspherical polynomial, and $a_h^{(m)}$ is the coefficient of z^h in the expansion of

$$\sum_{h=0}^{\infty} a_h^{(m)} z^h = \left[\sum_{h=0}^{\infty} \frac{\sin\{(h+3)(\theta - \frac{1}{2}\pi)\}}{h+3} z^h \right]^m.$$

The method of proof is essentially the method of 'steepest descents'. If we collect the first terms of each expression we find that

$$\frac{e^{-\frac{1}{2}x}}{\Gamma(b)} {}_1F_1[a; b; x] = \frac{\Gamma(1-a)}{\Gamma(b-a)} \frac{(2 \cos \theta)^{1-b}}{\sqrt{(\pi k \sin 2\theta)}} \left\{ \sin(\Theta + a\pi) - A_1(\theta) \frac{\cos(\Theta + a\pi)}{k \sin 2\theta} + O(k^{-2}) \right\},$$

where $\Theta = k(2\theta - \sin 2\theta) + \frac{1}{4}\pi$, and

$$A_1(\theta) = \frac{1}{12} \left\{ \frac{5}{4 \sin^2 \theta} + (3b^2 - 6b + 2) \sin^2 \theta - 1 \right\}.$$

Now

$$\frac{\Gamma(1-a)}{\Gamma(b-a)} = \frac{\Gamma(1-\frac{1}{2}b+k)}{\Gamma(\frac{1}{2}b+k)} = k^{1-b} \{1 + O(k^{-2})\},$$

since $k = \frac{1}{2}b - a$.

Hence we have finally

$${}_1F_1[a; b; x] = \frac{\Gamma(b) \exp(2k \cos^2 \theta) (2k \cos \theta)^{1-b}}{(\pi k \sin 2\theta)^{\frac{1}{2}}} \left\{ \sin(\Theta + a\pi) - A_1(\theta) \frac{\cos(\Theta + a\pi)}{k \sin 2\theta} + O(k^{-2}) \right\} \quad (4.5.11)$$

for $0 < x < 4k$, where $k = \frac{1}{2}b - a$, and $\cos \theta = \sqrt{\frac{x}{4k}}$.

In Tricomi's paper a numerical example shows that for $a = -10$, $b = 1$, so that $4k = 42$, the above approximation gives four-figure accuracy over the range $1 \leq x \leq 30$.

4.5.3 Whittaker functions when k and x are large

The asymptotic approximations for the Whittaker functions, when k and x are both large, are similar to those for the Kummer functions. We let $x/4k = \cosh^2 \alpha$ where α is real, and put

$$Q_1(\alpha) = \frac{1}{96 \tanh^3 \alpha} \{3(16m^2 - 1) \tanh^4 \alpha - 6 \tanh^2 \alpha + 5\},$$

$$Q_2(\alpha) = \frac{1}{18432 \tanh^6 \alpha} \{9(16m^2 - 9)(16m^2 - 1) \tanh^8 \alpha + 36(48m^2 - 7) \tanh^6 \alpha \\ + 6(121 - 112m^2) \tanh^4 \alpha - 924 \tanh^2 \alpha + 385\}.$$

There are four cases to consider,

$$1 < x/4k < \infty, \quad 0 < x/4k < 1, \quad -\infty < x/4k < 0 \quad \text{and} \quad x/4k \sim 1.$$

The proofs, mainly due to Buchholz (1950), are based on the 'steepest descents' method, and will not be given here.

In the first case, when $1 < x/4k < \infty$,

$$W_{k,m}(x) = W_{k,m}(4k \cosh^2 \alpha) \sim (\tanh \alpha)^{-\frac{1}{2}} \exp \left[k \{ \log k - 1 + 2\alpha - \sinh 2\alpha \} + \frac{4m^2 - \frac{1}{3}}{8k} \right] \\ \times \{ 1 - Q_1(\alpha) k^{-1} + Q_2(\alpha) k^{-2} + O(k^{-3}) \}, \quad (4.5.12)$$

where $|\arg x| = |\arg k| \leq \frac{1}{2}\pi$, and so it follows that

$$W_{-k,m}(x e^{\pm \pi i}) \\ = W_{-k,m}(4k \cosh^2 \alpha e^{\pm \pi i}) \sim (\tanh \alpha)^{-\frac{1}{2}} \exp \left[-k \{ \log(k e^{\pm \pi i}) - 1 + 2\alpha - \sinh 2\alpha \} - \frac{4m^2 - \frac{1}{3}}{8k} \right] \\ \times \{ 1 + Q_1(\alpha) k^{-1} + Q_2(\alpha) k^{-2} + O(k^{-3}) \}, \quad (4.5.13)$$

where for the upper signs $-\frac{1}{2}\pi \leq (\arg x, \arg k) \leq 0$, and for the lower signs $0 \leq (\arg x, \arg k) \leq \frac{1}{2}\pi$.

For $M_{k,m}(x)$, we deduce

$$M_{k,m}(x) = M_{k,m}(4k \cosh^2 \alpha) \sim \Gamma(1 + 2m) (2\pi \tanh \alpha)^{-\frac{1}{2}} \exp \left\{ -m \log k + \frac{m(4m^2 - 1)}{24k^2} \right\} \\ \times [2 \sin \pi(\frac{1}{2} + m - k) \exp \{ k(\sinh 2\alpha - 2\alpha) \} \{ 1 + Q_1(\alpha) k^{-1} + Q_2(\alpha) k^{-2} + O(k^{-3}) \} \\ + \exp \{ \mp \pi i(k - \frac{1}{2} - m) - k(\sinh 2\alpha - 2\alpha) \} \{ 1 - Q_1(\alpha) k^{-1} + Q_2(\alpha) k^{-2} + O(k^{-3}) \}], \quad (4.5.14)$$

where, for the upper sign $0 \leq (\arg x, \arg k) \leq \frac{1}{2}\pi$, and, for the lower sign, $-\frac{1}{2}\pi \leq (\arg x, \arg k) \leq 0$.

In the second case, $0 < x/4k < 1$, the results are

$$W_{-k,m}(x e^{\pm \pi i}) = W_{-k,m}(4k \cos^2 \alpha e^{\pm \pi i}) \\ \sim (\tan \alpha)^{-\frac{1}{2}} \exp \left\{ \pm ik(2\alpha - \sin 2\alpha) \pm \frac{1}{4}\pi i - k \log(k e^{\pm \pi i}) + k - \frac{4m^2 - \frac{1}{3}}{8k} \right\} \\ \times \{ 1 \mp Q_1(i\alpha) k^{-1} + Q_2(i\alpha) k^{-2} + O(k^{-3}) \}, \quad (4.5.15)$$

for $0 < \alpha < \frac{1}{2}\pi$, $-\frac{1}{2}\pi \leq (\arg x, \arg k) \leq \frac{1}{2}\pi$ ($\tanh i\alpha = i \tan \alpha$), and so, by the continuation formula (2.5.19)

$$W_{k,m}(x) = W_{k,m}(4k \cos^2 \alpha) \sim 2(\tan \alpha)^{-\frac{1}{2}} \exp \left\{ k \log k - k + \frac{4m^2 - \frac{1}{3}}{8k} + O(k^{-3}) \right\} \\ \times [\sin \{ k(2\alpha - \sin 2\alpha) + \frac{1}{4}\pi \} \{ (1 + Q_2(i\alpha) k^{-2} + O(k^{-4})) \} \\ + \cos \{ k(2\alpha - \sin 2\alpha) + \frac{1}{4}\pi \} \{ iQ_1(i\alpha) k^{-1} + O(k^{-3}) \}]. \quad (4.5.16)$$

From (2.5.18) we deduce that

$$M_{k,m}(x) = M_{k,m}(4k \cos^2 \alpha) \sim \sqrt{2} \Gamma(1 + 2m) (\pi \tan \alpha)^{-\frac{1}{2}} k^{-m} \exp \left\{ \frac{m(4m^2 - 1)}{24k^2} + O(k^{-4}) \right\} \\ \times [\sin \{ k(\pi - 2\alpha + \sin 2\alpha) - \pi m + \frac{1}{4}\pi \} \{ 1 + Q_2(i\alpha) k^{-2} + O(k^{-4}) \} \\ - \cos \{ k(\pi - 2\alpha + \sin 2\alpha) - \pi m + \frac{1}{4}\pi \} \{ iQ_1(i\alpha) k^{-1} + O(k^{-3}) \}], \quad (4.5.17)$$

for $0 < \alpha < \frac{1}{2}\pi$, $-\frac{1}{2}\pi \leq (\arg k, \arg x) \leq \frac{1}{2}\pi$.

In particular, if ix and ik are purely imaginary, we have

$$M_{-ik,m}(-ix) \sim -\pi^{-\frac{1}{2}} 2^{\frac{1}{2}} (\tan \alpha)^{-\frac{1}{2}} ik^{-m} e^{\frac{1}{2}\pi im} \sinh \{ k(\pi - 2\alpha + \sin 2\alpha) - \pi im + \frac{1}{4}\pi \}. \quad (4.5.18)$$

In the third case, $-\infty < x/4k < 0$, we have, on putting $-k$ for k , in the previous result (4.5.12) and $\alpha + \frac{1}{2}\pi i$ for α ,

$$W_{-k,m}(x) = W_{-k,m}(4k \sinh^2 \alpha) \sim (\tanh \alpha)^{-\frac{1}{2}} \exp \left\{ -k \log k + k - k(2\alpha + \sinh 2\alpha) - \frac{4m^2 - \frac{1}{3}}{8k} \right\} \\ \times \{1 + Q_1(\alpha + \frac{1}{2}\pi i)k^{-1} + Q_2(\alpha + \frac{1}{2}\pi i)k^{-2} + O(k^{-3})\}, \quad (4.5.19)$$

for $0 < \alpha < \infty$, $-\frac{1}{2}\pi \leq (\arg x, \arg k) \leq \frac{1}{2}\pi$, and so

$$W_{-k,m}(x e^{-2\pi i}) = W_{-k,m}(4k \sinh^2 \alpha e^{-2\pi i}) \sim -i(\tanh \alpha)^{\frac{1}{2}} \exp \left\{ -k \log k + k + k(2\alpha + \sinh 2\alpha) - \frac{4m^2 - \frac{1}{3}}{8k} \right\} \\ \times \{1 - Q_1(\alpha + \frac{1}{2}\pi i) + Q_2(\alpha + \frac{1}{2}\pi i)k^{-2} + O(k^{-3})\}, \quad (4.5.20)$$

for $0 < \alpha < \infty$, $-\frac{1}{2}\pi \leq (\arg x, \arg k) \leq \frac{1}{2}\pi$.

From these results we have, by (2.5.18),

$$M_{k,m}(4k \sinh^2 \alpha e^{-\pi i}) = e^{-\frac{1}{2}\pi i - \pi i m} M_{-k,m}(4k \sinh^2 \alpha) \\ \sim \Gamma(1 + 2m) (\tanh \alpha)^{\frac{1}{2}} (2\pi)^{-\frac{1}{2}} k^{-m} \exp \left\{ -\frac{1}{4}\pi i + \frac{m(4m^2 - 1)}{24k^2} \right\} \\ \times [\exp \{ \pi i m + \frac{1}{4}\pi i - k(2\alpha + \sinh 2\alpha) \} \{1 + Q_1(\alpha + \frac{1}{2}\pi i)k^{-1} + Q_2(\alpha + \frac{1}{2}\pi i)k^{-2} \\ + O(k^{-3})\} + \exp \{ -\pi i m - \frac{1}{4}\pi i + k(2\alpha + \sinh 2\alpha) \} \\ \times \{1 - Q_1(\alpha + \frac{1}{2}\pi i)k^{-1} + Q_2(\alpha + \frac{1}{2}\pi i)k^{-2} + O(k^{-3})\}]. \quad (4.5.21)$$

Also, by (2.5.19),

$$W_{k,m}(4k \sinh^2 \alpha e^{-i\pi}) \sim (\tanh \alpha)^{\frac{1}{2}} e^{-\frac{1}{4}\pi i - k} k^k [\exp \{ \pi i k - \frac{1}{4}\pi i - k(2\alpha + \sinh 2\alpha) \} \\ \times \{1 + Q_1(\alpha + \frac{1}{2}\pi i)k^{-1} + Q_2(\alpha + \frac{1}{2}\pi i)k^{-2} + O(k^{-3})\} \\ + \exp \{ -i\pi k + \frac{1}{4}\pi i + k(2\alpha + \sinh 2\alpha) \} \{1 - Q_1(\alpha + \frac{1}{2}\pi i)k^{-1} \\ + Q_2(\alpha + \frac{1}{2}\pi i)k^{-2} + O(k^{-3})\}], \quad (4.5.22)$$

and, in particular, when ik is purely imaginary,

$$M_{-ik,m}(4ik \sinh^2 \alpha) \sim 2^{\frac{1}{2}} \pi^{-\frac{1}{2}} (\tanh \alpha)^{-\frac{1}{2}} \Gamma(1 + 2m) k^{-m} \exp \left\{ \frac{1}{4}\pi i + \frac{1}{2}\pi i m - \frac{m(4m^2 - 1)}{24k^2} \right\} \\ \times [\sin \{ k(2\alpha + \sinh 2\alpha) + \frac{1}{4}\pi - m\pi \} \{1 - Q_2(\alpha + \frac{1}{2}\pi i)k^{-2} + O(k^{-4})\} \\ + \cos \{ k(2\alpha + \sinh 2\alpha) + \frac{1}{4}\pi - m\pi \} \{Q_1(\alpha + \frac{1}{2}\pi i)k^{-1} + O(k^{-3})\}]. \quad (4.5.23)$$

In the final case, $x/4k \sim 1$, and we suppose that $x - 4k = 2\epsilon = o[(\frac{1}{6}x)^{\frac{1}{3}}]$. Then we have

$$W_{k,m}(x) \sim \Gamma(\frac{1}{3}) \pi^{-\frac{1}{2}} (\frac{1}{6}x)^{\frac{1}{6}} \exp \left\{ k \log k - k + \frac{4m^2 - \frac{1}{3}}{8k} \right\} \\ \times \left[1 - \frac{\epsilon \Gamma(\frac{5}{6})}{(\frac{1}{3}x)^{\frac{1}{3}} \pi^{\frac{1}{2}}} + \frac{\epsilon}{3! \frac{1}{2}x} (\epsilon^2 + \frac{1}{5}) - \frac{\Gamma(\frac{5}{6})}{3 \cdot 3! (\frac{1}{3}x)^{\frac{1}{3}} \pi^{\frac{1}{2}}} (\epsilon^4 + 2\epsilon^2 + 12m^2 - \frac{33}{5}) + O\left(\frac{\epsilon^6}{(\frac{1}{3}x)^2}\right) \right], \quad (4.5.24)$$

so that

$$W_{-k,m}(x e^{\pm \pi i}) \sim \Gamma(\frac{1}{3}) \pi^{-\frac{1}{2}} (\frac{1}{6}x e^{\pm i\pi})^{\frac{1}{6}} \exp \left\{ -k \log (k e^{\pm \pi i}) + k - \frac{4m^2 - \frac{1}{3}}{8k} \right\} \\ \times \left\{ 1 - \frac{\epsilon e^{\pm \frac{2}{3}\pi i} \Gamma(\frac{5}{6})}{\pi^{\frac{1}{2}} (\frac{1}{3}x)^{\frac{1}{3}}} + \frac{\epsilon(\epsilon^2 + \frac{1}{5})}{3! \frac{1}{2}x} \right. \\ \left. - \frac{e^{\pm 2\pi i/3} \Gamma(\frac{5}{6})}{3 \cdot 3! (\frac{1}{3}x)^{\frac{1}{3}} \pi^{\frac{1}{2}}} (\epsilon^4 + 2\epsilon^2 + 12m^2 - \frac{33}{5}) + O\left(\frac{\epsilon^6}{(\frac{1}{3}x)^2}\right) \right\}, \quad (4.5.25)$$

and

$$\begin{aligned}
 M_{k,m}(x) &\sim \frac{2^{\frac{1}{2}}}{\pi} \Gamma(1+2m) \Gamma\left(\frac{1}{3}\right) \left(\frac{1}{6}x\right)^{\frac{1}{6}} k^{-m} \\
 &\times \left(\sin \pi\left(m-k+\frac{2}{3}\right) \left[1 + \frac{\epsilon(\epsilon^2+\frac{1}{5})}{3! \frac{1}{2}x} + O\left(\frac{\epsilon^6}{(\frac{1}{3}x)^2}\right) \right] \right. \\
 &\quad \left. - \sin \pi\left(m-k-\frac{2}{3}\right) \Gamma\left(\frac{5}{6}\right) \pi^{-\frac{1}{2}} \left[\frac{\epsilon}{(\frac{1}{3}x)^{\frac{1}{3}}} + \frac{1}{3 \cdot 3! (\frac{1}{3}x)^{\frac{4}{3}}} \right] \right. \\
 &\quad \left. \times (\epsilon^4 + 2\epsilon^2 + 12m^2 - \frac{33}{5}) + O\left(\frac{\epsilon^6}{(\frac{1}{3}x)^2}\right) \right], \tag{4.5.26}
 \end{aligned}$$

for $-\frac{1}{2}\pi \leq \arg x \leq \frac{1}{2}\pi$.

4.6 Olver's theorems

In a recent series of papers (1954, 1956) Olver has discussed the asymptotic expansions for large values of u , of solutions of the equation

$$\frac{d^2y}{dx^2} = \{u^2p(x) + q(x)\}y, \tag{4.6.1}$$

where x lies in a simply connected domain D , and $p(x)$ and $q(x)$ are functions of the complex variable x independent of u . It is enough to consider u real, since if $u = |u| e^{i\theta}$, for θ fixed and real, the transformation $X = e^{\frac{1}{2}i\theta} x$ converts (4.6.1) into an equation of the same form with u replaced by $|u|$.

Olver shows that (4.6.1) can be transformed into an equation of the form

$$\frac{d^2w}{dz^2} = \{u^2z^n + f(z)\}w \tag{4.6.2}$$

by putting

$$w = \left(\frac{dx}{dz}\right)^{-\frac{1}{2}} y, \tag{4.6.3}$$

where

$$\left(\frac{dx}{dz}\right)^2 p(x) = z^n. \tag{4.6.4}$$

If we integrate (4.6.4) we find that

$$z^{\frac{1}{2}n+\frac{1}{2}} = \left(\frac{1}{2}n+1\right) \int_{x_0}^x \{p(t)\}^{\frac{1}{2}} dt, \tag{4.6.5}$$

and also that

$$f(z) = \left(\frac{dx}{dz}\right)^2 q(x) + \left(\frac{dx}{dz}\right)^{\frac{1}{2}} \frac{d^2}{dz^2} \left\{ \left(\frac{dx}{dz}\right)^{-\frac{1}{2}} \right\}. \tag{4.6.6}$$

Next Olver shows that (4.6.2) can assume three forms given by $n = +1, 0$ and -1 , and he gives the asymptotic expansions for large u of the solutions in these three cases.

THEOREM I ($n = 0$). For large values of u , the equation

$$\frac{d^2w}{dz^2} = \{u^2 + f(z)\}w \tag{4.6.7}$$

has solutions $W_1(u^2, z, f(z))$ and $W_2(u^2, z, f(z))$ uniform with respect to z such that, if z lies in D_1 ,

$$W_1(z) = e^{uz} \left\{ \sum_{s=0}^{M-1} \frac{A_s(z)}{u^s} + O(u^{-M}) \right\}, \tag{4.6.8}$$

$$W_1'(z) = u e^{uz} \left\{ \sum_{s=0}^{M-1} \frac{B_s(z)}{u^s} + O(u^{-M}) \right\}, \tag{4.6.9}$$

and if z lies in D_2 ,

$$W_2(z) = e^{-uz} \left\{ \sum_{s=0}^{M-1} \frac{(-1)^s A_s(z)}{u^s} + O(u^{-M}) \right\}, \quad (4.6.10)$$

$$W_2'(z) = -u e^{-uz} \left\{ \sum_{s=0}^{M-1} \frac{(-1)^s B_s(z)}{u^s} + O(u^{-M}) \right\}, \quad (4.6.11)$$

where $W_1(z)$ and $W_2(z)$ are independent of the arbitrary unbounded integer M , D_1 is the domain comprising those points z which can be joined to an arbitrary fixed point a' by a contour which lies wholly to the left of the line through z parallel to the imaginary axis, D_2 is the domain comprising those points z which can be joined to an arbitrary fixed point b' by a contour which lies wholly to the right of the line through z parallel to the imaginary axis, and the coefficients $A_s(z)$, $B_s(z)$ can be determined from the relations

$$\left. \begin{aligned} A_0(z) &= 1, \\ B_s(z) &= A_s(z) + A'_{s-1}(z), \end{aligned} \right\} \quad (4.6.12)$$

and

$$A_{s+1}(z) = -\frac{1}{2}A'_s(z) + \frac{1}{2} \int f(z) A_s(z) dz + K_s, \quad (4.6.13)$$

where K_s is an arbitrary constant.

THEOREM II ($n = 1$). For large values of u , the equation

$$\frac{d^2 w}{dz^2} = \{u^2 z + f(z)\} w \quad (4.6.14)$$

has solutions $W_j\{u^2, z, f(z)\}$ ($j = 1, 2, 3$) uniform with respect to z , such that if z lies in D_j ,

$$W_j(z) = \text{Ai}(\rho_j u^{\frac{2}{3}} z) \sum_{s=0}^M \frac{A_s(z)}{u^{2s}} + \frac{\text{Ai}'(\rho_j u^{\frac{2}{3}} z)}{u^{\frac{4}{3}}} \sum_{s=0}^{M-1} \frac{B_s(z)}{u^{2s}} + \frac{\exp\{-\frac{2}{3}(\rho_j u^{\frac{2}{3}} z)^{\frac{3}{2}}\}}{1 + |u^{\frac{2}{3}} z|^{\frac{1}{4}}} O(u^{-2M-1}), \quad (4.6.15)$$

and

$$\begin{aligned} W_j'(z) &= \text{Ai}(\rho_j u^{\frac{2}{3}} z) \sum_{s=0}^{M-1} \frac{C_s(z)}{u^{2s}} + u^{\frac{2}{3}} \text{Ai}'(\rho_j u^{\frac{2}{3}} z) \sum_{s=0}^M \frac{D_s(z)}{u^{2s}} \\ &\quad + (1 + |u^{\frac{2}{3}} z|^{\frac{1}{4}}) \exp\{-\frac{2}{3}(\rho_j u^{\frac{2}{3}} z)^{\frac{3}{2}}\} O(u^{-2M}), \end{aligned} \quad (4.6.16)$$

where $\rho_1 = 1$, $\rho_2 = e^{\frac{2}{3}\pi i}$, $\rho_3 = e^{-\frac{2}{3}\pi i}$, the coefficients $A_s(z)$, $B_s(z)$, $C_s(z)$ and $D_s(z)$ can be determined by the relations

$$\left. \begin{aligned} A_0(z) &= D_0(z) = 1, \\ B_s(z) &= \frac{1}{2} z^{-\frac{1}{2}} \int_0^z t^{-\frac{1}{2}} \{f(t) A_s(t) - A_s'(t)\} dt, \end{aligned} \right\} \quad (4.6.17)$$

$$A_{s+1}(z) = -\frac{1}{2} B'_s(z) + \frac{1}{2} \int f(z) B_s(z) dz + K_s, \quad (4.6.18)$$

where K_s is an arbitrary constant, $C_s(z) = A'_s(z) + z B_s(z)$, $(4.6.19)$

$$D_s(z) = A_s(z) + B'_{s-1}(z), \quad (4.6.20)$$

and the domains D_j are such that, if S_1 , S_2 and S_3 are the sectors

$$-\frac{1}{3}\pi \leq \arg z \leq \frac{1}{3}\pi, \quad -\pi \leq \arg z < -\frac{1}{3}\pi, \quad \frac{1}{3}\pi < \arg z \leq \pi,$$

respectively, and a_j is an arbitrary fixed point of the sector S_j , then D_j is the domain comprising those points z for which

$$|\exp\{-\frac{2}{3}(\rho_j z)^{\frac{3}{2}}\}| \geq |\exp\{-\frac{2}{3}(\rho_j a_j)^{\frac{3}{2}}\}|,$$

such that a contour can be found joining z and a_j which lies in at most two of the sectors S_j , and does not cross that curve $\exp\{-\frac{2}{3}(\rho_j z)^{\frac{3}{2}}\} = \text{constant}$, which passes through z .

THEOREM III ($n = -1$). For large values of u , the equation

$$\frac{d^2w}{dz^2} = \{u^2z^{-1} + f(z)\}w \quad (4.6.21)$$

can be transformed into an equation of the general type

$$\frac{d^2w}{dz^2} = \frac{1}{z} \frac{dw}{dz} + \left\{ u^2 + \frac{\mu^2 - 1}{z^2} + \phi(z) \right\} w, \quad (4.6.22)$$

where $f(z) \equiv \frac{\gamma + zh(z)}{z^2}$, γ is a constant and $h(z)$ is a regular function of z , if we write z^2 for z , $\frac{1}{4}u^2$ for u^2 , $\phi(z)$ for $4h(z^2)$ and $\frac{\mu^2 - 1}{4} = \gamma$.

Also if $\text{Rl } \mu \geq 0$, this equation has solutions $W_j\{u, z, \mu, \phi(z)\}$ ($j = 1, 2$) uniform with respect to z , such that if z lies in D_1 ,

$$W_1(z) = zI_\mu(uz) \left\{ \sum_{s=0}^{M-1} \frac{A_s(z)}{u^{2s}} + O(u^{-2M}) \right\} + \frac{z}{u} I_{\mu+1}(uz) \left\{ \sum_{s=0}^{M-1} \frac{B_s(z)}{u^{2s}} + \frac{z}{1+|z|} O(u^{-2M}) \right\}, \quad (4.6.23)$$

$$W_1'(z) = I_\mu(uz) \left\{ \sum_{s=0}^{M-1} \frac{C_s(z)}{u^{2s}} + (1+|z|) O(u^{-2M}) \right\} + uz I_{\mu+1}(uz) \left\{ \sum_{s=0}^{M-1} \frac{D_s(z)}{u^{2s}} + O(u^{-2M}) \right\}, \quad (4.6.24)$$

and if z lies in D_2 ,

$$W_2(z) = zK_\mu(uz) \left\{ \sum_{s=0}^{M-1} \frac{A_s(z)}{u^{2s}} + O(u^{-2M}) \right\} - \frac{z}{u} K_{\mu+1}(uz) \left\{ \sum_{s=0}^{M-1} \frac{B_s(z)}{u^{2s}} + \frac{z}{1+|z|} O(u^{-2M}) \right\}, \quad (4.6.25)$$

$$W_2'(z) = K_\mu(uz) \left\{ \sum_{s=0}^{M-1} \frac{C_s(z)}{u^{2s}} + (1+|z|) O(u^{-2M}) \right\} - uz K_{\mu+1}(uz) \left\{ \sum_{s=0}^{M-1} \frac{D_s(z)}{u^{2s}} + O(u^{-2M}) \right\}, \quad (4.6.26)$$

where the coefficients $A_s(z)$, $B_s(z)$, $C_s(z)$ and $D_s(z)$ can be determined by the relations

$$\left. \begin{aligned} A_0(z) &= D_0(z) = 1, \\ B_s(z) &= -\frac{1}{2}A_s'(z) + \frac{1}{2} \int_0^z \left\{ \phi(t) A_s(t) - \frac{2\mu+1}{t} A_s'(t) \right\} dt, \end{aligned} \right\} \quad (4.6.27)$$

$$A_{s+1}(z) = \frac{1}{2} \left(\frac{2\mu+1}{z} \right) B_s(z) - \frac{1}{2} B_s'(z) + \frac{1}{2} \int \phi(z) B_s(z) dz + K_s, \quad (4.6.28)$$

where K_s is an arbitrary constant,

$$C_s(z) = (\mu+1) A_s(z) + z A_s'(z) + z B_s(z), \quad (4.6.29)$$

and

$$D_s(z) = A_s(z) - (\mu/z) B_{s-1}(z) + B_{s-1}'(z), \quad (4.6.30)$$

and the domains D_j are such that D_1 comprises those points z which can be joined to the origin by a contour which does not cross either the imaginary axis or the line through z parallel to the imaginary axis, and, if a' is an arbitrary fixed point of the sector $|\arg z| < \frac{1}{2}\pi$, D_2 comprises those points z which can be joined to a' by a contour, wholly to the right of a line through z parallel to the imaginary axis, which does not cross the negative imaginary axis, if $\frac{1}{2}\pi \leq \arg z \leq \frac{3}{2}\pi$, and does not cross the positive imaginary axis, if $-\frac{3}{2}\pi \leq \arg z \leq -\frac{1}{2}\pi$. D_2 excludes the point $z = 0$.

The restriction $\text{Rl } \mu > 0$ implies no loss of generality but merely selects the positive root of μ^2 , in terms of which we state our results.

4.6.1 Asymptotic expansions when a is large

We shall now apply theorem III to the normalized confluent hypergeometric equation

$$w'' = -\frac{\frac{1}{2}b-a}{x}w + \left\{ \frac{1}{4} + \frac{b(b-2)}{4x^2} \right\} w. \quad (4.6.31)$$

In (4.6.31) let us put $z^2 = x$ and $\frac{1}{4}u^2 = a - \frac{1}{2}b$. Then (4.6.31) becomes

$$\frac{d^2w}{dz^2} = \frac{1}{z} \frac{dw}{dz} + \left\{ u^2 + \frac{b(b-2)}{z^2} + z^2 \right\} w. \quad (4.6.32)$$

This equation is of the same form as (4.6.22) with $\mu = b-1$, and $\phi(z) = z^2$. Hence for large u , (4.6.32) will have solutions $W_1(u, z, b-1, z^2)$ and $W_2(u, z, b-1, z^2)$.

But this equation also has solutions

$$F \equiv e^{-\frac{1}{2}z^2} z^b {}_1F_1\left[\frac{1}{4}u^2 + \frac{1}{2}b; b; z^2\right] = M_{-\frac{1}{4}u^2, \frac{1}{2}b-\frac{1}{2}}(z^2) \quad (4.6.33)$$

and

$$U \equiv e^{-\frac{1}{2}z^2} z^b U\left(\frac{1}{4}u^2 + \frac{1}{2}b; b; z^2\right) = W_{-\frac{1}{4}u^2, \frac{1}{2}b-\frac{1}{2}}(z^2). \quad (4.6.34)$$

Hence

$$F = \alpha_1 W_1 + \alpha_2 W_2, \quad (4.6.35)$$

and

$$U = \beta_1 W_1 + \beta_2 W_2, \quad (4.6.36)$$

where $\alpha_1, \beta_1, \alpha_2, \beta_2$ are arbitrary constants, independent of z . Now the expansions W_1 and W_2 in z are uniform so that we may keep u and b fixed and let $z \rightarrow 0$ through positive values. Then, near $z = 0$,

$$F(z) \sim z^b e^{-\frac{1}{2}z^2} \quad (4.6.37)$$

and

$$W_1(z) \sim \frac{u^{b-1} z^b}{2^{b-1} \Gamma(b)}, \quad \text{since} \quad I_\mu(x) \sim \frac{x^\mu 2^{-\mu}}{\Gamma(1+\mu)}.$$

Also, since $K_\mu(x) \sim \Gamma(\mu) x^{-\mu} 2^{\mu-1}$

$$W_2(z) \sim \Gamma(b-1) u^{1-b} z^{2-b} 2^{b-2}. \quad (4.6.38)$$

Hence $\alpha_2 \equiv 0$, and $F(z) = \alpha_1 W_1(z)$. When u is large

$$\frac{1}{\alpha_1} \sim \frac{W_1(z)}{F(z)} \sim \frac{u^{b-1} z^b e^{\frac{1}{2}z^2}}{2^{b-1} \Gamma(b) z^b} \sim \frac{u^{b-1}}{2^{b-1} \Gamma(b)}, \quad (4.6.39)$$

since we can choose K_s in $W_1(z)$ so that $A_{s+1}(z) \rightarrow 0$ as $z \rightarrow 0$. Hence, near $z = 0$,

$$e^{-\frac{1}{2}z^2} z^b {}_1F_1\left[\frac{1}{4}u^2 + \frac{1}{2}b; b; z^2\right] \sim \frac{u^{1-b}}{2^{1-b}} \Gamma(b) W_1(u, z, b-1, z^2). \quad (4.6.40)$$

Also

$$U(z) \sim \frac{\Gamma(b-1)}{\Gamma(\frac{1}{4}u^2 + \frac{1}{2}b)} e^{-\frac{1}{2}z^2} z^{2-b},$$

so that, in (4.6.36), we can take $\beta_1 \equiv 0$, and then $\beta_2 \sim \frac{U(z)}{W_2(z)} \sim \frac{2^{2-b} u^{b-1}}{\Gamma(\frac{1}{4}u^2 + \frac{1}{2}b)}$ near $z = 0$. Hence

$$e^{-\frac{1}{2}z^2} z^b U\left(\frac{1}{4}u^2 + \frac{1}{2}b; b; z^2\right) \sim \frac{2^{2-b} u^{b-1}}{\Gamma(\frac{1}{4}u^2 + \frac{1}{2}b)} W_2(u, z, b-1, z^2). \quad (4.6.41)$$

If, in (4.6.40), we put $\frac{1}{2}u = i\sqrt{(\frac{1}{2}b-a)}$ and $x = z^2$, when a, b and x are real we can again deduce

Tricomi's approximation (4.4.15). Similarly, if we make the same substitution in (4.6.41) and use the fact that

$$\frac{1}{\Gamma(a)} \sim \frac{1}{\pi} \sin \left\{ \left(\frac{1}{2}b - k \right) \pi \right\} \Gamma \left(\frac{1}{2}b - a + \frac{1}{2} \right) k^{\frac{1}{2} - b},$$

where $k \equiv \frac{1}{2}b - a$, we can deduce (4.4.18) from (4.6.41). These results check the above identification of the two solutions.

Thus our final expansions are

$$e^{-\frac{1}{2}z^2} z^b {}_1F_1 \left[\frac{1}{4}u^2 + \frac{1}{2}b; b; z^2 \right] \sim \Gamma(b) u^{1-b} 2^{b-1} \\ \times \left[z I_{b-1}(uz) \left\{ \sum_{s=0}^{N-1} \frac{A_s(z)}{u^{2s}} + O\left(\frac{1}{u^{2N}} \right) \right\} + \frac{z}{u} I_b(uz) \left\{ \sum_{s=0}^{N-1} \frac{B_s(z)}{u^{2s}} + \frac{z}{1+|z|} O\left(\frac{1}{u^{2N}} \right) \right\} \right] \quad (4.6.42)$$

$$\text{and } e^{-\frac{1}{2}z^2} z^b U \left(\frac{1}{4}u^2 + \frac{1}{2}b; b; z^2 \right) \sim \frac{2^{2-b} u^{b-1}}{\Gamma \left(\frac{1}{4}u^2 + \frac{1}{2}b \right)} \\ \times \left[z K_{b-1}(uz) \left\{ \sum_{s=0}^{N-1} \frac{A_s(z)}{u^{2s}} + O\left(\frac{1}{u^{2N}} \right) \right\} - \frac{z}{u} K_b(uz) \left\{ \sum_{s=0}^{N-1} \frac{B_s(z)}{u^{2s}} + \frac{z}{1+|z|} O\left(\frac{1}{u^{2N}} \right) \right\} \right], \quad (4.6.43)$$

uniformly with respect to z , where $A_0 = 1$,

$$B_s(z) = -\frac{1}{2} A'_s(z) + \int_0^z \left\{ \frac{1}{2} t^2 A_s(t) - \frac{b-\frac{1}{2}}{t} A'_s(t) \right\} dt \quad (4.6.44)$$

$$\text{and } A_{s+1}(z) = \frac{b-\frac{1}{2}}{z} B_s(z) - \frac{1}{2} B'_s(z) + \int \frac{1}{2} t^2 B_s(t) dt + K_s, \quad (4.6.45)$$

and K_s is chosen so that $A_{s+1}(z) \rightarrow 0$ as $z \rightarrow 0$. In fact,

$$\left. \begin{aligned} A_0 &= 1, \\ B_0 &= \frac{x^3}{6}, \\ A_1 &= (b-2) \frac{x^2}{2 \cdot 3} + \frac{x^6}{6^2 2!}, \\ B_1 &= -b(b-2) \frac{x}{3} - \frac{x^5}{3 \cdot 5} + \frac{x^9}{6^3 3!}, \\ A_2 &= -(5b-12)(b+2) \frac{x^4}{5!} + \left(\frac{b}{6^4} - \frac{13}{2^2 \cdot 3^4 \cdot 5} \right) x^8 + \frac{x^{12}}{6^4 4!}, \\ B_2 &= \left(\frac{b^3}{18} + \frac{b^2}{30} - \frac{b}{9} - \frac{4}{15} \right) x^3 + \left(-\frac{5b^2}{6^4} + \frac{5b}{3 \cdot 6^3} + \frac{23}{3^2 \cdot 6 \cdot 5 \cdot 7} \right) x^7 - \frac{7}{2 \cdot 6^4 \cdot 5} x^{11} + \frac{x^{15}}{6^5 5!}. \end{aligned} \right\} \quad (4.6.46)$$

In terms of the Whittaker functions these results become

$$M_{k,m}(z^2) \sim \Gamma(1+2m) u^{-2m} 2^{2m} \left[z I_{2m}(uz) \left\{ \sum_{s=0}^{N-1} \frac{A_s(z)}{u^{2s}} + O(u^{-2N}) \right\} \right. \\ \left. + \frac{z}{u} I_{2m+1}(uz) \left\{ \sum_{s=0}^{N-1} \frac{B_s(z)}{u^{2s}} + \frac{z}{1+|z|} O(u^{-2N}) \right\} \right], \quad (4.6.47)$$

and

$$W_{k,m}(z^2) \sim \frac{2^{1-2m}u^{2m}}{\Gamma(\frac{1}{2}+m-k)} \left[zK_{2m}(uz) \left\{ \sum_{s=0}^{N-1} \frac{A_s(z)}{u^{2s}} + O(u^{-2N}) \right\} - \frac{z}{u} K_{2m+1}(uz) \left\{ \sum_{s=0}^{N-1} \frac{B_s(z)}{u^{2s}} + \frac{z}{1+|z|} O(u^{-2N}) \right\} \right], \quad (4.6.48)$$

uniformly with respect to z , where $-4k = u^2$ and, in the coefficients $A_s(z)$ and $B_s(z)$, $b = 1 + 2m$.

4.6.2 Asymptotic expansions when k and x are large

In the normalized confluent hypergeometric equation (4.6.31) let us put

$$t = x/4k, \quad (4.6.49)$$

where $k = \frac{1}{2}b - a$, and we suppose for the moment that k is real. We assume that x and k become large together. Then (4.6.31) becomes

$$\dot{y} = 4k^2(1 - 1/t)y + b(b-2)y/4t^2. \quad (4.6.50)$$

This is of the same form as Olver's general equation (4.6.1) and it can be analysed in the same way. We have

$$p(t) = 1 - 1/t, \quad q(t) = b(b-2)/4t^2, \quad \text{and} \quad u^2 = 4k^2.$$

Here $p(t)$ has one pole at $t = 0$ and one zero at $t = 1$, and $q(t)$ has a double pole at $t = 0$. We have already, in §4.6.1, considered the expansions in a domain near $t = 0$, using theorem III. There are two cases left to consider:

Case I. If we choose a domain consisting of the whole t plane excluding small circles about the points $t = 0$ and $t = 1$, (4.6.50) comes under theorem I, since $p(t)$ has no poles nor zeros in this domain. Hence this case will lead to expansions of which the leading terms are exponential functions.

Case II. If we choose a domain consisting of a circle about the point $t = 1$, (4.6.50) comes under theorem II, since at $t = 1$, $p(t)$ has a simple zero. Hence this case will lead to expansions in terms of Airy functions.

First let us transform (4.6.50) to new variables, w and z . We take

$$w = \left(\frac{dt}{dz} \right)^{-\frac{1}{2}} y = (t')^{-\frac{1}{2}} y. \quad (4.6.51)$$

Then

$$w'' = \{4k^2(t')^2(1 - 1/t) + f(z)\}w, \quad (4.6.52)$$

where

$$f(z) = (t')^2 \frac{b(b-2)}{4t^2} + \frac{3t''^2 - 2t't'''}{4t'^2} \quad (4.6.53)$$

and

$$t'^2(1 - 1/t) = z^n. \quad (4.6.54)$$

Hence

$$z^{\frac{1}{2}(n+2)} = \pm \frac{1}{2}(n+2) \int_{t_0}^t (1 - 1/t)^{\frac{1}{2}} dt. \quad (4.6.55)$$

We have a choice of signs in this equation, and we adopt the negative sign so that $\text{Re } z > 0$ when $\text{Re } t > 0$. The integration is from t_0 , any pole or zero of $p(t)$, to t .

Let

$$I \equiv - \int_{t_0}^t (1 - 1/t)^{\frac{1}{2}} dt$$

and put $-(1 - 1/t)^{\frac{1}{2}} = v$. Then

$$I = \frac{1}{2} [\text{Log} \{(v+1)/(v-1)\} + 2v/(v^2-1)],$$

that is,
$$I = \sqrt{\{(t-1)t\}} + \text{Log} \{\sqrt{t} - \sqrt{(t-1)}\}. \quad (4.6.56)$$

Also
$$t' = z^{\frac{1}{2}n}(1-t)^{-\frac{1}{2}}, \quad (4.6.57)$$

from (4.6.54), and so, differentiating this result twice, we find, from (4.6.53), that

$$f(z) = \frac{z^n b(b-2)}{4t(t-1)} + \frac{n(n+4)}{16z^2} - \frac{3(8t-9)z^n}{16t(t-1)^3}. \quad (4.6.58)$$

We consider first the transformation

$$z = \text{Log} \{\sqrt{t} - \sqrt{(t-1)}\} + \sqrt{\{t(t-1)\}}. \quad (4.6.59)$$

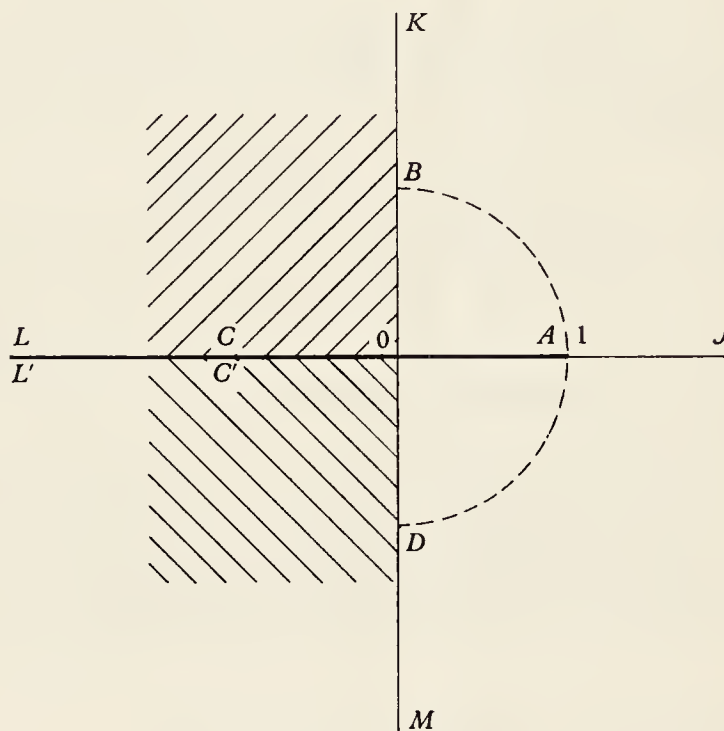


Fig. 4.2

This maps the whole of the t plane, excluding part of the real axis, on any half strip of the z plane, $\pi(n - \frac{1}{2}) < \text{Im } z < \pi(n + \frac{1}{2})$ for all n , and either $\text{Re } z > 0$ or $\text{Re } z < 0$. We choose that branch of the logarithmic function which makes z lie in the half strip $\text{Re } z > 0$, $-\frac{1}{2}\pi < \text{Im } z < \frac{1}{2}\pi$, so that the real axis $0 \leq t \leq 1$ maps into the imaginary axis $-\frac{1}{2}\pi i \leq z \leq \frac{1}{2}\pi i$.

So far we have assumed that k is real, but the transformations can easily be adapted to the case

$$k = |k| e^{i\theta},$$

where θ is a real fixed number. In (4.6.31) let us put

$$X = x e^{i\theta}.$$

Then we find that
$$\frac{d^2 y}{dX^2} = \left\{ \frac{-|k|}{X} + \frac{1}{4} e^{-2i\theta} + \frac{b(b-2)}{4X^2} \right\} y, \quad (4.6.60)$$

which is of the same form as the original equation in x . In (4.6.60), we now substitute $t = X/4|k|$ ($= x/4k$), and find that

$$\ddot{y} = \left\{ \frac{-4|k|^2}{t} + e^{-2i\theta} 4|k|^2 + \frac{b(b-2)}{4t^2} \right\} y. \quad (4.6.61)$$

This is of the same form as (4.6.50) with $p(t) = e^{-2i\theta} - 1/t$. Hence it has a pole at $t = 0$, and a zero at $t = e^{2i\theta}$. Also

$$I = -e^{i\theta} \int_{t_0}^t (1 - e^{2i\theta}/t)^{\frac{1}{2}} d(t e^{-2i\theta}), \quad (4.6.62)$$

so that

$$z^{\frac{1}{2}(n+2)} = \frac{1}{2}(n+2) I, \quad \text{as before,} \quad (4.6.63)$$

where now,

$$I = e^{i\theta} [\sqrt{\{t e^{-2i\theta} (t e^{-2i\theta} - 1)\}} + \text{Log} \{\sqrt{\{t e^{-2i\theta}\}} - \sqrt{\{t e^{-2i\theta} - 1\}}\}]. \quad (4.6.64)$$

The new transform

$$z = I, \quad (4.6.65)$$

is exactly the same as the previous transform rotated through an angle 2θ in the t plane and an angle θ in the z plane. Thus we have the same results, for complex k , as for real k , provided that we put $z e^{-i\theta}$ for z and $t e^{-2i\theta}$ for t throughout our results.

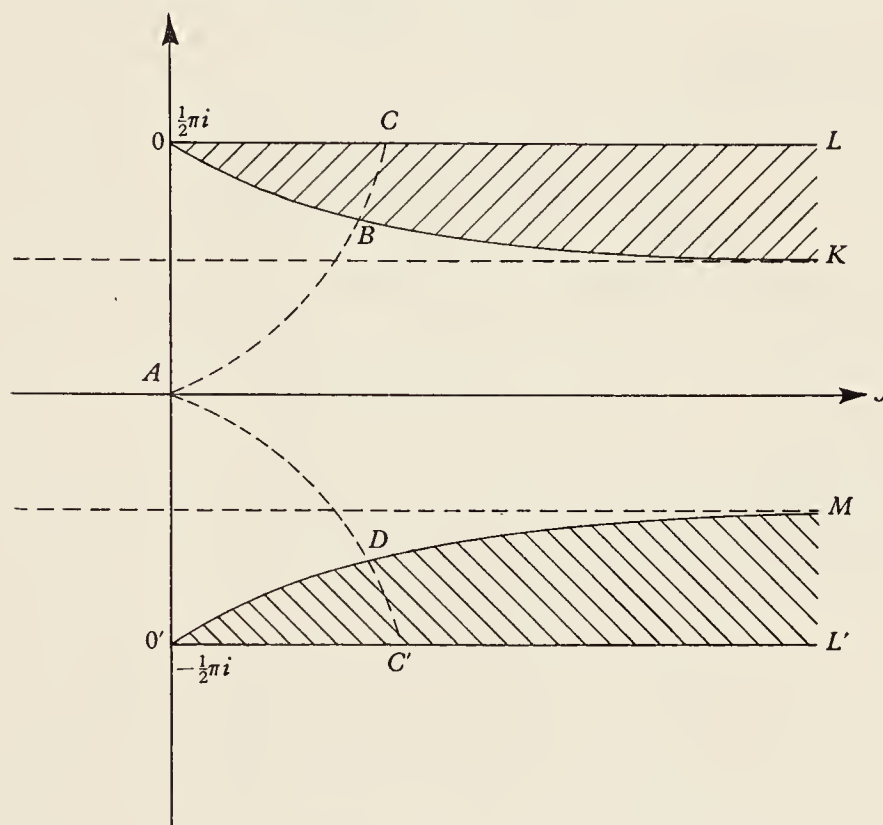


Fig. 4.3

4.6.3 Asymptotic expansions when $4k \neq x$

When $4k \neq x$, the equation (4.6.50) comes under case I, and we can apply theorem I. By this theorem we see that there exist asymptotic expansions for k and x large, $4k \neq x$, which are solutions of (4.6.50) for k real and for all values of $t = x/4k$, except the immediate neighbourhoods of the points $t = 0$, and $t = 1$. These solutions are

$$W_1(z) = e^{2kz} \left[\sum_{s=0}^{M-1} A_s(z) (2k)^{-s} + O\{(2k)^{-M}\} \right] \quad (4.6.66)$$

in the domain D_1 , $|\arg z| < \frac{1}{2}\pi$, and

$$W_2(z) = e^{-2kz} \left[\sum_{s=0}^{M-1} A_s(z) (-1)^s (2k)^{-s} + O\{(2k)^{-M}\} \right] \quad (4.6.67)$$

in the domain D_2 defined also by $|\arg z| < \frac{1}{2}\pi$. We see that $D_1 = D_2$ and that the solutions can be extended over the whole t -plane excluding $t = 0$ and $t = 1$, by the use of the continuation formulae (2.5.18) and (2.5.19).

In these solutions we have

$$\frac{1}{2}b - a = k, \quad \frac{1}{2}b - \frac{1}{2} = m, \quad (4.6.68)$$

$$z = t^{\frac{1}{2}}(t-1)^{\frac{1}{2}} + \log \{t^{\frac{1}{2}} - (t-1)^{\frac{1}{2}}\}, \quad (4.6.69)$$

$$t = x/4k, \quad (4.6.70)$$

$$A_0(z) = 1,$$

and
$$A_{s+1}(z) = -\frac{1}{2}A'_s(z) + \frac{1}{2} \int f(z) A_s(z) dz + K_s, \quad (4.6.71)$$

where
$$f(z) = \frac{b(b-2)}{4t(t-1)} + \frac{3(8t-9)}{16t(t-1)^3}, \quad (4.6.72)$$

and K_s is an arbitrary constant.

The leading terms of these two solutions are

$$W_1(z) = e^{2kz} \{1 + A_1(z) k^{-1} + O(k^{-2})\} \quad (4.6.73)$$

and
$$W_2(z) = e^{-2kz} \{1 - A_1(z) k^{-1} + O(k^{-2})\}. \quad (4.6.74)$$

But $(t')^{-\frac{1}{2}} W_{k,m}(x)$ is also a solution of (4.6.50). Hence there must exist a linear relation between these three solutions of the form

$$W_{k,m}(4kt) = \{B_1 W_1(z) + B_2 W_2(z)\} (t')^{\frac{1}{2}}. \quad (4.6.75)$$

Now from (4.6.57) we have
$$(t')^{-\frac{1}{2}} = e^{\frac{1}{4}\pi i} (t-1)^{\frac{1}{4}} t^{-\frac{1}{4}}, \quad (4.6.76)$$

and, from (4.6.69) above,
$$z = t - \frac{1}{2} + \log \left(\frac{1}{2}t^{\frac{1}{2}}\right) - O(t^{-1}). \quad (4.6.77)$$

Thus, when k is real, as $t \rightarrow \infty$ for $|\arg t| < \frac{1}{2}\pi$, we find that

$$(t')^{-\frac{1}{2}} W_{k,m}(x) \rightarrow 0.$$

Also if $k > 0$, $W_1(z) \rightarrow \infty$ and $W_2(z) \rightarrow 0$, as $z \rightarrow \infty$, with t . Hence we must have $B_1 \equiv 0$, and so

$$e^{\frac{1}{4}\pi i} (t-1)^{\frac{1}{4}} t^{-\frac{1}{4}} W_{k,m}(4kt) = B_2 W_2(z). \quad (4.6.78)$$

Let us expand this result, neglecting terms in t^{-1} and using the expansion of $W_{k,m}(4kt)$ for k fixed and t large,

$$W_{k,m}(4kt) \sim k^k (4t)^k e^{-2kt} \{1 + O(t^{-1})\}. \quad (4.6.79)$$

We find that

$$\begin{aligned} & e^{\frac{1}{4}\pi i} \{1 - \frac{1}{4}O(t^{-1})\} k^k (4t)^k e^{-2kt} \{1 + O(t^{-1})\} \\ &= B_2 \exp \left[-2k \left\{ t - \frac{1}{2} + \log \left(\frac{1}{2}t^{\frac{1}{2}} \right) - O(t^{-1}) \right\} \right] \left[1 + \sum_{s=1}^{M-1} A_s(z) (-2k)^{-s} + O\{(2k)^{-M}\} \right], \end{aligned}$$

so that if K_s , the constant of integration in $A_s(z)$, is chosen so that $A_s(\infty) = 0$, we can say that

$$B_2 \sim \lim_{t \rightarrow \infty} \frac{(t')^{-\frac{1}{2}} W_{k,m}(4kt)}{W_2(z)},$$

that is
$$B_2 \sim e^{\frac{1}{4}\pi i} k^k e^{-k}. \quad (4.6.80)$$

Our result is then
$$W_{k,m}(x) \sim k^k e^{-k} (1 - 4k/x)^{-\frac{1}{4}} W_2(z), \quad (4.6.81)$$

for k real and for $|\arg x/4k| < \frac{1}{2}\pi$.

We can immediately extend this result to complex values of k , and we find, for $k = |k|e^{i\theta}$, that

$$W_{k,m}(x) \sim k^k e^{-k} (1 - 4k/x)^{-\frac{1}{4}} W_2(z e^{-i\theta}), \quad (4.6.82)$$

where now, $z e^{-i\theta} = \sqrt{\{t e^{-2i\theta} (t e^{-2i\theta} - 1)\}} + \text{Log} \{\sqrt{(t e^{-2i\theta})} - \sqrt{(t e^{-2i\theta} - 1)}\}$

and
$$f(z e^{-i\theta}) = \frac{m^2 - \frac{1}{4}}{t e^{-2i\theta} (t e^{-2i\theta} - 1)} - \frac{3(8t e^{-2i\theta} - 9)}{16t e^{-2i\theta} (t e^{-2i\theta} - 1)^3}, \quad (4.6.83)$$

for $|\arg x/4|k| < \frac{1}{2}\pi$ and $|\arg k| < \frac{1}{2}\pi$. Again, these conditions can be relaxed still further by the use of continuation formulae, to cover the whole t plane with the exception of the immediate neighbourhoods of the points $t = 0$ and $t = e^{2i\theta}$. Thus

$$W_{-k,m}(x e^{\pm\pi i}) \sim e^{\mp\pi i k} k^{-k} e^k (1 - 4k/x)^{-\frac{1}{4}} W_1(z e^{-i\theta}), \quad (4.6.84)$$

where, for the upper signs $-\frac{1}{2}\pi \leq \arg(x/4|k|) \leq 0$, and for the lower signs $0 \leq \arg(x/4|k|) \leq \frac{1}{2}\pi$ and $|\arg k| < \frac{1}{2}\pi$.

From (4.6.82) and (4.6.84), using the continuation formula (2.5.18), we find that

$$M_{k,m}(x) \sim \Gamma(1 + 2m) \left\{ \frac{k^{-k} e^k (1 - 4k/x)^{-\frac{1}{4}}}{\Gamma(\frac{1}{2} + m - k)} W_1(z e^{-i\theta}) + \frac{e^{\pm\pi i(k - \frac{1}{2} - m)} k^k e^{-k} (1 - 4k/x)^{-\frac{1}{4}}}{\Gamma(\frac{1}{2} + m + k)} W_2(z e^{-i\theta}) \right\}, \quad (4.6.85)$$

where, for the upper signs $-\frac{1}{2}\pi \leq \arg(x/4|k|) \leq 0$, and for the lower signs $0 \leq \arg(x/4|k|) \leq \frac{1}{2}\pi$ and $|\arg k| < \frac{1}{2}\pi$.

If, in (4.6.82), (4.6.84) and (4.6.85), we put $z = 4k \cosh^2 \alpha$, we can deduce the results (4.5.12), (4.5.13) and (4.5.14). If we put $z = 4k \cos^2 \alpha$, we can deduce the results (4.5.16), (4.5.15) and (4.5.17) and if we put $z = 4k \cosh^2(\alpha + \frac{1}{2}\pi i)$, we can deduce the results (4.5.22), (4.5.19) and (4.5.21).

When we express these results in terms of Kummer's functions, we find that

$$U(a; b; x) \sim e^{\frac{1}{2}x} x^{-\frac{1}{2}b} (1 - 4k/x)^{-\frac{1}{4}} k^k e^{-k} W_2(z e^{-i\theta}), \quad (4.6.86)$$

for $|\arg x/4|k| < \frac{1}{2}\pi$ and $|\arg k| < \frac{1}{2}\pi$. Also

$${}_1F_1[a; b; x] \sim e^{\frac{1}{2}x} x^{-\frac{1}{2}b} \Gamma(b) (1 - 4k/x)^{-\frac{1}{4}} \left\{ \frac{k^{-k} e^k}{\Gamma(a)} W_1(z e^{-i\theta}) + \frac{e^{\mp\pi i a} k^k e^{-k}}{\Gamma(b-a)} W_2(z e^{-i\theta}) \right\}, \quad (4.6.87)$$

where, for the upper sign $-\frac{1}{2}\pi \leq \arg(x/4|k|) \leq 0$, and for the lower sign, $0 \leq \arg(x/4|k|) \leq \frac{1}{2}\pi$, and $|\arg k| < \frac{1}{2}\pi$. Here $k \equiv \frac{1}{2}b - a$, $b = 1 + 2m$.

The coefficients $A_s(z e^{-i\theta})$, in $W_1(z e^{-i\theta})$ and $W_2(z e^{-i\theta})$, are defined by

$$\left. \begin{aligned} A_0(z e^{-i\theta}) &= 1, \\ A_{s+1}(z e^{-i\theta}) &= -\frac{1}{2} A'_s(z e^{-i\theta}) + \frac{1}{2} \int f(z e^{-i\theta}) A_s(z e^{-i\theta}) \frac{dz}{dt} dt + K_s, \end{aligned} \right\} \quad (4.6.88)$$

where

$$\frac{dz}{dt} = (1 - e^{2i\theta} t^{-1})^{\frac{1}{2}},$$

$f(z e^{-i\theta})$ is defined by (4.6.83), and for each value of s , K_s , the constant of integration, is chosen so that $A_{s+1}(\infty) = 0$. These coefficients can be evaluated numerically in any given case for fixed values of b and θ .

4.6.4 Asymptotic expansions when $4k \doteq x$

When $4k \doteq x$, the equation (4.6.50) comes under case II and we can apply theorem II. By this theorem, we see that now there exist asymptotic expansions, for k and x large, $4k \doteq x$, which are solutions of (4.6.50) for k real and for all values of $\arg t$ in the immediate neighbourhood of the point $t = 1$. These solutions are

$$W_j(z) = \text{Ai}\{\rho_j(2k)^{\frac{2}{3}}z\} \sum_{s=0}^M A_s(z) (2k)^{-2s} + \frac{\text{Ai}'\{\rho_j(2k)^{\frac{2}{3}}z\}}{(2k)^{\frac{4}{3}}} \sum_{s=0}^{M-1} B_s(z) (2k)^{-2s} + \frac{\exp\{-\frac{2}{3}(\rho_j(2k)^{\frac{2}{3}}z)^{\frac{3}{2}}\}}{1 + |(2k)^{\frac{2}{3}}z|^{\frac{1}{2}}} O\{(2k)^{-2M-1}\}, \quad (4.6.89)$$

in the domains D_j , for $j = 1, 2, 3$, where $k = \frac{1}{2}b - a$, $t = x/4k$,

$$\rho_1 = 1, \quad \rho_2 = e^{\frac{2}{3}\pi i}, \quad \rho_3 = e^{-\frac{2}{3}\pi i}, \quad (4.6.90)$$

$$z = \left(\frac{3}{2}\right)^{\frac{2}{3}} [\log\{t^{\frac{1}{2}} - (t-1)^{\frac{1}{2}}\} + t^{\frac{1}{2}}(t-1)^{\frac{1}{2}}]^{\frac{2}{3}}, \quad (4.6.91)$$

$$A_0(z) = 1,$$

$$B_s(z) = \frac{1}{2}z^{-\frac{1}{2}} \int_0^z z^{-\frac{1}{2}} \{f(z) A_s(z) - A_s''(z)\} dz, \quad (4.6.92)$$

$$A_{s+1}(z) = -\frac{1}{2}B_s'(z) + \frac{1}{2} \int f(z) B_s(z) dz + K_s, \quad (4.6.93)$$

where K_s is an arbitrary constant of integration, and

$$f(z) = z \frac{b(b-2)}{4t(t-1)} + \frac{5}{16z^2} - \frac{3z(8t-9)}{16t(t-1)^3}. \quad (4.6.94)$$

Again one solution of (4.6.50) is $t'^{-\frac{1}{2}}W_{k,m}(x)$, so that we must have a linear relation connecting these four solutions, of the form

$$t'^{-\frac{1}{2}}W_{k,m}(x) = B_1W_1(z) + B_2W_2(z) + B_3W_3(z). \quad (4.6.95)$$

Here

$$t'^{-\frac{1}{2}} = (1 - 1/t)^{\frac{1}{2}} z^{-\frac{1}{2}}, \quad (4.6.96)$$

and the leading terms of the three solutions $W_j(z)$ are

$$W_1(z) \sim \frac{(2k)^{-\frac{1}{3}} z^{-\frac{1}{2}} \exp(-\frac{4}{3}kz^{\frac{3}{2}})}{2\sqrt{\pi}} [1 + O\{(2k)^{-1}\}], \quad (4.6.97)$$

$$W_2(z) \sim \frac{e^{-\frac{1}{3}\pi i} (2k)^{-\frac{1}{3}} z^{-\frac{1}{2}} \exp(\frac{4}{3}kz^{\frac{3}{2}})}{2\sqrt{\pi}} [1 + O\{(2k)^{-1}\}], \quad (4.6.98)$$

and

$$W_3(z) \sim \frac{e^{\frac{1}{3}\pi i} (2k)^{-\frac{1}{3}} z^{-\frac{1}{2}} \exp(\frac{4}{3}kz^{\frac{3}{2}})}{2\sqrt{\pi}} [1 + O\{(2k)^{-1}\}], \quad (4.6.99)$$

since

$$\text{Ai}(z) \sim \frac{z^{-\frac{1}{2}} \exp(-\frac{2}{3}z^{\frac{3}{2}})}{2\sqrt{\pi}} \{1 + O(\frac{3}{2}z^{-\frac{3}{2}})\}, \quad (4.6.100)$$

$$\text{Ai}'(z) \sim \frac{-z^{\frac{1}{2}} \exp(-\frac{2}{3}z^{\frac{3}{2}})}{2\sqrt{\pi}} \{1 + O(\frac{3}{2}z^{-\frac{3}{2}})\}, \quad (4.6.101)$$

and

$$z = \left(\frac{3}{2}\right)^{\frac{2}{3}} \{t - \frac{1}{2} + \log(\frac{1}{2}t^{\frac{1}{2}}) - O(t^{-1})\}^{\frac{2}{3}}. \quad (4.6.102)$$

Now as before, when $t \rightarrow \infty$,

$$W_{k,m}(x) \sim e^{-2kt} (4kt)^k \{1 + O(t^{-1})\} \rightarrow 0, \quad (4.6.103)$$

$W_1(z) \rightarrow 0$, and $W_2(z)$ and $W_3(z) \sim \exp(\frac{4}{3}kz^{\frac{3}{2}}) \rightarrow \infty$, for k real, and $|\arg x/4k| < \frac{1}{2}\pi$. Hence we must have $B_2 \equiv B_3 \equiv 0$, and

$$t'^{-\frac{1}{2}}W_{k,m}(x) = B_1W_1(z), \quad (4.6.104)$$

where

$$B_1 \sim \lim_{t \rightarrow \infty} \{t'^{-\frac{1}{2}}W_{k,m}(x)/W_1(z)\},$$

that is

$$B_1 \sim 2\sqrt{\pi}(2k)^{\frac{1}{6}}k^ke^{-k}. \quad (4.6.105)$$

Hence our result is $W_{k,m}(4kt) \sim z^{\frac{1}{4}}(1-1/t)^{-\frac{1}{4}}2\sqrt{\pi}(2k)^{\frac{1}{6}}k^ke^{-k}W_1(z)$, (4.6.106)

uniformly in z , for k real, and $|\arg z| < \frac{1}{2}\pi$. Again this result can be extended immediately to other ranges of $\arg z$ by use of the continuation formula (2.5.19).

It can also be extended to complex values of k , if we put $k = |k|e^{i\theta}$, $|\arg k| = |\theta| < \frac{1}{2}\pi$, and write $ze^{-\frac{2}{3}i\theta}$ for z , and $te^{-2i\theta}$ for t throughout the result. Thus finally, we have

$$W_{k,m}(x) \sim z^{\frac{1}{4}}e^{-\frac{1}{6}i\theta}(1-e^{2i\theta}/t)^{-\frac{1}{4}}2\sqrt{\pi}(2k)^{\frac{1}{6}}k^ke^{-k}W_1(ze^{-\frac{2}{3}i\theta}), \quad (4.6.107)$$

for $|\arg \frac{x}{4|k|}| < \frac{1}{2}\pi$ and $|\arg k| = |\theta| < \frac{1}{2}\pi$.

If we compare the leading terms of this result with (4.5.24) we see that the two expansions are in agreement.

Similarly, for the two functions $W_{k,m}(xe^{\pm\pi i})$, we have

$$W_{k,m}(xe^{\pi i}) \sim \frac{e^{\frac{1}{6}\pi i}z^{\frac{1}{4}}e^{-\frac{1}{6}i\theta}}{(1-e^{2i\theta}/t)^{\frac{1}{4}}}2\sqrt{\pi}(2k)^{\frac{1}{6}}e^{-k\pi i}k^{-k}e^k W_2(ze^{\frac{2}{3}\pi i-\frac{2}{3}i\theta}), \quad (4.6.108)$$

and

$$W_{k,m}(xe^{-\pi i}) \sim e^{-\frac{1}{6}\pi i}z^{\frac{1}{4}}e^{-\frac{1}{6}i\theta}2\sqrt{\pi}(2k)^{\frac{1}{6}}e^{k\pi i}k^{-k}e^k W_3(ze^{-\frac{2}{3}\pi i-\frac{2}{3}i\theta}), \quad (4.6.109)$$

for $|\arg \frac{x}{4|k|}| < \frac{1}{2}\pi$ and $|\arg k| = |\theta| < \frac{1}{2}\pi$. From these results, by the continuation formula (2.5.18), we find that

$$\begin{aligned} M_{k,m}(x) &\sim \frac{\Gamma(\frac{1}{2}-m+k)z^{\frac{1}{4}}e^{-\frac{1}{6}i\theta}}{\sqrt{\pi i}}(2k)^{\frac{1}{6}}k^{-k}e^k(1-e^{2i\theta}4k/t)^{-\frac{1}{4}}\Gamma(1+2m) \\ &\times [\exp\{\pi i(\frac{1}{2}+m+\frac{1}{6}-k)\}W_2\{ze^{-\frac{2}{3}(\theta-\pi)i}\} - \exp\{-\pi i(\frac{1}{2}+m+\frac{1}{6}-k)\}W_3\{ze^{-\frac{2}{3}(\theta+\pi)i}\}]. \end{aligned} \quad (4.6.110)$$

From these results, we deduce immediately that

$$\begin{aligned} {}_1F_1[a; b; x] &\sim \frac{e^{\frac{1}{2}x}x^{-\frac{1}{2}b}}{\sqrt{\pi i}}\Gamma(1-a)z^{\frac{1}{4}}e^{-\frac{1}{6}i\theta}(2k)^{\frac{1}{6}}k^{-k}e^k(1-e^{2i\theta}4k/t)^{-\frac{1}{4}} \\ &\times \Gamma(b)[\exp\{\pi i(a+\frac{1}{6})\}W_2\{ze^{-\frac{2}{3}(\theta-\pi)i}\} - \exp\{-\pi i(a+\frac{1}{6})\}W_3\{ze^{-\frac{2}{3}(\theta+\pi)i}\}], \end{aligned} \quad (4.6.111)$$

and

$$U(a; b; x) \sim e^{\frac{1}{2}x}x^{-\frac{1}{2}b}z^{\frac{1}{4}}e^{-\frac{1}{6}i\theta}2\sqrt{\pi}(2k)^{\frac{1}{6}}k^ke^{-k}(1-4ke^{2i\theta}/x)^{-\frac{1}{4}}W_1(ze^{-\frac{2}{3}i\theta}), \quad (4.6.112)$$

where

$$z = e^{\frac{2}{3}i\theta}(\frac{3}{2})^{\frac{2}{3}}[\text{Log}\{\sqrt{(te^{-2i\theta})}-\sqrt{(te^{-2i\theta})-1}}\} + \sqrt{\{(te^{-2i\theta})(te^{-2i\theta}-1)\}^{\frac{2}{3}}}], \quad (4.6.113)$$

for $|\arg z| < \frac{1}{2}\pi$ and $|\arg k| < \frac{1}{2}\pi$.

The coefficients $A_s(ze^{-\frac{2}{3}i\theta})$ and $B_s(ze^{-\frac{2}{3}i\theta})$, in the solutions

$$W_1(ze^{-\frac{2}{3}i\theta}), \quad W_2(ze^{-\frac{2}{3}i(\theta-\pi)}) \quad \text{and} \quad W_3(ze^{-\frac{2}{3}i(\theta+\pi)}),$$

are defined by

$$A_0(ze^{-\frac{2}{3}i\theta}) = 1,$$

$$B_s(ze^{-\frac{2}{3}i\theta}) = \frac{1}{2}z^{-\frac{1}{2}}e^{\frac{2}{3}i\theta} \int_0^{ze^{-\frac{2}{3}i\theta}} z^{-\frac{1}{2}}\{f(ze^{-\frac{2}{3}i\theta})A_s(ze^{-\frac{2}{3}i\theta}) - A_s''(ze^{-\frac{2}{3}i\theta})\} \left(\frac{dz}{dt}\right) dt, \quad (4.6.114)$$

and
$$A_{s+1}(z e^{-\frac{2}{3}i\theta}) = -\frac{1}{2}B'_s(z e^{-\frac{2}{3}i\theta}) + \frac{1}{2} \int f(z e^{-\frac{2}{3}i\theta}) B_s(z e^{-\frac{2}{3}i\theta}) \left(\frac{dz}{dt}\right) dt + K_s, \quad (4.6.115)$$

where
$$\frac{dz}{dt} = \left(1 - \frac{e^{2i\theta}}{t}\right)^{\frac{1}{2}} z^{-\frac{1}{2}} e^{\frac{1}{3}i\theta},$$

$$f(z e^{-\frac{2}{3}i\theta}) = \frac{z e^{\frac{4}{3}i\theta} b(b-2)}{4t(te^{-2i\theta}-1)} + \frac{5e^{\frac{4}{3}i\theta}}{16z^2} - \frac{3ze^{\frac{4}{3}i\theta}(8te^{-2i\theta}-9)}{16t(te^{-2i\theta}-1)^3}, \quad (4.6.116)$$

z is defined by (4.6.113), and for each value of s , K_s , the constant of integration is chosen so that $A_{s+1}(\infty) = 0$. These coefficients can be evaluated numerically in any given case for fixed values of b and θ .

CHAPTER 5

RELATED DIFFERENTIAL EQUATIONS AND
PARTICULAR CASES OF THE FUNCTIONS

5.1 General transforms of Kummer's equation

In Kummer's equation let us make the general transform

$$y = z^A e^{f(z)} Y(z), \quad x = Ch(z). \quad (5.1.1)$$

We find that $Y(z)$ now satisfies the equation

$$Y'' + \left(\frac{2A}{z} + 2f' + \frac{h'b}{h} - Ch' - \frac{h''}{h'} \right) Y' \\ + \left\{ \left(\frac{h'b}{h} - Ch' - \frac{h''}{h'} \right) \left(\frac{A}{z} + f' \right) + \frac{A(A-1)}{z^2} + \frac{2Af'}{z} + f'' + f'^2 - \frac{aCh'^2}{h} \right\} Y = 0, \quad (5.1.2)$$

where dashes denote differentiation with respect to z . We shall now consider various special cases of (5.1.2), for in general this equation will transform into an equation of Kummer's type.

First let us put

$$f(z) = Bz^m, \quad h(z) = z^m. \quad (5.1.3)$$

Then (5.1.2) becomes

$$Y'' + \left(\frac{2A}{z} + 2Bmz^{m-1} + \frac{mb}{z} - Cmz^{m-1} - \frac{m-1}{z} \right) Y' \\ + \left\{ \left(\frac{mb}{z} - Cmz^{m-1} - \frac{m-1}{z} \right) \left(\frac{A}{z} + Bmz^{m-1} \right) + \frac{A(A-1)}{z^2} + 2ABmz^{m-2} \right. \\ \left. + Bm(m-1)z^{m-2} + B^2m^2z^{2m-2} - aCm^2z^{m-2} \right\} Y = 0, \quad (5.1.4)$$

and its solutions have the general form

$$Y(z) = z^{-A} e^{-Bz^m} \mathfrak{F}[a; b; Cz^m]. \quad (5.1.5)$$

If $m = 1$, the equation has the general form

$$Y'' + \left(A_0 + \frac{B_0}{z} \right) Y' + \left(A_1 + \frac{B_1}{z} + \frac{C_1}{z^2} \right) Y = 0. \quad (5.1.6)$$

Every homogeneous linear differential equation of the second order, with a regular singularity at $z = 0$ and coefficients regular for all $z \neq 0$, can be put into this form (Tricomi, 1947), and this equation can always be integrated in terms of confluent hypergeometric functions, Bessel functions or elementary functions.

First let us suppose that

$$A = 0, \quad f(z) = Bz \quad \text{and} \quad Ch(z) = kz + n,$$

then our transform reduces to

$$y = e^{Bz} Y, \quad x = z + n/k, \quad (5.1.7)$$

and (5.1.2) becomes

$$Y'' + \left[2B + \frac{b}{z+n/k} - k \right] Y' + \left\{ \left(\frac{b}{z+n/k} - k \right) B + B^2 - \frac{ak}{z+n/k} \right\} Y = 0, \quad (5.1.8)$$

that is $(a_0 z + b_0) Y'' + (a_1 z + b_1) Y' + (a_2 z + b_2) Y = 0,$ (5.1.9)

where
$$\left. \begin{aligned} a_0 &= k, & b_0 &= n, \\ a_1 &= (2B - k)k, & b_1 &= bk + (2B - k)n, \\ a_2 &= Bk(B - k), & b_2 &= Bk(b - n) + B^2n - ak^2. \end{aligned} \right\} \quad (5.1.10)$$

and

This equation (5.1.9) is a homogeneous linear differential equation of the second order in which the coefficients are all linear functions of z . We now see that any equation of this type can always be integrated in terms of confluent hypergeometric functions or more elementary functions. We can exclude the case $a_0 = a_1 = a_2 = 0$, since then the equation has constant coefficients and can be integrated in terms of elementary functions. Equation (5.1.9) becomes another equation of the same type under the transform $y = e^{\lambda \xi} \eta$ and $z = \kappa \xi + \mu$ ($\kappa \neq 0$).

We find that $(\alpha_0 \xi + \beta_0) \frac{d^2 \eta}{d\xi^2} + (\alpha_1 \xi + \beta_1) \frac{d\eta}{d\xi} + (\alpha_2 \xi + \beta_2) \eta = 0,$ (5.1.11)

where
$$\left. \begin{aligned} \alpha_0 &= \frac{a_0}{\kappa}, & \alpha_1 &= 2a_0 \lambda + a_1, & \alpha_2 &= \kappa(a_0 \lambda^2 + a_1 \lambda + a_2), \\ \beta_0 &= \frac{a_0 \mu + b_0}{\kappa^2}, & \beta_1 &= \frac{\mu(2a_0 \lambda + a_1) + 2b_0 \lambda + b_1}{\kappa}, \\ \beta_2 &= \mu(a_0 \lambda^2 + a_1 \lambda + a_2) + b_0 \lambda^2 + b_1 \lambda + b_2. \end{aligned} \right\} \quad (5.1.12)$$

In those cases in which it is possible to choose κ , μ , and $\lambda \neq 0$ so that

$$\left. \begin{aligned} \beta_0 &= 0, & \text{that is} & & a_0 \mu + b_0 &= 0, \\ \alpha_1 &= -\alpha_0, & \text{that is} & & a_0 + \kappa(2a_0 \lambda + a_1) &= 0, \\ \alpha_2 &= 0, & \text{that is} & & a_0 \lambda^2 + a_1 \lambda + a_2 &= 0, \end{aligned} \right\} \quad (5.1.13)$$

and

(5.1.11) becomes an equation of Kummer's type, with solutions in terms of confluent hypergeometric functions. Tricomi (1954) has shown that there are four possible cases. First in general, we have in (5.1.9), $a_0 \neq 0$ and $a_1^2 - 4a_0 a_2 \neq 0$. Here we can take

$$\left. \begin{aligned} \lambda &= \frac{-a_1 \pm \sqrt{(a_1^2 - 4a_0 a_2)}}{2a_0}, \\ \kappa &= \mp \frac{a_0}{\sqrt{(a_1^2 - 4a_0 a_2)}}, \\ \mu &= -\frac{b_0}{a_0}, \end{aligned} \right\} \quad (5.1.14)$$

and we obtain solutions of the general form

$\eta = \mathfrak{F}[a; b; \xi],$ (5.1.15)

with parameters

$$\left. \begin{aligned} a &= \frac{b_0 \lambda^2 + b_1 \lambda + b_2}{2a_0 \lambda^2 + a_1}, \\ b &= \frac{a_0 b_1 - a_1 b_0}{a_0^2}. \end{aligned} \right\} \quad (5.1.16)$$

and

Secondly, if $a_0 = 0$, but $b_0 a_1 \neq 0$ then $\lambda = -a_2/a_1$, (5.1.17)

but we cannot determine κ and μ by (5.1.13). So we choose

$$\kappa = 1, \quad (5.1.18)$$

arbitrarily, then when $\beta_1 = 0$, we find by (5.1.12), that

$$\mu = \frac{2b_0 a_2 - b_1 a_1}{a_1^2}. \quad (5.1.19)$$

Equation (5.1.11) is now $b_0 \frac{d^2 \eta}{d\xi^2} + a_1 \xi \frac{d\eta}{d\xi} + (b_0^2 \lambda^2 + b_1 \lambda + b_2) \eta = 0$, (5.1.20)

which becomes $t \frac{d^2 \eta}{dt^2} + (\frac{1}{2} - t) \frac{d\eta}{dt} - \eta \frac{b_0 \lambda^2 + b_1 \lambda + b_2}{2a_1} = 0$, (5.1.21)

under the transform $\xi^2 = -\frac{2b_0 t}{a_1}$. (5.1.22)

Hence the solutions of (5.1.11) are of the form

$$\eta = \mathfrak{F}[a; \frac{1}{2}; A\xi^2], \quad (5.1.23)$$

where $a = (b_0 \lambda^2 + b_1 \lambda - b_2)/2a_1$, and $A = -a_1/2b_0$.

If, thirdly, $a_0 \neq 0$, but $a_1^2 = 4a_0 a_2$, then the equations (5.1.13) give

$$\mu = -\frac{b_0}{a_0}, \quad (5.1.24)$$

and $\lambda = -\frac{a_1}{2a_0}$. (5.1.25)

Again we are free to choose κ , so we take $\kappa = a_0$, (5.1.26)

to make $\alpha_0 = 1$. The equation (5.1.11) then assumes the form

$$\xi \frac{d^2 \eta}{d\xi^2} + \beta_1 \frac{d\eta}{d\xi} + \beta_2 \eta = 0, \quad (5.1.27)$$

where $\beta_1 = (2b_0 \lambda + b_1)/a_0$ and $\beta_2 = b_0 \lambda^2 + b_1 \lambda + b_2$. If in (5.1.27) we put

$$y = \xi^{-a} \eta, \quad \text{and} \quad \xi b^2 = x^2,$$

this equation reduces to one of Bessel's type,

$$\frac{d^2 y}{dx^2} + \frac{1}{x} \frac{dy}{dx} + \left(1 - \frac{n^2}{x^2}\right) y = 0. \quad (5.1.28)$$

We shall let the symbol

$$\mathfrak{C}_n(x),$$

indicate any general solution of Bessel's equation. So, in this case, solutions of (5.1.11) are of the form

$$\eta = \xi^a \mathfrak{C}_{2a}(b\sqrt{\xi}), \quad (5.1.29)$$

where $a = \frac{1}{2} - \frac{2b_0 \lambda^2 + b_1}{2a_0}$,

and $b = 2\sqrt{(b_0 \lambda^2 + b_1 \lambda + b_2)}$.

Fourthly, if $a_0 = 0$, but $a_2 \neq 0$ and $b_0 \neq 0$, we can choose

$$\kappa = 1, \quad (5.1.30)$$

then

$$b_1 = b_2 = 0,$$

and so, from (5.1.12),

$$\lambda = -b_1/2b_0, \quad (5.1.31)$$

and

$$\mu = 4b_0b_2 - b_1^2/4b_0a_2. \quad (5.1.32)$$

The equation (5.1.11) now assumes the form

$$\frac{d^2\eta}{d\xi^2} + \frac{a_2}{b_0}\xi\eta = 0. \quad (5.1.33)$$

If, in (5.1.33), we put $x^2 = b^2\xi^3$, and $y = \xi^{-\frac{1}{2}}\eta$, then this equation also reduces to one of the same form as Bessel's equation, and we see that in this case the solutions of (5.1.11) are of the form

$$\eta = \xi^{\frac{1}{2}}\mathfrak{C}_3(b\xi^{\frac{2}{3}}), \quad (5.1.34)$$

where $b = \frac{2}{3}\sqrt{(a_2/b_0)}$.

5.2 Kummer's second theorem and the connection with Bessel functions

Let us compare (5.1.6) with (5.1.4) when $m = 1$. We find the following relations between the coefficients,

$$\left. \begin{aligned} A_0 &= 2B - C, \\ B_0 &= 2A + b, \\ A_1 &= B(B - C), \\ B_1 &= Bb - AC + 2AB - Ca, \\ C_1 &= Ab + A(A - 1). \end{aligned} \right\} \quad (5.2.1)$$

and

But if in (5.1.6) we put

$$\left. \begin{aligned} C_1 &= -n^2, \\ A_0 &= B_1 = 0, \\ A_1 &= B_0 = 1, \end{aligned} \right\} \quad (5.2.2)$$

and

the equation becomes of the form of Bessel's equation (5.1.28) with solutions of the form $\mathfrak{C}_n(x)$. If we solve the equations (5.2.1), under the conditions (5.2.2), we find that if there is a relation between the coefficients of the confluent hypergeometric function,

$$b = 2a, \quad (5.2.3)$$

then the constants in the transform (5.1.5) can take the values

$$A = \frac{1}{2} - a, \quad B = i \quad \text{and} \quad C = 2i, \quad (5.2.4)$$

where $\frac{1}{2} - a = -n$.

Hence any solutions of Bessel's equation must also have the form

$$x^{-\frac{1}{2}+a} e^{-ix} \mathfrak{F}[a; 2a; 2ix],$$

that is to say there exists a relationship

$$\mathfrak{F}[\frac{1}{2} + n; 1 + 2n; 2ix] = x^{-n} e^{ix} \mathfrak{C}_n(x), \quad (5.2.5)$$

between the two types of solution and any Bessel function can be expressed as a special case of a confluent hypergeometric function. The result (5.2.5) is another form of Kummer's second theorem (1.8.3).

5.3 The Coulomb wave equation

We shall now consider briefly several functions which are special cases of the confluent hypergeometric functions. First, let us take $A_0 = B_0 = 0$,

in (5.1.6). Then, from (5.2.1), we have

$$2B = C \quad \text{and} \quad 2A = -b,$$

and (5.1.6) reduces to the form

$$y'' + \left\{ -B^2 + \frac{B(b-2a)}{z} + \frac{b(2-b)}{4z^2} \right\} y = 0. \quad (5.3.1)$$

This equation is of the same form as Whittaker's equation and, by (5.1.5), its solutions are of the general form

$$y = z^{\frac{1}{2}b} e^{-Bz} \mathfrak{F}[a; b; 2Bz] = \mathfrak{M}_{\frac{1}{2}b-a, \frac{1}{2}b-\frac{1}{2}}(2Bz). \quad (5.3.2)$$

In particular if $z = \rho > 0$, $B = i$, $b = 2L + 2$, and $a = L + 1 - i\eta$, (5.3.1) becomes Coulomb's wave equation

$$y'' + \left\{ 1 - \frac{2\eta}{\rho} - \frac{L(L+1)}{\rho^2} \right\} y = 0, \quad (5.3.3)$$

with solutions of the form

$$y = (2i\rho)^{L+1} e^{-i\rho} \mathfrak{F}[L+1-i\eta; 2L+2; 2i\rho], \quad (5.3.4)$$

$$= \mathfrak{M}_{i\eta, L+\frac{1}{2}}(2i\rho). \quad (5.3.5)$$

It is usually convenient to take as solutions the two Coulomb functions

$$F_L(\eta, \rho) = \frac{A}{\Gamma(2+2L)} M_{i\eta, L+\frac{1}{2}}(2i\rho), \quad (5.3.6)$$

and

$$G_L(\eta, \rho) = BW_{i\eta, L+\frac{1}{2}}(2i\rho) + \frac{C}{\Gamma(2+2L)} M_{i\eta, L+\frac{1}{2}}(2i\rho), \quad (5.3.7)$$

so that

$$F_L(\eta, \rho) \rightarrow \sin \phi \quad \text{and} \quad G_L(\eta, \rho) \rightarrow \cos \phi \quad \text{as} \quad \rho \rightarrow \infty,$$

where

$$\phi = \rho - \eta \log(2\rho) - \frac{1}{2}\pi L + \delta,$$

$$e^{\pm i\delta} |\Gamma(1+L \pm i\eta)| = \Gamma(1+L \pm i\eta),$$

and the constants A , B , C are defined by

$$A = -iC = \frac{1}{2} |\Gamma(L+1 \pm i\eta)| \exp \left\{ -\frac{1}{2}\pi\eta - \frac{1}{2}\pi i(L+1) \right\}$$

and

$$B = \exp \left(-i\delta + \frac{1}{2}\pi\eta + \frac{1}{2}\pi iL \right),$$

for $\eta > 0$, $L = 0, 1, 2, \dots$. For further details and some tabulation of these functions see Wheeler and Yost (1936), Lowen and Horenstein (1942), and 'Tables of Coulomb Wave Functions', National Bureau of Standards (1952).

5.4 Further forms of Whittaker's equation

The general form of (5.3.1) is

$$y'' + \left(-a + \frac{b}{z} - \frac{c}{z^2} \right) y = 0, \quad (5.4.1)$$

with solutions of the form

$$y = z^{\frac{1}{2}\{1+\sqrt{(4c+1)}\}} e^{-\sqrt{az}} \mathfrak{F}\left[\frac{1}{2} - \frac{b}{2\sqrt{a}} + \frac{1}{2}\sqrt{(4c+1)}; 1 + \sqrt{(4c+1)}; 2\sqrt{az}\right], \quad (5.4.2)$$

$$= \mathfrak{W}_{b/(2\sqrt{a}), \frac{1}{2}\sqrt{(4c+1)}}(2a^{\frac{1}{2}}z^{\frac{1}{2}}). \quad (5.4.3)$$

More generally, if in (5.2.1), $A_0 = 0$ and $B_0 = 1$, so that $C = 2B$ and $A = \frac{1}{2} - \frac{1}{2}b$ then the equation (5.1.6) assumes the form

$$zy'' + y' + \{-B^2z + B(b-2a) - \frac{1}{4}(b-1)^2/z\}y = 0, \quad (5.4.4)$$

with solutions of the form $y = z^{\frac{1}{2}b-\frac{1}{2}} e^{-Bz} \mathfrak{F}[a; b; 2Bz], \quad (5.4.5)$

$$= z^{-\frac{1}{2}} \mathfrak{W}_{\frac{1}{2}b-a, \frac{1}{2}b-\frac{1}{2}}(2Bz). \quad (5.4.6)$$

If in (5.2.1), $A_0 = 0$, $B_0 = b = 1 + \mu$ so that $C = 2B$ and $A = 0$, the equation (5.1.6) assumes the form

$$y'' + \frac{1+\mu}{z}y' + \left\{\frac{B(1+\mu-2a)}{z} - B^2\right\}y = 0, \quad (5.4.7)$$

with solutions of the form $y = e^{-Bz} \mathfrak{F}[a; 1+\mu; 2Bz], \quad (5.4.8)$

$$= z^{-\frac{1}{2}-\frac{1}{2}\mu} \mathfrak{W}_{\frac{1}{2}+\frac{1}{2}\mu-a, \frac{1}{2}\mu}(2Bz). \quad (5.4.9)$$

In particular, if $B = \pm i$, the equation (5.4.7) becomes Horn's equation

$$y'' + \frac{1+\mu}{z}y' + \left\{1 + \frac{i(1+\mu-2a)}{z}\right\}y = 0, \quad (5.4.10)$$

(J. Horn, series of papers in *Math Ann.* (1889)–(1940)), with solutions of the form

$$y = e^{-iz} \mathfrak{F}[a; 1+\mu; 2iz], \quad (5.4.11)$$

$$= z^{-\frac{1}{2}-\frac{1}{2}\mu} \mathfrak{W}_{\frac{1}{2}+\frac{1}{2}\mu-a, \frac{1}{2}\mu}(2iz). \quad (5.4.12)$$

If in (5.4.4), we take $B = i$, $b = 1$, we obtain Sharpe's equation

$$xy'' + y' + (x+C)y = 0, \quad (5.4.13)$$

(Sharpe, 1900). This equation occurs in the theory of paraboloidal reflectors and its solutions are

$$y = e^{\pm ix} {}_1F_1\left[\frac{1}{2} \mp \frac{1}{2}iC; 1; \mp 2ix\right]. \quad (5.4.14)$$

5.4.1 Watson's fourth-order equation

If u and v are solutions of the two equations

$$u'' + f(x)u = 0, \quad (5.4.15)$$

and

$$v'' + g(x)v = 0, \quad (5.4.16)$$

then the function $y = uv$ is a solution of the equation

$$\frac{d}{dx} \left\{ \frac{y''' + 2(f+g)y' + (f'+g')y}{f-g} \right\} + (f-g)y = 0. \quad (5.4.17)$$

This equation is discussed by Watson (1948, §5.4). We deduce that the fourth-order equation

$$y^{iv} + \frac{y'''}{x} + \left(1 + \frac{1-m^2}{x^2}\right)y'' + \left\{\frac{1}{x} - \frac{2(1-m^2)}{x^3}\right\}y' + 2\left(\frac{2t^2}{x^2} + \frac{1-m^2}{x^4}\right)y = 0, \quad (5.4.18)$$

has solutions of the form $y = \mathfrak{W}_{it, \frac{1}{2}m}(-ix) \mathfrak{W}_{-it, \frac{1}{2}m}(-ix). \quad (5.4.19)$

5.5 The Laguerre polynomials

If in (5.2.1), $A_0 = -1$, $B_0 = 1 + \mu$, $A_1 = C_1 = 0$, and $B_1 = \nu$ then $a = 1 + \mu + \nu$, $b = 1 + \mu$ and the equation (5.1.6) becomes

$$y'' + \left(\frac{1 + \mu}{z} - 1 \right) y' + \frac{\nu}{z} y = 0. \quad (5.5.1)$$

This is of the same form as Kummer's equation and it also arises in the theory of paraboloidal reflectors. It has as solutions, the Laguerre functions

$$y = L_\nu^{(\mu)}(z) = \frac{\Gamma(1 + \mu + \nu)}{\Gamma(1 + \mu) \Gamma(1 + \nu)} {}_1F_1[-\nu; 1 + \mu; z]. \quad (5.5.2)$$

Here ν is not necessarily an integer and the function is only an alternative notation for the Kummer function (see Pinny, 1946, and Mirimanov, 1948). From the definition of $L_\nu^{(\mu)}(z)$ it follows that

$$L_\nu^{(\mu)}(z) = \frac{\Gamma(1 + \mu + \nu)}{\Gamma(1 + \mu) \Gamma(1 + \nu)} z^{-\frac{1}{2} - \frac{1}{2}\mu} e^{\frac{1}{2}z} M_{\nu + \frac{1}{2} + \frac{1}{2}\mu, \frac{1}{2}\mu}(z). \quad (5.5.3)$$

A second solution is

$$U_\nu^{(\mu)}(z) = \frac{\epsilon i}{\sin \pi \mu} \left\{ e^{-\epsilon \pi i \mu} L_\nu^{(\mu)}(z) - \frac{\Gamma(1 + \mu + \nu)}{\Gamma(1 + \nu)} z^{-\mu} L_{\mu + \nu}^{(-\mu)}(z) \right\}, \quad (5.5.4)$$

where $\epsilon = +1$ for $0 \leq \arg z < \pi$ and $\epsilon = -1$ for $-\pi < \arg z < 0$. It follows that

$$U_\nu^{(\mu)}(z) = -e \frac{\Gamma(1 + \mu + \nu)}{\pi i} (z e^{\epsilon \pi i})^{-\frac{1}{2}\mu - \frac{1}{2}} e^{\frac{1}{2}z} W_{-\nu - \frac{1}{2} - \frac{1}{2}\mu, \frac{1}{2}\mu}(-z e^{-\epsilon i \pi}), \quad (5.5.5)$$

where $\epsilon = +1$ for $\text{Im } z \geq 0$ and $\epsilon = -1$ for $\text{Im } z < 0$.

In an alternative notation for these functions Mirimanov writes

$$S_\nu^\mu(z) = z^{\frac{1}{2}\mu} e^{-\frac{1}{2}z} L_\nu^{(\mu)}(z) \quad \text{and} \quad V_\nu^\mu(z) = z^{\frac{1}{2}\mu} e^{-\frac{1}{2}z} U_\nu^{(\mu)}(z). \quad (5.5.6)$$

The most useful cases arise when $\nu = n$, an integer. Then the function reduces to the Laguerre polynomial. We have

$$L_n^{(\mu)}(z) = \frac{(1 + \mu)_n}{n!} {}_1F_1[-n; 1 + \mu; z], \quad (5.5.7)$$

$$= \frac{(-1)^n}{n!} U(-n; 1 + \mu; z), \quad (5.5.8)$$

$$= \frac{(1 + \mu)_n}{n!} z^{-\frac{1}{2} - \frac{1}{2}\mu} e^{\frac{1}{2}z} M_{n + \frac{1}{2} + \frac{1}{2}\mu, \frac{1}{2}\mu}(z), \quad (5.5.9)$$

$$= \frac{(-1)^n}{n!} z^{-\frac{1}{2} - \frac{1}{2}\mu} e^{\frac{1}{2}z} W_{n + \frac{1}{2} + \frac{1}{2}\mu, \frac{1}{2}\mu}(z), \quad (5.5.10)$$

and in particular

$$L_n^{(\mu)}(0) = \frac{(1 + \mu)_n}{n!}, \quad (5.5.11)$$

$$L_{-n}^{(\mu)}(z) = 0, \quad \text{for } n = 1, 2, 3, \dots, \mu \neq 0, 1, 2, \dots, \text{Re } \mu > -1, \quad (5.5.12)$$

$$L_{-n}^{(m)}(z) = 0, \quad \text{for } n = 1, 2, 3, \dots, m \quad \text{and} \quad m = 1, 2, 3, \dots, \quad (5.5.13)$$

$$L_{-n}^{(m)}(z) = \frac{(-1)^m (n)_m}{m!} \quad \text{for } n = m + 1, m + 2, \dots, m = 0, 1, 2, \dots \quad (5.5.14)$$

By symmetry
$$n!(-z)^m L_n^{(m-n)}(z) = m!(-z)^n L_m^{(n-m)}(z), \quad (5.5.15)$$

for $n, m = 0, 1, 2, \dots$, and in particular

$$L_0^{(\mu)}(z) = 1, \quad L_n^{(-n)}(z) = \frac{(-z)^n}{n!}.$$

Two short tables are Slater (1955c) and Tricomi (1956).

Another alternative notation for a Laguerre polynomial is given by the Poisson–Charlier function which arises in the Calculus of Probability (Charlier, 1905) and Szego (1939). This is

$$\rho_n(\nu, a) = \sqrt{n!} a^{-\frac{1}{2}n} L_n^{(\nu-n)}(a) = \frac{(1+\nu-n)_n}{\sqrt{n!}} a^{-\frac{1}{2}n} {}_1F_1[-n; 1+\nu-n; a]. \quad (5.5.16)$$

Two related functions are the omega function of Cunningham

$$\omega_{n,m} = x^{-\frac{1}{2}-\frac{1}{2}m} \frac{e^{-\frac{1}{2}x - \pi i(n-\frac{1}{2}m)}}{\Gamma(1+n-\frac{1}{2}m)} W_{n+\frac{1}{2}, \frac{1}{2}m}(x), \quad (5.5.17)$$

(Cunningham, 1908) which is used as a normalizing function in statistics, and Bateman's function

$$k_\nu(x) = \frac{e^{-x}}{\Gamma(\frac{1}{2}\nu+1)} U(-\frac{1}{2}\nu; 0; 2x) \quad \text{for } x > 0, \quad (5.5.18)$$

(Bateman, 1931).

5.6 The incomplete gamma functions

If in Kummer's equation, we put $b = 1 + a$, and write $-x$ for x , we obtain

$$x \frac{d^2y}{dx^2} + (1+a+x) \frac{dy}{dx} + ay = 0. \quad (5.6.1)$$

The solutions of this differential equation are the incomplete gamma functions,

$$\gamma(a, x) = \frac{x^a}{a} {}_1F_1[a; 1+a; -x] \quad (5.6.2)$$

and
$$\Gamma(a, x) = e^{-x} U(1-a; 1-a; x). \quad (5.6.3)$$

From these definitions, we find that

$$\gamma(a, x) = e^{-x} x^a \sum_{n=0}^{\infty} \frac{x^n}{(a)_{n+1}}, \quad (5.6.4)$$

by Kummer's first theorem, and in particular,

$$\Gamma(a, 0) = \Gamma(a),$$

and
$$\gamma(1, x) = 1 - e^{-x}. \quad (5.6.5)$$

Also
$$\gamma(a, x) = -\Gamma(a, x) + \Gamma(a, 0). \quad (5.6.6)$$

In terms of Whittaker functions,

$$\gamma(a, x) = \frac{x^{\frac{1}{2}a-\frac{1}{2}} e^{\frac{1}{2}x}}{a} M_{\frac{1}{2}-\frac{1}{2}a, \frac{1}{2}a}(x) \quad (5.6.7)$$

and
$$\Gamma(a, x) = x^{\frac{1}{2}a-\frac{1}{2}} e^{-\frac{1}{2}x} W_{\frac{1}{2}a-\frac{1}{2}, \frac{1}{2}a}(x). \quad (5.6.8)$$

The functions have simple integral representations,

$$\gamma(a, x) = \int_0^x e^{-t} t^{a-1} dt, \quad \text{for } \operatorname{Re} a > 0, \quad (5.6.9)$$

and

$$\Gamma(a, x) = \int_x^\infty e^{-t} t^{a-1} dt. \quad (5.6.10)$$

For a detailed study of their properties see Tricomi (1954, ch. IV).

Alternative notations for the incomplete gamma functions are

(Neilson, 1906),

$$P(x, a) = \gamma(a, x) \quad \text{and} \quad Q(x, a) = \Gamma(a, x)$$

$$S(a, x) = x^{a-1} e^x Q(x, 1-a) = \int_0^\infty \frac{e^{-xt}}{(1+t)^a} dt, \quad (5.6.11)$$

(Schlömlich, 1859), and

$$E_n(x) = e^{-x} S(n, x) = \int_1^\infty e^{-xu} u^{-n} du, \quad (5.6.12)$$

(Placzek, 1946).

Another group of related functions is provided by the exponential integral,

$$-Ei(-x) = \int_x^\infty e^{-t} \frac{dt}{t} = e^{-x} U(1; 1; x) = e^{-\frac{1}{2}x} x^{-\frac{1}{2}} W_{-\frac{1}{2}, 0}(x), \quad (5.6.13)$$

the logarithmic integral,

$$li(x) = \int_0^x \frac{dt}{\log t} = Ei(\log x) = -xU(1; 1; -\log x)$$

$$= x^{\frac{1}{2}}(\log x)^{-\frac{1}{2}} W_{-\frac{1}{2}, 0}(\log x), \quad (5.6.14)$$

the integral sine,

$$Si(x) = \int_0^x \frac{\sin t}{t} dt = \frac{1}{2}\pi - \frac{1}{2}i e^{-ix} U(1; 1; ix) + \frac{1}{2}i e^{ix} U(1; 1; -ix)$$

$$= \frac{1}{2}\pi + \frac{1}{2i\sqrt{z}} \{e^{-\frac{1}{4}i\pi - \frac{1}{2}iz} W_{-\frac{1}{2}, 0}(iz) - e^{\frac{1}{4}i\pi + \frac{1}{2}iz} W_{-\frac{1}{2}, 0}(-iz)\}, \quad (5.6.15)$$

and the integral cosine,

$$Ci(x) = -\int_x^\infty \frac{\cos t}{t} dt = -\frac{1}{2}e^{-ix} U(1; 1; ix) - \frac{1}{2}e^{ix} U(1; 1; -ix)$$

$$= \frac{1}{2\sqrt{z}} \{e^{-\frac{1}{4}i\pi - \frac{1}{2}iz} W_{-\frac{1}{2}, 0}(iz) - e^{\frac{1}{4}i\pi + \frac{1}{2}iz} W_{-\frac{1}{2}, 0}(-iz)\}, \quad (5.6.16)$$

and the Fresnel integrals,

$$S(x) = \frac{1}{\sqrt{(2\pi)}} \int_0^x \frac{\sin t}{\sqrt{t}} dt = \sqrt{\frac{x}{2\pi}} i \{ {}_1F_1[\frac{1}{2}; \frac{3}{2}; -x e^{\frac{1}{2}\epsilon i\pi}] - {}_1F_1[\frac{1}{2}; \frac{3}{2}; -x e^{-\frac{1}{2}\epsilon i\pi}] \}, \quad (5.6.17)$$

and

$$C(x) = \frac{1}{\sqrt{(2\pi)}} \int_0^x \frac{\cos t}{\sqrt{t}} dt = \sqrt{\frac{x}{2\pi}} \{ {}_1F_1[\frac{1}{2}; \frac{3}{2}; -x e^{\frac{1}{2}\epsilon i\pi}] + {}_1F_1[\frac{1}{2}; \frac{3}{2}; -x e^{-\frac{1}{2}\epsilon i\pi}] \}, \quad (5.6.18)$$

where $\epsilon = +1$ for $\text{Im } x \geq 0$ and $\epsilon = -1$ for $\text{Im } x < 0$. Alternative notations for the integral sine and cosine are $si(x) = Si(x) - \frac{1}{2}\pi$ and $ci(x) = Ci(x)$.

Some other special functions which arise in a similar way are:

Chappell's function (Chappell, 1925),

$$C(z, k) = \frac{1}{2}\Gamma(\frac{1}{2} - k) W_{k, 0}(z/k). \quad (5.6.19)$$

Meixner's functions (Meixner, 1933),

$$F_1(a; b; x) = \frac{2 \exp(\pi ia + \frac{1}{2}x)}{\Gamma(b-a)} x^{-\frac{1}{2}b} W_{\frac{1}{2}b-a, \frac{1}{2}b-\frac{1}{2}}(x) \Gamma(b), \quad (5.6.20)$$

and

$$F_2(a; b; x) = \frac{2 \exp\{-\pi i(\frac{1}{2}b - a) + \frac{1}{2}x\}}{\Gamma(a)} x^{-\frac{1}{2}b} W_{a-\frac{1}{2}b, \frac{1}{2}b-\frac{1}{2}}(e^{-\pi i} x) \Gamma(b), \quad (5.6.21)$$

whence

$${}_1F_1[a; b; x] = \frac{1}{2}F_1(a; b; x) + \frac{1}{2}F_2(a; b; x), \quad (5.6.22)$$

and Krupp's functions (Krupp, 1950),

$${}_1R(\nu, \lambda; x) = \frac{\nu^{\lambda+1}}{2x\Gamma(\mathbf{I} + 2\lambda)} M_{\nu, \lambda+\frac{1}{2}}(2x/\nu) \quad (5.6.23)$$

and

$${}_2R(\nu, \lambda; x) = \frac{\nu^{\lambda+1}\Gamma(-\mathbf{I} - \nu)}{2x\pi} W_{\nu, \lambda+\frac{1}{2}}(2x/\nu). \quad (5.6.24)$$

Finally when $a = b$, Kummer's equation reduces to its simplest form

$$x \frac{d^2y}{dx^2} + (a-x) \frac{dy}{dx} - ay = 0, \quad (5.6.25)$$

and this elementary equation has the solutions e^x and $x^{1-a} {}_1F_1[\mathbf{I}; 2-a; x]$.

In particular

$$e^{\pm x} = {}_1F_1[a; a; \pm x] = W_{0, \frac{1}{2}}(\mp 2x), \quad (5.6.26)$$

whence it follows that

$$M_{0, \frac{1}{2}}(-ix) = -2i \sin(\frac{1}{2}x), \quad (5.6.27)$$

and

$$M_{0, \frac{1}{2}}(x) = 2 \sinh(\frac{1}{2}x). \quad (5.6.28)$$

5.7 Transformations of Kummer's equation when $m = 2$

When $m = 2$ the solutions of (5.1.4) become functions of z^2 and this equation assumes the form

$$y'' + (A_0 z + B_0/z) y' + (A_1 z^2 + B_1 + C_1/z^2) y = 0, \quad (5.7.1)$$

where

$$\left. \begin{aligned} A_0 &= 4B - 2C, \\ B_0 &= 2A + 2b - \mathbf{I}, \\ A_1 &= 4B^2 - 4BC, \\ B_1 &= -2CA + 2B(2b - \mathbf{I}) + 4AB + 2B - 4aC, \\ C_1 &= A(A + 2b - 2). \end{aligned} \right\} \quad (5.7.2)$$

and

If $C_1 = 0 = A_0 = B_0$, this becomes of the same form as Weber's equation

$$\frac{d^2y}{dx^2} + (n + \frac{1}{2} - \frac{1}{4}z^2) y = 0. \quad (5.7.3)$$

We can also deduce Weber's equation as a special case of Whittaker's equation under the transformation

$$y = X^{\frac{1}{2}} Y, \quad z = \frac{1}{2} X^2, \quad (5.7.4)$$

with $k = \frac{1}{4} + \frac{1}{2}n$ and $m = -\frac{1}{4}$ (Whittaker and Watson, 1946, ch. xvi). Any solution of (5.7.3) is called a Weber function or a parabolic cylinder function. Let the symbol

$$\mathfrak{D}_n(x)$$

be any general solution of Weber's equation. Then we have

$$\mathfrak{W}_{\frac{1}{2}n+\frac{1}{4}, \pm\frac{1}{4}}(\frac{1}{2}x^2) = 2^{-\frac{1}{2}n-\frac{1}{4}} x^{\frac{1}{2}} \mathfrak{D}_n(x), \quad (5.7.5)$$

and we see that all Weber functions are special cases of Whittaker functions. In particular we have the solutions

$$D_\nu(x) = 2^{\frac{1}{2}\nu} (\frac{1}{2}x^2)^{-\frac{1}{4}} W_{\frac{1}{2}\nu+\frac{1}{4}, \pm\frac{1}{4}}(\frac{1}{2}x^2) = 2^{\frac{1}{2}\nu} e^{-\frac{1}{4}x^2} U(-\frac{1}{2}\nu; \frac{1}{2}; \frac{1}{2}x^2), \quad (5.7.6)$$

$$E_\nu^{(0)}(x) = \frac{\sqrt{(2\pi)}}{\Gamma(\frac{1}{2})} (\frac{1}{2}x^2)^{-\frac{1}{4}} M_{\frac{1}{2}\nu+\frac{1}{4}, -\frac{1}{4}}(\frac{1}{2}x^2), \quad (5.7.7)$$

$$= \sqrt{2} e^{-\frac{1}{4}x^2} {}_1F_1[-\frac{1}{2}\nu; \frac{1}{2}; \frac{1}{2}x^2] \quad (5.7.8)$$

and

$$E_\nu^{(1)}(x) = \frac{\sqrt{(2\pi)}}{\Gamma(\frac{3}{2})} (\frac{1}{2}x^2)^{-\frac{1}{4}} M_{\frac{1}{2}\nu+\frac{1}{4}, \frac{1}{4}}(\frac{1}{2}x^2), \quad (5.7.9)$$

$$= 2x e^{-\frac{1}{4}x^2} {}_1F_1[\frac{1}{2} - \frac{1}{2}\nu; \frac{3}{2}; \frac{1}{2}x^2]. \quad (5.7.10)$$

For full formulae and tables of these functions see Miller (1955). When $\nu = n$, an integer, the Weber functions reduce to the Hermite polynomials

$$\text{He}_{2n}(x) = (-\frac{1}{2})^n \frac{2n!}{n!} {}_1F_1[-n; \frac{1}{2}; \frac{1}{2}x^2], \quad (5.7.11)$$

and

$$\text{He}_{2n+1}(x) = (-\frac{1}{2})^n \frac{(2n+1)!}{n!} {}_1F_1[-n; \frac{3}{2}; \frac{1}{2}x^2]. \quad (5.7.12)$$

The polynomial $\text{He}_n(x)$ is a solution of the equation

$$y'' - xy' + ny = 0, \quad (5.7.13)$$

so that

$$2^{\frac{1}{2}n} \text{He}_n(x) \equiv H_n(x/\sqrt{2}) = 2^{\frac{1}{2}n} e^{\frac{1}{4}x^2} D_n(x) = 2^n U(-\frac{1}{2}n; \frac{1}{2}; \frac{1}{2}x^2). \quad (5.7.14)$$

An alternative notation for the Weber functions is given by Magnus (1941)

$$\delta(\xi, \nu) = e^{\frac{1}{2}i\xi^2} D_\nu\{\sqrt{(2i)\xi}\} \quad (5.7.15)$$

and

$$\rho(\xi, \nu) = \Gamma(-\frac{1}{2}\nu) \exp(\frac{1}{4}\pi i\nu + \frac{1}{2}i\xi^2) 2^{-\nu-\frac{3}{2}} E_\nu^{(0)}\{\sqrt{(2i)\xi}\}. \quad (5.7.16)$$

A generalization of the Weber function is Heatley's Toronto function

$$T(m, n, r) = r^{2n+m+1} e^{-r^2} \frac{\Gamma(\frac{1}{2}m + \frac{1}{2})}{\Gamma(1+n)} {}_1F_1[\frac{1}{2}m + \frac{1}{2}; 1+n; r^2], \quad (5.7.17)$$

$$= \frac{\Gamma(\frac{1}{2} + \frac{1}{2}m)}{\Gamma(1+n)} r^{n+m} e^{-\frac{1}{2}r^2} M_{\frac{1}{2}m+\frac{1}{2}n, \frac{1}{2}n}(r^2), \quad (5.7.18)$$

(Heatley, 1943).

Another group of functions of x^2 , which are related to the incomplete gamma functions, are the error functions

$$\text{Erf}(x) = \frac{1}{2}\gamma(\frac{1}{2}, x^2) = x {}_1F_1[\frac{1}{2}; \frac{3}{2}; -x^2] \quad (5.7.19)$$

$$= x^{-\frac{1}{2}} e^{-\frac{1}{2}x^2} M_{-\frac{1}{4}, \frac{1}{4}}(x^2), \quad (5.7.20)$$

and

$$\text{Erfc}(x) = \frac{1}{2}\Gamma(\frac{1}{2}, x^2) = \frac{1}{2}e^{-x^2} U(\frac{1}{2}; \frac{1}{2}; x^2), \quad (5.7.21)$$

$$= \frac{1}{2\sqrt{x}} e^{-\frac{1}{2}x^2} W_{-\frac{1}{4}, \pm\frac{1}{4}}(x^2). \quad (5.7.22)$$

These are of particular importance in the field of statistics. Alternative notations are

$$\text{Erfi}(x) = x {}_1F_1(\frac{1}{2}; \frac{3}{2}; x^2), \quad (5.7.23)$$

$$\phi(x) = 1 - \text{Erfc}(x), \quad (5.7.24)$$

and
$$L(x) = \frac{\sqrt{\pi}}{2} \operatorname{Erfc}(x). \quad (5.7.25)$$

A full discussion of the properties of these functions is given in Tricomi (1954, ch. IV).

5.7.1 The Poiseuille functions

These functions arise in the theory of cylindrical flow. They are defined as

$$\operatorname{pe}(r, w) = \frac{1}{r\sqrt{(2zw)}} M_{\frac{1}{2}w, 0}(2zwr^2), \quad (5.7.26)$$

and
$$\operatorname{qe}(r, w) = \frac{-\Gamma(\frac{1}{2} - \frac{1}{2}zw)}{r\sqrt{(2zw)}} W_{\frac{1}{2}w, 0}(2zwr^2) - \frac{\psi(\frac{1}{2} - \frac{1}{2}zw)}{r\sqrt{(2zw)}} M_{\frac{1}{2}w, 0}(2zwr^2), \quad (5.7.27)$$

where $\psi(x) = \Gamma'(x)/\Gamma(x)$. These functions satisfy the equation

$$\frac{d^2y}{dr^2} + \frac{1}{r} \frac{dy}{dr} + 4w^2(1-r^2)y = 0, \quad (5.7.28)$$

where $0 \leq r \leq 1$, and w is either real or purely imaginary. If $w = 2n + 1$, where n is a positive integer, the pe -function reduces to a Laguerre polynomial. Integral representations and asymptotic expansions of the functions have been developed by Lauwerier (1951).

The general equation (5.7.1) reduces to the above form when $A_0 = C_1 = 0$.

5.8 The Schrödinger equation

In (5.7.1), if $A_0 = B_0 = C_1 = 0$, this equation reduces to the well-known Schrödinger equation for the harmonic oscillator,

$$y'' + (a^2 - b^2z^2)y = 0, \quad (5.8.1)$$

with solutions of the form

$$y = \frac{z^{-\frac{1}{2}} e^{-\frac{1}{2}bz^2}}{\Gamma(1 \pm \frac{1}{2})} (bz^2)^{\frac{1}{2} \pm \frac{1}{4}} {}_1F_1\left[\frac{1}{2} - \frac{1}{4}a^2/b \pm \frac{1}{4}; 1 \pm \frac{1}{2}; bz^2\right], \quad (5.8.2)$$

or

$$y = \frac{z^{-\frac{1}{2}}}{\Gamma(1 \pm \frac{1}{2})} M_{a^2/4b, \pm \frac{1}{4}}(bz^2). \quad (5.8.3)$$

Two other special forms of the general equation (5.7.1) which have received attention are

$$y'' + a_0xy' + (a_1x^2 + b_1)y = 0, \quad (5.8.4)$$

with solutions of the form

$$y = \frac{\exp(-\frac{1}{4}a_0x^2)}{\Gamma(1 + \frac{1}{2})\sqrt{x}} M_{k, \pm \frac{1}{4}}(x^2\sqrt{[a_0^2/4 - a_1]}), \quad (5.8.5)$$

where

$$k = \frac{\frac{1}{2}(b_1 - \frac{1}{2}a_0)}{\sqrt{(a_0^2 - 4a_1)}},$$

and

$$y'' + \left(-A^2x^2 + 4kA + \frac{1}{4} - \frac{m^2}{x^2}\right)y = 0, \quad (5.8.6)$$

with solutions of the form

$$y = x^{-\frac{1}{2}} \mathfrak{S}_{k, \frac{1}{2}m}(Ax^2). \quad (5.8.7)$$

5.9 Kamke's equation

If we return to Whittaker's equation (1.6.2) and make the exponential transformation

$$y = z^A e^{\alpha z} Y(z), \quad x = C e^{\gamma z}, \quad (5.9.1)$$

so that

$$f(z) = \alpha z \quad \text{and} \quad h(z) = e^{\gamma z},$$

the general equation assumes the form

$$Y'' - (2\alpha + \gamma + 2\beta/z) Y' + \left\{ \alpha(\alpha + \gamma) + \frac{1}{4}(1 - m^2)\gamma^2 + \beta(2\alpha + \gamma)/z + \beta(1 + \beta)/z^2 - \frac{1}{4}A^2\gamma^2 e^{2\gamma z} + Ak\gamma^2 e^{\gamma z} \right\} Y = 0, \quad (5.9.2)$$

with solutions of the form

$$y = e^{\alpha z} z^\beta \mathfrak{W}_{k, \frac{1}{2}m}(A e^{\gamma z}). \quad (5.9.3)$$

In particular, we have Kamke's equation,

$$y'' - (a^2 e^{2x} + b e^x + c^2)y = 0, \quad (5.9.4)$$

with solutions of the form

$$y = e^{-\frac{1}{2}x} \mathfrak{W}_{-\frac{1}{2}b/a, c}(2a e^x) \quad (5.9.5)$$

(Kamke, 1943). Many other similar functional transformations of (1.1.6) and of (1.6.2) are possible, but these have not yet received much attention in the literature.

DESCRIPTIVE PROPERTIES

6.1 The distribution of the zeros

Since e^x and x^a have no zeros except at $x = -\infty$, and $x = \infty$, it follows from their definitions that $M_{k,m}(x)$ has the same zeros as ${}_1F_1[a; b; x]$ and $W_{k,m}(x)$ has the same zeros as $U(a; b; x)$. If a and b are real, then, since

$${}_1F_1[a; b; x] = \frac{\Gamma(b)}{\Gamma(a)} e^x x^{a-b} \{1 + O(|x|^{-1})\} > 0 \text{ as } \operatorname{Rl} x \rightarrow +\infty \tag{6.1.1}$$

and
$${}_1F_1[a; b; x] = \frac{\Gamma(b)}{\Gamma(b-a)} (-x)^{-a} \{1 + O(|x|^{-1})\} > 0 \text{ for } \operatorname{Rl} x \rightarrow -\infty, \tag{6.1.2}$$

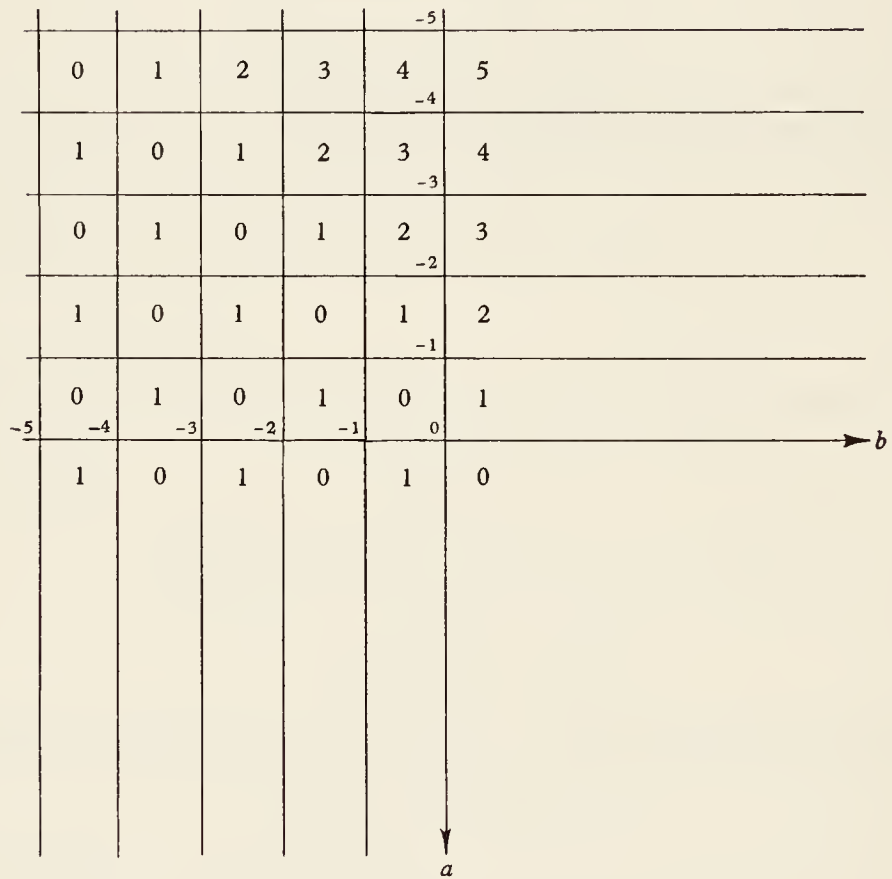


Fig. 6.1

${}_1F_1[a; b; x]$ has only a finite number of real zeros for any fixed values of a and b . Also $U(a; b; x)$ has only a finite number of positive zeros, for there are only a finite number of zeros in any finite interval and there can be no zero for x large, since

$$U(a; b; x) = x^{-a} + O(|x|^{-a-1}) > 0, \text{ for } \operatorname{Rl} x \rightarrow +\infty. \tag{6.1.3}$$

It has been proved that every confluent hypergeometric function has at most only a finite number of zeros for any fixed value of a and b .

If $a > 0$ and $b > 0$, then ${}_1F_1[a; b; x] \geq 1$, for $x \geq 0$, and this is still true if $a = -m + \theta$ and $b = -m + \theta'$, for $0 < \theta < 1$, $0 < \theta' < 1$. Thus, Kummer's function can have no zeros in these regions. Picone's theorem states that no integral of the differential equation

$$\frac{d}{dx} \left\{ p(x) \frac{dy}{dx} \right\} + P(x)y = 0, \tag{6.1.4}$$

can have more than one zero in regions where $p(x) \geq 0$ and $P(x) \leq 0$. If we apply this theorem to Kummer's equation, we see that ${}_1F_1[a; b; x]$ cannot have more than one zero in regions where $a > 0$. Also this result holds in regions where $b \leq a < 0$, since Kummer's equation can be written in the form

$$\frac{d}{dx} \left(x^b e^x \frac{dy}{dx} \right) + (b - a) x^{b-1} e^x y = 0. \tag{6.1.5}$$

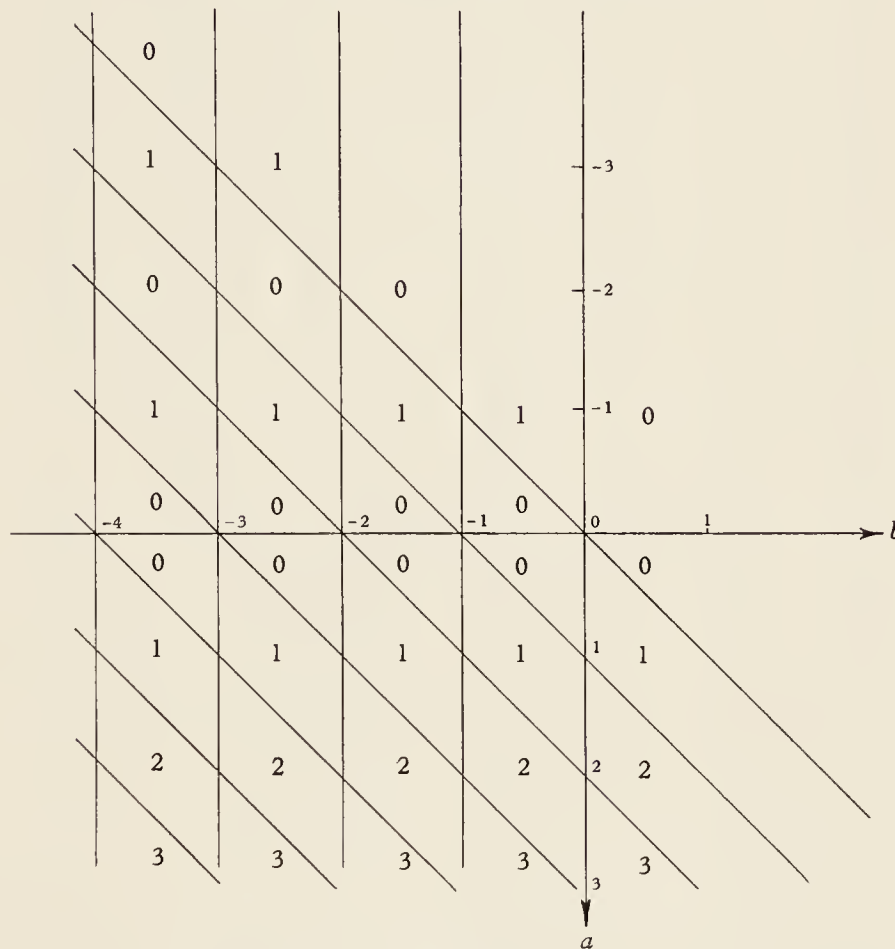


Fig. 6.2

Thus, if $a > 0$, we have no zeros when $[b]$ is an odd integer, and one zero when $[b]$ is an even integer. Also, when $a < 0$, and $b < a$, we have no zero if the integral parts of $-a$ and $-b$ are both even or both odd, and one zero otherwise.

If $a < 0$ and $b > 0$, let $a = -n + \theta$, $n = 1, 2, 3, \dots$, $0 < \theta < 1$, then, for $x > 0$, the functions ${}_1F_1[a+n; b; x]$ form a Sturmian chain, and ${}_1F_1[a+n; b; x] > 0$, for $a+n > 0$, $b > 0$. Hence ${}_1F_1[a; b; x]$ has n zeros for $b > 0$. Similarly, if $a = -n + \theta$, and $b = -m + \theta'$, for $0 < \theta < 1$, $0 < \theta' < 1$, $m, n = 1, 2, 3, \dots$, ${}_1F_1[a+n; b+m; x] > 0$, for $a+n > 0$, $b+m > 0$. Hence ${}_1F_1[a; b; x]$ has $n-m$ zeros for $b < a < 0$. These results, due to Kienast (1921), are illustrated in Fig. 6.1. In this diagram the number in each section represents the number of zeros of ${}_1F_1[a; b; x]$ in that section, the horizontal

boundaries belong to the sections above them, and, along the vertical lines $b = 0, -1, -2, \dots$, the function is not defined.

The functions ${}_1F_1[a+n; b; -x]$, $n = 0, 1, 2, \dots, N$, also, using Kummer's theorem, form a Sturmian chain and so the negative zeros of ${}_1F_1[a; b; x]$ are distributed in the way illustrated in Fig. 6.2. Here the oblique boundaries belong to the sections on their left.

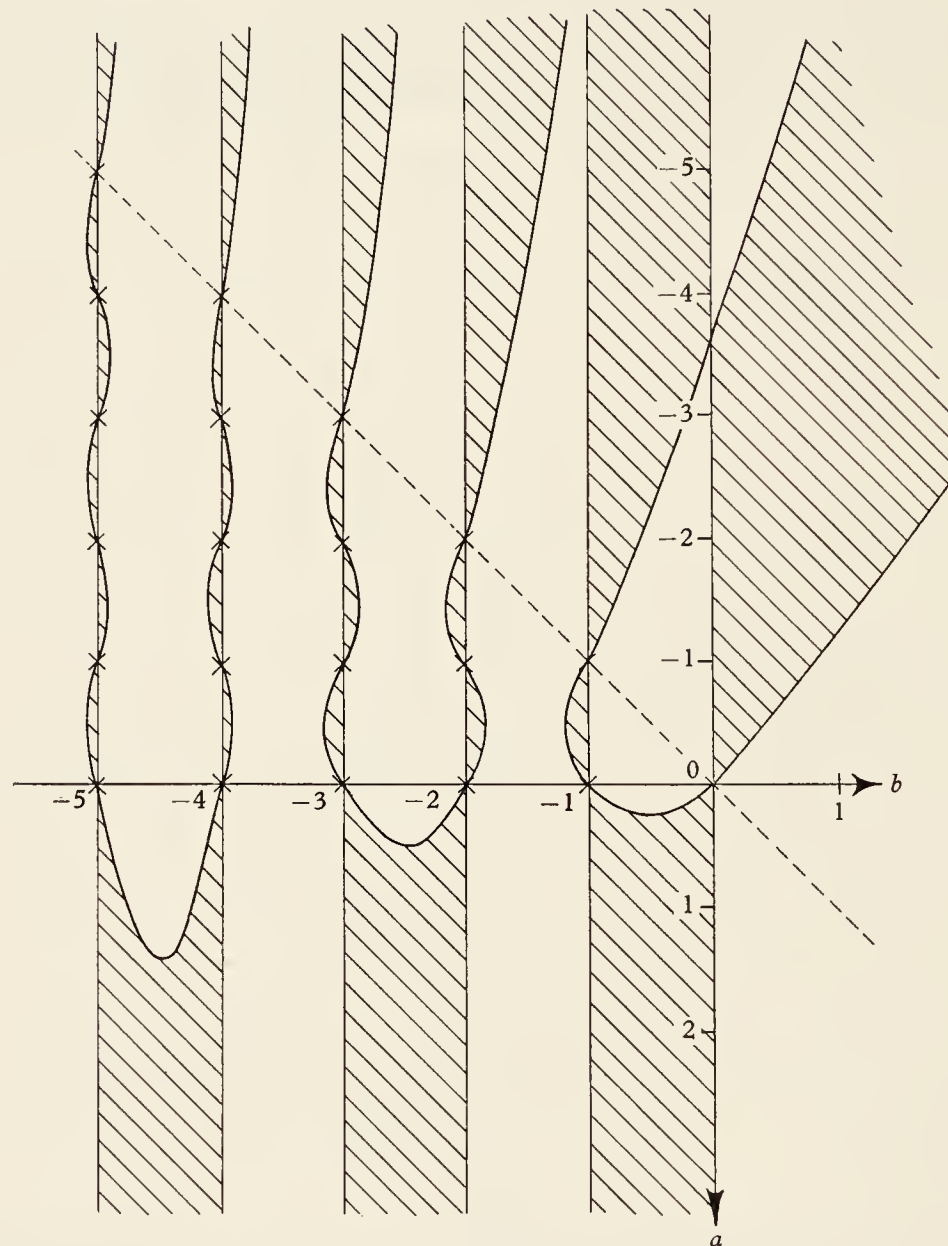


Fig. 6.3

6.1.1 The curves of zeros

The two sketches 6.3, 6.4 illustrate the curves

$${}_1F_1[a; b; 1] = 0 \quad \text{and} \quad {}_1F_1[a; b; 2] = 0.$$

The shaded areas are the regions in which the functions are negative in value. Fig. 6.3 shows the curve in which the surface ${}_1F_1[a; b; x] = 0$ cuts the plane $x = 1$. The function ${}_1F_1[a; b; x]$ is again not defined on the vertical planes $b = 0, -1, -2, \dots$, and it can assume any value on the vertical

lines through the points $(0, 0)$, $(0, 1)$, $(-1, -1)$, $(0, -2)$, $(-1, -2)$, $(-2, -2)$, ..., marked with crosses. On the vertical plane $a = 0$, ${}_1F_1[a; b; x] = 1$, for all values of b and x , and on the vertical plane $a = b$, ${}_1F_1[a; b; x] = e^x$, for all values of a and x . Fig. 6.4 shows the similar curves in which the surface ${}_1F_1[a; b; x] = 0$ cuts the plane $x = 2$. It can be seen, on comparing the two sketches, that the function has a saddle point between $x = 1$, and $x = 2$. By direct calculation on the electronic computer EDSAC I in the Mathematical Laboratory of Cambridge University this saddle point was found to be at

$$a = -0.34235, \quad b = -1.5708, \quad x = 1.4564.$$

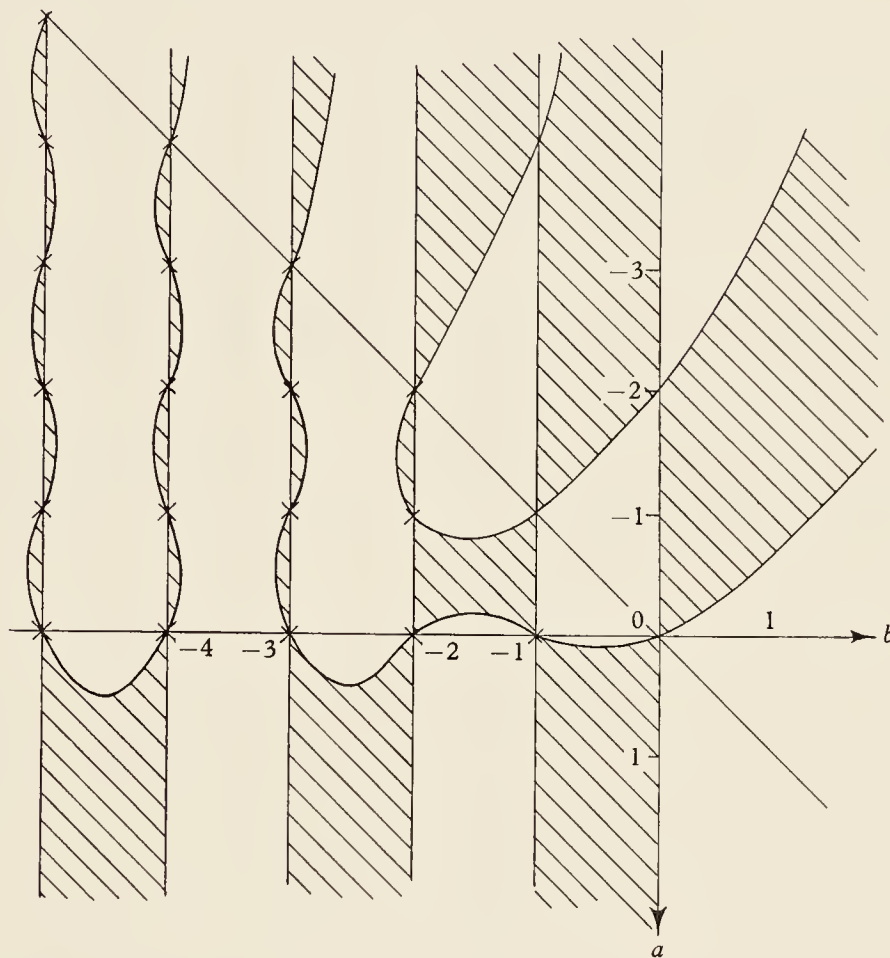


Fig. 6.4

At this point ${}_1F_1[a; b; x]$, has a maximum in b , a minimum in a and a zero in x . Similar saddle points will occur in each strip $-N < b < -N + 1$.

A table of further saddle points has been calculated on EDSAC II by M. Fieldhouse (1959).

6.1.2 The zeros of $U(a; b; x)$

Milne (1915), Tsvetkoff (1941 a, b) and Tricomi (1950) have discussed the real and complex zeros of $U(a; b; x)$. For $x \geq 0$, $U(a; b; x)$ cannot have positive zeros if $a > 0$, or if $1 + a - b > 0$. If, however, $-n < a < 1 - n$, $n = 1, 2, \dots$, the functions $U(a; b; x)$ form a Sturmian chain, since

$$U(a; b; x) > 0 \quad \text{for } x \rightarrow +\infty \text{ and, as } x \rightarrow 0,$$

$$U(a; b; x) \sim x^{1-b} \frac{\Gamma(b-1)}{\Gamma(a)} \quad \text{if } \operatorname{Rl} b > 1, \quad (6.1.6)$$

$$\sim \frac{\Gamma(1-b)}{\Gamma(1+a-b)} \quad \text{if } \operatorname{Rl} b < 1, \quad (6.1.7)$$

$$\sim \frac{\Gamma(1-b)}{\Gamma(1+a-b)} + x^{1-b} \frac{\Gamma(b-1)}{\Gamma(a)} \quad \text{if } \operatorname{Rl} b = 1 \quad (b \neq 1), \quad (6.1.8)$$

$$\sim -\{\log x + \psi(a) - 2\gamma\} / \Gamma(a) \quad \text{if } b = 1. \quad (6.1.9)$$

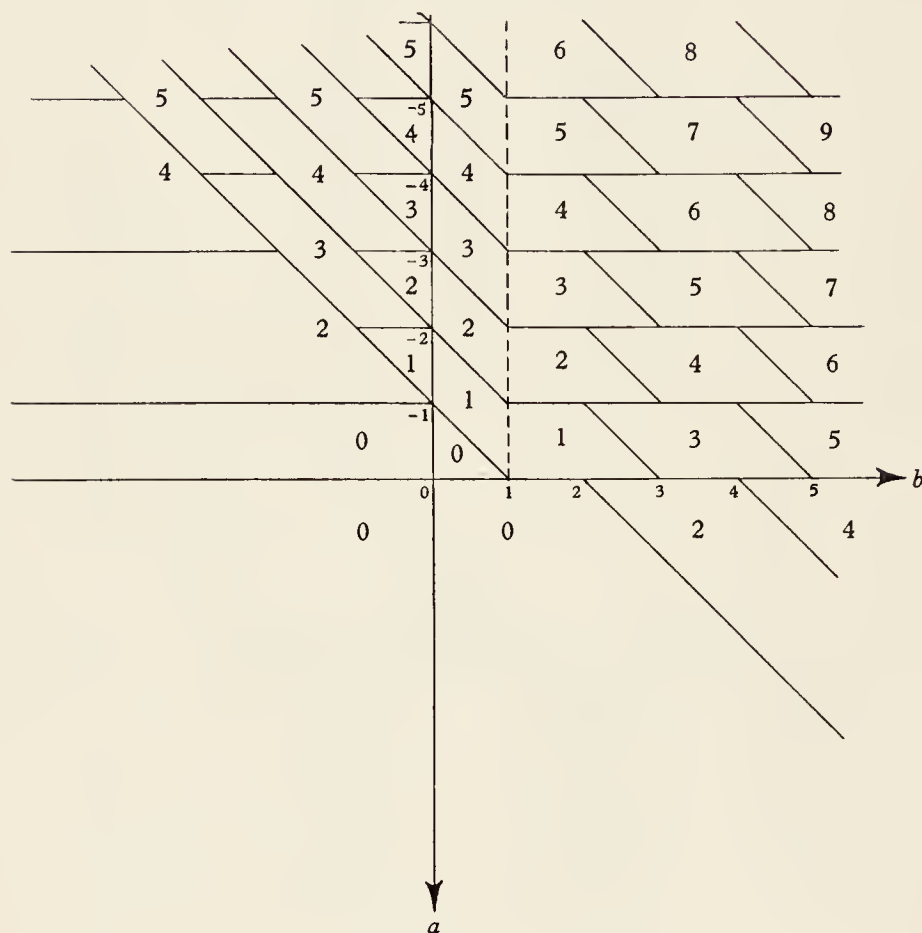


Fig. 6.5

These approximations determine the sign of $U(a; b; x)$ for x small. When x is real and negative $U(a; b; x)$ is complex, and we deduce the distribution of zeros illustrated in Fig. 6.5 for all real values of x . Here the number in each strip near the real axis represents the number of real zeros for x positive throughout that strip. The other zeros arise in conjugate complex pairs, from the negative values of x .

6.1.3 Approximations to the zeros

From the expansion of Kummer's function in terms of Bessel functions, (3.8.3), Tricomi (1947) has deduced that if X_r is the r th positive zero of ${}_1F_1[a; b; x]$ and $j_{b-1,r}$ is the r th positive zero of the Bessel function $J_{b-1}(x)$ then, for $k \equiv \frac{1}{2}b - a$ large,

$$X_r = \frac{j_{b-1,r}^2}{4k} \left\{ 1 + \frac{2b(b-2) + j_{b-1,r}^2}{48k^2} \right\} + O(k^{-4}). \quad (6.1.10)$$

If we use the known approximation for $j_{b-1,r}$ (Watson, 1948, § 15.53), we can deduce a very useful approximation for X_r from this result. We find that $j_{b-1,r} \doteq \pi(r + \frac{1}{2}b - \frac{3}{4})$, and so

$$X_r \doteq \frac{\pi^2(r + \frac{1}{2}b - \frac{3}{4})^2}{2b - 4a}. \tag{6.1.11}$$

This approximation is sufficiently accurate to provide a starting-point for the approximation processes now to be described, even in the case of the smallest positive zero X_0 .

Similarly, from (4.4.16), if η_r is the r th positive zero of $U(a; b; x)$ and λ_r is the r th positive zero of

$$\cos(-a\pi)J_{b-1}(x) + \sin(-a\pi)Y_{b-1}(x), \tag{6.1.12}$$

then
$$\eta_r = \frac{\lambda_r^2}{4k} + O(k^{-3}), \tag{6.1.13}$$

where
$$\lambda_r \doteq \pi(r + \frac{1}{2}b - \frac{3}{4} - a). \tag{6.1.14}$$

6.2 Expansions for the zeros

Let $y(x)$ be any solution of Kummer's equation and let k be a first approximation to a zero

$$x = c \tag{6.2.1}$$

of $y(x)$, Then a second approximation is $x = k + h$,
$$\tag{6.2.2}$$

where $h = -y(k)/y'(k) = -u$ say. By differentiating (6.2.2) with respect to k we find that

$$0 = 1 + \frac{\partial h}{\partial k} + \frac{\partial h}{\partial u} \frac{du}{dk}.$$

But $y/y' = u$, and so
$$\frac{du}{dk} = 1 - u^2 y''/y.$$

Also, from Kummer's equation, we have

$$y''/y = a/k + (k - b)/u.$$

Hence
$$\frac{du}{dk} = 1 - u^2 a/k - u(k - b),$$

and so
$$0 = 1 + \frac{\partial h}{\partial k} + \frac{\partial h}{\partial u} \{1 - u^2 a/h + u(b - k)\}. \tag{6.2.3}$$

Suppose now that
$$h = -u + a_2 u^2 + \dots + a_n u^n + \dots,$$

where $a_n \equiv a_n(k)$. Then
$$\frac{\partial h}{\partial k} = \frac{\partial a_2}{\partial k} u^2 + \dots + \frac{\partial a_n}{\partial k} u^n + \dots,$$

and
$$\frac{\partial h}{\partial u} = -1 + 2a_2 u + \dots + (n + 1) a_{n+1} u^n + \dots$$

If we substitute these values in (6.2.3) and equate the coefficients of u^n to zero, we find that

$$2a_2 + k - b = 0,$$

$$\frac{\partial a_2}{\partial k} + 3a_3 + \frac{a}{k} + 2a_2(b - k) = 0,$$

and in general
$$\frac{\partial a_n}{\partial k} + (n+1)a_{n+1} - (n-1)a_{n-1} \frac{a}{k} - na_n(k-b) = 0. \quad (6.2.4)$$

When we solve these difference equations, we have, after some reduction

where
$$c = k - u + a_2 u^2 + \dots + a_n u^n + \dots, \quad (6.2.5)$$

$$a_2 = \frac{1}{2}(b-k),$$

$$a_3 = \frac{1}{3} \left\{ \frac{1}{2} \frac{a}{k} - (b-k)^2 \right\},$$

$$a_4 = -\frac{1}{4} \left[\frac{a}{3k^2} + (b-k) \left\{ -\frac{2a}{k} + \frac{7}{6} - (b-k)^2 \right\} \right],$$

$$a_5 = -\frac{1}{5} \left\{ (k-b)^4 - (k-b)^2 \left(\frac{23}{12} - \frac{3a}{k} \right) + \frac{5}{6} (k-b) \frac{a}{k^2} + \frac{7}{24} + \frac{a^2}{k^2} + \frac{a}{6k^3} - \frac{a}{k} \right\},$$

and in general
$$a_{n+1} = \frac{1}{n+1} \left\{ n(k-b)a_n - \frac{\partial a_n}{\partial k} + \frac{a}{k}(n-1)a_{n-1} \right\}.$$

To find an expansion for the value of the derivative $y'(c)$, at any zero $x = c$, let us write

$$y'(c) = y'(k)(1 + \lambda).$$

Then λ satisfies the partial differential equation

$$0 = \left(\frac{a}{k} + \frac{k-b}{u} \right) (1 + \lambda) + \frac{1}{u} \frac{\partial \lambda}{\partial k} + \left(\frac{1}{u} - \frac{ua}{k} + b-k \right) \frac{\partial \lambda}{\partial u}. \quad (6.2.6)$$

Let

$$\lambda = a_1 u + a_2 u^2 + \dots + a_n u^n + \dots$$

Then

$$\frac{\partial \lambda}{\partial k} = \frac{\partial a_1}{\partial k} u + \frac{\partial a_2}{\partial k} u^2 + \dots + \frac{\partial a_n}{\partial k} u^n + \dots,$$

and

$$\frac{\partial \lambda}{\partial u} = a_1 + 2a_2 u + 3a_3 u^2 + \dots + (n+1)a_{n+1} u^n + \dots$$

When these values are substituted in (6.2.6) we find that

$$ka_1 + k(k-b) = 0,$$

$$a + k(k-b)a_1 + k \frac{\partial a_1}{\partial k} + 2ka_2 - k(k-b)a_1 = 0,$$

and in general
$$a_{n+1} = \frac{1}{n+1} \left\{ (k-b)(n-1)a_n + \frac{a}{k}(n-2)a_{n-1} - \frac{\partial a_n}{\partial k} \right\}. \quad (6.2.7)$$

Hence

$$y'(c) = y'(k)(1 + a_1 u + a_2 u^2 + \dots + a_n u^n + \dots), \quad (6.2.8)$$

where

$$a_1 = b-k,$$

$$a_2 = \frac{1}{2} \left(1 - \frac{a}{k} \right),$$

$$a_3 = \frac{1}{3} \left\{ \frac{1}{2} \left(1 - \frac{a}{k} \right) (k-b) - \frac{a}{2k^2} \right\},$$

$$a_4 = \frac{1}{4} \left\{ \frac{1}{3}(k-b)^2 \left(1 - \frac{a}{k} \right) - \frac{1}{2}(k-b) \frac{a}{k^2} - \frac{1}{2} \left(1 - \frac{a}{k} \right) \left(\frac{1}{3} - \frac{a}{k} \right) - \frac{a}{3k^3} \right\},$$

$$a_5 = \frac{1}{5} \left[\frac{1}{4}(k-b)^3 \left(1 - \frac{a}{k} \right) - \frac{1}{24}(k-b)^2 \frac{a}{k^2} + (k-b) \left\{ \left(1 - \frac{a}{k} \right) \right. \right. \\ \left. \left. \times \left(\frac{17a}{24k} - \frac{7}{24} \right) - \frac{a}{2k^2} \right\} - \frac{7a}{12k^2} \left(\frac{a}{k} - \frac{1}{2} + \frac{3}{7k^2} \right) \right].$$

This expansion is useful for general solutions of (6.2.1), but it is not necessary to use it for ${}_1F_1[a; b; c]$ when c is known, since

$${}_1F_1[a; b; c] = \frac{a}{b} {}_1F_1[a+1; b+1; c]. \quad (6.2.9)$$

Next let us suppose that c' is a turning value of $y(x)$, that is to say, c' is a zero of $y'(x)$, with a first approximation k' . Also let

$$v = \frac{y'(k')}{k'^2 y(k')}.$$

Then, proceeding in a similar way, we have

$$c' = k' + a_1 v + a_2 v^2 + \dots + a_n v^n + \dots, \quad (6.2.10)$$

where

$$a_1 = -k'^3/a,$$

$$a_2 = k'^5 \{ k'(k'-b) + 1 \} / 2a^2,$$

$$a_3 = -k'^7 \{ k'a + \frac{1}{2}(7k'^2 - 6k'b + 5) + (k'^2 - k'b - 2)(k'^2 - k'b + 1) \} / 3a^3,$$

$$a_4 = k'^9 \{ (3k'^2 - 3k'b - 6 + 7k')(ak' + \frac{5}{2}k'^2 - 2bk' + \frac{1}{2} + k'^4 - 2bk'^3 + b^2k'^2) \\ + ak' + 5k'^2 - 2bk' + 4k'^4 - 6k'^3b + 2k'^2b^2 + 3ak'(k'^2 - bk' + 1) \} / 12a^4,$$

$$\text{and in general} \quad a_{n+1} = k'^3 \left\{ k'^2(n-1)a_{n-1} + \left(b - k' + \frac{2}{k'} \right) na_n - \frac{\partial a_n}{\partial k'} \right\} / \{ (n+1)a \}. \quad (6.2.11)$$

Also, for the actual value of the solution at its turning point c' , we find that

$$y(c') = y(k') (1 + a_1 v + a_2 v^2 + \dots + a_n v^n + \dots), \quad (6.2.12)$$

where

$$a_1 = 0,$$

$$a_2 = k'^5 / 2a,$$

$$a_3 = k'^7 \{ k'(k'-b) + \frac{1}{2} \} / 3a^2,$$

$$a_4 = k'^9 \{ -9k'^2 + 8bk' - \frac{7}{2} - \frac{3}{2}ak' - (3k'^2 - 3bk' - 6)(k'^2 - bk' + \frac{1}{2}) \} / 12a^3,$$

$$\text{and in general} \quad a_{n+1} = k'^3 \left\{ k'^2(n-2)a_{n-1} - \frac{\partial a_n}{\partial k'} - n \left(k' - b - \frac{2}{k'} \right) a_n \right\} / \{ (n+1)a \}. \quad (6.2.13)$$

Again, for the solution ${}_1F_1[a; b; x]$, if a table of the zero c is available over a sufficiently wide range of a and b , the values of c' can be deduced directly from this table. A reasonable first approximation for the k of the solution ${}_1F_1[a; b; x]$ is given by (6.1.11).

The accuracy of the above approximations in series is limited and varies according to the values of a and b , and the accuracy of the initial approximation k . Two calculations should be made starting from different values of k , so that the number of significant figures can be determined.

6.3 Nesting processes

The expansion (6.2.5) is most useful in the evaluation of the zeros in x , of ${}_1F_1[a; b; x]$ with the help of an ordinary calculating machine, but, if high accuracy is required and an electronic calculator is available, it is simpler to draw up a programme based on the nesting processes which will now be described. For the solution ${}_1F_1[a; b; x]$, let us calculate an initial estimate of the zero k by (6.1.11), and using Newton's approximation, let

$$x_1 = k - y(k)/y'(k).$$

Now by (6.2.9), $y'(k)$ is known and we have

$$x_1 = k - \frac{b {}_1F_1[a; b; k]}{a {}_1F_1[a+1; b+1; k]}.$$

Similarly,

$$x_2 = x_1 - \frac{b {}_1F_1[a; b; x_1]}{a {}_1F_1[a+1; b+1; x_1]},$$

and so the cycle proceeds until

$$x_{n+1} = x_n - \frac{b {}_1F_1[a; b; x_n]}{a {}_1F_1[a+1; b+1; x_n]}, \quad (6.3.1)$$

and the latest correction made to x_n is negligible. This method gives about seven-figure accuracy after only four or five cycles with $N = 1$, $a = -4.0(0.1) - 0.1$, and $b = 0.1(0.1) 2.5$.

For $y(c)$, c' and $y(c')$, similar cyclic processes for the solution ${}_1F_1[a; b; x]$, which are suitable for use on electronic calculators, are given by

$$y'(x_{n+1}) = \frac{a}{b} {}_1F_1[a+1; b+1; x_n] + (b-x_n) {}_1F_1[a; b; x_n], \quad (6.3.2)$$

$$x'_{n+1} = x'_n - \frac{x'_n {}_1F_1[a+1; b+1; x'_n]}{b {}_1F_1[a; b; x'_n]} \quad (6.3.3)$$

and

$$y(x'_{n+1}) = {}_1F_1[a; b; x'_n] - \frac{x'_n a {}_1F_1[a+1; b+1; x'_n]}{2b^2 {}_1F_1[a; b; x'_n]}, \quad (6.3.4)$$

where

$$x'_0 = k' = \frac{\pi^2(N + \frac{1}{2}b - \frac{1}{4})^2}{2(b-2a-1)}. \quad (6.3.5)$$

6.4 Zeros in 'a'

We now seek an answer to the question 'For what values of a does ${}_1F_1[a; b; x] = 0$, when b and x are fixed?' Let us express ${}_1F_1[a+\epsilon; b; x]$ as a series in ϵ . We find by direct expansion that

$${}_1F_1[a+\epsilon; b; x] = \sum_{t=0}^{\infty} \epsilon^t \sum_{r=t}^{\infty} \frac{(a)_r x^r}{(b)_r r!} \Sigma_{r,t}, \quad (6.4.1)$$

where

$$\Sigma_{r,t} \equiv \sum_{p_0=0}^{r-t} \sum_{p_1=1+p_0}^{r-t+1} \dots \sum_{p_{t-1}=1+p_{t-2}}^{r-1} \frac{1}{(a+p_1)(a+p_2)\dots(a+p_t)}, \quad (6.4.2)$$

that is

$${}_1F_1[a+\epsilon; b; x] = {}_1F_1[a; b; x] + \epsilon \sum_{r=1}^{\infty} \frac{(a)_r x^r}{(b)_r r!} \sum_{n=1}^r \frac{1}{a+n-1} + O(\epsilon^2). \quad (6.4.3)$$

Hence, neglecting ϵ^2 , we have
$$\epsilon = \frac{{}_1F_1[a+\epsilon; b; x] - {}_1F_1[a; b; x]}{A(a)}, \quad (6.4.4)$$

where

$$A(a) \equiv \Sigma_{r,1} = \sum_{r=1}^{\infty} \frac{(a)_r x^r}{(b)_r r!} \sum_{n=1}^r \frac{1}{a+n-1},$$

and so

$$\frac{\partial}{\partial a}({}_1F_1[a; b; x]) = A(a).$$

Thus, given a first approximation a_0 to a zero a of ${}_1F_1[a; b; x]$, we have

$$a_1 = a_0 - \frac{{}_1F_1[a_0; b; x]}{\frac{\partial}{\partial a}({}_1F_1[a_0; b; x])}, \quad (6.4.5)$$

as our second approximation, and in general, the cyclic process is given by

$$a_n = a_{n-1} - \frac{{}_1F_1[a_{n-1}; b; x]}{A(a_{n-1})}. \quad (6.4.6)$$

For example, if $b = 2$, and $x = 1$, a first approximation to the real zero nearest to $a = 0$, is

$$a_0 = -2.8.$$

Also, by direct calculation, ${}_1F_1[a_0; b; x] = 0.00829$, and $A(a_0) = 0.1771$.

Hence $a_1 = -2.7532$. In fact $a = -2.75358$, so that only one cycle here has given two further significant figures. The value of the derivative at the zero a , is of course

$$\frac{\partial}{\partial a}({}_1F_1[a; b; x]) \quad (6.4.7)$$

that is, $y'(a) = A(a)$.

For the value of the zero a' of the derivative, it is necessary to calculate

$$\frac{\partial^2}{\partial a^2}({}_1F_1[a; b; x]).$$

We have, by direct expansion, $A(a+\epsilon) = A(a) + \epsilon A_1(a) + O(\epsilon^2)$, (6.4.8)

where $A_1(a) = \sum_{r=2}^{\infty} \frac{(a)_r x^r}{(b)_r r!} \left\{ \left(\frac{1}{a} + \dots + \frac{1}{a+r-1} \right)^2 - \frac{1}{a^2} - \dots - \frac{1}{(a+r-1)^2} \right\}$. (6.4.9)

Hence, neglecting ϵ^2 , we have $\epsilon = \frac{A(a+\epsilon) - A(a)}{A_1(a)}$, (6.4.10)

and so $\frac{\partial^2}{\partial a^2}({}_1F_1[a; b; x]) = \frac{\partial}{\partial a}\{A(a)\} = A_1(a)$. (6.4.11)

Thus, given a first approximation a'_0 to a zero of the derivative $A(a)$, we have

$$a'_1 = a'_0 - \frac{A(a'_0)}{A_1(a'_0)},$$

and, in general, the cyclic process is given by

$$a'_{n+1} = a'_n - \frac{A(a'_n)}{A_1(a'_n)}. \quad (6.4.12)$$

The value of the function ${}_1F_1[a; b; x]$ at a zero a' of its derivative with respect to a can then be calculated by direct evaluation of

$$y(a') = {}_1F_1[a'; b; x].$$

6.5 The zeros in 'b'

Next we require to find for what values of b , ${}_1F_1[a; b; x] = 0$, when a and x are fixed, so let us expand ${}_1F_1[a; b + \eta; x]$ in powers of η . Then

$${}_1F_1[a; b + \eta; x] = {}_1F_1[a; b; x] + \eta B(b) + O(\eta^2), \quad (6.5.1)$$

where

$$B(b) = - \sum_{r=1}^{\infty} \frac{(a)_r x^r}{(b)_r r!} \sum_{n=1}^r \frac{1}{b+n-1}, \quad (6.5.2)$$

so that

$$\frac{\partial}{\partial b} ({}_1F_1[a; b; x]) = B(b), \quad (6.5.3)$$

and the required cyclic process is given by

$$b_n = b_{n-1} - \frac{{}_1F_1[a; b_{n-1}; x]}{B(b_{n-1})}. \quad (6.5.4)$$

The value of the derivative $\frac{\partial}{\partial b} ({}_1F_1[a; b; x])$ at the zero is of course $y'(b) = B(b)$. (6.5.5)

For the zero b' of this derivative, it is necessary to calculate

$$\frac{\partial^2}{\partial b^2} ({}_1F_1[a; b; x]).$$

We have by direct expansion in series,

$$B(b + \eta) = B(b) + \eta B_1(b) + O(\eta^2), \quad (6.5.6)$$

where

$$B_1(b) = \sum_{r=1}^{\infty} \frac{(a)_r x^r}{(b)_r r!} \left\{ \left(\frac{1}{b} + \dots + \frac{1}{b+r-1} \right)^2 + \frac{1}{b^2} + \dots + \frac{1}{(b+r-1)^2} \right\}. \quad (6.5.7)$$

Then

$$\frac{\partial^2}{\partial b^2} ({}_1F_1[a; b; x]) = \frac{\partial}{\partial b} \{B(b)\} = B_1(b), \quad (6.5.8)$$

and so the cyclic process is given by $b'_n = b'_{n-1} - \frac{B(b'_{n-1})}{B_1(b'_{n-1})}$. (6.5.9)

The value of $y(b') = {}_1F_1[a; b'; x]$ can then be evaluated directly from its expansion in series.

6.5.1 The tabulation of zeros in x

Appendix I contains a table of the values of the smallest positive zero x_0 , for which ${}_1F_1[a; b; x_0] = 0$, over the range

$$a = -4.0(0.1) - 0.1, \quad b = 0.1(0.1) 2.5.$$

This table was calculated on EDSAC I to eight decimal places. Hence the error is not more than two units in the seventh decimal.

It is well known that any analytic function can be expressed as an infinite product in terms of its zeros (Watson, 1948, §7.5). Since the confluent hypergeometric function ${}_1F_1[a; b; x]$ is an analytic function of x except when b is zero or a negative integer, we have

$${}_1F_1[a; b; x] = e^{ax/b} \prod_{n=0}^{\infty} \left(1 - \frac{x}{x_n} \right) e^{x/x_n}, \quad (6.5.10)$$

where x_0, x_1, \dots, x_n are the real and imaginary zeros of the function. This fact leads us to suppose that

$$\left(1 - \frac{x}{x_0}\right) \exp\left(\frac{ax}{b} + \frac{x}{x_0}\right), \tag{6.5.11}$$

where x_0 is the real zero nearest to the plane $x = 0$, will provide a useful first approximation to the numerical value of ${}_1F_1[a; b; x]$ for real or complex values of x near x_0 .

No serious attempt has been made to study the next surface on which ${}_1F_1[a; b; x] = 0$. However, a single run on EDSAC I lasting about one hour, gave the following values of the next zero x_1 , and these should provide a useful starting-point for further study:

a	b = 0.1	b = 0.2	b = 0.3	b = 0.4	b = 0.5
-2.0	2.14880 88	2.29544 51	2.44017 54	2.58321 60	2.72474 49
-1.9	2.28349 16	2.43664 54	2.58761 05	2.73663 47	2.88392 26
-1.8	2.43848 89	2.59883 17	2.75665 80	2.91225 38	3.06585 48
-1.7	2.61942 09	2.78775 08	2.95318 16	3.11604 53	3.27661 55

6.6 The numerical evaluation of Kummer's function

We shall now consider the further practical problem of evaluating numerically Kummer's function, for real values of a, b and x . We have already seen, that for a single fixed value (a, b, x) , the function can be evaluated by direct summation of the series ${}_1F_1[a; b; x]$. This calculation is laborious by hand-machines but perfectly practicable with an electronic calculator, and the convergence is sufficiently smooth to give five or more accurate figures on a ten-figure machine over the range

$$a = -10.0(0.1)10.0, \quad b = -10.0(0.1)10.0, \quad x = -10.0(0.1)10.0.$$

However, it is only necessary to tabulate the function for positive values of x , since

$${}_1F_1[a; b; -x] = e^{-x} {}_1F_1[b-a; b; x].$$

Also it is only necessary to evaluate the function, by direct summation of series, over one unit in b , and two units in a , as the resulting table can be extended easily to other values of a and b by the use of recurrence relations, given in chapter 2.

For $|x|, |b|, |a| > 10$, the calculation should be attempted by the asymptotic formulae given in chapter 4, and these results can usually be improved by the use of converging factors, where these can be found. See also National Bureau of Standards (1959).

Appendix II contains a table of ${}_1F_1[a; b; x]$ over the range $a = -1.0(0.1)1.0, b = 0.0(0.1)1.0, x = 0.1(0.1)10.0$, to seven figures, calculated on EDSAC I, in the Cambridge University Mathematical Laboratory.

Several short tables of this function have already been published. Among these may be mentioned Airey and Webb (1918), Airey (*British Association Reports*, 1926 and 1927), P. Nath (1951), L. J. Slater (1953 and 1956), B. W. Connolly (1950), R. Gran Olssen (1937) and Rushton (1954). Further references will be found in Fletcher, Miller and Rosenhead (1946), and in Buchholz (1953).

It is possible to form quite a good idea of the relative numerical size of Kummer's function at any point, by considering the values assumed by the function at neighbouring points at which its value is already well known. Thus when $a = b, {}_1F_1[a; a; x] = e^x$, and so for $a = b + \epsilon$, we may expect the value to be near e^x . There are several networks over which the function reduces to a more elementary

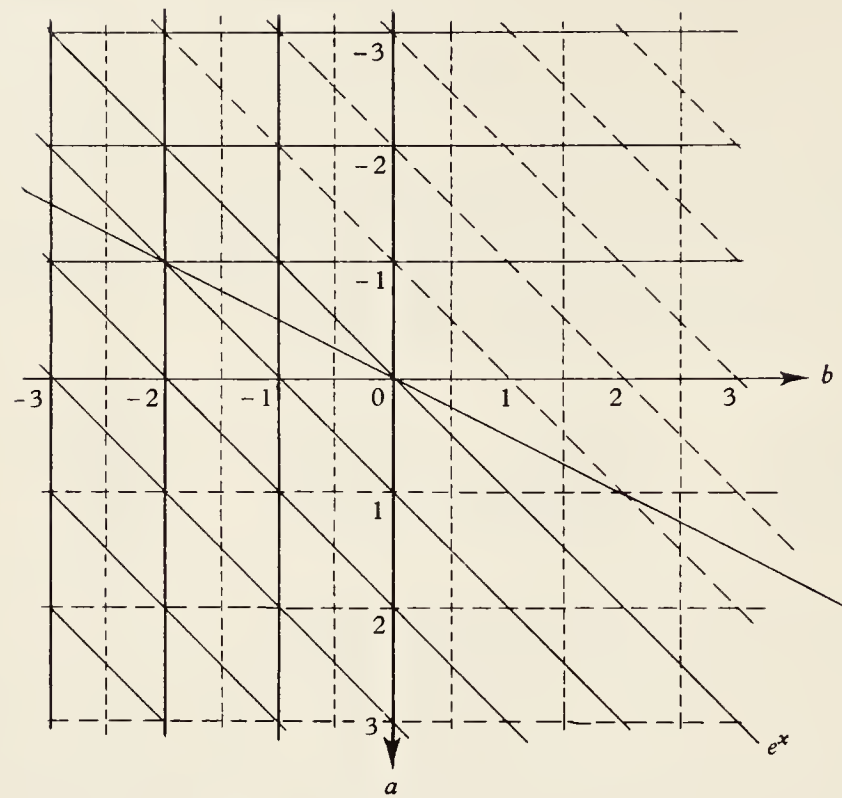


Fig. 6.6

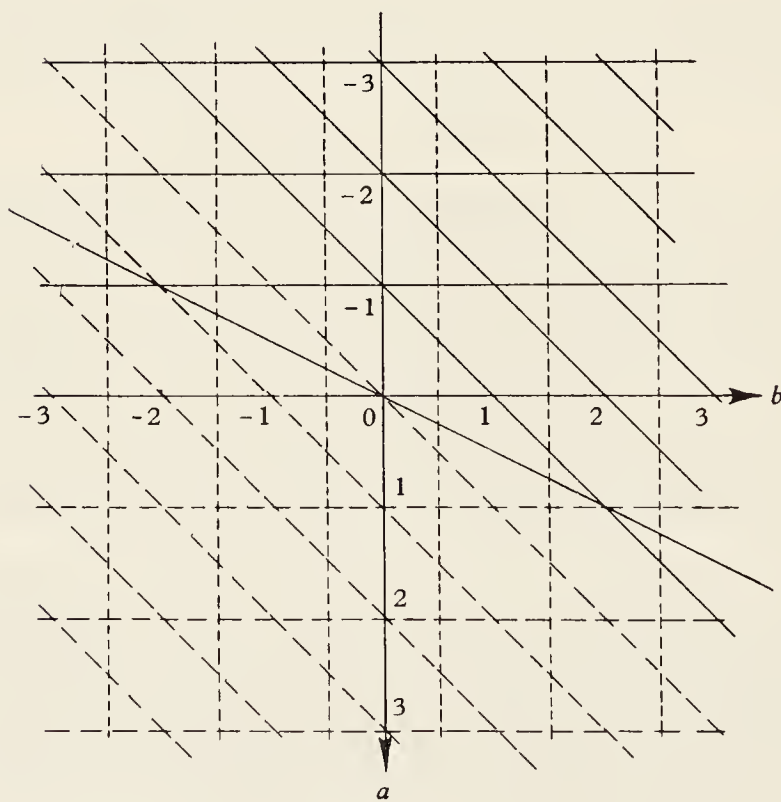


Fig. 6.7

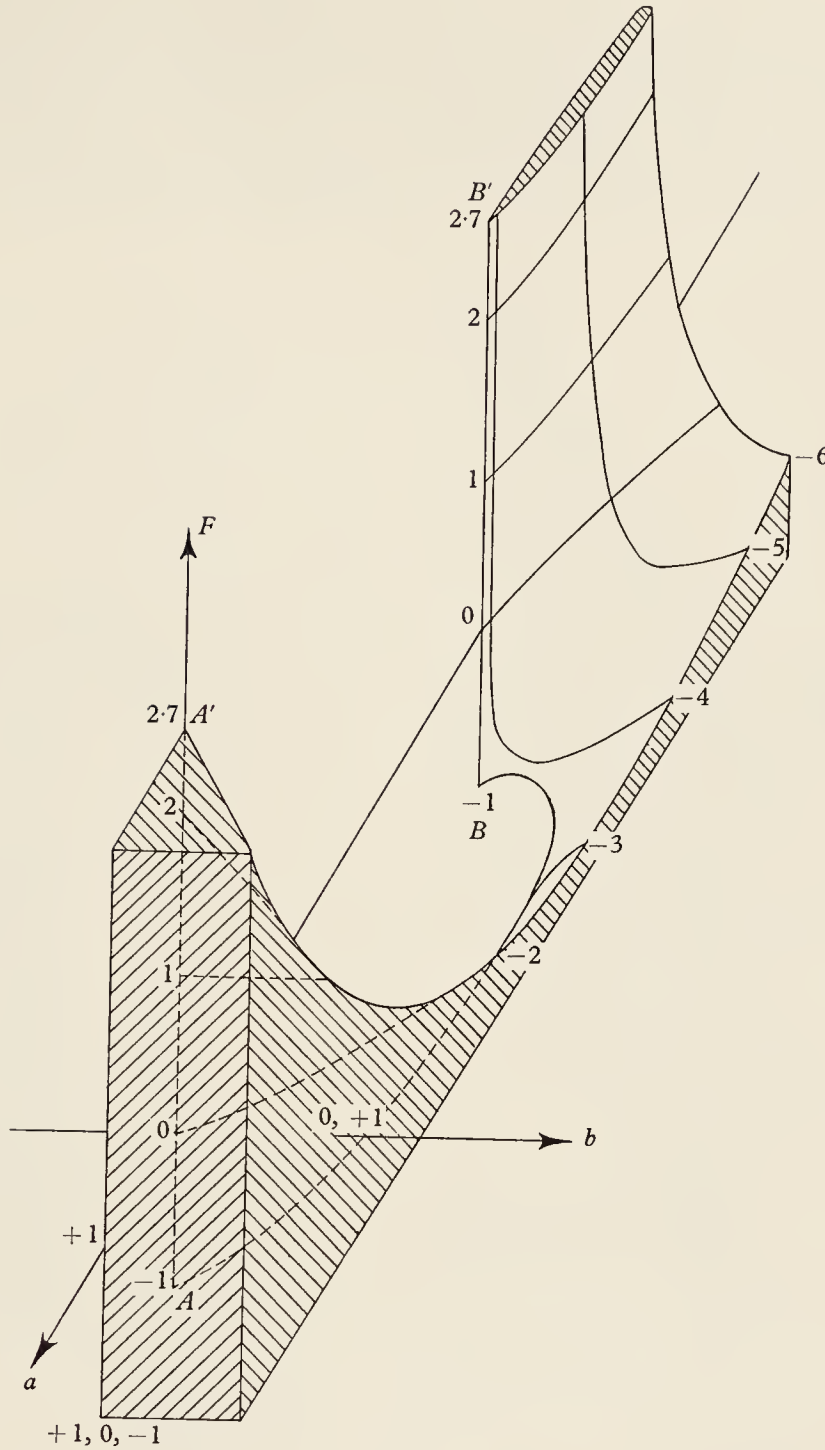


Fig. 6.8

function, which has already been tabulated. Thus when $a = b + m$, ${}_1F_1[a; b; x]$ is expressible as e^x multiplied by a polynomial, thus when $a = b + 1$,

$${}_1F_1[a; b; x] = e^x (1 + x/b).$$

If $a = -1$,

$${}_1F_1[-1; b; x] = 1 - x/b;$$

if $a = -2$,

$${}_1F_1[-2; b; x] = 1 - 2x/b + 2x^2/b(b+1);$$

and, in general, if $a = -n$, ${}_1F_1[-n; b; x]$ is a Laguerre polynomial. Also if $a = 1 + n$, ${}_1F_1[1 + n; b; x]$ is an incomplete gamma function; if $b = 2a$, ${}_1F_1[a; 2a; x]$ is a Bessel function; if $a = \frac{1}{2}$, and $b = \frac{3}{2}$,

${}_1F_1[\frac{1}{2}; \frac{3}{2}; x]$ is an error function, and if $b = \frac{1}{2}$ or if $b = \frac{3}{2}$, ${}_1F_1[a; \frac{1}{2}; x]$ and ${}_1F_1[a; \frac{3}{2}; x]$ are parabolic cylinder functions. When $x = 0$, ${}_1F_1[a; b; 0] = 1$; when $a = 0$, ${}_1F_1[a; b; x] = 1$; when b is a negative integer ${}_1F_1[a; -n; x]$ is not defined unless a is also a negative integer such that $|a| < |b|$, when the function can assume any value, and finally when $a \rightarrow \infty$ ${}_1F_1[a; b; x] \rightarrow +\infty$, if $-2n < b < -2n + 1$, and ${}_1F_1[a; b; x] \rightarrow -\infty$ if $-2n + 1 < b < -2n + 2$. These results are summed up in Fig. 6.6, due to Tricomi.

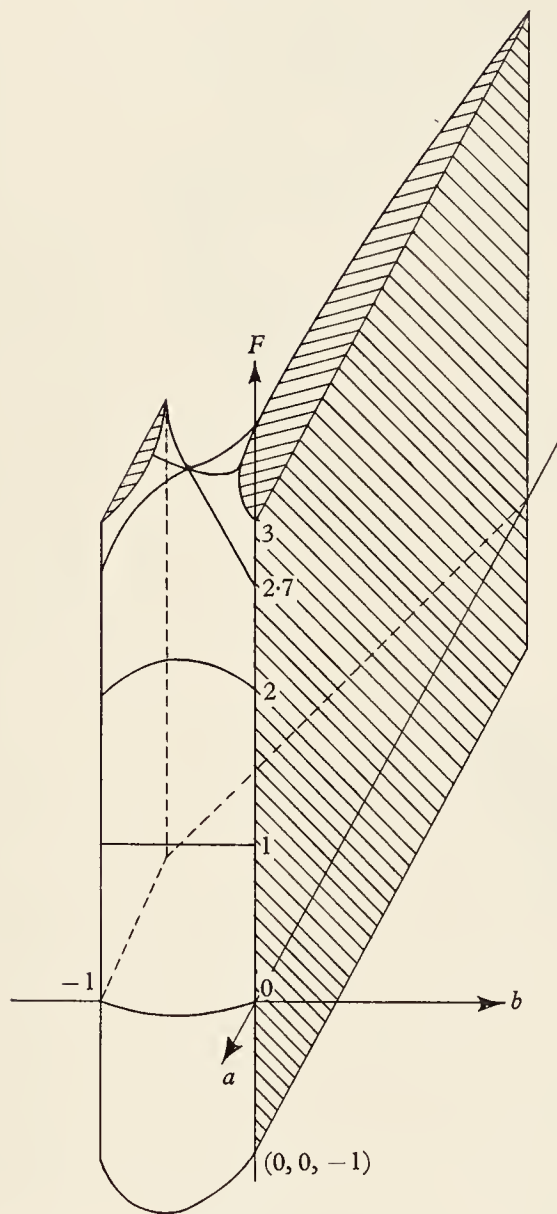


Fig. 6.9

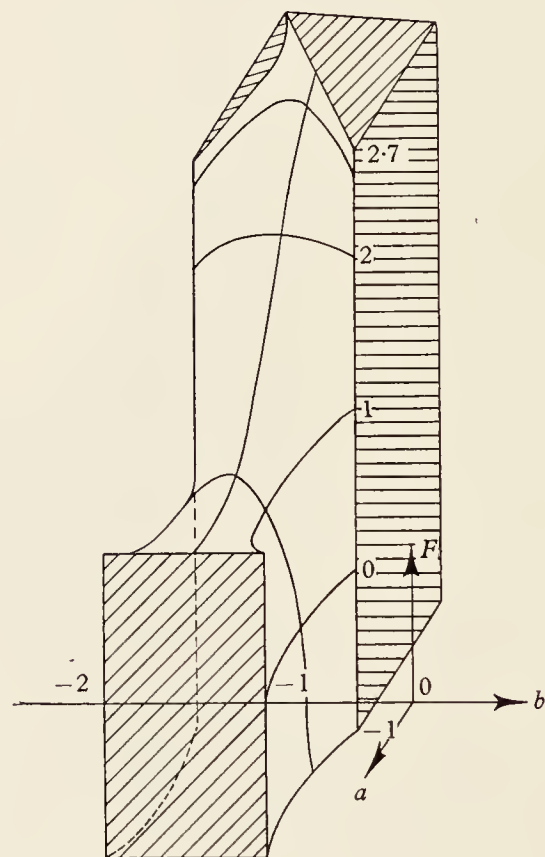


Fig. 6.10

Similar considerations hold for the second solution $U(a; b; x)$. These results are summed up in Fig. 6.7. In Figs. 6.6 and 6.7, the two functions are polynomials or exponential functions on the plain lines, error functions and incomplete gamma functions on the dashed lines and parabolic cylinder functions on the dotted lines.

Appendix III contains a table of ${}_1F_1[a; b; 1]$ over the range $a = -11.0(0.2)2.0$, $b = -4(0.2)1.0$, to eight figures. This table was calculated by the use of recurrence relations from the page for $x = 1$, of the table of Appendix II. The sketches of Figs. 6.8–6.12 are based on this tabulation. Fig. 6.8 shows the strip $0 < b < 1$. On the two vertical lines AA' , $a = b = 0$, and BB' , $a \doteq -3.7$, $b = 0$,

through the points where the line of zeros crosses the plane $b = 0$, the function F can assume any value as shown. Between $0 > a > -3.7$, $F \rightarrow -\infty$, as $b \rightarrow 0_+$, and for $a > 0$ or for $-3.7 > a$, $F \rightarrow +\infty$, as $b \rightarrow 0_+$. Fig. 6.9 shows the strip $-1 < b < 0$. Again the function F can assume all values on the vertical lines through the points $(0, 0)$, $(-3.7, 0)$, $(0, -1)$ and $(-1, -1)$. For $0 > a > -3.7$, $F \rightarrow +\infty$, as $b \rightarrow 0_-$, for $0 > a > -1$, also $F \rightarrow +\infty$, as $b \rightarrow -1_+$, and at $a = b = -0.5$, F has a saddle point.

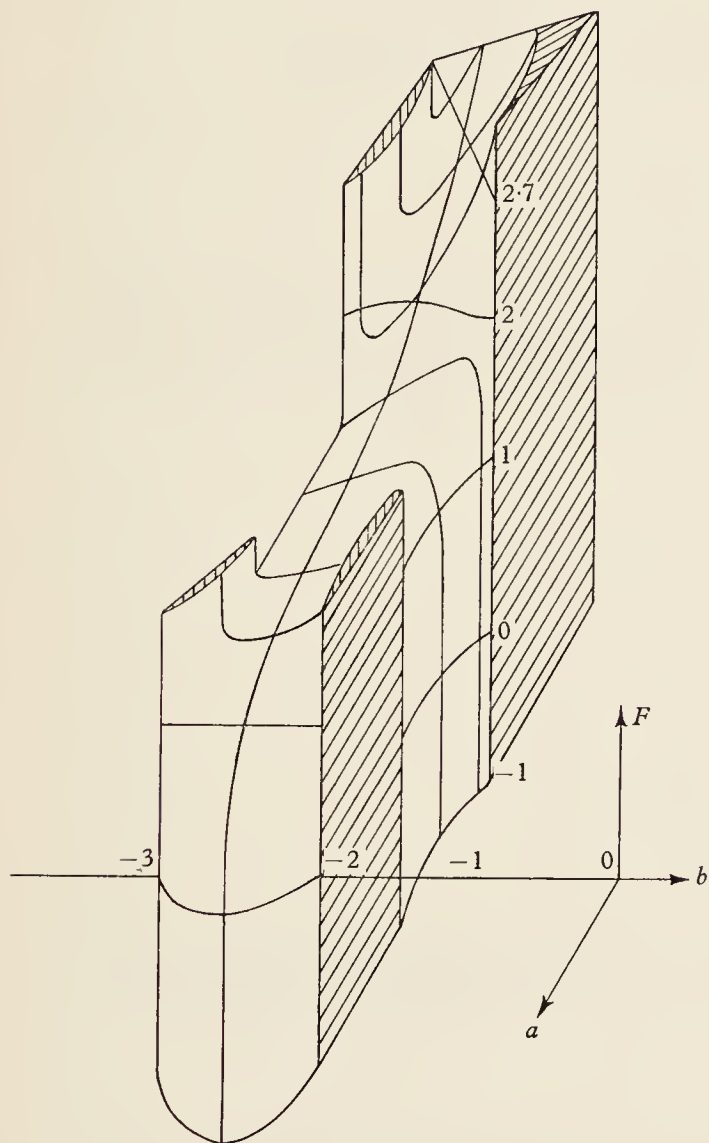


Fig. 6.11

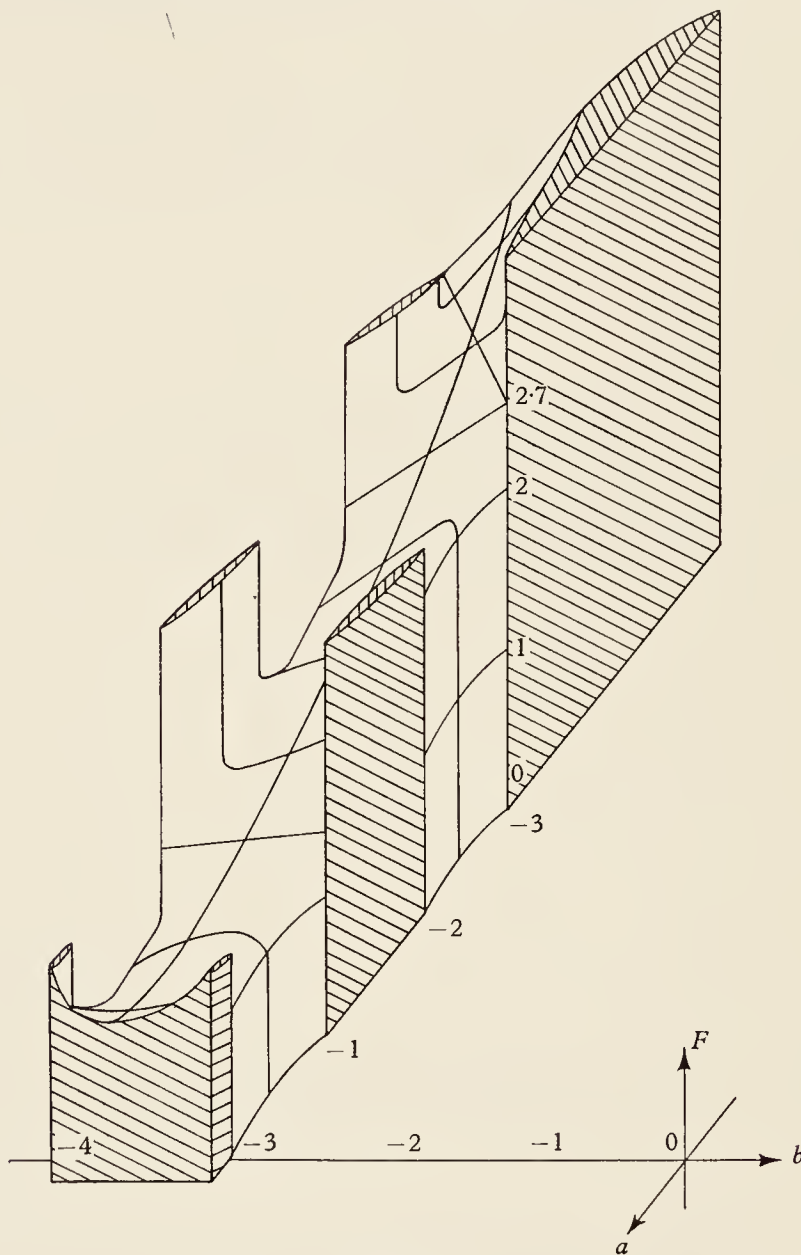


Fig. 6.12

Fig. 6.10 shows the strip $-2 < b < -1$. F can assume all values on the vertical lines through the points $(0, -1)$, $(0, -2)$, $(-1, -1)$, $(-1, -2)$ and $(-2, -2)$. For $0 > a > -1$, $F \rightarrow -\infty$, as $b \rightarrow -1_-$, and as $b \rightarrow -2_+$. For $-1 > a$, $F \rightarrow +\infty$, as $b \rightarrow -1_-$, and for $-1 > a > -2$, $F \rightarrow +\infty$, as $b \rightarrow -2_+$. For $-2 > a$, $F \rightarrow -\infty$, as $b \rightarrow -2_+$. Fig. 6.11 shows the strip $-3 < b < -2$. F can assume all values on the vertical lines through the points $(0, -2)$, $(0, -3)$, $(-1, -2)$, $(-1, -3)$. For $-1 > a > 0$, $F \rightarrow +\infty$, as $b \rightarrow -2_-$ and as $b \rightarrow -3_+$. For $-1 > a > -2$, $F \rightarrow -\infty$, as $b \rightarrow -2_-$ and as $b \rightarrow -3_+$. For $-2 > a$, $F \rightarrow +\infty$, as $b \rightarrow -2_-$, for $-2 > a > -3$, $F \rightarrow +\infty$, as $b \rightarrow -3_+$, and for $-3 > a$, $F \rightarrow -\infty$, as $b \rightarrow -3_+$. Fig. 6.12 shows the strip $-4 < b < -3$. F can assume all values on the vertical

lines through the points $(0, -3)$, $(0, -4)$, $(-1, -3)$, $(-1, -4)$, $(-2, -3)$, $(-2, -4)$, $(-3, -3)$, $(-3, -4)$ and $(-4, -4)$. For $0 > a > -1$, $F \rightarrow -\infty$ as $b \rightarrow -3_-$ and as $b \rightarrow -4_+$. For $-1 > a > -2$, $F \rightarrow +\infty$, as $b \rightarrow -3_-$, and as $b \rightarrow -4_+$. For $-2 > a > -3$, $F \rightarrow -\infty$, as $b \rightarrow -3_-$, and as $b \rightarrow -4_+$. For $-3 > a$, $F \rightarrow +\infty$, as $b \rightarrow -3_-$, for $-3 > a > -4$, $F \rightarrow +\infty$, as $b \rightarrow -4_+$, and for $-4 > a$, $F \rightarrow -\infty$, as $b \rightarrow -4_+$. Other graphs of the function will be found in Airey's papers (1918), (1926) and (1927), and in Jahnke and Emde (1948).

6.7 Exponential and oscillatory regions

We have seen that Kummer's equation can be transformed into the 'normal' form

$$y'' + I(x)y = 0, \quad (6.7.1)$$

where

$$I(x) = -\frac{1}{4} + \frac{\frac{1}{2}b - a}{x} + \frac{\frac{1}{4} - \frac{1}{4}(b-1)^2}{x^2}. \quad (6.7.2)$$

This is substantially Whittaker's equation. It has been proved (Miller 1950), that when $I(x)$ is a real function of x , in those regions where $I(x) < 0$, the solutions of (6.7.1) are all exponential in form, that is convex to the x -axis and where $I(x) > 0$, the solutions of (6.7.1) are all oscillatory in form, that is concave to the x -axis. In the present case

$$I(x) >, = \text{ or } < 0, \quad (6.7.3)$$

as

$$b(2-b) + 2(b-2a)x - x^2 >, = 0 \text{ or } < 0, \quad (6.7.4)$$

and so

$$I(x) > 0, \quad (6.7.5)$$

if

$$|b-2a - \sqrt{(4a^2 - 4ab + 2b)}| < |x| < |b-2a + \sqrt{(4a^2 - 4ab + 2b)}|, \quad (6.7.6)$$

and

$$I(x) < 0, \quad (6.7.7)$$

if

$$|x| < |b-2a - \sqrt{(4a^2 - 4ab + 2b)}|, \quad (6.7.8)$$

and if

$$|x| > |b-2a + \sqrt{(4a^2 - 4ab + 2b)}|. \quad (6.7.9)$$

From these facts we deduce that the solutions (6.7.1) have a finite oscillatory region in a , b and x and that this region must always lie outside the cylindrical hyperboloid standing on the base

$$4a^2 - 4ab + 2b = 0. \quad (6.7.10)$$

$I(x)$ can never be positive within this hyperboloid although it may be negative outside it.

The asymptotic planes of the hyperboloid are the planes vertical to the a, b plane

$$4a^2 - 4ab + 2b = 1, \quad (6.7.11)$$

the axial plane is the plane

$$2a = b, \quad (6.7.12)$$

and the central axis of the hyperboloid is the line, vertical to the a, b plane, through the point

$$a = \frac{1}{2}, \quad b = 1. \quad (6.7.13)$$

The exponential plane $a = b$ cuts this hyperboloid in the line $a = 0$, only.

In general the point (a, b, x) at which $I(x) = 0$, traces out the paraboloid

$$1 + 2x - 4ax = (b - x - 1)^2. \tag{6.7.14}$$

This paraboloid, within which $I(x) > 0$, always falls outside the previous hyperboloid. Its axial plane is $b = 1 + x$. A cross-section of the hyperboloid and paraboloid when $x = 1$, is given in Fig. 6.13.

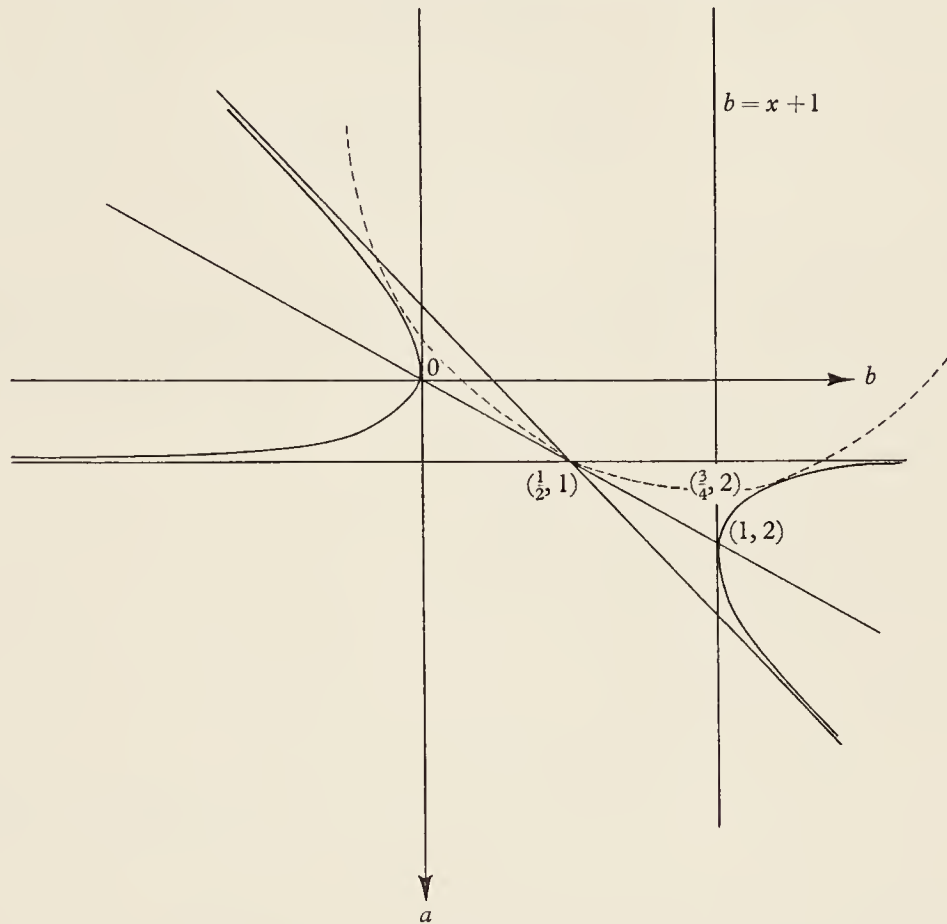


Fig. 6.13

6.7.1 The Sonine-Polya theorem

We can write Kummer's equation in the form

$$\frac{d}{dx} \left(x^b e^{-x} \frac{dy}{dx} \right) = ax^{b-1} e^{-x} y. \tag{6.7.15}$$

This is called the self-adjoint form of the equation. Now the Sonine-Polya theorem (Szego, 1939) states that the moduli of the maxima and minima of any solution of the equation

$$\frac{d}{dx} \left\{ f(x) \frac{dy}{dx} \right\} + F(x)y = 0, \tag{6.7.16}$$

form an increasing or a decreasing sequence according as $f(x)F(x)$ is a decreasing or an increasing function of x .

Hence the successive maxima of $|y|$, for (6.7.15), are decreasing or increasing, if $-ax^{2b-1}e^{-2x}$ is an increasing or a decreasing function of x , that is they are increasing if $a > 0$, $x < b - \frac{1}{2}$, or if $a < 0$ and $x > b - \frac{1}{2}$, and they are decreasing if $a > 0$, $x > b - \frac{1}{2}$ or if $a < 0$ and $x < b - \frac{1}{2}$.

For Whittaker's functions we have the corresponding results that the relative maxima of $|y|$ are increasing, if $k > 0$ and x is not between 0 and $2(m^2 - \frac{1}{4})/k$, or if $k < 0$ and x is between 0 and $2(m^2 - \frac{1}{4})/k$ and the relative maxima of $|y|$ are decreasing, if $k > 0$ and x is between 0 and $2(m^2 - \frac{1}{4})/k$, or if $k < 0$ and x is not between 0 and $2(m^2 - \frac{1}{4})/k$.

From the asymptotic formula (4.4.14) it can be seen that the maxima of $|y|$ lie on curves which have as their first approximations

$$y = \pm \frac{\Gamma(b)}{\sqrt{\pi}} e^{\frac{1}{2}x} (kx)^{\frac{1}{4} - \frac{1}{2}b}. \quad (6.7.17)$$

Closer approximations are

$$y = \pm \frac{\Gamma(b)}{\sqrt{\pi}} e^{\frac{1}{2}x} (kx)^{\frac{1}{4} - \frac{1}{2}b} \left(\frac{4k}{4k - x} \right)^{\frac{1}{4}}, \quad (6.7.18)$$

where $k \equiv \frac{1}{2}b - a$.

6.7.2 Graphing Kummer's function

Suppose that we wish to form a rough idea of the numerical behaviour of a confluent hypergeometric function, for example of $y = {}_1F_1[-4.5; 1; x]$ for x real. This example has been considered by Tricomi (Erdélyi, 1953) and by Middleton (1952).

Here $I(x) = -\frac{1}{4} + 5/x + 1/4x^2$. Hence from § 6.7, $I(x) > 0$ if $10 - \sqrt{101} < x < 10 + \sqrt{101}$, and so the solution y is oscillatory in form for x within this range of values, and exponential in form for x outside this range of values.

We have
$$y' = -4.5 {}_1F_1[-3.5; 2; x].$$

Now $y(0) = 1$, and $y'(0) = -4.5$. Also

$${}_1F_1[-4.5; 1; x] = \frac{\Gamma(1)}{\Gamma(-4.5)} e^x x^{-4.5-1} \{1 + O(1/|x|)\}.$$

Hence, since $\Gamma(-4.5) < 0$,

$$y(-\infty) = +\infty \quad \text{and} \quad y'(-\infty) = +\infty.$$

$$\text{Also} \quad y(\infty) = -\infty \quad \text{and} \quad y'(\infty) = -\infty.$$

From our diagram, Fig. 6.1, of the distribution of the zeros, when $-5 < a < -4$ and $b = 1$, we see that y has five positive zeros and no negative zeros. From the approximation (6.1.11), we can calculate the first approximations to these five zeros, namely

$$\xi_0 = 0.3, \quad \xi_1 = 1.5, \quad \xi_2 = 3.7, \quad \xi_3 = 6.9 \quad \text{and} \quad \xi_4 = 10.6.$$

Now the turning points of y are given by the zeros of y' . Hence first approximations to these four turning points are

$$\eta_1 = 0.9, \quad \eta_2 = 2.8, \quad \eta_3 = 5.8 \quad \text{and} \quad \eta_4 = 9.9.$$

Further, at all these points the conditions of the Sonine-Polya theorem (§ 6.7.1) are satisfied, $a < 0$, and $x > b - \frac{1}{2} = \frac{1}{2}$, and so the maxima of $|y|$ form increasing sequences and lie near the curves

$$y = \pm C e^{\frac{1}{2}x} x^{-\frac{1}{2}}.$$

From these facts we can make the sketch of the function ${}_1F_1[-4.5; 1; x]$ given in Fig. 6.14.

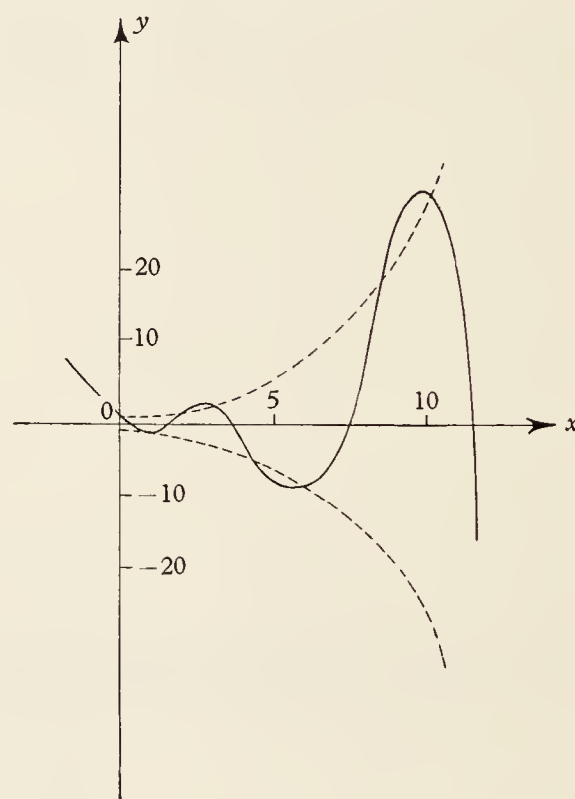


Fig. 6.14

REFERENCES

- AIREY, J. R. (1926). The confluent hypergeometric function. *British Association Report (Oxford)*, pp. 276–94.
- AIREY, J. R. (1927). The confluent hypergeometric function. *British Association Report (Leeds)*, pp. 220–44.
- AIREY, J. R. and WEBB, H. A. (1918). The practical importance of the confluent hypergeometric function. *Phil. Mag.* **36**, 129–41.
- ARCHIBALD, W. J. (1938). The complete solution of the differential equation for the confluent hypergeometric function. *Phil. Mag.* VII, **26**, 415–19.
- BAILEY, W. N. (1930). Some classes of functions which are their own reciprocals in the Fourier–Bessel integral transforms. *J. Lond. Math. Soc.* **5**, 258–65.
- BAILEY, W. N. (1935). *Generalized Hypergeometric Series*. Cambridge Mathematical Tract, no. 32. Cambridge.
- BAILEY, W. N. (1936). On the product of two Legendre polynomials with different arguments. *Proc. Lond. Math. Soc.* II, **41**, 215–20.
- BAILEY, W. N. (1937). An integral representation for the product of two Whittaker functions. *Quart. J. Math.* **8**, 51–3.
- BAILEY, W. N. (1938). Self-reciprocal functions involving confluent hypergeometric functions. *Proc. Lond. Math. Soc.* II, **13**, 111–12.
- BAILEY, W. N. (1939). On the product of two Laguerre polynomials. *Quart. J. Math.* **10**, 60–6.
- BARNES, E. W. (1908*a*). On functions defined by simple hypergeometric series. *Trans. Camb. Phil. Soc.* **20**, 253–79.
- BARNES, E. W. (1908*b*). A new development of the theory of hypergeometric functions. *Proc. Lond. Math. Soc.* **6**, 141–77.
- BATEMAN, H. (1931). The k -function, a particular case of the confluent hypergeometric function. *Trans. Amer. Math. Soc.* **33**, 817–31.
- BUCHHOLZ, H. (1943). Die konfluente hypergeometrische Funktion. *Z. angew. Math. Mech.* **23**, 47–58, 100–18.
- BUCHHOLZ, H. (1950). Die asymptotischen Entwicklungen für die beiden parabolischen Funktionen $M_{k, \frac{1}{2}m}(z)$ und $W_{k, \frac{1}{2}m}(z)$ bei grossen Werten von k und z für $-\infty < z/4k < +\infty$. *Z. angew. Math. Mech.* **30**, 133–48.
- BUCHHOLZ, H. (1953). *Die konfluente hypergeometrische Funktion*. Ergebnisse der Angewandten Mathematik. Berlin. (On the Whittaker functions, with a large bibliography.)
- CHAPPELL, G. E. (1925). The properties of a new orthogonal function associated with the confluent hypergeometric function. *Proc. Edinb. Math. Soc.* **43**, 117–30.
- CHARLIER, C. V. L. (1905). Über die Darstellung willkürlicher Funktionen. *Ark. Mat. Astr. Fys.* **2**, 20–35.
- CONNOLLY, B. W. (1950). A short table of the confluent hypergeometric function. *Quart. J. Appl. Math.* **3**, 236–40.
- CUNNINGHAM, E. (1908). The ω -function, a class of normalizing functions in statistics. *Proc. Roy. Soc. A*, **81**, 310–31.
- ERDÉLYI, A. (1936). Über einige bestimmte Integrale. *Math. Z.* **40**, 693–702.
- ERDÉLYI, A., MAGNUS, W., OBERHETTINGER, F. and TRICOMI, G. F. (1953). *Higher Transcendental Functions*. Bateman project. Vol. I. New York. (Chapter VI on Kummer functions.)
- ERDÉLYI, A., MAGNUS, W., OBERHETTINGER, F. and TRICOMI, F. G. (1954). *Tables of Integral Transforms*. Bateman project. Vols. I and II. New York.
- FIELDHOUSE, M. (1959). Saddle points of the confluent hypergeometric functions. *Proc. Camb. Phil. Soc.* **55** (in course of publication).

- FLETCHER, A., MILLER, J. C. P. and ROSENHEAD, L. (1946). *An Index of Mathematical Tables*. London.
- GOLDSTEIN, S. (1932). Operational representations of Whittaker's confluent hypergeometric function and Weber's parabolic cylinder function. *Proc. Lond. Math. Soc.* II, **34**, 103-25.
- GRAN OLSSON, R. (1937). Tabellen der konfluenten hypergeometrischen Funktion. *Ingenieur Arch.* **8**, 99-103, 270-5, 373-80.
- HARTREE, D. (1929). The wave mechanics of an atom with a non-Coulomb central field. *Proc. Camb. Phil. Soc.* **25**, 310-14.
- HEATLEY, A. H. (1943). A short table of the Toronto function. *Trans. Roy. Soc. Can.* (3), **37**, 13-29.
- HORN, J. (1940). Hypergeometrische Funktionen zweier Veränderlicher im Schnittpunkt dreier Singularitäten. *Math. Ann.* **117**, 579-86.
- HORN, J. (1889). Über die Konvergenz der hypergeometrischen Reihe zweier und dreier Veränderlichen. *Math. Ann.* **34**, 577-600.
- HUMBERT, P. (1920). Sur les fonctions hypercylindriques. *C.R. Acad. Sci., Paris*, **171**, 490-2.
- JAHNKE, E. and EMDE, F. (1948). *Tafeln höheren Funktionen*. Leipzig.
- JEFFREYS, H. and JEFFREYS, B. S. (1950). *Methods of Mathematical Physics*. Cambridge. (Chap. 23, on Kummer's functions.)
- KAMKE, E. (1943). *Differentialgleichungen, Lösungsmethoden und Lösungen*. Leipzig.
- KIENAST, A. (1921). Untersuchungen über die Lösungen der Differentialgleichung $xy'' + (\gamma - x)y' - \beta y = 0$. *Denkschr. schweiz. naturf. Ges.* **57**, 247-325.
- KRUPP, H. (1950). Bestimmung der allgemeinen Lösungen der Schrödinger-Gleichung für Coulomb-Potentiale. *Ber. Verh. Sachs. Akad. Wiss. Leipzig*, **97**, 1-28.
- KUMMER, E. E. (1836). Über die hypergeometrische Reihe $F(a; b; x)$. *J. reine angew. Math.* **15**, 39-83.
- LANGER, R. E. (1935). On the asymptotic solutions of ordinary differential equations with reference to the Stokes phenomenon about a singular point. *Trans. Amer. Math. Soc.* **37**, 357-416.
- LAUWERIER, H. A. (1950). The use of confluent hypergeometric functions in mathematical physics and the solution of an eigenvalue problem. *Appl. Sci. Res. A*, **2**, 184-204.
- LAUWERIER, H. A. (1951). Poiseuille Functions. *Appl. Sci. Res. A*, **3**, 58-72.
- LOWEN, A. and HORENSTEIN, W. (1942). On the function $H(m, a, x)$. *J. Math. Phys.* **21**, 264-83.
- MACROBERT, T. M. (1942). Some integrals involving E -functions. *Quart. J. Math.* **13**, 65-8.
- MAGNUS, W. (1941). Zur Theorie des zylindrisch-parabolischen Speigels. *Z. Phys.* **11**, 343-56.
- MAGNUS, W. and OBERHETTINGER, F. (1948). *Formeln und Lehrsätze für die speziellen Funktionen der mathematischen Physik*. Berlin.
- MEIJER, C. S. (1936a). Einige Integraldarstellungen aus der Theorie der Besselschen und der Whittaker Funktionen. *K. Akad. Wet. Amst. Proc.* **39**, 394-403, 519-27.
- MEIJER, C. S. (1936b). Über die Integraldarstellungen der Whittaker Funktionen und der Hankelschen und Besselschen Funktionen. *Nieuw Arch. Wisk.* (2), **18**, 10-39.
- MEIXNER, J. (1933). Die Greensche Funktion des Wellenmechanischen Kepler-problems. *Math. Z.* **36**, 677-707.
- MIDDLETON, D. and JOHNSON, V. (1952). A tabulation of selected confluent hypergeometric functions. *Harvard University Cruft Laboratory Technical Report*, **140**. Cambridge, Mass.
- MILLER, J. C. P. (1950). On the choice of standard solutions for a homogeneous linear differential equation of the second order. *Quart. J. Appl. Math.* **3**, 225-35.
- MILLER, J. C. P. (1952). A method for the determination of converging factors. *Proc. Camb. Phil. Soc.* **48**, 243-54.
- MILLER, J. C. P. (1955). *Tables of Weber Parabolic Cylinder Functions*. London: H.M.S.O.
- MILLER, J. C. P. (1957). The general solution of the confluent hypergeometric equation. *Math. Tables and Aids (Washington)*, XI, **58**, 97-9.
- MILNE, A. (1915). On the roots of the confluent hypergeometric functions. *Proc. Edinb. Math. Soc.* **33**, 48-64.

- MIRIMANOV, R. G. (1948). Lösung der Aufgabe der Diffraktion mit Hilfe der Laguerreschen Funktionen. *Akad. Nauk. S.S.S.R. (Doklady)*, **60**, 203-6.
- NATH, P. (1951). Confluent hypergeometric functions. *Sankhyā*, **11**, 153-66.
- NATIONAL BUREAU OF STANDARDS (1952). *Tables of Coulomb Wave Functions*. Vol. 1. Appl. Math. Series, **17**. Washington.
- NATIONAL BUREAU OF STANDARDS (1959). *Handbook of Mathematical Tables* (ch. 13, Confluent hypergeometric functions). Washington.
- NIELSEN, N. (1906). *Handbuch der Theorie der Zylinderfunktionen*. Leipzig.
- OLVER, F. W. J. (1954). The asymptotic solution of linear differential equations of the second order for large values of a parameter. *Phil. Trans. A*, **247**, 307-68.
- OLVER, F. W. J. (1957). The asymptotic solutions of linear differential equations of the second order in a domain containing one transition point. *Phil. Trans. A*, **249**, 65-97.
- PERRON, O. (1921). Über das Verhalten einer ausgearteten hypergeometrischen Reihe bei unbegrenztem Wachstum eines Parameters. *J. reine angew. Math. Crelles J.* **151**, 63-78.
- PINNEY, E. (1946). Laguerre functions in the mathematical foundations of electromagnetic theory of the paraboloidal reflector. *J. Math. Phys.* **25**, 49-79.
- PLACZEK, C. (1946). The function $E_n(x) = \int_1^\infty e^{-xu} u^{-n} du$. *National Research Council of Canada*, M.T./1.
- POCHHAMMER, L. (1890). Über eine Klasse von Integralen mit geschlossen Integrationskurven. *Math. Ann.* **37**, 500-11.
- RUSHTON, S. (1954). Tables of the Confluent Hypergeometric Function. *Sankhyā, Indian Journal of Statistics*, **13**, 369-411.
- SCHLÖMILCH, O. (1859). Über Fakultätenreihen. *Z. Math. Phys.* **4**, 390-415.
- SHARPE, H. J. (1900). On the reflexion of sound at a paraboloid. *Proc. Camb. Phil. Soc.* **10**, 101-36.
- SLATER, L. J. (1953). On the evaluation of the confluent hypergeometric function. *Proc. Camb. Phil. Soc.* **49**, 612-22.
- SLATER, L. J. (1954a). Some new results on equivalent products. *Proc. Camb. Phil. Soc.* **50**, 394-403.
- SLATER, L. J. (1954b). The evaluation of the basic confluent hypergeometric function. *Proc. Camb. Phil. Soc.* **50**, 404-13.
- SLATER, L. J. (1954c). Expansions of generalized Whittaker functions. *Proc. Camb. Phil. Soc.* **50**, 628-31.
- SLATER, L. J. (1955a). Integrals for asymptotic expansions of hypergeometric functions. *Proc. Amer. Math. Soc.* **6**, 226-31.
- SLATER, L. J. (1955b). The integration of hypergeometric functions. *Proc. Camb. Phil. Soc.* **51**, 288-96.
- SLATER, L. J. (1955c). A short table of the Laguerre polynomials. *Inst. Electric. Eng. Monogr.* **136**.
- SLATER, L. J. (1955d). Hypergeometric Mellin transforms. *Proc. Camb. Phil. Soc.* **51**, 577-89.
- SLATER, L. J. (1956). The real zeros of the confluent hypergeometric function. *Proc. Camb. Phil. Soc.* **52**, 626-35.
- SZEGO, G. (1939). *Orthogonal polynomials*. American Math. Soc. Colloquium **23**.
- TITCHMARSH, E. C. (1937). *Introduction to the Theory of Fourier Integrals*. Oxford.
- TRICOMI, F. G. (1947). Sulle funzioni ipergeometriche confluenti. *Ann. Mat. Pura Appl.* (IV), **36**, 141-75.
- TRICOMI, F. G. (1949a). Sul comportamento asintotico dell n'esimo polinomio di Laguerre. *Comm. Math. Helvetici*, **22**, 150-67.
- TRICOMI, F. G. (1949b). Sul comportamento asintotico di polinomi di Laguerre. *Ann. Mat.* (4), **28**, 263-89.
- TRICOMI, F. G. (1950). Über die Abzählung der Nullstellen der konfluenten hypergeometrischen Funktionen. *Math. Z.* **52**, 669-75.
- TRICOMI, F. G. (1954). *Funzioni Ipergeometriche Confluenti*. Monografie Matematiche Rome I. (On Kummer Functions.)
- TRICOMI, F. G. (1956). Valori Numerici di Funzioni Ortogonali di Laguerre. *Atti Accad. Torino*, **90**, 1-8.

- TSVETKOFF, G. E. (1941*a*). On roots of Whittaker's functions. *C.R. Acad. Sci. U.R.S.S.* **32**, 10-12.
- TSVETKOFF, G. E. (1941*b*). Sur les racines complexes des fonctions de Whittaker. *C.R. Acad. Sci. U.R.S.S.* **33**, 290-91.
- WATSON, G. N. (1948). *A Treatise on the Theory of Bessel Functions*, 2nd edition. Cambridge.
- WHEELER, J. A. and YOST, F. L. (1936). Coulomb wave functions in repulsive fields. *Phys. Rev.* **49**, 174-89.
- WHITTAKER, E. T. (1904). An expression of certain known functions as generalized hypergeometric series. *Bull. Amer. Math. Soc.* **10**, 125-34.
- WHITTAKER, E. T. and WATSON, G. N. (1946). *Modern Analysis*, 4th edition. Cambridge. (Chap. XVI on Whittaker functions.)
- WIDDER, D. V. (1941). *The Laplace Transform*. Princeton.

APPENDIX I

Table of the smallest positive zeros of ${}_1F_1[a; b; x]$ over the range

$$a = -4.0(0.1) - 0.1, \quad b = 0.1(0.1) 2.5$$

x_0 for ${}_1F_1[a; b; x] = 0$

a	b = 0.1	0.2	0.3	0.4	0.5
-4.0	0.02590 78	0.05352 31	0.08270 43	0.11333 19	0.14530 35
-3.9	0.02656 37	0.05486 17	0.08474 82	0.11610 04	0.14881 29
-3.8	0.02725 38	0.05626 90	0.08689 57	0.11900 75	0.15249 63
-3.7	0.02798 06	0.05775 04	0.08915 49	0.12206 41	0.15636 68
-3.6	0.02874 73	0.05931 20	0.09153 49	0.12528 21	0.16043 92
-3.5	0.02955 72	0.06096 03	0.09404 54	0.12867 44	0.16472 98
-3.4	0.03041 41	0.06270 30	0.09669 76	0.13225 58	0.16925 64
-3.3	0.03132 21	0.06454 82	0.09950 39	0.13604 24	0.17403 93
-3.2	0.03228 60	0.06650 54	0.10247 80	0.14005 26	0.17910 09
-3.1	0.03331 11	0.06858 50	0.10563 54	0.14430 66	0.18446 62
-3.0	0.03440 34	0.07079 89	0.10899 38	0.14882 74	0.19016 35
-2.9	0.03556 99	0.07316 06	0.11257 30	0.15364 11	0.19622 47
-2.8	0.03681 82	0.07568 53	0.11639 53	0.15877 71	0.20268 59
-2.7	0.03815 73	0.07839 06	0.12048 67	0.16426 88	0.20958 81
-2.6	0.03959 76	0.08129 65	0.12487 64	0.17015 47	0.21697 82
-2.5	0.04115 08	0.08442 63	0.12959 84	0.17647 89	0.22490 99
-2.4	0.04283 09	0.08780 68	0.13469 20	0.18329 22	0.23344 53
-2.3	0.04465 40	0.09146 95	0.14020 28	0.19065 39	0.24265 61
-2.2	0.04663 93	0.09545 13	0.14618 45	0.19863 31	0.25262 62
-2.1	0.04880 93	0.09979 58	0.15270 01	0.20731 10	0.26345 39
-2.0	0.05119 11	0.10455 49	0.15982 46	0.21678 40	0.27525 51
-1.9	0.05381 74	0.10979 10	0.16764 76	0.22716 69	0.28816 81
-1.8	0.05672 79	0.11557 98	0.17627 75	0.23859 77	0.30235 84
-1.7	0.05997 12	0.12201 36	0.18584 60	0.25124 43	0.31802 65
-1.6	0.06360 79	0.12920 67	0.19651 56	0.26531 23	0.33541 75
-1.5	0.06771 44	0.13730 23	0.20848 85	0.28105 66	0.35483 39
-1.4	0.07238 79	0.14648 18	0.22201 97	0.29879 77	0.37665 43
-1.3	0.07775 47	0.15697 88	0.23743 57	0.31894 34	0.40135 86
-1.2	0.08398 14	0.16909 96	0.25516 18	0.34202 15	0.42956 49
-1.1	0.09129 29	0.18325 34	0.27576 16	0.36872 80	0.46208 38
-1.0	0.10000 00	0.20000 00	0.30000 00	0.40000 00	0.50000 00
-0.9	0.11054 47	0.22012 64	0.32894 15	0.43713 15	0.54480 16
-0.8	0.12357 83	0.24477 52	0.36411 44	0.48196 35	0.59858 98
-0.7	0.14010 11	0.27567 24	0.40779 72	0.53721 21	0.66443 91
-0.6	0.16173 42	0.31555 72	0.46354 99	0.60707 04	0.74705 02
-0.5	0.19128 98	0.36906 09	0.53728 03	0.69839 96	0.85403 26
-0.4	0.23411 73	0.44470 78	0.63961 58	0.82334 00	0.99868 55
-0.3	0.30182 31	0.56019 88	0.79200 44	1.00591 69	1.20695 84
-0.2	0.42537 31	0.75993 80	1.04632 32	1.30289 37	1.53918 36
-0.1	0.72703 16	1.20342 40	1.58016 05	1.90320 51	2.19258 90

x_0 for ${}_1F_1[a; b; x]=0$

a	b = 0.6	0.7	0.8	0.9	1.0
-4.0	0.17853 00	0.21293 33	0.24844 41	0.28500 07	0.32254 76
-3.9	0.18279 45	0.21796 50	0.25425 35	0.29159 68	0.32993 79
-3.8	0.18726 81	0.22324 09	0.26034 19	0.29850 64	0.33767 63
-3.7	0.19196 66	0.22877 92	0.26673 00	0.30575 28	0.34578 81
-3.6	0.19690 74	0.23459 99	0.27344 05	0.31336 12	0.35430 11
-3.5	0.20210 97	0.24072 54	0.28049 85	0.32135 94	0.36324 60
-3.4	0.20759 50	0.24718 02	0.28793 19	0.32977 86	0.37265 66
-3.3	0.21338 70	0.25399 18	0.29577 14	0.33865 27	0.38257 06
-3.2	0.21951 23	0.26119 07	0.30405 16	0.34802 01	0.39302 97
-3.1	0.22600 06	0.26881 09	0.31281 07	0.35792 32	0.40408 03
-3.0	0.23288 51	0.27689 08	0.32209 17	0.36840 95	0.41577 45
-2.9	0.24020 35	0.28547 32	0.33194 30	0.37953 25	0.42817 06
-2.8	0.24799 82	0.29460 70	0.34241 91	0.39135 23	0.44133 41
-2.7	0.25631 74	0.30434 70	0.35358 15	0.40393 70	0.45533 92
-2.6	0.26521 61	0.31475 62	0.36550 06	0.41736 38	0.47027 00
-2.5	0.27475 74	0.32590 62	0.37825 64	0.43172 07	0.48622 22
-2.4	0.28501 36	0.33787 94	0.39194 08	0.44710 88	0.50330 54
-2.3	0.29606 86	0.35077 11	0.40665 97	0.46364 43	0.52164 55
-2.2	0.30801 99	0.36469 17	0.42253 61	0.48146 16	0.54138 80
-2.1	0.32098 19	0.37977 06	0.43971 32	0.50071 74	0.56270 24
-2.0	0.33508 89	0.39615 95	0.45835 92	0.52159 51	0.58578 64
-1.9	0.35050 08	0.41403 83	0.47867 27	0.54431 12	0.61087 33
-1.8	0.36740 86	0.43362 18	0.50089 05	0.56912 30	0.63823 96
-1.7	0.38604 28	0.45516 81	0.52529 68	0.59633 87	0.66821 62
-1.6	0.40668 45	0.47899 14	0.55223 59	0.62633 11	0.70120 23
-1.5	0.42967 94	0.50547 71	0.58212 99	0.65955 57	0.73768 47
-1.4	0.45545 87	0.53510 39	0.61550 11	0.69657 55	0.77826 37
-1.3	0.48456 67	0.56847 46	0.65300 53	0.73809 46	0.82368 84
-1.2	0.51770 18	0.60635 92	0.69547 76	0.78500 75	0.87490 75
-1.1	0.55577 50	0.64975 86	0.74399 95	0.83846 91	0.93314 33
-1.0	0.60000 00	0.70000 00	0.80000 00	0.90000 00	1.00000 00
-0.9	0.65203 19	0.75888 50	0.86541 05	0.97164 85	1.07763 19
-0.8	0.71419 38	0.82892 89	0.94291 59	1.05625 10	1.16901 22
-0.7	0.78986 07	0.91376 55	1.03637 62	1.15786 85	1.27838 33
-0.6	0.88415 45	1.01887 44	1.15158 21	1.28256 70	1.41205 79
-0.5	1.00529 53	1.15298 99	1.29771 21	1.43991 63	1.57995 68
-0.4	1.16751 37	1.33112 03	1.49044 27	1.64618 10	1.79887 13
-0.3	1.39828 59	1.58200 88	1.75960 56	1.93215 19	2.10045 49
-0.2	1.76075 91	1.97114 63	2.17271 84	2.36714 89	2.55566 24
-0.1	2.45881 88	2.70808 56	2.94434 51	3.17028 02	3.38779 57

x_0 for ${}_1F_1[a; b; x]=0$

a	b = 1.1	1.2	1.3	1.4	1.5
-4.0	0.36103 46	0.40041 59	0.44064 99	0.48169 84	0.52352 61
-3.9	0.36922 57	0.40941 35	0.45045 87	0.49232 23	0.53496 84
-3.8	0.37779 91	0.41882 72	0.46071 70	0.50342 89	0.54692 64
-3.7	0.38678 22	0.42868 66	0.47145 69	0.51505 24	0.55943 61
-3.6	0.39620 54	0.43902 45	0.48271 32	0.52723 00	0.57253 70
-3.5	0.40610 20	0.44987 69	0.49452 44	0.54000 25	0.58627 24
-3.4	0.41650 88	0.46128 33	0.50693 30	0.55341 51	0.60069 01
-3.3	0.42746 65	0.47328 76	0.51998 58	0.56751 75	0.61584 27
-3.2	0.43902 04	0.48593 85	0.53373 49	0.58236 51	0.63178 84
-3.1	0.45122 10	0.49929 01	0.54823 79	0.59801 90	0.64859 22
-3.0	0.46412 43	0.51340 28	0.56355 93	0.61454 76	0.66632 59
-2.9	0.47779 35	0.52834 42	0.57977 10	0.63202 72	0.68507 01
-2.8	0.49229 95	0.54419 03	0.59695 41	0.65054 33	0.70491 48
-2.7	0.50772 22	0.56102 67	0.61519 94	0.67019 23	0.72596 15
-2.6	0.52415 22	0.57895 02	0.63461 00	0.69108 28	0.74832 45
-2.5	0.54169 26	0.59807 10	0.65530 26	0.71333 83	0.77213 34
-2.4	0.56046 12	0.61851 47	0.67741 06	0.73709 93	0.79753 58
-2.3	0.58059 33	0.64042 55	0.70108 66	0.76252 65	0.82470 03
-2.2	0.60224 49	0.66396 96	0.72650 63	0.78980 50	0.85382 06
-2.1	0.62559 73	0.68933 96	0.75387 33	0.81914 87	0.88512 08
-2.0	0.65086 23	0.71676 03	0.78342 49	0.85080 66	0.91886 12
-1.9	0.67828 86	0.74649 54	0.81543 89	0.88507 03	0.95534 61
-1.8	0.70817 12	0.77885 71	0.85024 38	0.92228 37	0.99493 43
-1.7	0.74086 20	0.81421 74	0.88823 06	0.96285 56	1.03805 17
-1.6	0.77678 52	0.85302 37	0.92986 85	1.00727 62	1.08520 83
-1.5	0.81645 68	0.89581 94	0.97572 70	1.05613 94	1.13702 10
-1.4	0.86051 15	0.94327 18	1.02650 37	1.11017 15	1.19424 36
-1.3	0.90974 03	0.99621 06	1.08306 49	1.17027 32	1.25780 90
-1.2	0.96514 25	1.05568 22	1.14650 09	1.23757 59	1.32888 76
-1.1	1.02800 20	1.12302 80	1.21820 64	1.31352 47	1.40897 16
-1.0	1.10000 00	1.20000 00	1.30000 00	1.40000 00	1.50000 00
-0.9	1.18338 79	1.28893 94	1.39430 58	1.49950 39	1.60454 79
-0.8	1.28126 39	1.39305 95	1.50444 42	1.61545 64	1.72612 90
-0.7	1.39803 52	1.51691 93	1.63511 46	1.75268 81	1.86969 69
-0.6	1.54023 94	1.66726 23	1.79325 15	1.91831 19	2.04253 23
-0.5	1.71811 55	1.85462 01	1.98965 79	2.12338 42	2.25592 96
-0.4	1.94893 39	2.09670 44	2.24245 51	2.38641 01	2.52875 63
-0.3	2.26513 62	2.42668 47	2.58549 24	2.74187 86	2.89610 67
-0.2	2.73918 28	2.91842 47	3.09395 31	3.26622 24	3.43560 39
-0.1	3.59829 52	3.80284 33	4.00226 84	4.19722 81	4.38825 42

x_0 for ${}_1F_1[a; b; x] = 0$

a	b = 1.6	1.7	1.8	1.9	2.0
-4.0	0.56610 04	0.60939 12	0.65337 06	0.69801 22	0.74329 19
-3.9	0.57836 37	0.62247 80	0.66728 24	0.71275 04	0.75885 75
-3.8	0.59117 55	0.63614 53	0.68180 66	0.72813 25	0.77509 81
-3.7	0.60457 35	0.65043 30	0.69698 50	0.74420 22	0.79205 92
-3.6	0.61859 93	0.66538 48	0.71286 33	0.76100 71	0.80979 05
-3.5	0.63329 88	0.68104 87	0.72949 17	0.77859 97	0.82834 66
-3.4	0.64872 20	0.69747 74	0.74692 55	0.79703 76	0.84778 70
-3.3	0.66492 45	0.71472 91	0.76522 53	0.81638 41	0.86817 83
-3.2	0.68196 76	0.73286 82	0.78445 85	0.83670 92	0.88959 28
-3.1	0.69991 94	0.75196 59	0.80469 94	0.85809 04	0.91211 09
-3.0	0.71885 56	0.77210 14	0.82603 08	0.88061 37	0.93582 21
-2.9	0.73886 06	0.79336 32	0.84854 47	0.90437 49	0.96082 56
-2.8	0.76002 91	0.81584 99	0.87234 42	0.92948 12	0.98723 25
-2.7	0.78246 72	0.83967 28	0.89754 48	0.95605 24	1.01516 70
-2.6	0.80629 48	0.86495 66	0.92427 68	0.98422 37	1.04476 90
-2.5	0.83164 74	0.89184 31	0.95268 70	1.01414 76	1.07619 64
-2.4	0.85867 94	0.92049 31	0.98294 27	1.04599 72	1.10962 80
-2.3	0.88756 71	0.95108 97	1.01523 43	1.07996 97	1.14526 76
-2.2	0.91851 24	0.98384 35	1.04978 00	1.11629 11	1.18334 85
-2.1	0.95174 92	1.01899 72	1.08683 11	1.15522 12	1.22413 88
-2.0	0.98754 84	1.05683 23	1.12667 99	1.19706 14	1.26794 92
-1.9	1.02622 69	1.09767 75	1.16966 57	1.24216 24	1.31514 08
-1.8	1.06815 75	1.14191 91	1.21618 80	1.29093 58	1.36613 76
-1.7	1.11378 23	1.19001 48	1.26671 94	1.34386 93	1.42144 02
-1.6	1.16363 04	1.24251 15	1.32182 40	1.40154 28	1.48164 49
-1.5	1.21834 03	1.30006 90	1.38218 17	1.46465 55	1.54746 96
-1.4	1.27869 20	1.36349 17	1.44862 03	1.53405 78	1.61978 59
-1.3	1.34564 89	1.43377 23	1.52216 05	1.61079 70	1.69966 67
-1.2	1.42041 84	1.51215 31	1.60407 79	1.69618 05	1.78845 01
-1.1	1.50453 75	1.60021 36	1.69599 24	1.79186 71	1.88783 15
-1.0	1.60000 00	1.70000 00	1.80000 00	1.90000 00	2.00000 00
-0.9	1.70945 03	1.81422 20	1.91887 26	2.02341 07	2.12784 39
-0.8	1.83649 04	1.94656 54	2.05637 59	2.16594 10	2.27527 78
-0.7	1.98619 02	2.10221 05	2.21779 49	2.33297 62	2.44778 31
-0.6	2.16598 84	2.28874 58	2.41086 13	2.53238 46	2.65335 95
-0.5	2.38740 53	2.51790 61	2.64751 44	2.77630 17	2.90433 06
-0.4	2.66965 12	2.80922 90	2.94760 48	3.08487 85	3.22113 69
-0.3	3.04839 74	3.19893 69	3.34788 42	3.49537 62	3.64153 18
-0.2	3.60240 56	3.76688 56	3.92926 25	4.08972 39	4.24843 13
-0.1	4.57578 31	4.76017 79	4.94174 41	5.12074 20	5.29739 42

x_0 for ${}_1F_1[a; b; x] = 0$

	b = 2·1	2·2	2·3	2·4	2·5
a					
-4·0	0·78918 67	0·83567 52	0·88273 73	0·93035 39	0·97850 72
-3·9	0·80558 02	0·85289 69	0·90078 72	0·94923 18	0·99821 27
-3·8	0·82267 95	0·87085 49	0·91960 35	0·96890 59	1·01874 38
-3·7	0·84053 19	0·88959 81	0·93923 69	0·98942 86	1·04015 46
-3·6	0·85918 90	0·90918 03	0·95974 29	1·01085 71	1·06250 41
-3·5	0·87870 75	0·92965 99	0·98118 21	1·03325 40	1·08585 68
-3·4	0·89914 95	0·95110 13	1·00362 09	1·05668 79	1·11028 32
-3·3	0·92058 33	0·97357 53	1·02713 24	1·08123 40	1·13586 11
-3·2	0·94308 43	0·99715 97	1·05179 70	1·10697 54	1·16267 56
-3·1	0·96673 59	1·02194 09	1·07770 39	1·13400 38	1·19082 13
-3·0	0·99163 05	1·04801 44	1·10495 15	1·16242 06	1·22040 23
-2·9	1·01787 09	1·07548 63	1·13364 93	1·19233 86	1·25153 46
-2·8	1·04557 18	1·10447 52	1·16391 94	1·22388 34	1·28434 72
-2·7	1·07486 21	1·13511 38	1·19589 85	1·25719 53	1·31898 44
-2·6	1·10588 61	1·16755 08	1·22973 98	1·29243 21	1·35560 78
-2·5	1·13880 67	1·20195 43	1·26561 61	1·32977 11	1·39439 94
-2·4	1·17380 84	1·23851 41	1·30372 29	1·36941 29	1·43556 49
-2·3	1·21110 15	1·27744 71	1·34428 23	1·41158 57	1·47933 81
-2·2	1·25092 61	1·31900 00	1·38754 82	1·45654 96	1·52598 53
-2·1	1·29355 86	1·36345 68	1·43381 16	1·50460 32	1·57581 22
-2·0	1·33931 83	1·41114 56	1·48340 96	1·55609 11	1·62917 13
-1·9	1·38857 67	1·46244 76	1·53673 27	1·61141 31	1·68647 14
-1·8	1·44176 92	1·51780 94	1·59423 82	1·67103 75	1·74819 03
-1·7	1·49940 98	1·57775 77	1·65646 51	1·73551 48	1·81489 10
-1·6	1·56211 01	1·64291 89	1·72405 41	1·80549 97	1·88724 10
-1·5	1·63060 53	1·71404 53	1·79777 38	1·88177 66	1·96604 02
-1·4	1·70578 82	1·79204 96	1·87855 65	1·96529 63	2·05225 75
-1·3	1·78875 61	1·87805 29	1·96754 58	2·05722 47	2·14708 00
-1·2	1·88087 64	1·97345 07	2·06616 46	2·15901 05	2·25198 22
-1·1	1·98388 02	2·08000 82	2·17621 09	2·27248 44	2·36882 48
-1·0	2·10000 00	2·20000 00	2·30000 00	2·40000 00	2·50000 00
-0·9	2·23217 88	2·33642 18	2·44057 81	2·54465 28	2·64865 05
-0·8	2·38440 15	2·49332 55	2·60206 24	2·71062 30	2·81901 74
-0·7	2·56224 14	2·67637 40	2·79020 13	2·90374 20	3·01701 27
-0·6	2·77382 48	2·89381 47	3·01336 04	3·13248 94	3·25122 67
-0·5	3·03165 65	3·15832 83	3·28438 96	3·40987 98	3·53483 38
-0·4	3·35645 66	3·49090 48	3·62454 16	3·75742 03	3·88958 89
-0·3	3·78645 45	3·93023 60	4·07295 67	4·21468 86	4·35549 59
-0·2	4·40552 52	4·56112 85	4·71534 98	4·86828 53	5·02002 05
-0·1	5·47189 41	5·64441 02	5·81509 01	5·98406 45	6·15144 93

APPENDIX II

Table of ${}_1F_1[a; b; x]$ over the range

$$a = -1.0(0.1)1.0,$$

$$b = 0.1(0.1)1.0,$$

$$x = 0.1(0.1)10.0$$

$x=0.1$

$$10^{-t} {}_1F_1[a; b; x]$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t	t	t	t	t	t	t	t	t	t	t
-1.0	o	o.00000 00	o	o.50000 00	o	o.66666 67	o	o.75000 00	o	o.80000 00	o
-0.9	o	o.09583 64	o	o.54809 32	o	o.69882 74	o	o.77418 39	o	o.81939 11	o
-0.8	o	o.19258 62	o	o.59660 50	o	o.73124 58	o	o.79854 72	o	o.83891 60	o
-0.7	o	o.29025 38	o	o.64553 72	o	o.76392 27	o	o.82309 06	o	o.85857 53	o
-0.6	o	o.38884 37	o	o.69489 19	o	o.79685 95	o	o.84781 47	o	o.87836 96	o
-0.5	o	o.48836 02	o	o.74467 09	o	o.83005 72	o	o.87272 05	o	o.89829 94	o
-0.4	o	o.58880 79	o	o.79487 63	o	o.86351 70	o	o.89780 86	o	o.91836 52	o
-0.3	o	o.69019 13	o	o.84550 99	o	o.89724 00	o	o.92307 98	o	o.93856 76	o
-0.2	o	o.79251 47	o	o.89657 37	o	o.93122 74	o	o.94853 50	o	o.95890 72	o
-0.1	o	o.89578 28	o	o.94806 98	o	o.96548 03	o	o.97417 48	o	o.97938 45	o
0.0	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o
0.1	o	1.10517 09	o	1.05236 64	o	1.03478 75	o	1.02601 14	o	1.02075 43	o
0.2	o	1.21130 01	o	1.10517 09	o	1.06984 41	o	1.05220 99	o	1.04164 80	o
0.3	o	1.31839 20	o	1.15841 56	o	1.10517 09	o	1.07859 61	o	1.06268 16	o
0.4	o	1.42645 14	o	1.21210 24	o	1.14076 91	o	1.10517 09	o	1.08385 58	o
0.5	o	1.53548 28	o	1.26623 34	o	1.17663 99	o	1.13193 50	o	1.10517 09	o
0.6	o	1.64549 07	o	1.32081 05	o	1.21278 44	o	1.15888 93	o	1.12662 77	o
0.7	o	1.75647 99	o	1.37583 59	o	1.24920 38	o	1.18603 45	o	1.14822 66	o
0.8	o	1.86845 49	o	1.43131 14	o	1.28589 94	o	1.21337 14	o	1.16996 83	o
0.9	o	1.98142 05	o	1.48723 92	o	1.32287 23	o	1.24090 08	o	1.19185 33	o
1.0	o	2.09538 12	o	1.54362 12	o	1.36012 38	o	1.26862 36	o	1.21388 23	o
b =		0.6		0.7		0.8		0.9		1.0	
a	t	t	t	t	t	t	t	t	t	t	t
-1.0	o	o.83333 33	o	o.85714 29	o	o.87500 00	o	o.88888 89	o	o.90000 00	o
-0.9	o	o.84952 45	o	o.87104 52	o	o.88718 33	o	o.89973 35	o	o.90977 22	o
-0.8	o	o.86582 03	o	o.88503 19	o	o.89943 64	o	o.91063 67	o	o.91959 46	o
-0.7	o	o.88222 11	o	o.89910 33	o	o.91175 94	o	o.92159 89	o	o.92946 73	o
-0.6	o	o.89872 72	o	o.91325 96	o	o.92415 25	o	o.93262 01	o	o.93939 05	o
-0.5	o	o.91533 91	o	o.92750 12	o	o.93661 62	o	o.94370 06	o	o.94936 44	o
-0.4	o	o.93205 72	o	o.94182 85	o	o.94915 05	o	o.95484 07	o	o.95938 91	o
-0.3	o	o.94888 19	o	o.95624 16	o	o.96175 58	o	o.96604 04	o	o.96946 49	o
-0.2	o	o.96581 37	o	o.97074 11	o	o.97443 23	o	o.97730 01	o	o.97959 19	o
-0.1	o	o.98285 29	o	o.98532 71	o	o.98718 03	o	o.98861 99	o	o.98977 02	o
0.0	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o
0.1	o	1.01725 53	o	1.01476 01	o	1.01289 17	o	1.01144 07	o	1.01028 15	o
0.2	o	1.03461 94	o	1.02960 78	o	1.02585 56	o	1.02294 21	o	1.02061 50	o
0.3	o	1.05209 25	o	1.04454 34	o	1.03889 21	o	1.03450 45	o	1.03100 04	o
0.4	o	1.06967 52	o	1.05956 72	o	1.05200 13	o	1.04612 80	o	1.04143 81	o
0.5	o	1.08736 79	o	1.07467 94	o	1.06518 35	o	1.05781 30	o	1.05192 82	o
0.6	o	1.10517 09	o	1.08988 06	o	1.07843 90	o	1.06955 95	o	1.06247 09	o
0.7	o	1.12308 47	o	1.10517 09	o	1.09176 81	o	1.08136 79	o	1.07306 64	o
0.8	o	1.14110 98	o	1.12055 07	o	1.10517 09	o	1.09323 83	o	1.08371 47	o
0.9	o	1.15924 65	o	1.13602 05	o	1.11864 78	o	1.10517 09	o	1.09441 62	o
1.0	o	1.17749 53	o	1.15158 03	o	1.13219 91	o	1.11176 60	o	1.10517 09	o

$$10^{-t} {}_1F_1[a; b; x]$$

$$x = 0.2$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	0	-1.00000 00	0	0.00000 00	0	0.33333 33	0	0.50000 00	0	0.60000 00	0
-0.9	0	-0.81695 50	0	0.09224 15	0	0.39523 26	0	0.54668 44	0	0.63752 74	0
-0.8	0	-0.63023 97	0	0.18616 46	0	0.45816 63	0	0.59408 89	0	0.67559 24	0
-0.7	0	-0.43981 80	0	0.28178 50	0	0.52214 37	0	0.64221 97	0	0.71419 93	0
-0.6	0	-0.24565 34	0	0.37911 86	0	0.58717 41	0	0.69108 31	0	0.75335 26	0
-0.5	0	-0.04770 94	0	0.47818 14	0	0.65326 69	0	0.74068 53	0	0.79305 68	0
-0.4	0	0.15405 09	0	0.57898 95	0	0.72043 16	0	0.79103 26	0	0.83331 64	0
-0.3	0	0.35966 45	0	0.68155 91	0	0.78867 76	0	0.84213 13	0	0.87413 60	0
-0.2	0	0.56916 88	0	0.78590 64	0	0.85801 46	0	0.89398 78	0	0.91552 01	0
-0.1	0	0.78260 14	0	0.89204 79	0	0.92845 22	0	0.94660 86	0	0.95747 32	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	1.22140 28	0	1.10977 94	0	1.07266 78	0	1.05416 86	0	1.04310 51	0
0.2	0	1.44684 79	0	1.22140 28	0	1.14646 54	0	1.10912 08	0	1.08679 33	0
0.3	0	1.67637 41	0	1.33488 69	0	1.22140 28	0	1.16486 34	0	1.13106 91	0
0.4	0	1.91002 01	0	1.45024 87	0	1.29748 97	0	1.22140 28	0	1.17593 74	0
0.5	0	2.14782 49	0	1.56750 53	0	1.37473 61	0	1.27874 56	0	1.22140 28	0
0.6	0	2.38982 79	0	1.68667 37	0	1.45315 23	0	1.33689 87	0	1.26747 01	0
0.7	0	2.63606 85	0	1.80777 11	0	1.53274 81	0	1.39586 86	0	1.31414 41	0
0.8	0	2.88658 67	0	1.93081 50	0	1.61353 38	0	1.45566 22	0	1.36142 97	0
0.9	0	3.14142 24	0	2.05582 28	0	1.69551 97	0	1.51628 63	0	1.40933 17	0
1.0	0	3.40061 61	0	2.18281 20	0	1.77871 60	0	1.57774 76	0	1.45785 50	0

b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	0.66666 67	0	0.71428 57	0	0.75000 00	0	0.77777 78	0	0.80000 00	0
-0.9	0	0.69807 05	0	0.74130 23	0	0.77371 63	0	0.79892 00	0	0.81907 74	0
-0.8	0	0.72989 42	0	0.76865 74	0	0.79771 24	0	0.82029 78	0	0.83835 61	0
-0.7	0	0.76214 10	0	0.79635 37	0	0.82199 02	0	0.84191 27	0	0.85783 75	0
-0.6	0	0.79481 43	0	0.82439 37	0	0.84655 19	0	0.86376 64	0	0.87752 30	0
-0.5	0	0.82791 75	0	0.85278 01	0	0.87139 96	0	0.88586 07	0	0.89741 40	0
-0.4	0	0.86145 39	0	0.88151 55	0	0.89653 52	0	0.90819 73	0	0.91751 18	0
-0.3	0	0.89542 69	0	0.91060 26	0	0.92196 09	0	0.93077 78	0	0.93781 79	0
-0.2	0	0.92984 00	0	0.94004 39	0	0.94767 89	0	0.95360 39	0	0.95833 37	0
-0.1	0	0.96469 65	0	0.96984 22	0	0.97369 12	0	0.97667 74	0	0.97906 06	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	1.03575 39	0	1.03052 02	0	1.02660 74	0	1.02357 34	0	1.02115 34	0
0.2	0	1.07196 17	0	1.06140 54	0	1.05351 55	0	1.04739 95	0	1.04252 22	0
0.3	0	1.10862 70	0	1.09265 84	0	1.08072 66	0	1.07147 98	0	1.06410 78	0
0.4	0	1.14575 32	0	1.12428 18	0	1.10824 28	0	1.09581 63	0	1.08591 18	0
0.5	0	1.18334 39	0	1.15627 85	0	1.13606 63	0	1.12041 06	0	1.10793 56	0
0.6	0	1.22140 28	0	1.18865 12	0	1.16419 94	0	1.14526 47	0	1.13018 06	0
0.7	0	1.25993 33	0	1.22140 28	0	1.19264 41	0	1.17038 02	0	1.15264 83	0
0.8	0	1.29893 91	0	1.25453 59	0	1.22140 28	0	1.19575 89	0	1.17534 02	0
0.9	0	1.33842 39	0	1.28805 34	0	1.25047 76	0	1.22140 28	0	1.19825 79	0
1.0	0	1.37839 12	0	1.32195 81	0	1.27987 08	0	1.24731 35	0	1.22140 28	0

$x=0.3$

$$10^{-t} {}_1F_1[a; b; x]$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	0	-2.00000 00	0	-0.50000 00	0	0.00000 00	0	0.25000 00	0	0.40000 00	0
-0.9	0	-1.73884 94	0	-0.36776 22	0	0.08909 40	0	0.31742 04	0	0.45435 13	0
-0.8	0	-1.46940 36	0	-0.23172 48	0	0.18052 49	0	0.38646 74	0	0.50991 65	0
-0.7	0	-1.19153 81	0	-0.09183 33	0	0.27432 46	0	0.45716 24	0	0.56671 10	0
-0.6	0	-0.90512 71	0	0.05196 71	0	0.37052 56	0	0.52952 68	0	0.62475 02	0
-0.5	0	-0.61004 34	0	0.19973 19	0	0.46916 02	0	0.60358 25	0	0.68404 95	0
-0.4	0	-0.30615 88	0	0.35151 71	0	0.57026 15	0	0.67935 10	0	0.74462 45	0
-0.3	0	0.00665 63	0	0.50737 92	0	0.67386 24	0	0.75685 47	0	0.80649 11	0
-0.2	0	0.32853 28	0	0.66737 52	0	0.77999 66	0	0.83611 58	0	0.86966 51	0
-0.1	0	0.65960 29	0	0.83156 28	0	0.88869 78	0	0.91715 66	0	0.93416 27	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	1.34985 88	0	1.17274 56	0	1.11393 77	0	1.08466 87	0	1.06719 33	0
0.2	0	1.70931 54	0	1.34985 88	0	1.23054 56	0	1.17118 59	0	1.13575 92	0
0.3	0	2.07850 72	0	1.53139 94	0	1.34985 88	0	1.25957 48	0	1.20571 42	0
0.4	0	2.45757 27	0	1.71742 78	0	1.47191 26	0	1.34985 88	0	1.27707 51	0
0.5	0	2.84665 23	0	1.90800 49	0	1.59674 26	0	1.44206 18	0	1.34985 88	0
0.6	0	3.24588 71	0	2.10319 22	0	1.72438 49	0	1.53620 75	0	1.42408 24	0
0.7	0	3.65542 00	0	2.30305 18	0	1.85487 58	0	1.63232 02	0	1.49976 30	0
0.8	0	4.07539 50	0	2.50764 63	0	1.98825 19	0	1.73042 41	0	1.57691 80	0
0.9	0	4.50595 77	0	2.71703 90	0	2.12455 03	0	1.83054 38	0	1.65556 49	0
1.0	0	4.94725 50	0	2.93129 36	0	2.26380 82	0	1.93270 41	0	1.73572 13	0
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	0.50000 00	0	0.57142 86	0	0.62500 00	0	0.66666 67	0	0.70000 00	0
-0.9	0	0.54559 46	0	0.61073 76	0	0.65957 22	0	0.69753 80	0	0.72789 77	0
-0.8	0	0.59213 73	0	0.65081 10	0	0.69477 60	0	0.72894 09	0	0.75624 98	0
-0.7	0	0.63963 94	0	0.69165 79	0	0.73061 84	0	0.76088 12	0	0.78506 11	0
-0.6	0	0.68811 25	0	0.73328 70	0	0.76710 65	0	0.79336 46	0	0.81433 62	0
-0.5	0	0.73756 83	0	0.77570 75	0	0.80424 74	0	0.82639 70	0	0.84408 00	0
-0.4	0	0.78801 84	0	0.81892 83	0	0.84204 84	0	0.85998 42	0	0.87429 73	0
-0.3	0	0.83947 46	0	0.86295 87	0	0.88051 68	0	0.89413 21	0	0.90499 31	0
-0.2	0	0.89194 89	0	0.90780 79	0	0.91965 99	0	0.92884 67	0	0.93617 21	0
-0.1	0	0.94545 33	0	0.95348 52	0	0.95948 52	0	0.96413 40	0	0.96783 94	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	1.05560 11	0	1.04736 18	0	1.04121 19	0	1.03645 08	0	1.03265 88	0
0.2	0	1.11226 90	0	1.09558 01	0	1.08312 85	0	1.07349 27	0	1.06582 10	0
0.3	0	1.17001 62	0	1.14466 45	0	1.12575 75	0	1.11113 16	0	1.09949 16	0
0.4	0	1.22885 51	0	1.19462 48	0	1.16910 65	0	1.14937 40	0	1.13367 58	0
0.5	0	1.28879 84	0	1.24547 07	0	1.21318 32	0	1.18822 61	0	1.16837 88	0
0.6	0	1.34985 88	0	1.29721 20	0	1.25799 56	0	1.22769 42	0	1.20360 57	0
0.7	0	1.41204 93	0	1.34985 88	0	1.30355 15	0	1.26778 47	0	1.23936 18	0
0.8	0	1.47538 27	0	1.40342 10	0	1.34985 88	0	1.30850 41	0	1.27565 25	0
0.9	0	1.53987 22	0	1.45790 88	0	1.39692 56	0	1.34985 88	0	1.31248 30	0
1.0	0	1.60553 08	0	1.51333 23	0	1.44475 99	0	1.39185 54	0	1.34985 88	0

$$10^{-t} {}_1F_1[a; b; x]$$

$$x=0.4$$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	0	-3.000000	00	0	-1.000000	00	0	-0.333333	33	0	0.000000	00	0	0.200000	00
-0.9	0	-2.67035	54	0	-0.83213	94	0	-0.21971	83	0	0.08630	57	0	0.26980	11
-0.8	0	-2.32590	02	0	-0.65749	60	0	-0.10193	21	0	0.17551	44	0	0.34176	83
-0.7	0	-1.96633	24	0	-0.47593	79	0	0.02010	24	0	0.26767	75	0	0.41593	86
-0.6	0	-1.59134	63	0	-0.28733	19	0	0.14646	36	0	0.36284	71	0	0.49234	91
-0.5	0	-1.20063	20	0	-0.09154	28	0	0.27723	08	0	0.46107	60	0	0.57103	76
-0.4	0	-0.79387	53	0	0.11156	62	0	0.41248	42	0	0.56241	75	0	0.65204	22
-0.3	0	-0.37075	83	0	0.32213	37	0	0.55230	51	0	0.66692	56	0	0.73540	15
-0.2	0	0.06904	15	0	0.54030	02	0	0.69677	56	0	0.77465	51	0	0.82115	45
-0.1	0	0.52585	07	0	0.76620	76	0	0.84597	92	0	0.88566	12	0	0.90934	07
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	0	1.49182	47	0	1.24182	32	0	1.15892	34	0	1.11772	81	0	1.09317	29
0.2	0	2.00166	43	0	1.49182	47	0	1.32283	59	0	1.23890	27	0	1.18890	02
0.3	0	2.52986	27	0	1.75015	41	0	1.49182	47	0	1.36358	21	0	1.28722	34
0.4	0	3.07676	82	0	2.01696	26	0	1.66597	85	0	1.49182	47	0	1.38818	41
0.5	0	3.64273	38	0	2.29240	35	0	1.84538	67	0	1.62369	00	0	1.49182	47
0.6	0	4.22811	67	0	2.57663	20	0	2.03014	00	0	1.75923	82	0	1.59818	80
0.7	0	4.83327	91	0	2.86980	51	0	2.22033	03	0	1.89852	99	0	1.70731	73
0.8	0	5.45858	73	0	3.17208	17	0	2.41605	03	0	2.04162	67	0	1.81925	64
0.9	0	6.10441	27	0	3.48362	30	0	2.61739	39	0	2.18859	07	0	1.93404	94
1.0	0	6.77113	12	0	3.80459	19	0	2.82445	63	0	2.33948	51	0	2.05174	12

b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	0	0.333333	33	0	0.42857	14	0	0.500000	00	0	0.555555	56	0	0.600000	00
-0.9	0	0.39205	09	0	0.47931	55	0	0.54472	28	0	0.59556	45	0	0.63621	43
-0.8	0	0.45245	97	0	0.53142	34	0	0.59057	21	0	0.63652	15	0	0.67323	89
-0.7	0	0.51458	76	0	0.58491	64	0	0.63756	49	0	0.67844	05	0	0.71108	52
-0.6	0	0.57846	24	0	0.63981	62	0	0.68571	83	0	0.72133	55	0	0.74976	48
-0.5	0	0.64411	23	0	0.69614	47	0	0.73504	98	0	0.76522	04	0	0.78928	92
-0.4	0	0.71156	59	0	0.75392	39	0	0.78557	69	0	0.81010	97	0	0.82967	03
-0.3	0	0.78085	21	0	0.81317	64	0	0.83731	74	0	0.85601	77	0	0.87091	98
-0.2	0	0.85200	01	0	0.87392	46	0	0.89028	93	0	0.90295	89	0	0.91304	99
-0.1	0	0.92503	95	0	0.93619	14	0	0.94451	07	0	0.95094	80	0	0.95607	25
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	0	1.07691	20	0	1.06537	37	0	1.05677	57	0	1.05012	98	0	1.04484	47
0.2	0	1.15580	59	0	1.13233	62	0	1.11485	65	0	1.10135	26	0	1.09061	91
0.3	0	1.23671	28	0	1.20091	13	0	1.17426	15	0	1.15368	38	0	1.13733	59
0.4	0	1.31966	37	0	1.27112	31	0	1.23500	97	0	1.20713	88	0	1.18500	76
0.5	0	1.40469	04	0	1.34299	62	0	1.29712	04	0	1.26173	33	0	1.23364	74
0.6	0	1.49182	47	0	1.41655	50	0	1.36061	33	0	1.31748	31	0	1.28326	80
0.7	0	1.58109	90	0	1.49182	47	0	1.42550	81	0	1.37440	41	0	1.33388	28
0.8	0	1.67254	59	0	1.56883	04	0	1.49182	47	0	1.43251	25	0	1.38550	48
0.9	0	1.76619	84	0	1.64759	75	0	1.55958	33	0	1.49182	47	0	1.43814	76
1.0	0	1.86208	99	0	1.72815	18	0	1.62880	44	0	1.55235	70	0	1.49182	47

$x=0.5$ $10^{-t} {}_1F_1[a; b; x]$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	0	-4.00000 00	0	-1.50000 00	0	-0.66666 67	0	-0.25000 00	0	0.00000 00	0
-0.9	0	-3.61201 86	0	-1.30112 70	0	-0.53134 25	0	-0.14675 13	0	0.08381 14	0
-0.8	0	-3.20079 90	0	-1.09161 33	0	-0.38947 59	0	-0.03894 99	0	0.17101 97	0
-0.7	0	-2.76573 85	0	-0.87119 62	0	-0.24091 28	0	0.07350 67	0	0.26169 80	0
-0.6	0	-2.30622 47	0	-0.63960 86	0	-0.08549 65	0	0.19072 26	0	0.35592 08	0
-0.5	0	-1.82163 45	0	-0.39657 94	0	0.07693 19	0	0.31280 36	0	0.45376 36	0
-0.4	0	-1.31133 45	0	-0.14183 26	0	0.24653 41	0	0.43985 71	0	0.55530 31	0
-0.3	0	-0.77468 10	0	0.12491 18	0	0.42347 41	0	0.57199 21	0	0.66061 70	0
-0.2	0	-0.21101 94	0	0.40393 84	0	0.60791 85	0	0.70931 90	0	0.76978 42	0
-0.1	0	0.38031 55	0	0.69553 66	0	0.80003 65	0	0.85195 04	0	0.88288 48	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	1.64872 13	0	1.31762 72	0	1.20798 34	0	1.15358 36	0	1.12121 22	0
0.2	0	2.32717 78	0	1.64872 13	0	1.42416 39	0	1.31281 87	0	1.24660 50	0
0.3	0	3.03607 92	0	1.99359 02	0	1.64872 13	0	1.47782 43	0	1.37626 32	0
0.4	0	3.77614 69	0	2.35254 68	0	1.88183 81	0	1.64872 13	0	1.51027 29	0
0.5	0	4.54811 36	0	2.72590 86	0	2.12369 98	0	1.82563 24	0	1.64872 13	0
0.6	0	5.35272 38	0	3.11399 83	0	2.37449 45	0	2.00868 23	0	1.79169 69	0
0.7	0	6.19073 39	0	3.51714 35	0	2.63441 32	0	2.19799 70	0	1.93928 94	0
0.8	0	7.06291 26	0	3.93567 68	0	2.90364 98	0	2.39370 49	0	2.09159 01	0
0.9	0	7.97004 04	0	4.36993 59	0	3.18240 09	0	2.59593 60	0	2.24869 11	0
1.0	0	8.91291 03	0	4.82026 39	0	3.47086 63	0	2.80482 21	0	2.41068 61	0

b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	0.16666 67	0	0.28571 43	0	0.37500 00	0	0.44444 44	0	0.50000 00	0
-0.9	0	0.23739 04	0	0.34699 84	0	0.42913 82	0	0.49297 53	0	0.54400 72	0
-0.8	0	0.31076 59	0	0.41042 05	0	0.48504 22	0	0.54299 22	0	0.58928 44	0
-0.7	0	0.38684 84	0	0.47602 32	0	0.54274 57	0	0.59452 27	0	0.63585 42	0
-0.6	0	0.46569 33	0	0.54384 96	0	0.60228 31	0	0.64759 46	0	0.68373 95	0
-0.5	0	0.54735 74	0	0.61394 34	0	0.66368 92	0	0.70223 61	0	0.73296 36	0
-0.4	0	0.63189 79	0	0.68634 91	0	0.72699 92	0	0.75847 57	0	0.78355 00	0
-0.3	0	0.71937 30	0	0.76111 17	0	0.79224 89	0	0.81634 24	0	0.83552 26	0
-0.2	0	0.80984 17	0	0.83827 67	0	0.85947 43	0	0.87586 54	0	0.88890 54	0
-0.1	0	0.90336 38	0	0.91789 06	0	0.92871 23	0	0.93707 46	0	0.94372 30	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	1.09981 19	0	1.08465 27	0	1.07337 51	0	1.06467 21	0	1.05776 16	0
0.2	0	1.20286 18	0	1.17189 67	0	1.14887 58	0	1.13112 17	0	1.11703 33	0
0.3	0	1.30921 31	0	1.26178 10	0	1.22654 08	0	1.19938 02	0	1.17784 06	0
0.4	0	1.41892 99	0	1.35435 51	0	1.30640 94	0	1.26947 93	0	1.24020 96	0
0.5	0	1.53207 73	0	1.44966 91	0	1.38852 11	0	1.34145 10	0	1.30416 68	0
0.6	0	1.64872 13	0	1.54777 40	0	1.47291 64	0	1.41532 79	0	1.36973 88	0
0.7	0	1.76892 87	0	1.64872 13	0	1.55963 60	0	1.49114 29	0	1.43695 27	0
0.8	0	1.89276 74	0	1.75256 33	0	1.64872 13	0	1.56892 95	0	1.50583 59	0
0.9	0	2.02030 62	0	1.85935 29	0	1.74021 40	0	1.64872 13	0	1.57641 61	0
1.0	0	2.15161 47	0	1.96914 38	0	1.83415 67	0	1.73055 26	0	1.64872 13	0

$$10^{-t} {}_1F_1[a; b; x]$$

$$x=0.6$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	0	-5.00000 00	0	-2.00000 00	0	-1.00000 00	0	-0.50000 00	0	-0.20000 00	0
-0.9	0	-4.56442 36	0	-1.77497 83	0	-0.84592 65	0	-0.38184 85	0	-0.10368 71	0
-0.8	0	-4.09525 03	0	-1.53457 51	0	-0.68239 71	0	-0.25711 78	0	-0.00246 61	0
-0.7	0	-3.59141 57	0	-1.27832 65	0	-0.50913 97	0	-0.12562 70	0	0.10379 24	0
-0.6	0	-3.05183 34	0	-1.00575 96	0	-0.32587 73	0	0.01280 81	0	0.21521 99	0
-0.5	0	-2.47539 54	0	-0.71639 21	0	-0.13232 74	0	0.15837 51	0	0.33195 02	0
-0.4	0	-1.86097 11	0	-0.40973 24	0	0.07179 79	0	0.31126 51	0	0.45411 97	0
-0.3	0	-1.20740 73	0	-0.08527 91	0	0.28679 17	0	0.47167 27	0	0.58186 70	0
-0.2	0	-0.51352 78	0	0.25747 85	0	0.51295 29	0	0.63979 59	0	0.71533 33	0
-0.1	0	0.22186 69	0	0.61906 13	0	0.75058 57	0	0.81583 66	0	0.85466 22	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	1.82211 88	0	1.40083 54	0	1.26151 16	0	1.19249 52	0	1.15149 54	0
0.2	0	2.68949 49	0	1.82211 88	0	1.53544 21	0	1.39353 50	0	1.30929 97	0
0.3	0	3.60342 49	0	2.26441 15	0	1.82211 88	0	1.60333 61	0	1.47356 68	0
0.4	0	4.56523 01	0	2.72828 58	0	2.12187 52	0	1.82211 88	0	1.64445 34	0
0.5	0	5.57625 77	0	3.21432 45	0	2.43505 08	0	2.05010 75	0	1.82211 88	0
0.6	0	6.63788 04	0	3.72312 11	0	2.76199 12	0	2.28753 06	0	2.00672 51	0
0.7	0	7.75149 75	0	4.25528 05	0	3.10304 83	0	2.53462 03	0	2.19843 71	0
0.8	0	8.91853 48	0	4.81141 85	0	3.45858 04	0	2.79161 30	0	2.39742 24	0
0.9	0	10.14044 47	0	5.39216 24	0	3.82895 20	0	3.05874 92	0	2.60385 15	0
1.0	0	11.41870 77	0	5.99815 10	0	4.21453 43	0	3.33627 38	0	2.81789 78	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	1.12443 77	0	1.10530 38	0	1.09109 32	0	1.08014 45	0	1.07146 45	0
0.2	0	1.25375 33	0	1.21450 50	0	1.18537 84	0	1.16295 44	0	1.14519 01	0
0.3	0	1.38806 15	0	1.32769 20	0	1.28292 55	0	1.24848 64	0	1.22122 33	0
0.4	0	1.52747 91	0	1.44495 47	0	1.38380 56	0	1.33679 79	0	1.29961 13	0
0.5	0	1.67212 47	0	1.56638 46	0	1.48809 10	0	1.42794 70	0	1.38040 19	0
0.6	0	1.82211 88	0	1.69207 45	0	1.59585 51	0	1.52199 31	0	1.46364 36	0
0.7	0	1.97758 41	0	1.82211 88	0	1.70717 25	0	1.61899 63	0	1.54938 57	0
0.8	0	2.13864 53	0	1.95661 34	0	1.82211 88	0	1.71901 75	0	1.63767 83	0
0.9	0	2.30542 91	0	2.09565 57	0	1.94077 10	0	1.82211 88	0	1.72857 22	0
1.0	0	2.47806 43	0	2.23934 48	0	2.06320 72	0	1.92836 31	0	1.82211 88	0

$x=0.7$

$$10^{-t} {}_1F_1[a; b; x]$$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	0	-6.000000	00	0	-2.500000	00	0	-1.333333	33	0	-0.750000	00	0	-0.400000	00
-0.9	0	-5.52819	79	0	-2.25396	47	0	-1.16362	83	0	-0.61909	03	0	-0.29276	88
-0.8	0	-5.01049	23	0	-1.98691	64	0	-0.98100	71	0	-0.47919	49	0	-0.17883	48
-0.7	0	-4.44515	47	0	-1.69810	26	0	-0.78502	86	0	-0.33002	06	0	-0.05798	87
-0.6	0	-3.83041	49	0	-1.38675	31	0	-0.57524	18	0	-0.17126	79	0	0.06998	32
-0.5	0	-3.16446	06	0	-1.05207	99	0	-0.35118	57	0	-0.00263	08	0	0.20529	90
-0.4	0	-2.44543	68	0	-0.69327	71	0	-0.11238	89	0	0.17620	33	0	0.34818	16
-0.3	0	-1.67144	46	0	-0.30952	03	0	0.14163	03	0	0.36555	38	0	0.49885	85
-0.2	0	-0.84054	10	0	0.10003	36	0	0.41136	43	0	0.56574	68	0	0.65756	17
-0.1	0	0.04926	24	0	0.53624	65	0	0.69731	61	0	0.77711	55	0	0.82452	82
0.0	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00
0.1	0	2.01375	27	0	1.49219	50	0	1.31994	11	0	1.23474	77	0	1.18422	38
0.2	0	3.09264	92	0	2.01375	27	0	1.65767	60	0	1.48171	31	0	1.37745	14
0.3	0	4.23886	64	0	2.56561	44	0	2.01375	27	0	1.74125	83	0	1.57993	98
0.4	0	5.45463	05	0	3.14874	21	0	2.38873	10	0	2.01375	27	0	1.79195	11
0.5	0	6.74221	79	0	3.76411	90	0	2.78318	26	0	2.29957	36	0	2.01375	27
0.6	0	8.10395	55	0	4.41274	94	0	3.19769	11	0	2.59910	58	0	2.24561	75
0.7	0	9.54222	24	0	5.09565	95	0	3.63285	27	0	2.91274	21	0	2.48782	35
0.8	0	11.05945	01	0	5.81389	76	0	4.08927	57	0	3.24088	34	0	2.74065	46
0.9	0	12.65812	36	0	6.56853	43	0	4.56758	14	0	3.58393	85	0	3.00440	00
1.0	0	14.34078	25	0	7.36066	30	0	5.06840	37	0	3.94232	46	0	3.27935	49
b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	0	-0.16666	67	0	0.00000	00	0	0.12500	00	0	0.22222	22	0	0.30000	00
-0.9	0	-0.07549	15	0	0.07951	66	0	0.19563	47	0	0.28584	61	0	0.35793	69
-0.8	0	0.02091	55	0	0.16325	02	0	0.26975	16	0	0.35240	02	0	0.41837	74
-0.7	0	0.12271	09	0	0.25132	21	0	0.34744	70	0	0.42196	25	0	0.48138	58
-0.6	0	0.23005	45	0	0.34385	60	0	0.42881	90	0	0.49461	25	0	0.54702	76
-0.5	0	0.34310	95	0	0.44097	79	0	0.51396	77	0	0.57043	13	0	0.61536	94
-0.4	0	0.46204	24	0	0.54281	65	0	0.60299	50	0	0.64950	14	0	0.68647	91
-0.3	0	0.58702	28	0	0.64950	29	0	0.69600	49	0	0.73190	69	0	0.76042	60
-0.2	0	0.71822	42	0	0.76117	10	0	0.79310	34	0	0.81773	33	0	0.83728	05
-0.1	0	0.85582	31	0	0.87795	70	0	0.89439	84	0	0.90706	81	0	0.91711	41
0.0	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00
0.1	0	1.15093	86	0	1.12744	18	0	1.11002	02	0	1.09661	96	0	1.08601	24
0.2	0	1.30882	66	0	1.26042	67	0	1.22457	33	0	1.19701	89	0	1.17522	70
0.3	0	1.47385	50	0	1.39910	20	0	1.34377	57	0	1.30129	20	0	1.26772	07
0.4	0	1.64621	90	0	1.54361	79	0	1.46774	58	0	1.40953	43	0	1.36357	19
0.5	0	1.82611	74	0	1.69412	73	0	1.59660	44	0	1.52184	32	0	1.46286	04
0.6	0	2.01375	27	0	1.85078	59	0	1.73047	45	0	1.63831	77	0	1.56566	73
0.7	0	2.20933	17	0	2.01375	27	0	1.86948	15	0	1.75905	87	0	1.67207	52
0.8	0	2.41306	50	0	2.18318	94	0	2.01375	27	0	1.88416	89	0	1.78216	81
0.9	0	2.62516	73	0	2.35926	09	0	2.16341	82	0	2.01375	27	0	1.89603	16
1.0	0	2.84585	75	0	2.54213	50	0	2.31861	02	0	2.14791	66	0	2.01375	27

$$10^{-t} {}_1F_1[a; b; x]$$

$$x = 0.8$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	0	-7.00000 00	0	-3.00000 00	0	-1.66666 67	0	-1.00000 00	0	-0.60000 00	0
-0.9	0	-6.50401 49	0	-2.73837 67	0	-1.48461 68	0	-0.85858 80	0	-0.48351 24	0
-0.8	0	-5.94785 78	0	-2.44921 23	0	-1.28563 99	0	-0.70540 12	0	-0.35824 23	0
-0.7	0	-5.32888 96	0	-2.13135 83	0	-1.06906 32	0	-0.53999 28	0	-0.22387 11	0
-0.6	0	-4.64439 77	0	-1.78363 55	0	-0.83419 71	0	-0.36190 51	0	-0.08007 23	0
-0.5	0	-3.89159 56	0	-1.40483 36	0	-0.58033 36	0	-0.17066 85	0	0.07348 86	0
-0.4	0	-3.06762 06	0	-0.99371 02	0	-0.30674 70	0	0.03419 77	0	0.23715 39	0
-0.3	0	-2.16953 29	0	-0.54899 02	0	-0.01269 31	0	0.25318 65	0	0.41127 43	0
-0.2	0	-1.19431 35	0	-0.06936 56	0	0.30259 13	0	0.48680 28	0	0.59620 90	0
-0.1	0	-0.13886 31	0	0.44650 56	0	0.63988 84	0	0.73556 41	0	0.79232 54	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	2.22554 09	0	1.59252 93	0	1.38374 79	0	1.28065 33	0	1.21961 77	0
0.2	0	3.54111 04	0	2.22554 09	0	1.79197 39	0	1.57807 97	0	1.45157 27	0
0.3	0	4.95014 62	0	2.90051 91	0	2.22554 09	0	1.89284 81	0	1.69626 83	0
0.4	0	6.45617 50	0	3.61898 52	0	2.68533 25	0	2.22554 09	0	1.95411 70	0
0.5	0	8.06281 37	0	4.38249 84	0	3.17225 39	0	2.57675 45	0	2.22554 09	0
0.6	0	9.77377 18	0	5.19265 68	0	3.68723 21	0	2.94709 89	0	2.51097 18	0
0.7	0	11.59285 27	0	6.05109 78	0	4.23121 64	0	3.33719 88	0	2.81085 12	0
0.8	0	13.52395 58	0	6.95949 89	0	4.80517 86	0	3.74769 30	0	3.12563 06	0
0.9	0	15.57107 80	0	7.91957 87	0	5.41011 38	0	4.17923 55	0	3.45577 20	0
1.0	0	17.73831 61	0	8.93309 73	0	6.04704 06	0	4.63249 51	0	3.80174 73	0
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-0.33333 33	0	-0.14285 71	0	0.00000 00	0	0.11111 11	0	0.20000 00	0
-0.9	0	-0.23382 66	0	-0.05573 57	0	0.07764 68	0	0.18125 04	0	0.26402 80	0
-0.8	0	-0.12746 55	0	0.03691 02	0	0.15985 49	0	0.25522 77	0	0.33133 51	0
-0.7	0	-0.01401 16	0	0.13526 50	0	0.24677 09	0	0.33316 17	0	0.40201 88	0
-0.6	0	0.10677 92	0	0.23951 73	0	0.33854 42	0	0.41517 33	0	0.47617 88	0
-0.5	0	0.23515 64	0	0.34986 02	0	0.43532 80	0	0.50138 66	0	0.55391 71	0
-0.4	0	0.37137 59	0	0.46649 09	0	0.53727 86	0	0.59192 79	0	0.63533 77	0
-0.3	0	0.51569 93	0	0.58961 15	0	0.64455 59	0	0.68692 65	0	0.72054 67	0
-0.2	0	0.66839 41	0	0.71942 84	0	0.75732 33	0	0.78651 44	0	0.80965 26	0
-0.1	0	0.82973 43	0	0.85615 26	0	0.87574 78	0	0.89082 63	0	0.90276 61	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	1.17947 78	0	1.15119 12	0	1.13025 42	0	1.11417 60	0	1.10146 98	0
0.2	0	1.36846 08	0	1.30995 18	0	1.26668 86	0	1.23349 80	0	1.20729 31	0
0.3	0	1.56724 87	0	1.47651 22	0	1.40948 49	0	1.35811 24	0	1.31758 99	0
0.4	0	1.77614 79	0	1.65110 80	0	1.55882 92	0	1.48816 89	0	1.43248 29	0
0.5	0	1.99547 19	0	1.83397 98	0	1.71491 10	0	1.62382 02	0	1.55209 71	0
0.6	0	2.22554 09	0	2.02537 37	0	1.87792 43	0	1.76522 23	0	1.67656 00	0
0.7	0	2.46668 24	0	2.22554 09	0	2.04806 69	0	1.91253 43	0	1.80600 17	0
0.8	0	2.71923 11	0	2.43473 81	0	2.22554 09	0	2.06591 86	0	1.94055 51	0
0.9	0	2.98352 90	0	2.65322 75	0	2.41055 26	0	2.22554 09	0	2.08035 55	0
1.0	0	3.25992 56	0	2.88127 67	0	2.60331 27	0	2.39157 03	0	2.22554 09	0

$x=0.9$

$$10^{-t} {}_1F_1[a; b; x]$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	0	-8.00000 00	0	-3.50000 00	0	-2.00000 00	0	-1.25000 00	0	-0.80000 00	0
-0.9	0	-7.49259 77	0	-3.22852 60	0	-1.80907 26	0	-1.10046 06	0	-0.67600 20	0
-0.8	0	-6.90878 25	0	-2.92208 05	0	-1.59665 35	0	-0.93597 23	0	-0.54085 52	0
-0.7	0	-6.24470 96	0	-2.57899 21	0	-1.36176 42	0	-0.75588 59	0	-0.39409 65	0
-0.6	0	-5.49641 35	0	-2.19753 81	0	-1.10339 79	0	-0.55953 36	0	-0.23525 02	0
-0.5	0	-4.65980 55	0	-1.77594 43	0	-0.82051 80	0	-0.34622 85	0	-0.06382 73	0
-0.4	0	-3.73067 11	0	-1.31238 34	0	-0.51205 81	0	-0.11526 47	0	0.12067 45	0
-0.3	0	-2.70466 65	0	-0.80497 39	0	-0.17692 10	0	0.13408 38	0	0.31877 11	0
-0.2	0	-1.57731 62	0	-0.25177 88	0	0.18602 19	0	0.40256 28	0	0.53099 24	0
-0.1	0	-0.34401 01	0	0.34919 54	0	0.57793 11	0	0.69093 90	0	0.75788 25	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	2.45960 31	0	1.70274 56	0	1.45345 52	0	1.33055 47	0	1.25791 83	0
0.2	0	4.03983 23	0	2.45960 31	0	1.93955 77	0	1.68343 42	0	1.53222 60	0
0.3	0	5.74586 78	0	3.27280 51	0	2.45960 31	0	2.05949 16	0	1.82352 69	0
0.4	0	7.58304 05	0	4.14464 74	0	3.01492 28	0	2.45960 31	0	2.13244 07	0
0.5	0	9.55683 50	0	5.07749 00	0	3.60688 44	0	2.88466 81	0	2.45960 31	0
0.6	0	11.67289 29	0	6.07375 88	0	4.23689 26	0	3.33560 96	0	2.80566 62	0
0.7	0	13.93701 65	0	7.13594 68	0	4.90639 02	0	3.81337 52	0	3.17129 88	0
0.8	0	16.35517 22	0	8.26661 58	0	5.61685 85	0	4.31893 68	0	3.55718 66	0
0.9	0	18.93349 40	0	9.46839 74	0	6.36981 80	0	4.85329 20	0	3.96403 29	0
1.0	0	21.67828 70	0	10.74399 49	0	7.16683 00	0	5.41746 38	0	4.39255 83	0

b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-0.50000 00	0	-0.28571 43	0	-0.12500 00	0	0.00000 00	0	0.10000 00	0
-0.9	0	-0.39350 64	0	-0.19205 84	0	-0.04121 49	0	0.07592 74	0	0.16950 40	0
-0.8	0	-0.27831 23	0	-0.09139 07	0	0.04835 93	0	0.15672 55	0	0.24316 90	0
-0.7	0	-0.15407 15	0	0.01655 65	0	0.14393 49	0	0.24256 62	0	0.32113 65	0
-0.6	0	-0.02042 85	0	0.13205 79	0	0.24572 95	0	0.33362 57	0	0.40355 13	0
-0.5	0	0.12298 15	0	0.25539 51	0	0.35396 65	0	0.43008 44	0	0.49056 20	0
-0.4	0	0.27653 32	0	0.38685 73	0	0.46887 47	0	0.53212 73	0	0.58232 05	0
-0.3	0	0.44061 11	0	0.52674 09	0	0.59068 88	0	0.63994 39	0	0.67898 24	0
-0.2	0	0.61560 98	0	0.67535 01	0	0.71964 90	0	0.75372 83	0	0.78070 70	0
-0.1	0	0.80193 43	0	0.83299 65	0	0.85600 20	0	0.87367 91	0	0.88765 72	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	1.21023 31	0	1.17668 82	0	1.15190 18	0	1.13289 93	0	1.11790 61	0
0.2	0	1.43307 07	0	1.36339 71	0	1.31197 24	0	1.27259 03	0	1.24155 02	0
0.3	0	1.66896 10	0	1.56047 10	0	1.48048 31	0	1.41929 15	0	1.37111 10	0
0.4	0	1.91836 37	0	1.76826 25	0	1.65771 19	0	1.57322 64	0	1.50677 14	0
0.5	0	2.18175 01	0	1.98713 34	0	1.84394 34	0	1.73462 38	0	1.64871 85	0
0.6	0	2.45960 31	0	2.21745 38	0	2.03946 90	0	1.90371 79	0	1.79714 36	0
0.7	0	2.75241 79	0	2.45960 31	0	2.24458 71	0	2.08074 81	0	1.95224 22	0
0.8	0	3.06070 20	0	2.71396 99	0	2.45960 31	0	2.26595 96	0	2.11421 46	0
0.9	0	3.38497 52	0	2.98095 21	0	2.68482 96	0	2.45960 31	0	2.28326 51	0
1.0	0	3.72577 04	0	3.26095 72	0	2.92058 65	0	2.66193 51	0	2.45960 31	0

$$10^{-t} {}_1F_1[a; b; x]$$

$$x = 1.0$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	0	-9.000000 00	0	-4.000000 00	0	-2.333333 33	0	-1.500000 00	0	-1.000000 00	0
-0.9	0	-8.49472 34	0	-3.72474 63	0	-2.13718 90	0	-1.34483 48	0	-0.87032 73	0
-0.8	0	-7.89481 34	0	-3.40618 57	0	-1.91443 23	0	-1.17116 05	0	-0.72685 14	0
-0.7	0	-7.19487 27	0	-3.04197 32	0	-1.66369 18	0	-0.97806 74	0	-0.56892 41	0
-0.6	0	-6.38931 43	0	-2.62968 42	0	-1.38355 11	0	-0.76461 68	0	-0.39587 72	0
-0.5	0	-5.47235 71	0	-2.16681 22	0	-1.07254 74	0	-0.52984 05	0	-0.20702 17	0
-0.4	0	-4.43802 02	0	-1.65076 69	0	-0.72917 04	0	-0.27273 93	0	-0.00164 75	0
-0.3	0	-3.28011 86	0	-1.07887 24	0	-0.35186 13	0	0.00771 68	0	0.22097 68	0
-0.2	0	-1.99225 77	0	-0.44836 46	0	0.06098 84	0	0.31259 00	0	0.46160 48	0
-0.1	0	-0.56782 81	0	0.24361 07	0	0.51103 83	0	0.64297 49	0	0.72101 28	0
0.0	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0
0.1	0	2.71828 18	0	1.82384 44	0	1.52963 87	0	1.38482 77	0	1.29938 93	0
0.2	0	4.59430 40	0	2.71828 18	0	2.10177 40	0	1.79865 55	0	1.62002 78	0
0.3	0	6.63559 00	0	3.68654 95	0	2.71828 18	0	2.24271 69	0	1.96278 70	0
0.4	0	8.84990 62	0	4.73198 60	0	3.38109 51	0	2.71828 18	0	2.32856 41	0
0.5	0	11.24526 75	0	5.85803 42	0	4.09220 54	0	3.22665 80	0	2.71828 18	0
0.6	0	13.82994 37	0	7.06824 32	0	4.85366 43	0	3.76919 11	0	3.13288 93	0
0.7	0	16.61246 51	0	8.36627 12	0	5.66758 48	0	4.34726 66	0	3.57336 27	0
0.8	0	19.60162 96	0	9.75588 81	0	6.53614 27	0	4.96230 95	0	4.04070 57	0
0.9	0	22.80650 83	0	11.24097 77	0	7.46157 79	0	5.61578 63	0	4.53595 02	0
1.0	0	26.23645 24	0	12.82554 07	0	8.44619 59	0	6.30920 51	0	5.06015 69	0

b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-0.66666 67	0	-0.42857 14	0	-0.25000 00	0	-0.11111 11	0	0.00000 00	0
-0.9	0	-0.55459 74	0	-0.32950 25	0	-0.16099 03	0	-0.03015 50	0	0.07433 86	0
-0.8	0	-0.43175 67	0	-0.22175 34	0	-0.06481 47	0	0.05682 99	0	0.15382 72	0
-0.7	0	-0.29766 05	0	-0.10495 00	0	0.03882 37	0	0.15008 37	0	0.23866 34	0
-0.6	0	-0.15180 98	0	0.02129 30	0	0.15022 99	0	0.24985 32	0	0.32905 02	0
-0.5	0	0.00630 91	0	0.15737 20	0	0.26971 79	0	0.35639 20	0	0.42519 58	0
-0.4	0	0.17722 54	0	0.30369 49	0	0.39761 04	0	0.46996 09	0	0.52731 44	0
-0.3	0	0.36148 37	0	0.46068 14	0	0.53423 91	0	0.59082 74	0	0.63562 57	0
-0.2	0	0.55964 47	0	0.62876 31	0	0.67994 51	0	0.71926 65	0	0.75035 51	0
-0.1	0	0.77228 56	0	0.80838 38	0	0.83507 87	0	0.85556 08	0	0.87173 40	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	1.24339 88	0	1.20408 08	0	1.17507 90	0	1.15288 19	0	1.13539 67	0
0.2	0	1.50311 02	0	1.42110 86	0	1.36069 55	0	1.31451 22	0	1.27817 41	0
0.3	0	1.77978 05	0	1.65157 89	0	1.55723 98	0	1.48520 44	0	1.42858 86	0
0.4	0	2.07407 40	0	1.89600 10	0	1.76511 25	0	1.66528 05	0	1.58690 33	0
0.5	0	2.38667 38	0	2.15489 81	0	1.98472 52	0	1.85507 07	0	1.75338 77	0
0.6	0	2.71828 18	0	2.42880 78	0	2.21650 00	0	2.05491 38	0	1.92831 84	0
0.7	0	3.06961 97	0	2.71828 18	0	2.46087 06	0	2.26515 76	0	2.11197 89	0
0.8	0	3.44142 89	0	3.02388 72	0	2.71828 18	0	2.48615 83	0	2.30465 98	0
0.9	0	3.83447 12	0	3.34620 60	0	2.98919 01	0	2.71828 18	0	2.50665 90	0
1.0	0	4.24952 89	0	3.68583 55	0	3.27406 39	0	2.96190 29	0	2.71828 18	0

$x = 1 \cdot 1$ $10^{-t} {}_1F_1[a; b; x]$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	o	-10.00000	oo	o	-4.50000	oo	o	-2.66666	67	o	-1.75000	oo	o	-1.20000	oo
-0.9	o	-9.51122	70	o	-4.22739	59	o	-2.46917	29	o	-1.59184	65	o	-1.06658	38
-0.8	o	-8.90761	82	o	-3.90224	24	o	-2.23938	89	o	-1.41123	60	o	-0.91642	13
-0.7	o	-8.18182	02	o	-3.52134	71	o	-1.97544	90	o	-1.20693	20	o	-0.74863	19
-0.6	o	-7.32619	29	o	-3.08139	71	o	-1.67541	95	o	-0.97765	40	o	-0.56230	44
-0.5	o	-6.33280	19	o	-2.57895	60	o	-1.33729	66	o	-0.72207	64	o	-0.35649	67
-0.4	o	-5.19341	00	o	-2.01045	98	o	-0.95900	44	o	-0.43882	77	o	-0.13023	48
-0.3	o	-3.89946	81	o	-1.37221	42	o	-0.53839	33	o	-0.12648	85	o	0.11748	81
-0.2	o	-2.44210	75	o	-0.66039	03	o	-0.07323	77	o	0.21640	87	o	0.38771	25
-0.1	o	-0.81213	00	o	0.12897	83	o	0.43876	55	o	0.59138	19	o	0.68151	31
0.0	o	1.00000	oo	o	1.00000	oo	o	1.00000	oo	o	1.00000	oo	o	1.00000	oo
0.1	o	3.00416	60	o	1.95692	83	o	1.61293	09	o	1.44388	43	o	1.34431	95
0.2	o	5.21060	79	o	3.00416	60	o	2.28010	77	o	1.92471	03	o	1.71565	49
0.3	o	7.62993	21	o	4.14626	92	o	3.00416	60	o	2.44420	85	o	2.11522	76
0.4	o	10.27312	11	o	5.38795	10	o	3.78783	04	o	3.00416	60	o	2.54429	79
0.5	o	13.15154	41	o	6.73408	63	o	4.63391	58	o	3.60642	82	o	3.00416	60
0.6	o	16.27696	69	o	8.18971	52	o	5.54533	08	o	4.25289	97	o	3.49617	31
0.7	o	19.66156	23	o	9.76004	79	o	6.52507	91	o	4.94554	63	o	4.02170	20
0.8	o	23.31792	16	o	11.45046	89	o	7.57626	24	o	5.68639	63	o	4.58217	87
0.9	o	27.25906	42	o	13.26654	13	o	8.70208	32	o	6.47754	21	o	5.17907	31
1.0	o	31.49845	05	o	15.21401	20	o	9.90584	63	o	7.32114	14	o	5.81390	00
b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	o	-0.83333	33	o	-0.57142	85	o	-0.37500	00	o	-0.22222	22	o	-0.10000	00
-0.9	o	-0.71717	00	o	-0.46812	19	o	-0.28172	18	o	-0.13703	08	o	-0.02149	59
-0.8	o	-0.58793	93	o	-0.35428	55	o	-0.17975	13	o	-0.04452	68	o	0.06325	46
-0.7	o	-0.44498	38	o	-0.22941	14	o	-0.06868	57	o	0.05561	54	o	0.15451	93
-0.6	o	-0.28762	39	o	-0.09297	52	o	0.05189	01	o	0.16373	16	o	0.25257	41
-0.5	o	-0.11515	74	o	0.05556	43	o	0.18240	46	o	0.28016	76	o	0.35770	30
-0.4	o	0.07314	09	o	0.21676	61	o	0.32329	93	o	0.40527	97	o	0.47019	84
-0.3	o	0.27802	00	o	0.39120	65	o	0.47502	95	o	0.53943	51	o	0.59036	14
-0.2	o	0.50025	32	o	0.57948	02	o	0.63806	48	o	0.68301	21	o	0.71850	19
-0.1	o	0.74063	86	o	0.78220	06	o	0.81288	89	o	0.83640	01	o	0.85493	87
0.0	o	1.00000	oo	o	1.00000	oo	o	1.00000	oo	o	1.00000	oo	o	1.00000	oo
0.1	o	1.27918	71	o	1.23353	06	o	1.19991	18	o	1.17422	47	o	1.15402	33
0.2	o	1.57907	66	o	1.48346	43	o	1.41315	29	o	1.35949	90	o	1.31735	58
0.3	o	1.90057	26	o	1.75049	37	o	1.64026	82	o	1.55626	02	o	1.49035	45
0.4	o	2.24460	73	o	2.03533	25	o	1.88181	82	o	1.76495	82	o	1.67338	66
0.5	o	2.61214	16	o	2.33871	56	o	2.13838	02	o	1.98605	57	o	1.86682	94
0.6	o	3.00416	60	o	2.66140	03	o	2.41054	83	o	2.22002	87	o	2.07107	10
0.7	o	3.42170	11	o	3.00416	60	o	2.69893	39	o	2.46736	69	o	2.28651	00
0.8	o	3.86579	86	o	3.36781	56	o	3.00416	60	o	2.72857	35	o	2.51355	63
0.9	o	4.33754	16	o	3.75317	52	o	3.32689	17	o	3.00416	60	o	2.75263	08
1.0	o	4.83804	56	o	4.16109	54	o	3.66777	66	o	3.29467	65	o	3.00416	60

$$10^{-t} {}_1F_1[a; b; x]$$

$$x = 1.2$$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	0	-11.000000	00	0	-5.000000	00	0	-3.000000	00	0	-2.000000	00	0	-1.400000	00
-0.9	0	-10.543000	70	0	-4.736850	92	0	-2.805240	60	0	-1.841640	07	0	-1.264870	38
-0.8	0	-9.948990	60	0	-4.411020	00	0	-2.571960	66	0	-1.656480	90	0	-1.109760	90
-0.7	0	-9.208190	24	0	-4.018240	22	0	-2.297680	55	0	-1.442900	46	0	-0.933510	82
-0.6	0	-8.310400	34	0	-3.554100	95	0	-1.979820	72	0	-1.199180	33	0	-0.734900	98
-0.5	0	-7.245000	35	0	-3.014020	44	0	-1.615710	33	0	-0.923550	50	0	-0.512680	65
-0.4	0	-6.000920	21	0	-2.393200	26	0	-1.202550	99	0	-0.614180	18	0	-0.265540	45
-0.3	0	-4.566620	92	0	-1.686660	69	0	-0.737470	42	0	-0.269150	61	0	0.007860	80
-0.2	0	-2.930120	19	0	-0.889240	24	0	-0.217450	19	0	0.113500	12	0	0.308950	28
-0.1	0	-1.078900	96	0	0.004450	00	0	0.360620	67	0	0.535840	37	0	0.639160	20
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	0	3.320110	69	0	2.103210	29	0	1.704020	78	0	1.508170	70	0	1.393020	50
0.2	0	5.895480	33	0	3.320110	69	0	2.476190	54	0	2.062660	19	0	1.819850	01
0.3	0	8.740680	97	0	4.656960	90	0	3.320110	69	0	2.665820	38	0	2.282140	52
0.4	0	11.870890	13	0	6.120260	19	0	4.239530	91	0	3.320110	69	0	2.781630	83
0.5	0	15.301820	37	0	7.716730	05	0	5.238340	48	0	4.028080	21	0	3.320110	69
0.6	0	19.049820	09	0	9.453350	90	0	6.320550	69	0	4.792340	97	0	3.899430	02
0.7	0	23.131830	16	0	11.337380	79	0	7.490340	22	0	5.615640	18	0	4.521480	99
0.8	0	27.565430	78	0	13.376320	13	0	8.752010	54	0	6.500770	46	0	5.188270	26
0.9	0	32.368870	16	0	15.577930	39	0	10.110040	30	0	7.450660	14	0	5.901820	12
1.0	0	37.561030	57	0	17.950270	89	0	11.569040	80	0	8.468310	44	0	6.664240	65

b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	0	-1.000000	00	0	-0.714280	57	0	-0.500000	00	0	-0.333330	33	0	-0.200000	00
-0.9	0	-0.881290	98	0	-0.607970	44	0	-0.403450	48	0	-0.244730	64	0	-0.118020	91
-0.8	0	-0.747010	07	0	-0.489100	19	0	-0.296540	06	0	-0.147410	67	0	-0.028600	75
-0.7	0	-0.596260	16	0	-0.356990	57	0	-0.178720	47	0	-0.040940	35	0	0.068610	87
-0.6	0	-0.428140	92	0	-0.210950	91	0	-0.049450	58	0	0.075120	82	0	0.174010	53
-0.5	0	-0.241730	75	0	-0.050270	12	0	0.091830	62	0	0.201250	89	0	0.287950	99
-0.4	0	-0.036050	64	0	0.125810	48	0	0.245740	12	0	0.337920	40	0	0.410840	26
-0.3	0	0.189890	88	0	0.318070	17	0	0.412860	91	0	0.485610	48	0	0.543060	57
-0.2	0	0.437160	88	0	0.527290	88	0	0.593850	02	0	0.644830	88	0	0.685040	50
-0.1	0	0.706830	09	0	0.754320	32	0	0.789330	61	0	0.816120	02	0	0.837200	90
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	0	1.317820	99	0	1.265210	34	0	1.226530	74	0	1.197030	67	0	1.173870	44
0.2	0	1.661510	42	0	1.550870	71	0	1.469660	66	0	1.407800	68	0	1.359300	26
0.3	0	2.032280	71	0	1.857930	57	0	1.730120	93	0	1.632900	50	0	1.556760	97
0.4	0	2.431420	50	0	2.187360	46	0	2.008690	11	0	1.872940	49	0	1.766770	58
0.5	0	2.860240	71	0	2.540170	18	0	2.306140	23	0	2.128550	91	0	1.989830	64
0.6	0	3.320110	69	0	2.917390	77	0	2.623290	84	0	2.400400	03	0	2.226480	25
0.7	0	3.812440	31	0	3.320110	69	0	2.961000	06	0	2.689140	11	0	2.477260	15
0.8	0	4.338680	10	0	3.749430	85	0	3.320110	69	0	2.995470	51	0	2.742730	69
0.9	0	4.900330	35	0	4.206500	70	0	3.701540	22	0	3.320110	69	0	3.023480	94
1.0	0	5.498950	25	0	4.692500	36	0	4.106190	95	0	3.663800	29	0	3.320110	69

$x = 1.3$

$10^{-t} {}_1F_1[a; b; x]$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	o	-12.00000 00	o	-5.50000 00	o	-3.33333 33	o	-2.25000 00	o	-1.60000 00	o
-0.9	o	-11.59102 97	o	-5.25354 92	o	-3.14564 59	o	-2.09437 26	o	-1.46530 61	o
-0.8	o	-11.02088 73	o	-4.93334 70	o	-2.91264 17	o	-1.90723 01	o	-1.30711 25	o
-0.7	o	-10.27684 66	o	-4.53387 68	o	-2.63110 14	o	-1.68644 14	o	-1.12390 32	o
-0.6	o	-9.34558 54	o	-4.04937 20	o	-2.29766 45	o	-1.42978 48	o	-0.91409 97	o
-0.5	o	-8.21316 49	o	-3.47380 68	o	-1.90882 40	o	-1.13494 45	o	-0.67605 90	o
-0.4	o	-6.86500 89	o	-2.80088 91	o	-1.46092 25	o	-0.79950 82	o	-0.40807 11	o
-0.3	o	-5.28588 28	o	-2.02405 03	o	-0.95014 64	o	-0.42096 38	o	-0.10835 73	o
-0.2	o	-3.45987 20	o	-1.13643 77	o	-0.37252 22	o	0.00330 34	o	0.22493 15	o
-0.1	o	-1.37035 87	o	-0.13090 43	o	0.27608 98	o	0.47601 47	o	0.59371 82	o
0.0	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o
0.1	o	3.66929 67	o	2.26403 84	o	1.80369 47	o	1.57820 21	o	1.44585 19	o
0.2	o	6.65639 69	o	3.66929 67	o	2.69184 18	o	2.21367 96	o	1.93342 84	o
0.3	o	9.98096 46	o	5.22419 36	o	3.66929 67	o	2.90961 07	o	2.46496 63	o
0.4	o	13.66348 74	o	6.93749 12	o	4.74110 71	o	3.66929 67	o	3.04278 63	o
0.5	o	17.72530 22	o	8.81830 56	o	5.91251 98	o	4.49616 53	o	3.66929 67	o
0.6	o	22.18862 26	o	10.87611 78	o	7.18898 63	o	5.39377 49	o	4.34699 45	o
0.7	o	27.07656 64	o	13.12078 46	o	8.57616 89	o	6.36581 83	o	5.07846 93	o
0.8	o	32.41318 28	o	15.56255 05	o	10.07994 70	o	7.41612 62	o	5.86640 50	o
0.9	o	38.22348 23	o	18.21205 95	o	11.70642 40	o	8.54867 18	o	6.71358 29	o
1.0	o	44.53346 54	o	21.08036 63	o	13.46193 28	o	9.76757 46	o	7.62288 43	o
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	o	-1.16666 67	o	-0.85714 28	o	-0.62500 00	o	-0.44444 44	o	-0.30000 00	o
-0.9	o	-1.04706 70	o	-0.74912 11	o	-0.52623 72	o	-0.35330 99	o	-0.21529 21	o
-0.8	o	-0.90913 14	o	-0.62632 49	o	-0.41527 85	o	-0.25191 62	o	-0.12182 14	o
-0.7	o	-0.75172 91	o	-0.48788 20	o	-0.29143 36	o	-0.13970 54	o	-0.01912 95	o
-0.6	o	-0.57368 46	o	-0.33288 62	o	-0.15398 58	o	-0.01609 89	o	0.09325 84	o
-0.5	o	-0.37377 51	o	-0.16039 62	o	-0.00219 14	o	0.11950 30	o	0.21583 44	o
-0.4	o	-0.15072 96	o	0.03056 51	o	0.16472 10	o	0.26772 20	o	0.34910 77	o
-0.3	o	0.09677 22	o	0.24101 28	o	0.34755 17	o	0.42920 21	o	0.49360 57	o
-0.2	o	0.37010 25	o	0.47199 95	o	0.54713 00	o	0.60461 05	o	0.64987 40	o
-0.1	o	0.67068 55	o	0.72461 76	o	0.76431 58	o	0.79463 80	o	0.81847 74	o
0.0	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o
0.1	o	1.35958 05	o	1.29932 12	o	1.25510 54	o	1.22143 68	o	1.19504 60	o
0.2	o	1.75101 90	o	1.62379 87	o	1.53058 77	o	1.45971 46	o	1.40423 99	o
0.3	o	2.17596 65	o	1.97469 39	o	1.82743 65	o	1.71562 58	o	1.62822 76	o
0.4	o	2.63613 50	o	2.35331 37	o	2.14667 60	o	1.98999 02	o	1.86767 63	o
0.5	o	3.13329 88	o	2.76101 16	o	2.48936 63	o	2.28365 55	o	2.12327 57	o
0.6	o	3.66929 67	o	3.19918 91	o	2.85660 38	o	2.59749 79	o	2.39573 81	o
0.7	o	4.24603 35	o	3.66929 67	o	3.24952 30	o	2.93242 31	o	2.68579 93	o
0.8	o	4.86548 22	o	4.17283 57	o	3.66929 67	o	3.28936 70	o	2.99421 90	o
0.9	o	5.52968 55	o	4.71135 93	o	4.11713 75	o	3.66929 67	o	3.32178 16	o
1.0	o	6.24075 81	o	5.28647 42	o	4.59429 91	o	4.07321 07	o	3.66929 67	o

$$10^{-t} {}_1F_1[a; b; x]$$

$$x = 1.4$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	o	-13.00000 00	o	-6.00000 00	o	-3.66666 67	o	-2.50000 00	o	-1.80000 00	o
-0.9	o	-12.65633 49	o	-5.77790 91	o	-3.49062 72	o	-2.35020 82	o	-1.66799 73	o
-0.8	o	-12.12538 69	o	-5.47011 55	o	-3.26192 61	o	-2.16379 30	o	-1.50868 54	o
-0.7	o	-11.39087 49	o	-5.06956 71	o	-2.97645 10	o	-1.93803 38	o	-1.32013 06	o
-0.6	o	-10.43568 90	o	-4.56886 39	o	-2.62989 33	o	-1.67008 43	o	-1.10031 21	o
-0.5	o	-9.24186 01	o	-3.96024 62	o	-2.21774 10	o	-1.35696 77	o	-0.84711 95	o
-0.4	o	-7.79052 87	o	-3.23558 14	o	-1.73527 16	o	-0.99557 27	o	-0.55834 94	o
-0.3	o	-6.06191 28	o	-2.38635 08	o	-1.17754 51	o	-0.58264 82	o	-0.23170 23	o
-0.2	o	-4.03527 48	o	-1.40363 60	o	-0.53939 63	o	-0.11479 93	o	0.13522 01	o
-0.1	o	-1.68888 77	o	-0.27810 54	o	0.18457 24	o	0.41151 76	o	0.54491 87	o
0.0	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o
0.1	o	4.05520 00	o	2.44088 22	o	1.91277 34	o	1.65450 41	o	1.50317 93	o
0.2	o	7.50162 09	o	4.05520 00	o	2.92903 68	o	2.37905 01	o	2.05728 47	o
0.3	o	11.36530 68	o	5.85408 51	o	4.05520 00	o	3.17782 73	o	2.66526 01	o
0.4	o	15.67347 51	o	7.84915 84	o	5.29794 66	o	4.05520 00	o	3.33016 94	o
0.5	o	20.45455 77	o	10.05254 52	o	6.66424 38	o	5.01571 26	o	4.05520 00	o
0.6	o	25.73824 16	o	12.47689 29	o	8.16135 11	o	6.06409 58	o	4.84366 66	o
0.7	o	31.55551 11	o	15.13538 81	o	9.79682 97	o	7.20527 22	o	5.69901 55	o
0.8	o	37.93869 09	o	18.04177 40	o	11.57855 27	o	8.44436 24	o	6.62482 80	o
0.9	o	44.92149 15	o	21.21036 84	o	13.51471 41	o	9.78669 13	o	7.62482 54	o
1.0	o	52.53905 45	o	24.65608 28	o	15.61384 03	o	11.23779 43	o	8.70287 26	o

b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	o	-1.33333 33	o	-1.00000 00	o	-0.75000 00	o	-0.55555 56	o	-0.40000 00	o
-0.9	o	-1.21455 74	o	-0.89162 73	o	-0.65011 98	o	-0.46279 23	o	-0.31331 80	o
-0.8	o	-1.07447 30	o	-0.76608 52	o	-0.53606 70	o	-0.35810 69	o	-0.21645 32	o
-0.7	o	-0.91163 90	o	-0.62226 23	o	-0.40696 24	o	-0.24078 96	o	-0.10882 23	o
-0.6	o	-0.72455 12	o	-0.45900 03	o	-0.26189 03	o	-0.11010 15	o	0.01018 05	o
-0.5	o	-0.51164 06	o	-0.27509 21	o	-0.09989 77	o	0.03472 54	o	0.14118 49	o
-0.4	o	-0.27127 07	o	-0.06928 03	o	0.08000 68	o	0.19448 94	o	0.28484 46	o
-0.3	o	-0.00173 60	o	0.15974 43	o	0.27885 51	o	0.37002 04	o	0.44183 81	o
-0.2	o	0.29874 06	o	0.41334 45	o	0.49771 94	o	0.56218 01	o	0.61287 02	o
-0.1	o	0.63201 02	o	0.69293 82	o	0.73771 49	o	0.77186 35	o	0.79867 14	o
0.0	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o
0.1	o	1.40471 57	o	1.33606 34	o	1.28577 81	o	1.24755 39	o	1.21764 21	o
0.2	o	1.84824 41	o	1.70272 23	o	1.59629 90	o	1.51552 59	o	1.45241 27	o
0.3	o	2.33275 55	o	2.10163 28	o	1.93286 02	o	1.80495 38	o	1.70515 65	o
0.4	o	2.86050 65	o	2.53451 50	o	2.29680 81	o	2.11691 40	o	1.97674 86	o
0.5	o	3.43384 23	o	3.00315 56	o	2.68954 00	o	2.45252 23	o	2.26809 58	o
0.6	o	4.05520 00	o	3.50940 66	o	3.11250 49	o	2.81293 51	o	2.58013 68	o
0.7	o	4.72711 04	o	4.05520 00	o	3.56720 56	o	3.19935 08	o	2.91384 38	o
0.8	o	5.45220 23	o	4.64252 54	o	4.05520 00	o	3.61301 06	o	3.27022 31	o
0.9	o	6.23320 39	o	5.27345 64	o	4.57810 27	o	4.05520 00	o	3.65031 60	o
1.0	o	7.07294 70	o	5.95014 12	o	5.13758 69	o	4.52724 99	o	4.05520 00	o

$x=1.5$

$10^{-t} {}_1F_1[a; b; x]$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	o	-14.00000	00	o	-6.50000	00	o	-4.00000	00	o	-2.75000	00	o	-2.00000	00
-0.9	o	-13.74004	28	o	-6.31041	56	o	-3.84046	32	o	-2.60932	55	o	-1.87307	17
-0.8	o	-13.26475	72	o	-6.02228	71	o	-3.62037	06	o	-2.42653	58	o	-1.71473	83
-0.7	o	-12.55362	57	o	-5.62673	46	o	-3.33454	80	o	-2.19821	03	o	-1.52256	98
-0.6	o	-11.58500	26	o	-5.11440	49	o	-2.97755	41	o	-1.92075	72	o	-1.29401	84
-0.5	o	-10.33606	86	o	-4.47545	35	o	-2.54366	98	o	-1.59040	70	o	-1.02641	36
-0.4	o	-8.78278	43	o	-3.69952	56	o	-2.02688	89	o	-1.20320	61	o	-0.71695	85
-0.3	o	-6.89984	43	o	-2.77573	70	o	-1.42090	69	o	-0.75501	01	o	-0.36272	46
-0.2	o	-4.66062	75	o	-1.69265	41	o	-0.71910	97	o	-0.24147	69	o	0.03935	22
-0.1	o	-2.03714	73	o	-0.43827	33	o	0.08543	70	o	0.34194	01	o	0.49247	69
0.0	o	1.00000	00	o	1.00000	00	o	1.00000	00	o	1.00000	00	o	1.00000	00
0.1	o	4.48168	91	o	2.63537	33	o	2.03219	00	o	1.73768	10	o	1.56542	24
0.2	o	8.44031	96	o	4.48168	91	o	3.18997	43	o	2.56018	86	o	2.19240	09
0.3	o	12.90987	32	o	6.55344	80	o	4.48168	91	o	3.47296	34	o	2.88475	33
0.4	o	17.92597	14	o	8.86583	29	o	5.91605	28	o	4.48168	91	o	3.64646	39
0.5	o	23.52593	59	o	11.43473	24	o	7.50217	95	o	5.59230	10	o	4.48168	91
0.6	o	29.74885	01	o	14.27676	69	o	9.24959	24	o	6.81099	47	o	5.39476	32
0.7	o	36.63562	13	o	17.40931	37	o	11.16823	84	o	8.14423	47	o	6.39020	45
0.8	o	44.22904	84	o	20.85053	43	o	13.26850	19	o	9.59876	34	o	7.47272	09
0.9	o	52.57388	73	o	24.61940	04	o	15.56122	09	o	11.18161	09	o	8.64721	65
1.0	o	61.71691	93	o	28.73572	39	o	18.05770	13	o	12.90010	41	o	9.91879	80
b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	o	-1.50000	00	o	-1.14285	71	o	-0.87500	00	o	-0.66666	67	o	-0.50000	00
-0.9	o	-1.38386	22	o	-1.03556	24	o	-0.77515	70	o	-0.57322	64	o	-0.41214	20
-0.8	o	-1.24321	95	o	-0.90852	24	o	-0.65901	50	o	-0.46607	56	o	-0.31257	31
-0.7	o	-1.07626	21	o	-0.76034	24	o	-0.52547	11	o	-0.34432	36	o	-0.20056	29
-0.6	o	-0.88109	46	o	-0.58956	36	o	-0.37337	32	o	-0.20704	18	o	-0.07534	97
-0.5	o	-0.65573	34	o	-0.39466	14	o	-0.20151	83	o	-0.05326	13	o	0.06386	00
-0.4	o	-0.39810	32	o	-0.17404	23	o	-0.00865	07	o	0.11802	81	o	0.21789	29
-0.3	o	-0.10603	39	o	0.07395	78	o	0.20653	94	o	0.30787	92	o	0.38760	94
-0.2	o	0.22274	25	o	0.35107	65	o	0.44541	85	o	0.51738	89	o	0.57390	53
-0.1	o	0.59059	57	o	0.65912	67	o	0.70941	06	o	0.74769	95	o	0.77771	24
0.0	o	1.00000	00	o	1.00000	00	o	1.00000	00	o	1.00000	00	o	1.00000	00
0.1	o	1.45353	81	o	1.37566	84	o	1.31873	27	o	1.27552	81	o	1.24177	58
0.2	o	1.95390	48	o	1.78818	77	o	1.66721	87	o	1.57557	15	o	1.50408	75
0.3	o	2.50391	08	o	2.23969	96	o	2.04713	38	o	1.90146	90	o	1.78802	33
0.4	o	3.10648	64	o	2.73243	51	o	2.46022	20	o	2.25461	29	o	2.09471	42
0.5	o	3.76468	52	o	3.26871	71	o	2.90829	74	o	2.63645	03	o	2.42533	43
0.6	o	4.48168	91	o	3.85096	31	o	3.39324	67	o	3.04848	47	o	2.78110	27
0.7	o	5.26081	12	o	4.48168	91	o	3.91703	11	o	3.49227	77	o	3.16328	48
0.8	o	6.10550	11	o	5.16351	15	o	4.48168	91	o	3.96945	13	o	3.57319	35
0.9	o	7.01934	88	o	5.89915	15	o	5.08933	84	o	4.48168	91	o	4.01219	08
1.0	o	8.00608	93	o	6.69143	75	o	5.74217	90	o	5.03073	85	o	4.48168	91

$$10^{-t} {}_1F_1[a; b; x]$$

$$x = 1.6$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	0	-15.00000 00	0	-7.00000 00	0	-4.33333 33	0	-3.00000 00	0	-2.20000 00	0
-0.9	0	-14.84336 02	0	-6.85158 08	0	-4.19544 68	0	-2.87191 48	0	-2.08066 25	0
-0.8	0	-14.44144 14	0	-6.59089 83	0	-3.98856 77	0	-2.69584 31	0	-1.92553 96	0
-0.7	0	-13.76872 59	0	-6.20691 66	0	-3.70627 03	0	-2.46754 02	0	-1.73161 79	0
-0.6	0	-12.79818 43	0	-5.68796 69	0	-3.34177 18	0	-2.18253 24	0	-1.49572 65	0
-0.5	0	-11.50121 21	0	-5.02172 18	0	-2.88791 79	0	-1.83610 83	0	-1.21453 04	0
-0.4	0	-9.84756 43	0	-4.19516 77	0	-2.33716 79	0	-1.42330 95	0	-0.88452 53	0
-0.3	0	-7.80528 64	0	-3.19457 73	0	-1.68157 96	0	-0.93892 06	0	-0.50202 97	0
-0.2	0	-5.34064 46	0	-2.00548 09	0	-0.91279 31	0	-0.37745 97	0	-0.06317 93	0
-0.1	0	-2.41805 34	0	-0.61263 66	0	-0.02201 46	0	0.26683 17	0	0.43608 01	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	4.95303 24	0	2.84930 79	0	2.16296 33	0	1.82838 91	0	1.63303 61	0
0.2	0	9.48264 47	0	4.95303 24	0	3.47707 29	0	2.75865 26	0	2.33985 57	0
0.3	0	14.63260 38	0	7.32981 93	0	4.95303 24	0	3.79776 48	0	3.12534 60	0
0.4	0	20.44892 87	0	9.99924 92	0	6.60206 63	0	4.95303 24	0	3.99462 15	0
0.5	0	26.97997 68	0	12.98187 34	0	8.43594 35	0	6.23210 75	0	4.95303 24	0
0.6	0	34.27653 45	0	16.29924 99	0	10.46699 57	0	7.64299 90	0	6.00617 30	0
0.7	0	42.39191 18	0	19.97398 25	0	12.70813 76	0	9.19408 65	0	7.15989 02	0
0.8	0	51.38203 65	0	24.02975 91	0	15.17288 98	0	10.89413 30	0	8.42029 26	0
0.9	0	61.30555 32	0	28.49139 17	0	17.87539 97	0	12.75229 91	0	9.79376 00	0
1.0	0	72.22392 82	0	33.38485 84	0	20.83046 49	0	14.77815 65	0	11.28695 18	0

b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-1.66666 67	0	-1.28571 43	0	-1.00000 00	0	-0.77777 78	0	-0.60000 00	0
-0.9	0	-1.55507 90	0	-1.18100 05	0	-0.90140 64	0	-0.68465 86	0	-0.51180 09	0
-0.8	0	-1.41556 75	0	-1.05378 56	0	-0.78423 86	0	-0.57591 44	0	-0.41025 60	0
-0.7	0	-1.24588 88	0	-0.90234 22	0	-0.64713 12	0	-0.45044 38	0	-0.29446 16	0
-0.6	0	-1.04368 61	0	-0.72485 74	0	-0.48865 33	0	-0.30709 37	0	-0.16347 29	0
-0.5	0	-0.80648 41	0	-0.51942 99	0	-0.30730 62	0	-0.14465 80	0	-0.01630 26	0
-0.4	0	-0.53168 49	0	-0.28406 70	0	-0.10152 05	0	0.03812 48	0	0.14808 06	0
-0.3	0	-0.21656 30	0	-0.01668 04	0	0.13034 58	0	0.24257 35	0	0.33075 40	0
-0.2	0	0.14173 87	0	0.28491 61	0	0.39001 06	0	0.47006 57	0	0.53284 14	0
-0.1	0	0.54621 43	0	0.62301 15	0	0.67926 97	0	0.72204 05	0	0.75551 57	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	1.50637 85	0	1.41838 54	0	1.35416 20	0	1.30551 19	0	1.26756 97	0
0.2	0	2.06878 51	0	1.88078 47	0	1.74380 28	0	1.64021 17	0	1.55955 37	0
0.3	0	2.69081 25	0	2.38993 24	0	2.17105 94	0	2.00580 46	0	1.87733 67	0
0.4	0	3.37621 69	0	2.94868 44	0	2.63816 12	0	2.40406 81	0	2.22236 06	0
0.5	0	4.12892 35	0	3.56002 17	0	3.14743 35	0	2.83685 47	0	2.59612 68	0
0.6	0	4.95303 24	0	4.22705 59	0	3.70130 06	0	3.30609 37	0	3.00019 75	0
0.7	0	5.85282 48	0	4.95303 24	0	4.30228 94	0	3.81379 44	0	3.43619 87	0
0.8	0	6.83276 91	0	5.74133 58	0	4.95303 24	0	4.36204 85	0	3.90582 12	0
0.9	0	7.89752 79	0	6.59549 42	0	5.65627 15	0	4.95303 24	0	4.41082 32	0
1.0	0	9.05196 37	0	7.51918 39	0	6.41486 16	0	5.58901 06	0	4.95303 24	0

$x = 1.7$

$10^{-t} {}_1F_1[a; b; x]$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	o	-16.000000 00	o	-7.500000 00	o	-4.666666 67	o	-3.250000 00	o	-2.400000 00	o
-0.9	o	-15.96758 70	o	-7.40195 60	o	-4.55589 30	o	-3.13818 02	o	-2.29091 19	o
-0.8	o	-15.65808 05	o	-7.17706 71	o	-4.36715 52	o	-2.97212 81	o	-2.14137 72	o
-0.7	o	-15.04010 52	o	-6.81177 48	o	-4.09256 50	o	-2.74663 66	o	-1.94770 17	o
-0.6	o	-14.08029 35	o	-6.29168 78	o	-3.72376 38	o	-2.45619 71	o	-1.70598 39	o
-0.5	o	-12.74319 63	o	-5.60154 40	o	-3.25190 32	o	-2.09498 81	o	-1.41210 68	o
-0.4	o	-10.99118 98	o	-4.72517 34	o	-2.66762 34	o	-1.65686 10	o	-1.06172 89	o
-0.3	o	- 8.78437 95	o	-3.64545 87	o	-1.96103 23	o	-1.13532 71	o	-0.65027 48	o
-0.2	o	- 6.08049 99	o	-2.34429 42	o	-1.12168 21	o	-0.52354 38	o	-0.17292 63	o
-0.1	o	- 2.83481 28	o	-0.80254 43	o	-0.13854 72	o	0.18570 03	o	0.37538 73	o
0.0	o	1.000000 00	o	1.000000 00	o	1.000000 00	o	1.000000 00	o	1.000000 00	o
0.1	o	5.47394 74	o	3.08466 65	o	2.30621 41	o	1.92734 89	o	1.70651 84	o
0.2	o	10.63984 15	o	5.47394 74	o	3.79300 15	o	2.97615 55	o	2.50083 31	o
0.3	o	16.55341 68	o	8.19154 76	o	5.47394 74	o	4.15525 91	o	3.38912 91	o
0.4	o	23.27345 07	o	11.26243 43	o	7.36334 22	o	5.47394 74	o	4.37789 75	o
0.5	o	30.86189 01	o	14.71288 82	o	9.47621 05	o	6.94197 32	o	5.47394 74	o
0.6	o	39.38398 12	o	18.57055 54	o	11.82833 86	o	8.56957 32	o	6.68441 75	o
0.7	o	48.90840 29	o	22.86450 35	o	14.43630 61	o	10.36748 57	o	8.01678 88	o
0.8	o	59.50740 71	o	27.62527 67	o	17.31751 52	o	12.34697 05	o	9.47889 69	o
0.9	1	7.12569 61	o	32.88495 40	o	20.49022 35	o	14.51982 83	o	11.07894 66	o
1.0	1	8.42368 94	o	38.67720 93	o	23.97357 56	o	16.89842 13	o	12.82552 38	o
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	o	- 1.83333 33	o	-1.42857 14	o	-1.12500 00	o	-0.88888 89	o	-0.70000 00	o
-0.9	o	- 1.72831 21	o	-1.32802 05	o	-1.02892 96	o	-0.79713 74	o	-0.61233 46	o
-0.8	o	- 1.59172 74	o	-1.20203 44	o	-0.91186 18	o	-0.68772 20	o	-0.50958 15	o
-0.7	o	- 1.42083 11	o	-1.04849 78	o	-0.77212 59	o	-0.55929 55	o	-0.39063 57	o
-0.6	o	- 1.21272 55	o	-0.86518 36	o	-0.60796 53	o	-0.41044 31	o	-0.25433 90	o
-0.5	o	- 0.96435 71	o	-0.64974 84	o	-0.41753 35	o	-0.23968 03	o	-0.09947 67	o
-0.4	o	- 0.67251 05	o	-0.39972 77	o	-0.19889 19	o	-0.04544 95	o	0.07522 32	o
-0.3	o	- 0.33380 20	o	-0.11253 14	o	0.04999 47	o	0.17388 19	o	0.27109 37	o
-0.2	o	0.05532 73	o	0.21456 09	o	0.33126 17	o	0.42002 52	o	0.48952 93	o
-0.1	o	0.49861 82	o	0.58440 59	o	0.64714 80	o	0.69477 25	o	0.73198 93	o
0.0	o	1.000000 00	o	1.000000 00	o	1.000000 00	o	1.000000 00	o	1.000000 00	o
0.1	o	1.56359 87	o	1.46448 62	o	1.39227 59	o	1.33767 11	o	1.29515 68	o
0.2	o	2.19374 38	o	1.98115 88	o	1.82654 95	o	1.70983 93	o	1.61912 67	o
0.3	o	2.89497 61	o	2.55346 92	o	2.30551 45	o	2.11865 20	o	1.97365 12	o
0.4	o	3.67205 61	o	3.18503 17	o	2.83198 90	o	2.56635 30	o	2.36054 83	o
0.5	o	4.52997 18	o	3.87962 97	o	3.40891 98	o	3.05528 69	o	2.78171 59	o
0.6	o	5.47394 74	o	4.64122 17	o	4.03938 70	o	3.58790 15	o	3.23913 38	o
0.7	o	6.50945 19	o	5.47394 74	o	4.72660 90	o	4.16675 28	o	3.73486 71	o
0.8	o	7.64220 82	o	6.38213 49	o	5.47394 74	o	4.79450 74	o	4.27106 86	o
0.9	o	8.87820 23	o	7.37030 68	o	6.28491 15	o	5.47394 74	o	4.84998 28	o
1.0	o	10.22369 33	o	8.44318 75	o	7.16316 43	o	6.20797 35	o	5.47394 74	o

$$10^{-t} {}_1F_1[a; b; x]$$

$$x = 1.8$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	o	-17.00000 00	o	-8.00000 00	o	-5.00000 00	o	-3.50000 00	o	-2.60000 00	o
-0.9	o	-17.11412 53	o	-7.96213 42	o	-4.92213 96	o	-3.40834 05	o	-2.50397 24	o
-0.8	o	-16.91752 89	o	-7.78199 95	o	-4.75682 00	o	-3.25583 51	o	-2.36256 02	o
-0.7	o	-16.37202 30	o	-7.44310 73	o	-4.49445 49	o	-3.03616 02	o	-2.17128 03	o
-0.6	o	-15.43682 89	o	-6.92788 59	o	-4.12484 78	o	-2.74260 18	o	-1.92538 12	o
-0.5	o	-14.06845 45	o	-6.21762 99	o	-3.63716 44	o	-2.36803 84	o	-1.61983 09	o
-0.4	o	-12.22056 42	o	-5.29244 78	o	-3.01990 34	o	-1.90492 20	o	-1.24930 50	o
-0.3	o	-9.84384 57	o	-4.13120 72	o	-2.26086 64	o	-1.34526 05	o	-0.80817 39	o
-0.2	o	-6.88587 11	o	-2.71147 87	o	-1.34712 68	o	-0.68059 74	o	-0.29048 98	o
-0.1	o	-3.29095 39	o	-1.00947 69	o	-0.26499 81	o	0.09800 75	o	0.31002 64	o
0.0	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o
0.1	o	6.04964 75	o	3.34363 31	o	2.46317 57	o	2.03535 17	o	1.78641 58	o
0.2	o	11.92436 67	o	6.04964 75	o	4.14070 08	o	3.21458 30	o	2.67663 35	o
0.3	o	18.69442 35	o	9.14787 99	o	6.04964 75	o	4.54878 58	o	3.67840 34	o
0.4	o	26.43414 26	o	12.66984 83	o	8.20802 68	o	6.04964 75	o	4.79988 18	o
0.5	o	35.22208 54	o	16.64882 39	o	10.63482 79	o	7.72947 51	o	6.04964 75	o
0.6	o	45.14123 20	o	21.11990 54	o	13.35005 87	o	9.60122 10	o	7.43671 91	o
0.7	o	56.27917 44	o	26.12009 70	o	16.37478 87	o	11.67850 92	o	8.97057 24	o
0.8	I	6.87283 11	o	31.68838 74	o	19.73119 18	o	13.97566 21	o	10.66115 82	o
0.9	I	8.25860 51	o	37.86583 24	o	23.44259 16	o	16.50772 86	o	12.51892 16	o
1.0	I	9.79550 25	o	44.69564 05	o	27.53350 85	o	19.29051 32	o	14.55482 06	o
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	o	-2.00000 00	o	-1.57142 86	o	-1.25000 00	o	-1.00000 00	o	-0.80000 00	o
-0.9	o	-1.90367 27	o	-1.47670 67	o	-1.15779 19	o	-0.91071 50	o	-0.71378 50	o
-0.8	o	-1.77192 49	o	-1.35343 91	o	-1.04201 70	o	-0.80160 34	o	-0.61063 41	o
-0.7	o	-1.60142 39	o	-1.19906 20	o	-0.90065 16	o	-0.67103 42	o	-0.48921 07	o
-0.6	o	-1.38864 30	o	-1.01086 71	o	-0.73156 05	o	-0.51728 92	o	-0.34810 85	o
-0.5	o	-1.12985 34	o	-0.78599 46	o	-0.53249 28	o	-0.33855 95	o	-0.18584 88	o
-0.4	o	-0.82111 54	o	-0.52142 77	o	-0.30107 69	o	-0.13294 16	o	-0.00087 77	o
-0.3	o	-0.45826 96	o	-0.21398 57	o	-0.03481 57	o	0.10156 61	o	0.20843 69	o
-0.2	o	-0.03692 77	o	0.13968 29	o	0.26891 83	o	0.36706 74	o	0.44380 83	o
-0.1	o	0.44753 72	o	0.54310 69	o	0.61288 87	o	0.66577 21	o	0.70703 43	o
0.0	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o
0.1	o	1.62559 43	o	1.51426 78	o	1.43330 33	o	1.37218 57	o	1.32468 15	o
0.2	o	2.32972 23	o	2.09001 58	o	1.91600 24	o	1.78488 22	o	1.68314 92	o
0.3	o	3.11806 63	o	2.73155 70	o	2.45145 92	o	2.24076 63	o	2.07757 13	o
0.4	o	3.99659 90	o	3.44342 03	o	3.04320 02	o	2.74264 25	o	2.51021 73	o
0.5	o	4.97159 61	o	4.23035 88	o	3.69492 25	o	3.29344 83	o	2.98346 21	o
0.6	o	6.04964 75	o	5.09735 86	o	4.41050 04	o	3.89625 89	o	3.49978 96	o
0.7	o	7.23766 95	o	6.04964 75	o	5.19399 23	o	4.55429 24	o	4.06179 68	o
0.8	o	8.54291 84	o	7.09270 43	o	6.04964 75	o	5.27091 53	o	4.67219 76	o
0.9	o	9.97300 27	o	8.23226 86	o	6.98191 27	o	6.04964 75	o	5.33382 74	o
1.0	o	11.53589 71	o	9.47435 04	o	7.99544 06	o	6.89416 80	o	6.04964 75	o

$x = 1.9$

$$10^{-t} {}_1F_1[a; b; x]$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	o	-18.00000 00	o	-8.50000 00	o	-5.33333 33	o	-3.75000 00	o	-2.80000 00	o
-0.9	o	-18.28448 78	o	-8.53275 35	o	-5.29455 00	o	-3.68263 04	o	-2.72000 70	o
-0.8	o	-18.22287 44	o	-8.40699 75	o	-5.15830 19	o	-3.54744 15	o	-2.58942 00	o
-0.7	o	-17.76909 79	o	-8.10285 93	o	-4.91304 44	o	-3.33682 33	o	-2.40284 79	o
-0.6	o	-16.87376 78	o	-7.59907 62	o	-4.54645 00	o	-3.04266 54	o	-2.15455 52	o
-0.5	o	-15.48399 81	o	-6.87292 64	o	-4.04537 13	o	-2.65633 38	o	-1.83844 68	o
-0.4	o	-13.54322 89	o	-5.90015 74	o	-3.39580 04	o	-2.16864 53	o	-1.44805 10	o
-0.3	o	-10.99104 63	o	-4.65491 10	o	-2.58282 84	o	-1.56984 27	o	-0.97650 23	o
-0.2	o	-7.76299 01	o	-3.10964 63	o	-1.59060 32	o	-0.84956 80	o	-0.41652 37	o
-0.1	o	-3.79035 80	o	-1.23505 95	o	-0.40228 48	o	0.00316 50	o	0.23959 16	o
0.0	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o
0.1	o	6.68589 44	o	3.62861 59	o	2.63520 53	o	2.15326 62	o	1.87332 64	o
0.2	o	13.35001 66	o	6.68589 44	o	4.52341 51	o	3.47601 02	o	2.86868 53	o
0.3	o	21.08016 88	o	10.20905 78	o	6.68589 44	o	4.98202 73	o	3.99570 18	o
0.4	o	29.96950 06	o	14.23754 08	o	9.14514 19	o	6.68589 44	o	5.26453 38	o
0.5	o	40.11675 75	o	18.81309 05	o	11.92494 50	o	8.60300 40	o	6.68589 44	o
0.6	o	51.62653 44	o	23.97987 02	o	15.05043 60	o	10.74959 91	o	8.27107 61	o
0.7	I	6.46095 44	o	29.78456 71	o	18.54815 16	o	13.14281 03	o	10.03197 42	o
0.8	I	7.91828 88	o	36.27650 33	o	22.44609 30	o	15.80069 36	o	11.98111 27	o
0.9	I	9.54703 48	o	43.50775 13	o	26.77378 95	o	18.74226 79	o	14.13167 04	o
1.0	I	11.36026 72	o	51.53325 42	o	31.56236 29	o	21.98755 79	o	16.49750 76	o
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	o	-2.16666 67	o	-1.71428 57	o	-1.37500 00	o	-1.11111 11	o	-0.90000 00	o
-0.9	o	-2.08127 98	o	-1.62714 90	o	-1.28806 31	o	-1.02544 65	o	-0.81619 66	o
-0.8	o	-1.95640 16	o	-1.50818 21	o	-1.17484 56	o	-0.91767 04	o	-0.71350 43	o
-0.7	o	-1.78802 68	o	-1.35430 62	o	-1.03291 81	o	-0.78582 61	o	-0.59032 04	o
-0.6	o	-1.57190 18	o	-1.16225 67	o	-0.85970 92	o	-0.62784 56	o	-0.44495 33	o
-0.5	o	-1.30351 37	o	-0.92857 55	o	-0.65249 87	o	-0.44154 43	o	-0.27561 85	o
-0.4	o	-0.97807 84	o	-0.64960 23	o	-0.40841 17	o	-0.22461 69	o	-0.08043 55	o
-0.3	o	-0.59052 84	o	-0.32146 58	o	-0.12441 18	o	0.02536 87	o	0.14257 69	o
-0.2	o	-0.13550 05	o	0.05992 58	o	0.20270 59	o	0.31097 59	o	0.39550 50	o
-0.1	o	0.39267 74	o	0.49889 41	o	0.57632 21	o	0.63490 53	o	0.68054 33	o
0.0	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o
0.1	o	1.69279 80	o	1.56805 43	o	1.47749 37	o	1.40925 18	o	1.35630 09	o
0.2	o	2.47775 27	o	2.20812 80	o	2.01275 54	o	1.86580 69	o	1.75199 44	o
0.3	o	3.36191 14	o	2.92556 34	o	2.60994 41	o	2.37297 24	o	2.18975 62	o
0.4	o	4.35270 21	o	3.72598 49	o	3.27343 37	o	2.93422 24	o	2.67239 44	o
0.5	o	5.45795 02	o	4.61531 11	o	4.00782 14	o	3.55320 49	o	3.20285 46	o
0.6	o	6.68589 44	o	5.59976 66	o	4.81793 76	o	4.23374 86	o	3.78422 53	o
0.7	o	8.04520 42	o	6.68589 44	o	5.70885 41	o	4.97987 00	o	4.41974 32	o
0.8	o	9.54499 74	o	7.88056 89	o	6.68589 44	o	5.79578 05	o	5.11279 93	o
0.9	o	11.19485 87	o	9.19100 88	o	7.75464 34	o	6.68589 44	o	5.86694 41	o
1.0	o	13.00485 78	o	10.62479 10	o	8.92095 76	o	7.65483 61	o	6.68589 44	o

$$10^{-t} {}_1F_1[a; b; x]$$

$$x = 2.0$$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	o	-19.000000	00	o	-9.000000	00	o	-5.666666	67	o	-4.000000	00	o	-3.000000	00
-0.9	o	-19.48030	55	o	-9.11450	17	o	-5.67351	46	o	-3.96130	19	o	-2.93919	07
-0.8	o	-19.57745	66	o	-9.05346	66	o	-5.57239	86	o	-3.84746	13	o	-2.82231	33
-0.7	o	-19.23633	87	o	-8.79313	67	o	-5.34952	70	o	-3.64939	40	o	-2.64293	65
-0.6	o	-18.39760	92	o	-8.30798	80	o	-4.99011	58	o	-3.35738	15	o	-2.39419	33
-0.5	o	-16.99746	84	o	-7.57063	95	o	-4.47833	69	o	-2.96103	91	o	-2.06875	95
-0.4	o	-14.96742	39	o	-6.55175	56	o	-3.79726	53	o	-2.44928	29	o	-1.65883	15
-0.3	o	-12.23404	35	o	-5.21994	53	o	-2.92882	35	o	-1.81029	53	o	-1.15610	28
-0.2	o	-8.71869	86	o	-3.54165	85	o	-1.85372	47	o	-1.03148	91	o	-0.55174	05
-0.1	o	-4.33729	58	o	-1.48107	68	o	-0.55141	27	o	-0.09947	04	o	0.16363	98
0.0	o	1.000000	00	o	1.000000	00	o	1.000000	00	o	1.000000	00	o	1.000000	00
0.1	o	7.38905	61	o	3.94227	08	o	2.82379	65	o	2.28204	65	o	1.96790	63
0.2	o	14.93207	26	o	7.38905	61	o	4.94472	25	o	3.76272	10	o	3.07855	70
0.3	o	23.73789	60	o	11.38642	43	o	7.38905	61	o	5.45904	52	o	4.34381	16
0.4	o	33.92234	38	o	15.98332	46	o	10.18467	94	o	7.38905	61	o	5.77622	05
0.5	o	45.60854	33	o	21.23172	33	o	13.36115	38	o	9.57185	21	o	7.38905	61
0.6	o	58.92728	34	o	27.18674	62	o	16.94979	76	o	12.02764	18	o	9.19634	52
0.7	I	7.40173	79	o	33.90682	71	o	20.98376	67	o	14.77779	32	o	11.21290	24
0.8	I	9.10260	50	o	41.45386	02	o	25.49813	83	o	17.84488	62	o	13.45436	45
0.9	I	11.01093	18	o	49.89335	97	o	30.52999	75	o	21.25276	63	o	15.93722	65
1.0	I	13.14324	07	o	59.29462	61	o	36.11852	77	o	25.02659	99	o	18.67887	84
b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	o	-2.33333	33	o	-1.85714	29	o	-1.50000	00	o	-1.22222	22	o	-1.00000	00
-0.9	o	-2.26126	09	o	-1.77944	34	o	-1.41981	77	o	-1.14139	10	o	-0.91961	70
-0.8	o	-2.14541	69	o	-1.66645	90	o	-1.31049	88	o	-1.03604	28	o	-0.81828	84
-0.7	o	-1.98102	67	o	-1.51452	15	o	-1.16915	08	o	-0.90384	92	o	-0.69410	79
-0.6	o	-1.76300	12	o	-1.31972	80	o	-0.99270	14	o	-0.74234	11	o	-0.54505	72
-0.5	o	-1.48592	22	o	-1.07793	01	o	-0.77789	00	o	-0.54890	19	o	-0.36900	05
-0.4	o	-1.14402	63	o	-0.78472	21	o	-0.52125	94	o	-0.32076	13	o	-0.16367	96
-0.3	o	-0.73118	88	o	-0.43542	96	o	-0.21914	64	o	-0.05498	80	o	0.07329	14
-0.2	o	-0.24090	68	o	-0.02509	64	o	0.13232	70	o	0.25151	67	o	0.34443	19
-0.1	o	0.33371	85	o	0.45152	76	o	0.53726	33	o	0.60202	71	o	0.65240	03
0.0	o	1.00000	00	o	1.00000	00	o	1.00000	00	o	1.00000	00	o	1.00000	00
0.1	o	1.76568	31	o	1.62619	96	o	1.52511	88	o	1.44908	29	o	1.39018	53
0.2	o	2.63896	62	o	2.33634	06	o	2.11745	71	o	1.95312	22	o	1.82606	83
0.3	o	3.62852	02	o	3.13698	76	o	2.78211	92	o	2.51617	15	o	2.31092	49
0.4	o	4.74350	98	o	4.03507	07	o	3.52448	69	o	3.14250	04	o	2.84820	18
0.5	o	5.99361	56	o	5.03790	12	o	4.35023	19	o	3.83660	33	o	3.44152	38
0.6	o	7.38905	61	o	6.15318	83	o	5.26532	81	o	4.60320	94	o	4.09470	06
0.7	o	8.94061	14	o	7.38905	61	o	6.27606	40	o	5.44729	14	o	4.81173	45
0.8	o	10.65964	79	o	8.75406	09	o	7.38905	61	o	6.37407	65	o	5.59682	82
0.9	o	12.55814	26	o	10.25720	96	o	8.61126	20	o	7.38905	61	o	6.45439	28
1.0	o	14.64870	93	o	11.90797	86	o	9.94999	52	o	8.49799	64	o	7.38905	61

$x=2 \cdot 1$

$$10^{-t} {}_1F_1[a; b; x]$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	o	-20.00000 00	o	-9.50000 00	o	-6.00000 00	o	-4.25000 00	o	-3.20000 00	o
-0.9	o	-20.70333 98	o	-9.70811 91	o	-6.05945 31	o	-4.24462 51	o	-3.16171 03	o
-0.8	o	-20.98488 95	o	-9.72292 51	o	-5.99996 90	o	-4.15644 72	o	-3.06162 19	o
-0.7	o	-20.77918 02	o	-9.51621 92	o	-5.80519 20	o	-3.97470 08	o	-2.89211 83	o
-0.6	o	-20.01542 10	o	-9.05758 33	o	-5.45751 82	o	-3.68782 39	o	-2.64503 69	o
-0.5	o	-18.61719 70	o	-8.31425 69	o	-4.93802 93	o	-3.28341 62	o	-2.31163 94	o
-0.4	o	-16.50215 35	o	-7.25100 78	o	-4.22642 10	o	-2.74819 47	o	-1.88258 33	o
-0.3	o	-13.58166 80	o	-5.82999 93	o	-3.30092 93	o	-2.06794 77	o	-1.34789 05	o
-0.2	o	-9.76050 68	o	-4.01064 99	o	-2.13825 50	o	-1.22748 80	o	-0.69691 56	o
-0.1	o	-4.93646 62	o	-1.74948 78	o	-0.71348 30	o	-0.21060 23	o	0.08168 74	o
0.0	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o
0.1	o	8.16616 99	o	4.28752 69	o	3.03059 28	o	2.42274 09	o	2.07087 46	o
0.2	o	16.68745 89	o	8.16616 99	o	5.40856 88	o	4.07722 93	o	3.30797 23	o
0.3	o	26.69784 18	o	12.69254 15	o	8.16616 99	o	5.98431 91	o	4.72580 15	o
0.4	o	38.34029 38	o	17.92696 77	o	11.33770 40	o	8.16616 99	o	6.33975 88	o
0.5	o	51.76724 70	o	23.93367 38	o	14.95964 65	o	10.64630 99	o	8.16616 99	o
0.6	I	6.71410 69	o	30.78097 64	o	19.07074 45	o	13.44970 09	o	10.22233 44	o
0.7	I	8.46345 57	o	38.54148 57	o	23.71212 71	o	16.60280 60	o	12.52656 98	o
0.8	I	10.44314 58	o	47.29231 30	o	28.92741 99	o	20.13366 04	o	15.09825 93	o
0.9	I	12.67270 03	o	57.11528 66	o	34.76286 06	o	24.07194 39	o	17.95790 02	o
1.0	I	15.17284 64	o	68.09718 08	o	41.26742 40	o	28.44905 77	o	21.12715 47	o
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	o	-2.50000 00	o	-2.00000 00	o	-1.62500 00	o	-1.33333 33	o	-1.10000 00	o
-0.9	o	-2.44375 17	o	-1.93369 21	o	-1.55313 46	o	-1.25861 07	o	-1.02409 62	o
-0.8	o	-2.33924 82	o	-1.82847 82	o	-1.44913 78	o	-1.15684 75	o	-0.92508 81	o
-0.7	o	-2.18083 87	o	-1.68001 97	o	-1.30959 02	o	-1.02529 31	o	-0.80072 53	o
-0.6	o	-1.96247 85	o	-1.48368 41	o	-1.13084 80	o	-0.86102 07	o	-0.64861 67	o
-0.5	o	-1.67770 90	o	-1.23453 07	o	-0.90903 08	o	-0.66091 87	o	-0.46622 42	o
-0.4	o	-1.31963 68	o	-0.92729 48	o	-0.64001 07	o	-0.42168 22	o	-0.25085 59	o
-0.3	o	-0.88091 22	o	-0.55637 22	o	-0.31940 02	o	-0.13980 34	o	0.00034 07	o
-0.2	o	-0.35370 63	o	-0.11580 27	o	0.05745 95	o	0.18843 70	o	0.29038 70	o
-0.1	o	0.27031 18	o	0.40074 66	o	0.49551 25	o	0.56698 05	o	0.62247 99	o
0.0	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o
0.1	o	1.84476 79	o	1.68909 02	o	1.57647 50	o	1.49191 12	o	1.42652 01	o
0.2	o	2.81460 33	o	2.47557 85	o	2.23081 62	o	2.04738 29	o	1.90581 32	o
0.3	o	3.92009 96	o	3.36747 41	o	2.96924 32	o	2.67134 84	o	2.44186 18	o
0.4	o	5.17248 37	o	4.37325 49	o	3.79833 13	o	3.36901 76	o	3.03886 64	o
0.5	o	6.58364 59	o	5.50188 87	o	4.72502 78	o	4.14588 87	o	3.70125 57	o
0.6	o	8.16616 99	o	6.76285 54	o	5.75666 82	o	5.00776 08	o	4.43369 54	o
0.7	o	9.93336 48	o	8.16616 99	o	6.90099 41	o	5.96074 75	o	5.24109 93	o
0.8	o	11.89929 80	o	9.72240 60	o	8.16616 99	o	7.01128 96	o	6.12863 92	o
0.9	o	14.07882 88	o	11.44272 12	o	9.56080 16	o	8.16616 99	o	7.10175 58	o
1.0	o	16.48764 42	o	13.33888 17	o	11.09395 64	o	9.43252 68	o	8.16616 99	o

$$10^{-t} {}_1F_1[a; b; x]$$

$$x = 2.2$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	o	-21.00000 00	o	-10.00000 00	o	-6.33333 33	o	-4.50000 00	o	-3.40000 00	o
-0.9	o	-21.95549 32	o	-10.31440 47	o	-6.45281 75	o	-4.53289 07	o	-3.38776 72	o
-0.8	o	-22.44908 73	o	-10.41701 35	o	-6.44194 11	o	-4.47499 48	o	-3.30775 76	o
-0.7	o	-22.40352 09	o	-10.27457 71	o	-6.28143 41	o	-4.31363 82	o	-3.15101 11	o
-0.6	o	-21.73489 40	o	-9.85107 90	o	-5.95047 18	o	-4.03515 53	o	-2.90788 75	o
-0.5	o	-20.35226 95	o	-9.10757 47	o	-5.42658 77	o	-3.62483 42	o	-2.56802 94	o
-0.4	o	-18.15726 05	o	-8.00202 25	o	-4.68557 99	o	-3.06685 89	o	-2.12032 39	o
-0.3	o	-15.04359 49	o	-6.48910 72	o	-3.70141 42	o	-2.34424 90	o	-1.55286 16	o
-0.2	o	-10.89666 23	o	-4.52005 57	o	-2.44612 38	o	-1.43879 65	o	-0.85289 48	o
-0.1	o	-5.59304 05	o	-2.04244 49	o	-0.88970 36	o	-0.33100 08	o	-0.00679 36	o
0.0	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o
0.1	o	9.02501 35	o	4.66761 43	o	3.25740 35	o	2.57650 05	o	2.18301 97	o
0.2	o	18.63491 85	o	9.02501 35	o	5.91930 60	o	4.42230 20	o	3.55882 42	o
0.3	o	29.99356 18	o	14.14132 08	o	9.02501 35	o	6.56278 96	o	5.14504 93	o
0.4	o	43.27630 72	o	20.09040 57	o	12.61646 94	o	9.02501 35	o	6.96046 15	o
0.5	o	58.67064 71	o	26.95113 21	o	16.73838 95	o	11.83777 20	o	9.02501 35	o
0.6	I	7.63768 45	o	34.80761 63	o	21.43840 29	o	15.03169 93	o	11.35990 32	o
0.7	I	9.66086 03	o	43.74949 80	o	26.76719 93	o	18.63935 69	o	13.98763 39	o
0.8	I	11.95937 52	o	53.87221 92	o	32.77868 06	o	22.69532 61	o	16.93207 64	o
0.9	I	14.55749 85	I	6.52773 17	o	39.53012 12	o	27.23630 94	o	20.21853 60	o
1.0	I	17.48106 03	I	7.80727 29	o	47.08233 38	o	32.30122 98	o	23.87381 90	o

b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	o	-2.66666 67	o	-2.14285 71	o	-1.75000 00	o	-1.44444 44	o	-1.20000 00	o
-0.9	o	-2.62889 84	o	-2.09000 53	o	-1.68809 90	o	-1.37717 30	o	-1.12968 82	o
-0.8	o	-2.53819 42	o	-1.99446 44	o	-1.59093 57	o	-1.28022 15	o	-1.03401 36	o
-0.7	o	-2.38791 03	o	-1.85113 68	o	-1.45449 55	o	-1.15036 21	o	-0.91033 65	o
-0.6	o	-2.17091 38	o	-1.65456 03	o	-1.27448 43	o	-0.98414 83	o	-0.75584 31	o
-0.5	o	-1.87955 60	o	-1.39888 84	o	-1.04631 42	o	-0.77790 40	o	-0.56753 70	o
-0.4	o	-1.50564 49	o	-1.07787 05	o	-0.76508 85	o	-0.52771 18	o	-0.34223 03	o
-0.3	o	-1.04041 74	o	-0.68483 11	o	-0.42558 62	o	-0.22940 13	o	-0.07653 39	o
-0.2	o	-0.47450 90	o	-0.21264 87	o	-0.02224 57	o	0.12146 28	o	0.23315 15	o
-0.1	o	0.20207 66	o	0.34626 74	o	0.45085 18	o	0.52959 48	o	0.59064 59	o
0.0	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o
0.1	o	1.93061 91	o	1.75714 86	o	1.63188 55	o	1.53798 95	o	1.46550 64	o
0.2	o	3.00602 42	o	2.62685 41	o	2.35360 70	o	2.14919 40	o	1.99171 07	o
0.3	o	4.23907 43	o	3.61882 61	o	3.17269 43	o	2.83957 94	o	2.58342 26	o
0.4	o	5.64343 45	o	4.74336 95	o	4.09712 93	o	3.61546 18	o	3.24572 89	o
0.5	o	7.23361 54	o	6.01141 30	o	5.13536 73	o	4.48352 40	o	3.98400 54	o
0.6	o	9.02501 35	o	7.43453 89	o	6.29635 83	o	5.45083 22	o	4.80393 01	o
0.7	o	11.03395 40	o	9.02501 35	o	7.58957 04	o	6.52485 31	o	5.71149 64	o
0.8	o	13.27773 44	o	10.79581 94	o	9.02501 35	o	7.71347 24	o	6.71302 77	o
0.9	o	15.77467 05	o	12.76068 81	o	10.61326 35	o	9.02501 35	o	7.81519 12	o
1.0	o	18.54414 38	o	14.93413 45	o	12.36548 87	o	10.46825 62	o	9.02501 35	o

$x = 2.3$

$$10^{-t} {}_1F_1[a; b; x]$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	o	-22.00000 00	o	-10.50000 00	o	-6.66666 67	o	-4.75000 00	o	-3.60000 00	o
-0.9	o	-23.23882 05	o	-10.93421 88	o	-6.85409 39	o	-4.82641 04	o	-3.61757 64	o
-0.8	o	-23.97428 67	o	-11.13750 51	o	-6.89931 51	o	-4.80374 52	o	-3.56116 25	o
-0.7	o	-24.11576 32	o	-11.07088 63	o	-6.77976 13	o	-4.66717 15	o	-3.42027 96	o
-0.6	o	-23.56439 52	o	-10.69196 83	o	-6.47094 28	o	-4.40063 28	o	-3.18361 03	o
-0.5	o	-22.21259 59	o	-9.95472 55	o	-5.94633 55	o	-3.98677 71	o	-2.83894 98	o
-0.4	o	-19.94350 56	o	-8.80928 20	o	-5.17725 94	o	-3.40688 22	o	-2.37315 63	o
-0.3	o	-16.63042 48	o	-7.20168 00	o	-4.13275 31	o	-2.64077 67	o	-1.77209 84	o
-0.2	o	-12.13622 14	o	-5.07364 06	o	-2.77944 14	o	-1.66675 83	o	-1.02059 92	o
-0.1	o	-6.31270 97	o	-2.36231 08	o	-1.08139 77	o	-0.46150 78	o	-0.10237 99	o
0.0	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o
0.1	o	9.97418 25	o	5.08609 59	o	3.50622 00	o	2.74459 02	o	2.30520 61	o
0.2	o	20.79520 90	o	9.97418 25	o	6.48173 42	o	4.80098 52	o	3.83319 31	o
0.3	o	33.66229 26	o	15.74816 79	o	9.97418 25	o	7.19990 68	o	5.60527 37	o
0.4	o	48.78926 63	o	22.49797 87	o	14.03454 41	o	9.97418 25	o	7.64418 91	o
0.5	I	6.64053 96	o	30.31988 69	o	18.71731 71	o	13.15885 58	o	9.97418 25	o
0.6	I	8.67562 33	o	39.31685 40	o	24.08070 49	o	16.79130 27	o	12.62107 69	o
0.7	I	11.01044 99	o	49.59889 04	o	30.18681 18	o	20.91135 13	o	15.61235 49	o
0.8	I	13.67310 13	o	61.28342 68	o	37.10184 47	o	25.56140 89	o	18.97724 21	o
0.9	I	16.69356 66	I	7.44957 07	o	44.89632 81	o	30.78659 13	o	22.74679 45	o
1.0	I	20.10384 22	I	8.93691 92	o	53.64532 18	o	36.63485 94	o	26.95398 82	o

b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	o	-2.83333 33	o	-2.28571 43	o	-1.87500 00	o	-1.55555 56	o	-1.30000 00	o
-0.9	o	-2.81685 71	o	-2.24849 97	o	-1.82480 10	o	-1.49714 86	o	-1.23645 00	o
-0.8	o	-2.74257 47	o	-2.16465 71	o	-1.73607 74	o	-1.40630 98	o	-1.14518 11	o
-0.7	o	-2.60272 23	o	-2.02823 28	o	-1.60414 35	o	-1.27927 39	o	-1.02311 57	o
-0.6	o	-2.38893 21	o	-1.83282 40	o	-1.42396 95	o	-1.11200 62	o	-0.86696 23	o
-0.5	o	-2.09219 93	o	-1.57155 33	o	-1.19016 27	o	-0.90018 94	o	-0.67320 42	o
-0.4	o	-1.70284 58	o	-1.23704 29	o	-0.89694 85	o	-0.63920 76	o	-0.43808 78	o
-0.3	o	-1.21048 40	o	-0.82138 81	o	-0.53815 01	o	-0.32413 15	o	-0.15761 12	o
-0.2	o	-0.60397 75	o	-0.31612 84	o	-0.10716 74	o	0.05029 77	o	0.17248 93	o
-0.1	o	0.12859 85	o	0.28778 10	o	0.40304 49	o	0.48968 52	o	0.55675 12	o
0.0	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o
0.1	o	2.02385 71	o	1.83083 59	o	1.69170 30	o	1.58759 30	o	1.50736 33	o
0.2	o	3.21472 07	o	2.79127 57	o	2.48667 55	o	2.25921 59	o	2.08428 51	o
0.3	o	4.58810 89	o	3.89302 18	o	3.39398 13	o	3.02204 09	o	2.73654 12	o
0.4	o	6.16055 81	o	5.14852 67	o	4.42325 35	o	3.88368 12	o	3.47025 53	o
0.5	o	7.94967 32	o	6.57103 17	o	5.58472 23	o	4.85221 18	o	4.29191 53	o
0.6	o	9.97418 25	o	8.17460 49	o	6.88924 37	o	5.93619 26	o	5.20839 01	o
0.7	o	12.25399 31	o	9.97418 25	o	8.34833 04	o	7.14469 00	o	6.22694 81	o
0.8	o	14.81025 02	o	11.98561 07	o	9.97418 25	o	8.48730 20	o	7.35527 50	o
0.9	o	17.66539 63	o	14.22568 96	o	11.77972 05	o	9.97418 25	o	8.60149 34	o
1.0	o	20.84323 60	o	16.71221 90	o	13.77861 96	o	11.61606 66	o	9.97418 25	o

$$10^{-t} {}_1F_1[a; b; x]$$

$$x = 2.4$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	0	-23.000000 00	0	-11.000000 00	0	-7.000000 00	0	-5.000000 00	0	-3.800000 00	0
-0.9	0	-24.555554 10	0	-11.56848 97	0	-7.26380 57	0	-5.12551 97	0	-3.85136 91	0
-0.8	0	-25.56507 92	0	-11.88631 72	0	-7.37317 06	0	-5.14338 90	0	-3.82231 20	0
-0.7	0	-25.92286 03	0	-11.90804 75	0	-7.30180 48	0	-5.03634 31	0	-3.70064 07	0
-0.6	0	-25.51303 37	0	-11.58404 65	0	-7.02106 32	0	-4.78561 76	0	-3.47314 00	0
-0.5	0	-24.20898 72	0	-10.86020 93	0	-6.49979 78	0	-4.37085 54	0	-3.12550 54	0
-0.4	0	-21.87265 48	0	-9.67767 76	0	-5.70420 18	0	-3.77001 14	0	-2.64227 82	0
-0.3	0	-18.35377 46	0	-7.97254 23	0	-4.59764 78	0	-2.95925 03	0	-2.00677 83	0
-0.2	0	-13.48913 00	0	-5.67553 09	0	-3.14051 60	0	-1.91284 09	0	-1.20103 27	0
-0.1	0	-7.10173 71	0	-2.71168 17	0	-1.29001 60	0	-0.60304 46	0	-0.20570 14	0
0.0	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0
0.1	0	11.02317 64	0	5.54690 11	0	3.77923 43	0	2.92839 94	0	2.43838 18	0
0.2	0	23.19131 72	0	11.02317 64	0	7.10114 80	0	5.21663 23	0	4.13336 48	0
0.3	0	37.74535 19	0	17.53014 28	0	11.02317 64	0	7.90168 21	0	6.11056 90	0
0.4	0	54.94463 55	0	25.17669 20	0	15.60695 29	0	11.02317 64	0	8.39740 61	0
0.5	1	7.50680 19	0	34.07972 13	0	20.91854 43	0	14.62353 20	0	11.02317 64	0
0.6	1	9.84149 63	0	44.36458 49	0	27.02869 64	0	18.74810 72	0	14.01916 97	0
0.7	1	12.53067 15	0	56.16556 77	0	34.01308 99	0	23.44535 89	0	17.41877 02	0
0.8	1	15.60875 35	1	6.96263 74	0	41.95260 91	0	28.76700 92	0	21.25756 72	0
0.9	1	19.11259 67	1	8.49006 51	0	50.93362 14	0	34.76821 87	0	25.57346 94	0
1.0	1	23.08161 90	1	10.21525 18	0	61.04826 94	0	41.50776 41	0	30.40682 37	0

b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-3.000000 00	0	-2.42857 14	0	-2.000000 00	0	-1.66666 67	0	-1.400000 00	0
-0.9	0	-3.00779 48	0	-2.40930 09	0	-1.96333 70	0	-1.61861 37	0	-1.34444 24	0
-0.8	0	-2.95273 34	0	-2.33931 37	0	-1.88476 07	0	-1.53526 82	0	-1.25871 50	0
-0.7	0	-2.82579 21	0	-2.21169 47	0	-1.75883 19	0	-1.41226 18	0	-1.13924 96	0
-0.6	0	-2.61720 76	0	-2.01897 89	0	-1.57969 04	0	-1.24489 77	0	-0.98221 64	0
-0.5	0	-2.31643 50	0	-1.75311 96	0	-1.34103 21	0	-1.02813 19	0	-0.78351 03	0
-0.4	0	-1.91210 14	0	-1.40545 49	0	-1.03608 31	0	-0.75655 48	0	-0.53873 58	0
-0.3	0	-1.39195 86	0	-0.96667 36	0	-0.65757 48	0	-0.42437 11	0	-0.24319 17	0
-0.2	0	-0.74283 32	0	-0.42677 84	0	-0.19771 60	0	-0.02537 90	0	0.10814 51	0
-0.1	0	0.04942 55	0	0.22495 13	0	0.35183 50	0	0.44705 13	0	0.52063 63	0
0.0	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0
0.1	0	2.12516 14	0	1.91065 64	0	1.75631 19	0	1.64102 16	0	1.55232 86	0
0.2	0	3.44232 89	0	2.97005 69	0	2.63094 67	0	2.37816 90	0	2.18410 78	0
0.3	0	4.97013 18	0	4.19223 09	0	3.63475 68	0	3.22001 92	0	2.90223 69	0
0.4	0	6.72847 41	0	5.59214 73	0	4.77930 67	0	4.17570 01	0	3.71404 88	0
0.5	0	8.73860 25	0	7.18576 34	0	6.07690 84	0	5.25491 77	0	4.62733 19	0
0.6	0	11.02317 64	0	8.99007 65	0	7.54066 02	0	6.46798 57	0	5.65035 23	0
0.7	0	13.60634 17	0	11.02317 64	0	9.18448 60	0	7.82585 49	0	6.79187 69	0
0.8	0	16.51380 79	0	13.30430 15	0	11.02317 64	0	9.34014 49	0	8.06119 80	0
0.9	0	19.77292 73	0	15.85389 62	0	13.07243 15	0	11.02317 64	0	9.46815 83	0
1.0	0	23.41277 89	0	18.69367 12	0	15.34890 60	0	12.88800 50	0	11.02317 64	0

$x=2.5$

$$10^{-t} {}_1F_1[a; b; x]$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	o	-24.00000 00	o	-11.50000 00	o	-7.33333 33	o	-5.25000 00	o	-4.00000 00	o
-0.9	o	-25.90805 55	o	-12.21821 97	o	-7.68251 69	o	-5.43057 89	o	-4.08939 34	o
-0.8	o	-27.22643 91	o	-12.66552 33	o	-7.86467 35	o	-5.49467 08	o	-4.09171 79	o
-0.7	o	-27.83236 51	o	-12.78920 52	o	-7.84933 05	o	-5.42227 97	o	-3.99286 78	o
-0.6	o	-27.59072 62	o	-12.53143 77	o	-7.60314 61	o	-5.19158 35	o	-3.77748 76	o
-0.5	o	-26.35324 38	o	-11.82892 68	o	-7.08972 02	o	-4.77881 65	o	-3.42889 33	o
-0.4	o	-23.95757 62	o	-10.61254 93	o	-6.26939 51	o	-4.15814 63	o	-2.92898 93	o
-0.3	o	-20.22637 77	o	-8.80697 16	o	-5.09904 85	o	-3.30154 37	o	-2.25818 23	o
-0.2	o	-14.96631 32	o	-6.33025 02	o	-3.53187 37	o	-2.17864 71	o	-1.39529 00	o
-0.1	o	-7.96701 65	o	-3.09340 93	o	-1.51714 96	o	-0.75661 98	o	-0.31744 60	o
0.0	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o
0.1	o	12.18249 39	o	6.05436 41	o	4.07885 96	o	3.12945 53	o	2.58358 68	o
0.2	o	25.84869 74	o	12.18249 39	o	7.78338 74	o	5.67293 50	o	4.46185 13	o
0.3	o	42.28858 30	o	19.50613 57	o	12.18249 39	o	8.67474 62	o	6.66544 30	o
0.4	o	61.81518 96	o	28.15650 66	o	17.35033 23	o	12.18249 39	o	9.22724 38	o
0.5	I	8.47660 17	o	38.27485 65	o	23.36660 46	o	16.24727 03	o	12.18249 39	o
0.6	I	11.15044 87	o	50.01305 51	o	30.31688 18	o	20.92385 15	o	15.56892 25	o
0.7	I	14.24214 72	o	63.53421 12	o	38.29293 67	o	26.27090 46	o	19.42688 58	o
0.8	I	17.79368 99	I	7.90133 16	o	47.39309 73	o	32.35120 37	o	23.79950 93	o
0.9	I	21.85014 37	I	9.66379 17	o	57.72260 96	o	39.23185 57	o	28.73283 78	o
1.0	I	26.45982 53	I	11.66088 33	o	69.39402 46	o	46.98453 50	o	34.27599 35	o

b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	o	-3.16666 67	o	-2.57142 86	o	-2.12500 00	o	-1.77777 78	o	-1.50000 00	o
-0.9	o	-3.20189 06	o	-2.57254 28	o	-2.10381 02	o	-1.74164 92	o	-1.45373 01	o
-0.8	o	-3.16903 98	o	-2.51871 03	o	-2.03719 77	o	-1.66726 29	o	-1.37474 84	o
-0.7	o	-3.05767 72	o	-2.40193 89	o	-1.91887 98	o	-1.54957 66	o	-1.25893 79	o
-0.6	o	-2.85646 81	o	-2.21356 80	o	-1.74206 31	o	-1.38314 86	o	-1.10186 52	o
-0.5	o	-2.55312 42	o	-1.94422 88	o	-1.49941 35	o	-1.16211 60	o	-0.89876 17	o
-0.4	o	-2.13434 58	o	-1.58380 16	o	-1.18302 42	o	-0.88016 95	o	-0.64450 52	o
-0.3	o	-1.58576 08	o	-1.12137 16	o	-0.78438 28	o	-0.53052 85	o	-0.33360 02	o
-0.2	o	-0.89186 11	o	-0.54518 23	o	-0.29433 68	o	-0.10591 48	o	0.03984 28	o
-0.1	o	-0.03593 55	o	0.15741 26	o	0.29694 29	o	0.40147 53	o	0.48212 86	o
0.0	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o
0.1	o	2.23527 55	o	1.99716 12	o	1.82613 19	o	1.69860 18	o	1.60066 11	o
0.2	o	3.69064 40	o	3.16452 66	o	2.78743 16	o	2.50683 95	o	2.29180 16	o
0.3	o	5.38836 12	o	4.51883 45	o	3.89683 09	o	3.43492 06	o	3.08162 25	o
0.4	o	7.35226 96	o	6.07799 12	o	5.16814 50	o	4.49373 62	o	3.97886 27	o
0.5	o	9.60788 63	o	7.86113 47	o	6.61612 05	o	5.69489 62	o	4.99282 64	o
0.6	o	12.18249 39	o	9.88870 14	o	8.25648 42	o	7.05076 73	o	6.13341 27	o
0.7	o	15.10523 69	o	12.18249 39	o	10.10599 41	o	8.57451 20	o	7.41114 52	o
0.8	o	18.40722 15	o	14.76575 50	o	12.18249 39	o	10.28012 84	o	8.83720 34	o
0.9	o	22.12161 95	o	17.66324 15	o	14.50496 81	o	12.18249 39	o	10.42345 59	o
1.0	o	26.28377 83	o	20.90130 35	o	17.09360 01	o	14.29740 82	o	12.18249 39	o

$$10^{-t} {}_1F_1[a; b; x]$$

$$x=2.6$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	0	-25.000000 00	0	-12.000000 00	0	-7.666666 67	0	-5.500000 00	0	-4.200000 00	0
-0.9	0	-27.29896 02	0	-12.88449 02	0	-8.11083 48	0	-5.74197 61	0	-4.33191 57	0
-0.8	0	-28.96375 85	0	-13.47736 79	0	-8.37508 26	0	-5.85839 34	0	-4.36993 07	0
-0.7	0	-29.85248 35	0	-13.71777 00	0	-8.42424 95	0	-5.82619 81	0	-4.29779 48	0
-0.6	0	-29.80827 23	0	-13.53862 48	0	-8.21970 10	0	-5.62012 67	0	-4.09774 56	0
-0.5	0	-28.65824 05	0	-12.86621 71	0	-7.71908 79	0	-5.21255 72	0	-3.75041 07	0
-0.4	0	-26.21234 49	0	-11.61972 61	0	-6.87609 51	0	-4.57335 34	0	-3.23470 14	0
-0.3	0	-22.26219 13	0	-9.71074 26	0	-5.64017 71	0	-3.66969 96	0	-2.52770 43	0
-0.2	0	-16.57977 13	0	-7.04275 78	0	-3.95627 87	0	-2.46592 80	0	-1.60456 46	0
-0.1	0	-8.91613 67	0	-3.51062 61	0	-1.76454 26	0	-0.92333 71	0	-0.43836 54	0
0.0	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0
0.1	0	13.46373 80	0	6.61326 57	0	4.40775 27	0	3.34943 56	0	2.74196 18	0
0.2	0	28.79553 69	0	13.46373 80	0	8.53489 47	0	6.17395 84	0	4.82141 36	0
0.3	0	47.34284 61	0	21.69706 37	0	13.46373 80	0	9.52641 42	0	7.27485 96	0
0.4	1	6.94818 19	0	31.47066 01	0	19.28310 31	0	13.46373 80	0	10.14156 92	0
0.5	1	9.56191 12	0	42.95443 96	0	26.08871 30	0	18.04719 75	0	13.46373 80	0
0.6	1	12.61937 76	0	56.33157 15	0	33.98360 60	0	23.34264 74	0	17.28665 75	0
0.7	1	16.16792 47	1	7.17992 79	0	43.07856 96	0	29.42079 67	0	21.65889 18	0
0.8	1	20.25854 23	1	8.95696 84	0	53.49259 41	0	36.35748 90	0	26.63246 40	0
0.9	1	24.94608 65	1	10.98706 74	1	6.53533 50	0	44.23399 47	0	32.26304 96	0
1.0	1	30.28950 78	1	13.29468 38	1	7.87976 85	0	53.13732 09	0	38.61018 10	0
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-3.33333 33	0	-2.71428 57	0	-2.25000 00	0	-1.88888 89	0	-1.60000 00	0
-0.9	0	-3.39933 69	0	-2.73836 90	0	-2.24633 07	0	-1.86634 14	0	-1.56438 22	0
-0.8	0	-3.39189 02	0	-2.70314 30	0	-2.19361 52	0	-1.80247 20	0	-1.49342 31	0
-0.7	0	-3.29897 77	0	-2.59941 26	0	-2.08463 00	0	-1.69148 62	0	-1.38239 46	0
-0.6	0	-3.10749 89	0	-2.41717 61	0	-1.91153 51	0	-1.52710 90	0	-1.22618 74	0
-0.5	0	-2.80319 81	0	-2.14557 40	0	-1.66583 66	0	-1.30255 53	0	-1.01928 74	0
-0.4	0	-2.37059 15	0	-1.77283 58	0	-1.33834 63	0	-1.01049 99	0	-0.75575 24	0
-0.3	0	-1.79288 93	0	-1.28622 36	0	-0.91914 00	0	-0.64304 49	0	-0.42918 68	0
-0.2	0	-1.05191 55	0	-0.67197 35	0	-0.39751 29	0	-0.19168 61	0	-0.03271 61	0
-0.1	0	-0.12802 19	0	0.08476 62	0	0.23806 53	0	0.35272 11	0	0.44104 08	0
0.0	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0
0.1	0	2.35501 37	0	2.09095 26	0	1.90162 09	0	1.76068 96	0	1.65264 25	0
0.2	0	3.96163 49	0	3.37614 02	0	2.95723 59	0	2.64608 59	0	2.40804 50	0
0.3	0	5.84633 45	0	4.87544 47	0	4.18218 59	0	3.66828 33	0	3.27591 30	0
0.4	0	8.03754 64	0	6.61019 16	0	5.59290 28	0	4.84021 91	0	4.26661 47	0
0.5	0	10.56577 97	0	8.60323 21	0	7.20697 04	0	6.17571 92	0	5.39121 62	0
0.6	0	13.46373 79	0	10.87902 72	0	9.04318 73	0	7.68954 63	0	6.66151 93	0
0.7	0	16.76644 10	0	13.46373 80	0	11.12163 31	0	9.39745 01	0	8.09009 98	0
0.8	0	20.51135 60	0	16.38531 86	0	13.46373 79	0	11.31621 95	0	9.69034 80	0
0.9	0	24.73853 14	0	19.67361 33	0	16.09235 45	0	13.46373 79	0	11.47651 14	0
1.0	0	29.49073 80	0	23.36045 89	0	19.03183 37	0	15.85903 97	0	13.46373 80	0

$x=2.7$

$$10^{-t} {}_1F_1[a; b; x]$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	o	-26.00000 00	o	-12.50000 00	o	-8.00000 00	o	-5.75000 00	o	-4.40000 00	o
-0.9	o	-28.73106 17	o	-13.56846 87	o	-8.54941 48	o	-6.06012 85	o	-4.57922 22	o
-0.8	o	-30.78288 60	o	-14.32427 93	o	-8.90575 85	o	-6.23542 28	o	-4.65754 41	o
-0.7	o	-31.99213 46	o	-14.69744 16	o	-9.02863 20	o	-6.24941 46	o	-4.61632 22	o
-0.6	o	-32.17743 60	o	-14.61048 28	o	-8.87345 16	o	-6.07297 69	o	-4.43509 62	o
-0.5	o	-31.13803 52	o	-13.97789 85	o	-8.39114 84	o	-5.67413 74	o	-4.09146 35	o
-0.4	o	-28.65236 09	o	-12.70557 45	o	-7.52785 10	o	-5.01788 16	o	-3.56094 82	o
-0.3	o	-24.47651 78	o	-10.69017 41	o	-6.22455 25	o	-4.06594 53	o	-2.81686 18	o
-0.2	o	-18.34268 87	o	-7.81849 24	o	-4.41675 69	o	-2.77659 65	o	-1.83015 86	o
-0.1	o	-9.95745 15	o	-3.96677 50	o	-2.03410 84	o	-1.10440 49	o	-0.56928 13	o
0.0	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o
0.1	o	14.87973 16	o	7.22887 93	o	4.76883 88	o	3.59018 46	o	2.91475 81	o
0.2	o	32.06304 61	o	14.87973 18	o	9.36277 58	o	6.72417 80	o	5.21508 58	o
0.3	o	52.96456 60	o	24.12608 74	o	14.87973 16	o	10.46475 51	o	7.94428 47	o
0.4	1	7.80344 30	o	35.15602 29	o	21.42566 08	o	14.87973 18	o	11.14906 08	o
0.5	1	10.77606 03	o	48.17308 92	o	29.11505 61	o	20.04227 69	o	14.87973 16	o
0.6	1	14.26713 08	o	63.39729 19	o	38.07147 76	o	26.03123 84	o	19.19044 35	o
0.7	1	18.33375 87	1	8.10661 17	o	48.42811 24	o	32.93148 76	o	24.13939 78	o
0.8	1	23.03759 88	1	10.14356 13	o	60.32835 66	o	40.83427 79	o	29.78909 02	o
0.9	1	28.44514 01	1	12.47815 26	1	7.39264 32	o	49.83762 23	o	36.20656 09	o
1.0	1	34.62800 25	1	15.14004 89	1	8.93880 28	o	60.04669 21	o	43.46365 66	o
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	o	-3.50000 00	o	-2.85714 28	o	-2.37500 00	o	-2.00000 00	o	-1.70000 00	o
-0.9	o	-3.60033 97	o	-2.90693 33	o	-2.39101 66	o	-1.99278 30	o	-1.67647 25	o
-0.8	o	-3.62171 17	o	-2.89292 97	o	-2.35425 70	o	-1.94108 60	o	-1.61489 09	o
-0.7	o	-3.55034 07	o	-2.80459 75	o	-2.25645 08	o	-1.83827 88	o	-1.50984 88	o
-0.6	o	-3.37114 84	o	-2.63043 38	o	-2.08858 83	o	-1.67715 52	o	-1.35548 23	o
-0.5	o	-3.06766 49	o	-2.35790 37	o	-1.84087 29	o	-1.44989 60	o	-1.14544 18	o
-0.4	o	-2.62193 68	o	-1.97337 21	o	-1.50267 11	o	-1.14803 07	o	-0.87286 17	o
-0.3	o	-2.01442 99	o	-1.46203 44	o	-1.06245 92	o	-0.76239 75	o	-0.53033 00	o
-0.2	o	-1.22392 71	o	-0.80784 14	o	-0.50776 87	o	-0.28310 09	o	-0.10985 47	o
-0.1	o	-0.22742 07	o	0.00657 82	o	0.17487 22	o	0.30053 23	o	0.39716 99	o
0.0	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o
0.1	o	2.48526 80	o	2.19268 94	o	1.98327 88	o	1.82767 27	o	1.70857 90	o
0.2	o	4.25746 26	o	3.60649 19	o	3.14156 84	o	2.79684 52	o	2.53357 76	o
0.3	o	6.34794 05	o	5.26492 82	o	4.49299 40	o	3.92178 94	o	3.48643 51	o
0.4	o	8.79047 31	o	7.19329 27	o	6.05702 03	o	5.21781 11	o	4.57940 27	o
0.5	o	11.62139 03	o	9.41875 81	o	7.85452 86	o	6.70130 78	o	5.82558 86	o
0.6	o	14.87973 17	o	11.97048 41	o	9.90789 75	o	8.38983 02	o	7.23900 55	o
0.7	o	18.60740 47	o	14.87973 18	o	12.24108 80	o	10.30214 71	o	8.83462 01	o
0.8	o	22.84935 09	o	18.17998 31	o	14.87973 16	o	12.45831 14	o	10.62840 51	o
0.9	o	27.65372 08	o	21.90706 62	o	17.85122 36	o	14.87973 16	o	12.63739 26	o
1.0	o	33.07205 19	o	26.09928 66	o	21.18482 02	o	17.58924 40	o	14.87973 17	o

$$10^{-t} {}_1F_1[a; b; x]$$

$$x = 2.8$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	o	-27.00000 00	o	-13.00000 00	o	-8.33333 33	o	-6.00000 00	o	-4.60000 00	o
-0.9	o	-30.20739 92	o	-14.27141 68	o	-8.99896 28	o	-6.38548 56	o	-4.83162 05	o
-0.8	o	-32.69016 26	o	-15.20888 77	o	-9.45817 17	o	-6.62669 32	o	-4.95519 64	o
-0.7	o	-34.26101 35	o	-15.73223 52	o	-9.66472 00	o	-6.69335 18	o	-4.94942 20	o
-0.6	o	-34.71103 52	o	-15.75231 16	o	-9.56735 47	o	-6.55200 83	o	-4.79081 81	o
-0.5	o	-33.80797 81	o	-15.17031 11	o	-9.10943 43	o	-6.16579 37	o	-4.45357 64	o
-0.4	o	-31.29447 41	o	-13.87704 67	o	-8.22853 23	o	-5.49417 79	o	-3.90939 53	o
-0.3	o	-26.88613 55	o	-11.75218 16	o	-6.85601 94	o	-4.49270 98	o	-3.12730 65	o
-0.2	o	-20.26955 03	o	-8.66341 77	o	-4.91662 00	o	-3.11274 30	o	-2.07349 21	o
-0.1	o	-11.10015 73	o	-4.46563 82	o	-2.32794 53	o	-1.30114 66	o	-0.71109 18	o
0.0	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o
0.1	o	16.44464 67	o	7.90702 22	o	5.16533 88	o	3.85372 90	o	3.10334 87	o
0.2	o	35.68578 35	o	16.44464 68	o	10.27486 98	o	7.32852 26	o	5.64620 39	o
0.3	o	59.21633 38	o	26.81885 03	o	16.44464 66	o	11.49867 08	o	8.67973 76	o
0.4	I	8.75729 37	o	39.25349 19	o	23.80058 73	o	16.44464 67	o	12.25929 29	o
0.5	I	12.13388 97	o	53.99149 83	o	32.47911 57	o	22.25350 26	o	16.44464 70	o
0.6	I	16.11472 85	I	7.12961 09	o	42.62784 01	o	29.01928 57	o	21.30028 58	o
0.7	I	20.76842 24	I	9.14521 94	o	54.40626 03	o	36.84347 58	o	26.89569 60	o
0.8	I	26.16923 24	I	11.47676 35	I	6.79865 21	o	45.83544 68	o	33.30566 72	o
0.9	I	32.39743 22	I	14.15747 84	I	8.35541 94	o	56.11294 85	o	40.61061 30	o
1.0	I	39.53969 12	I	17.22319 87	I	10.13091 09	I	6.78026 14	o	48.89690 70	o
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	o	-3.66666 67	o	-3.00000 00	o	-2.50000 00	o	-2.11111 11	o	-1.80000 00	o
-0.9	o	-3.80512 04	o	-3.07840 06	o	-2.53799 39	o	-2.12107 24	o	-1.79007 94	o
-0.8	o	-3.85896 28	o	-3.08841 18	o	-2.51938 38	o	-2.08330 86	o	-1.73931 41	o
-0.7	o	-3.81246 32	o	-3.01801 15	o	-2.43473 79	o	-1.99026 30	o	-1.64154 58	o
-0.6	o	-3.64833 26	o	-2.85402 14	o	-2.27374 15	o	-1.83369 17	o	-1.49007 09	o
-0.5	o	-3.34761 54	o	-2.58202 69	o	-2.02513 89	o	-1.60461 79	o	-1.27760 54	o
-0.4	o	-2.88957 27	o	-2.18629 30	o	-1.67667 00	o	-1.29328 42	o	-0.99624 72	o
-0.3	o	-2.25156 21	o	-1.64967 70	o	-1.21500 46	o	-0.88910 25	o	-0.63743 83	o
-0.2	o	-1.40890 89	o	-0.95353 49	o	-0.62567 34	o	-0.38060 11	o	-0.19192 26	o
-0.1	o	-0.33477 17	o	-0.07762 39	o	0.10700 47	o	0.24463 04	o	0.35029 61	o
0.0	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o
0.1	o	2.62701 51	o	2.30309 15	o	2.07165 13	o	1.89997 38	o	1.76880 40	o
0.2	o	4.58049 78	o	3.85732 82	o	3.34175 14	o	2.96014 07	o	2.66920 59	o
0.3	o	6.89745 39	o	5.69043 11	o	4.83163 51	o	4.19727 93	o	3.71463 80	o
0.4	o	9.61784 37	o	7.83229 02	o	6.56427 33	o	5.62942 96	o	4.91952 24	o
0.5	o	12.78476 53	o	10.31509 54	o	8.56437 02	o	7.27596 59	o	6.29932 78	o
0.6	o	16.44464 66	o	13.17347 63	o	10.85846 60	o	9.15767 55	o	7.87062 96	o
0.7	o	20.64744 67	o	16.44464 70	o	13.47504 46	o	11.29683 99	o	9.65117 27	o
0.8	o	25.44686 77	o	20.16855 73	o	16.44464 66	o	13.71732 03	o	11.65993 74	o
0.9	o	30.90057 59	o	24.38805 52	o	19.79998 76	o	16.44464 66	o	13.91720 85	o
1.0	o	37.07043 74	o	29.14905 21	o	23.57608 14	o	19.50611 19	o	16.44464 68	o

$x=2.9$

$$10^{-t} {}_1F_1[a; b; x]$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	o	-28.000000 00	o	-13.500000 00	o	-8.666666 67	o	-6.250000 00	o	-4.800000 00	o
-0.9	o	-31.73126 17	o	-14.99469 74	o	-9.46024 07	o	-6.71853 11	o	-5.08944 16	o
-0.8	o	-34.69247 10	o	-16.13404 03	o	-10.03391 17	o	-7.03321 28	o	-5.26357 64	o
-0.7	o	-36.66966 46	o	-16.82650 88	o	-10.33494 30	o	-7.15954 92	o	-5.29814 37	o
-0.6	o	-37.42303 62	o	-16.96987 82	o	-10.30462 12	o	-7.05925 28	o	-5.16629 50	o
-0.5	o	-36.68483 16	o	-16.45036 74	o	-9.87778 99	o	-6.68995 61	o	-4.83840 23	o
-0.4	o	-34.15712 41	o	-15.14173 78	o	-8.98235 82	o	-6.00490 66	o	-4.28185 36	o
-0.3	o	-29.50944 41	o	-12.90433 50	o	-7.53877 86	o	-4.95264 26	o	-3.46083 72	o
-0.2	o	-22.37627 32	o	-9.58407 54	o	-5.45949 43	o	-3.47665 22	o	-2.33611 44	o
-0.1	o	-12.35437 95	o	-5.01137 25	o	-2.64835 54	o	-1.51501 08	o	-0.86477 83	o
0.0	o	1.000000 00	o	1.000000 00	o	1.000000 00	o	1.000000 00	o	1.000000 00	o
0.1	o	18.17414 53	o	8.65411 14	o	5.60079 93	o	4.14229 67	o	3.30924 01	o
0.2	o	39.70201 27	o	18.17414 53	o	11.27982 49	o	7.99241 96	o	6.11843 45	o
0.3	1	6.61675 77	o	29.80374 42	o	18.17414 53	o	12.63798 21	o	9.48784 73	o
0.4	1	9.82083 67	o	43.80842 97	o	26.43288 26	o	18.17414 53	o	13.48282 86	o
0.5	1	13.65191 98	o	60.47710 22	o	36.21802 61	o	24.70411 80	o	18.17414 55	o
0.6	1	18.18561 18	1	8.01236 21	o	47.70528 95	o	32.33968 23	o	23.63833 11	o
0.7	1	23.50405 66	1	10.30884 80	o	61.08501 02	o	41.20175 09	o	29.95805 22	o
0.8	1	29.69637 80	1	12.97405 63	1	7.65631 06	o	51.42094 85	o	37.22249 40	o
0.9	1	36.85914 19	1	16.04790 01	1	9.43620 73	o	63.13822 98	o	45.52776 84	o
1.0	1	45.09684 68	1	19.57351 24	1	11.47220 45	1	7.65055 24	o	54.97734 01	o
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	o	-3.83333 33	o	-3.14285 71	o	-2.62500 00	o	-2.22222 22	o	-1.90000 00	o
-0.9	o	-4.01391 62	o	-3.25294 74	o	-2.68739 75	o	-2.25131 52	o	-1.90528 67	o
-0.8	o	-4.10413 72	o	-3.28995 58	o	-2.68927 56	o	-2.22935 84	o	-1.86686 61	o
-0.7	o	-4.08609 66	o	-3.24021 24	o	-2.61991 64	o	-2.14777 03	o	-1.77774 87	o
-0.6	o	-3.94004 17	o	-3.08867 26	o	-2.46755 36	o	-1.99715 36	o	-1.63029 83	o
-0.5	o	-3.64423 09	o	-2.81881 82	o	-2.21930 01	o	-1.76723 87	o	-1.41618 82	o
-0.4	o	-3.17479 16	o	-2.41255 41	o	-1.86106 96	o	-1.44682 50	o	-1.12635 56	o
-0.3	o	-2.50556 85	o	-1.85009 88	o	-1.37749 56	o	-1.02371 84	o	-0.75095 31	o
-0.2	o	-1.60796 48	o	-1.10986 86	o	-0.75184 50	o	-0.48466 60	o	-0.27929 82	o
-0.1	o	-0.45077 35	o	-0.16835 56	o	0.03407 24	o	0.18471 29	o	0.30018 06	o
0.0	o	1.000000 00	o	1.000000 00	o	1.000000 00	o	1.000000 00	o	1.000000 00	o
0.1	o	2.78132 50	o	2.42294 67	o	2.16733 43	o	1.97805 37	o	1.83368 06	o
0.2	o	4.93334 28	o	4.13056 27	o	3.55923 11	o	3.13708 99	o	2.81580 92	o
0.3	o	7.49957 47	o	6.15540 69	o	5.20071 71	o	4.49676 66	o	3.96210 47	o
0.4	o	10.52714 07	o	8.53267 71	o	7.11880 70	o	6.07827 21	o	5.28948 58	o
0.5	o	14.06699 04	o	11.30037 52	o	9.34262 62	o	7.90441 94	o	6.81614 32	o
0.6	o	18.17414 53	o	14.49948 29	o	11.90354 07	o	9.99974 29	o	8.56161 58	o
0.7	o	22.90795 32	o	18.17414 54	o	14.83529 32	o	12.39060 24	o	10.54686 88	o
0.8	o	28.33235 82	o	22.37186 33	o	18.17414 53	o	15.10529 03	o	12.79437 83	o
0.9	o	34.51618 12	o	27.14369 68	o	21.95902 78	o	18.17414 53	o	15.32821 67	o
1.0	o	41.53341 63	o	32.54447 83	o	26.23169 82	o	21.62967 07	o	18.17414 54	o

$$10^{-t} {}_1F_1[a; b; x]$$

$$x=3.0$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	o	-29.00000 00	o	-14.00000 00	o	-9.00000 00	o	-6.50000 00	o	-5.00000 00	o
-0.9	o	-33.30621 16	o	-15.73978 46	o	-9.93407 08	o	-7.05978 64	o	-5.35304 12	o
-0.8	o	-36.79727 80	o	-17.10282 33	o	-10.63469 80	o	-7.45607 06	o	-5.58342 64	o
-0.7	o	-39.22955 51	o	-17.98499 40	o	-11.04193 46	o	-7.64967 22	o	-5.66362 13	o
-0.6	o	-40.32866 49	o	-18.26945 68	o	-11.08873 86	o	-7.59691 35	o	-5.56302 56	o
-0.5	o	-39.78690 70	o	-17.82560 47	o	-10.70039 99	o	-7.24926 52	o	-5.24773 51	o
-0.4	o	-37.26049 47	o	-16.50794 66	o	-9.79393 10	o	-6.55296 83	o	-4.68029 12	o
-0.3	o	-32.36662 44	o	-14.15492 18	o	-8.27742 11	o	-5.44863 44	o	-3.81941 32	o
-0.2	o	-24.68034 89	o	-10.58764 06	o	-6.04935 06	o	-3.87082 14	o	-2.61971 68	o
-0.1	o	-13.73126 66	o	-5.60854 66	o	-2.99786 42	o	-1.74758 44	o	-1.03141 45	o
0.0	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o
0.1	o	20.08553 69	o	9.47722 59	o	6.07912 54	o	4.45833 69	o	3.53408 59	o
0.2	o	44.15409 90	o	20.08553 69	o	12.38718 15	o	8.72184 58	o	6.63580 89	o
0.3	I	7.38953 06	o	33.11220 35	o	20.08553 69	o	13.89352 31	o	10.37591 45	o
0.4	I	11.00640 87	o	48.87114 63	o	29.35022 61	o	20.08553 69	o	14.83132 12	o
0.5	I	15.34853 86	I	6.77048 22	o	40.37297 04	o	27.41985 47	o	20.08553 69	o
0.6	I	20.50591 44	I	8.99862 22	o	53.36225 72	o	36.02890 68	o	26.22909 71	o
0.7	I	26.57655 56	I	11.61209 77	I	6.85444 79	o	46.05628 63	o	33.36002 71	o
0.8	I	33.66706 53	I	14.65495 92	I	8.61651 36	o	57.65748 59	o	41.58433 07	o
0.9	I	41.89321 92	I	18.17497 90	I	10.64901 07	I	7.10006 77	o	51.01650 20	o
1.0	I	51.38057 94	I	22.22390 06	I	12.98069 89	I	8.62675 29	o	61.78006 74	o

b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	o	-4.00000 00	o	-3.28571 43	o	-2.75000 00	o	-2.33333 33	o	-2.00000 00	o
-0.9	o	-4.22698 22	o	-3.43076 30	o	-2.83937 21	o	-2.38362 40	o	-2.02218 42	o
-0.8	o	-4.35776 62	o	-3.49795 60	o	-2.86423 29	o	-2.37946 94	o	-1.99773 28	o
-0.7	o	-4.37205 21	o	-3.47180 10	o	-2.81244 39	o	-2.31115 69	o	-1.91873 97	o
-0.6	o	-4.24734 55	o	-3.33517 92	o	-2.67062 70	o	-2.16800 93	o	-1.77653 51	o
-0.5	o	-3.95879 10	o	-3.06922 35	o	-2.42407 50	o	-1.93831 66	o	-1.56163 16	o
-0.4	o	-3.47899 59	o	-2.65319 13	o	-2.05665 60	o	-1.60926 30	o	-1.26366 86	o
-0.3	o	-2.77784 39	o	-2.06432 90	o	-1.55071 24	o	-1.16684 99	o	-0.87135 18	o
-0.2	o	-1.82229 73	o	-1.27772 90	o	-0.88695 48	o	-0.59581 55	o	-0.37239 14	o
-0.1	o	-0.57618 87	o	-0.26617 84	o	-0.04434 96	o	0.12045 11	o	0.24656 45	o
0.0	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o	1.00000 00	o
0.1	o	2.94937 02	o	2.55311 64	o	2.27097 83	o	2.06241 48	o	1.90360 36	o
0.2	o	5.31885 34	o	4.42829 20	o	3.79559 00	o	3.32891 38	o	2.97434 69	o
0.3	o	8.15947 04	o	6.66364 61	o	5.60309 83	o	4.82245 41	o	4.23056 48	o
0.4	o	11.52660 56	o	9.30049 38	o	7.72517 18	o	6.56784 35	o	5.69204 17	o
0.5	o	15.48029 63	o	12.38355 42	o	10.19603 85	o	8.59185 66	o	7.38010 13	o
0.6	o	20.08553 69	o	15.96116 96	o	13.05264 80	o	10.92335 76	o	9.31770 08	o
0.7	o	25.41259 97	o	20.08553 69	o	16.33484 26	o	13.59343 00	o	11.52953 10	o
0.8	o	31.53737 44	o	24.81295 02	o	20.08553 69	o	16.63551 15	o	14.04211 98	o
0.9	o	38.54172 24	o	30.20405 68	o	24.35090 60	o	20.08553 69	o	16.88394 24	o
1.0	o	46.51385 21	o	36.32412 59	o	29.18058 51	o	23.98208 78	o	20.08553 69	o

$x=3.1$

$$10^{-t} {}_1F_1[a; b; x]$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	0	-30.00000 00	0	-14.50000 00	0	-9.33333 33	0	-6.75000 00	0	-5.20000 00	0
-0.9	0	-34.93610 48	0	-16.50827 03	0	-10.42133 98	0	-7.40981 29	0	-5.62280 17	0
-0.8	0	-39.01269 22	0	-18.11857 93	0	-11.26239 00	0	-7.89644 23	0	-5.91554 60	0
-0.7	0	-41.95316 08	0	-19.21282 93	0	-11.78854 90	0	-8.16552 33	0	-6.04707 99	0
-0.6	0	-43.44452 08	0	-19.65787 58	0	-11.92349 59	0	-8.16738 01	0	-5.98263 15	0
-0.5	0	-43.13420 75	0	-19.30424 16	0	-11.58181 93	0	-7.84659 04	0	-5.68352 03	0
-0.4	0	-40.62668 28	0	-17.98474 06	0	-10.66827 03	0	-7.14152 18	0	-5.10684 75	0
-0.3	0	-35.47981 09	0	-15.51301 64	0	-9.07696 45	0	-5.98383 89	0	-4.20516 84	0
-0.2	0	-27.20100 00	0	-11.68198 53	0	-6.69053 70	0	-4.29798 08	0	-2.92614 43	0
-0.1	0	-15.24309 34	0	-6.26218 19	0	-3.37924 22	0	-2.00060 53	0	-1.21217 35	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	22.19795 13	0	10.38417 55	0	6.60461 84	0	4.80454 21	0	3.77970 06	0
0.2	0	49.08894 65	0	22.19795 11	0	13.60746 45	0	9.52338 44	0	7.20275 98	0
0.3	1	8.24849 28	0	36.77902 90	0	22.19795 13	0	15.27724 76	0	11.35198 08	0
0.4	1	12.32771 54	0	54.49743 70	0	32.58325 95	0	22.19795 13	0	16.31762 90	0
0.5	1	17.24421 02	1	7.57578 92	0	44.98961 70	0	30.42919 91	0	22.19795 12	0
0.6	1	23.10477 10	1	10.10023 03	0	59.66364 75	0	40.12740 88	0	29.09972 44	0
0.7	1	30.02600 07	1	13.07124 22	1	7.68738 05	0	51.46258.96	0	37.13883 45	0
0.8	1	38.13501 37	1	16.54126 52	1	9.69118 82	1	6.46192 69	0	46.44088 91	0
0.9	1	47.57017 65	1	20.56729 99	1	12.00946 12	1	7.97974 69	0	57.14186 14	0
1.0	1	58.48189 55	1	25.21122 31	1	14.67653 51	1	9.72137 49	1	6.93887 72	0
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-4.16666 67	0	-3.42857 14	0	-2.87500 00	0	-2.44444 45	0	-2.10000 00	0
-0.9	0	-4.44459 20	0	-3.61205 03	0	-2.99407 20	0	-2.51811 91	0	-2.14086 70	0
-0.8	0	-4.62042 14	0	-3.71283 54	0	-3.04457 76	0	-2.53389 21	0	-2.13211 23	0
-0.7	0	-4.67120 38	0	-3.71342 38	0	-3.01281 18	0	-2.48080 43	0	-2.06482 08	0
-0.6	0	-4.57140 07	0	-3.59439 55	0	-2.88361 02	0	-2.34676 21	0	-1.92917 91	0
-0.5	0	-4.29268 13	0	-3.33426 54	0	-2.64023 93	0	-2.11845 28	0	-1.71441 04	0
-0.4	0	-3.80370 77	0	-2.90932 65	0	-2.26427 93	0	-1.78125 65	0	-1.40870 54	0
-0.3	0	-3.06990 45	0	-2.29348 42	0	-1.73550 03	0	-1.31915 10	0	-0.99914 96	0
-0.2	0	-2.05321 62	0	-1.45807 95	0	-1.03173 16	0	-0.71461 28	0	-0.47164 57	0
-0.1	0	-0.71184 94	0	-0.37170 35	0	-0.12872 33	0	0.05148 83	0	0.18916 73	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	3.13243 53	0	2.69454 32	0	2.38329 44	0	2.15360 53	0	1.97900 36	0
0.2	0	5.74016 44	0	4.75281 42	0	4.05255 96	0	3.53694 60	0	3.14586 51	0
0.3	0	8.88282 31	0	7.21931 02	0	6.04191 14	0	5.17675 33	0	4.52190 84	0
0.4	0	12.62531 62	0	10.14238 27	0	8.38836 40	0	7.10198 59	0	6.13019 99	0
0.5	0	17.03817 75	0	13.57449 82	0	11.13202 18	0	9.34397 48	0	7.99566 11	0
0.6	0	22.19795 11	0	17.57251 08	0	14.31628 19	0	11.93657 68	0	10.14518 55	0
0.7	0	28.18759 35	0	22.19795 12	0	17.98804 70	0	14.91633 47	0	12.60776 35	0
0.8	0	35.09689 70	0	27.51733 23	0	22.19795 11	0	18.32264 80	0	15.41461 19	0
0.9	0	43.02293 64	0	33.60246 82	0	27.00059 55	0	22.19795 11	0	18.59931 18	0
1.0	0	52.07054 16	0	40.53081 38	0	32.45479 82	0	26.58790 04	0	22.19795 13	0

$$10^{-t} {}_1F_1[a; b; x]$$

$$x=3.2$$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	0	-31.000000	00	0	-15.000000	00	0	-9.666666	67	0	-7.000000	00	0	-5.400000	00
-0.9	0	-36.62512	21	0	-17.30187	83	0	-10.92300	64	0	-7.76921	66	0	-5.89913	53
-0.8	0	-41.34751	66	0	-19.18493	37	0	-11.91900	09	0	-8.35559	85	0	-6.26079	89
-0.7	0	-44.85405	86	0	-20.51559	69	0	-12.57788	30	0	-8.70905	47	0	-6.44984	41
-0.6	0	-46.78870	50	0	-21.14256	90	0	-12.81301	10	0	-8.77324	55	0	-6.42687	07
-0.5	0	-46.74859	39	0	-20.89524	48	0	-12.52700	96	0	-8.48505	19	0	-6.14787	05
-0.4	0	-44.27988	02	0	-19.58203	20	0	-11.61085	44	0	-7.77400	82	0	-5.56384	99
-0.3	0	-38.87328	71	0	-16.98855	60	0	-9.94289	51	0	-6.56169	88	0	-4.62042	85
-0.2	0	-29.95935	51	0	-12.87574	65	0	-7.38781	65	0	-4.76111	63	0	-3.25741	21
-0.1	0	-16.90337	79	0	-6.97779	83	0	-3.79553	08	0	-2.27597	77	0	-1.40833	93
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	0	24.53253	02	0	11.38357	57	0	7.18201	52	0	5.18387	24	0	4.04807	60
0.2	0	54.55848	48	0	24.53253	00	0	14.95228	64	0	10.40428	54	0	7.82416	07
0.3	1	9.20311	67	0	40.84274	94	0	24.53253	00	0	16.80234	32	0	12.42490	31
0.4	1	13.79998	39	0	60.74917	84	0	36.16590	66	0	24.53253	02	0	17.95593	80
0.5	1	19.36170	70	1	8.47287	68	0	50.11860	92	0	33.76368	61	0	24.53253	00
0.6	1	26.01465	50	1	11.33035	92	1	6.66815	54	0	44.68004	08	0	32.28025	85
0.7	1	33.89713	91	1	14.70441	75	1	8.61721	60	0	57.48230	85	0	41.33573	59
0.8	1	43.16029	08	1	18.65729	87	1	10.89362	49	1	7.23888	46	0	51.84737	62
0.9	2	5.39689	94	1	23.25681	79	1	13.53500	38	1	8.96368	79	0	63.97619	77
1.0	2	6.65028	76	1	28.57675	15	1	16.58222	73	1	10.94837	97	1	7.78966	80

b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	0	-4.33333	33	0	-3.57142	86	0	-3.00000	00	0	-2.55555	56	0	-2.20000	00
-0.9	0	-4.66704	02	0	-3.79702	75	0	-3.15166	33	0	-2.65492	98	0	-2.26143	76
-0.8	0	-4.89271	91	0	-3.93505	01	0	-3.23065	62	0	-2.69289	57	0	-2.27021	78
-0.7	0	-4.98449	62	0	-3.96577	85	0	-3.22154	97	0	-2.65712	38	0	-2.21631	69
-0.6	0	-4.91345	83	0	-3.86724	49	0	-3.10720	36	0	-2.53395	48	0	-2.08865	89
-0.5	0	-4.64740	41	0	-3.61505	11	0	-2.86863	18	0	-2.30829	69	0	-1.87503	67
-0.4	0	-4.15058	03	0	-3.18217	69	0	-2.48486	05	0	-1.96351	82	0	-1.56202	76
-0.3	0	-3.38340	11	0	-2.53877	81	0	-1.93277	66	0	-1.48133	08	0	-1.13490	46
-0.2	0	-2.30215	03	0	-1.65196	98	0	-1.18696	77	0	-0.84166	98	0	-0.57754	21
-0.1	0	-0.85866	50	0	-0.48559	85	0	-0.21955	23	0	-0.02256	41	0	0.12768	37
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	0	3.33192	79	0	2.84825	82	0	2.50505	88	0	2.25222	26	0	2.06034	91
0.2	0	6.20071	75	0	5.10664	78	0	4.33203	44	0	3.76264	36	0	3.33150	55
0.3	0	9.67588	18	0	7.82696	79	0	6.52059	06	0	5.56230	30	0	4.83820	08
0.4	0	13.83327	16	0	11.06564	92	0	9.11387	00	0	7.68491	17	0	6.60725	41
0.5	0	18.75552	29	0	14.88407	61	0	12.15873	20	0	10.16703	11	0	8.66771	10
0.6	0	24.53253	00	0	19.34892	60	0	15.70600	36	0	13.04826	27	0	11.05098	88
0.7	0	31.26194	60	0	24.53253	00	0	19.81074	52	0	16.37144	87	0	13.79103	12
0.8	0	39.04971	17	0	30.51325	17	0	24.53253	02	0	20.18288	19	0	16.92446	91
0.9	0	48.01061	43	0	37.37588	67	0	29.93573	97	0	24.53253	00	0	20.49079	27
1.0	0	58.26887	71	0	45.21208	69	0	36.08987	52	0	29.47426	84	0	24.53253	02

$x=3.3$ $10^{-t} {}_1F_1[a; b; x]$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	0	-32.00000 00	0	-15.50000 00	0	-10.00000 00	0	-7.25000 00	0	-5.60000 00	0
-0.9	0	-38.37779 46	0	-18.12247 36	0	-11.44010 57	0	-8.13865 01	0	-6.18248 56	0
-0.8	0	-43.81131 25	0	-20.30581 61	0	-12.60670 97	0	-8.83491 22	0	-6.62011 64	0
-0.7	0	-47.94702 63	0	-21.89936 35	0	-13.41329 50	0	-9.28238 00	0	-6.87334 63	0
-0.6	0	-50.38096 06	0	-22.73163 03	0	-13.76176 01	0	-9.41732 33	0	-6.89764 72	0
-0.5	0	-50.65395 51	0	-22.60839 62	0	-13.54137 37	0	-9.16804 22	0	-6.64307 89	0
-0.4	0	-48.24657 91	0	-21.31065 54	0	-12.62766 12	0	-8.45417 68	0	-6.05382 98	0
-0.3	0	-42.57369 48	0	-18.59242 46	0	-10.88121 13	0	-7.18597 21	0	-5.06772 79	0
-0.2	0	-32.97863 99	0	-14.17840 17	0	-8.14640 70	0	-5.26349 29	0	-3.61571 99	0
-0.1	0	-18.72700 50	0	-7.76146 37	0	-4.25006 67	0	-2.57578 81	0	-1.62131 60	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	27.11263 89	0	12.48493 21	0	7.81653 24	0	5.59958 31	0	4.34139 87	0
0.2	0	60.62020 30	0	27.11263 87	0	16.43445 88	0	11.37253 46	0	8.50537 10	0
0.3	1	10.26390 71	0	45.34601 64	0	27.11263 87	0	18.48336 09	0	13.60443 80	0
0.4	1	15.44012 73	1	6.76949 85	0	40.13571 89	0	27.11263 89	0	19.76190 06	0
0.5	1	21.72636 58	1	9.47201 43	0	55.81610 36	0	37.45822 45	0	27.11263 91	0
0.6	1	29.27175 81	1	12.70366 44	1	7.44960 44	0	49.73653 41	0	35.80395 66	0
0.7	1	38.23992 02	1	16.53183 26	1	9.65498 64	1	6.41839 05	0	45.99647 73	0
0.8	1	48.81005 35	1	21.03020 89	1	12.23863 12	1	8.10580 38	0	57.86509 50	0
0.9	2	6.11781 03	1	26.27925 33	1	15.24510 60	1	10.06395 19	1	7.15999 76	1
1.0	2	7.55580 00	1	32.36669 02	1	18.72293 59	1	12.32334 44	1	8.74076 27	1
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-4.50000 00	0	-3.71428 57	0	-3.12500 00	0	-2.66666 67	0	-2.30000 00	0
-0.9	0	-4.89464 33	0	-3.98592 84	0	-3.31232 40	0	-2.79419 41	0	-2.38400 52	0
-0.8	0	-5.17532 28	0	-4.16508 97	0	-3.42284 05	0	-2.85676 86	0	-2.41227 71	0
-0.7	0	-5.31294 83	0	-4.22961 65	0	-3.43922 74	0	-2.84055 68	0	-2.37357 62	0
-0.6	0	-5.27487 20	0	-4.15472 44	0	-3.34216 17	0	-2.73017 16	0	-2.25543 50	0
-0.5	0	-5.02458 75	0	-3.91277 84	0	-3.11015 84	0	-2.50854 93	0	-2.04406 20	0
-0.4	0	-4.52140 88	0	-3.47306 19	0	-2.71939 65	0	-2.15681 78	0	-1.72424 06	0
-0.3	0	-3.72012 88	0	-2.80152 90	0	-2.14353 65	0	-1.65415 69	0	-1.27921 99	0
-0.2	0	-2.57065 62	0	-1.86054 16	0	-1.35352 43	0	-0.97765 00	0	-0.69060 15	0
-0.1	0	-1.01762 80	0	-0.60859 09	0	-0.31738 44	0	-0.10212 77	0	0.06178 30	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	3.54939 12	0	3.01538 98	0	2.63711 59	0	2.35891 91	0	2.14815 04	0
0.2	0	6.70429 08	0	5.49255 36	0	4.63608 78	0	4.00759 95	0	3.53251 37	0
0.3	0	10.54551 80	0	8.49163 62	0	7.04290 09	0	5.98199 22	0	5.18169 98	0
0.4	0	15.16148 75	0	12.07832 78	0	9.90771 44	0	8.32123 99	0	7.12681 12	0
0.5	0	20.64876 19	0	16.32426 06	0	13.28513 91	0	11.06789 68	0	9.40161 27	0
0.6	0	27.11263 87	0	21.30742 99	0	17.23455 00	0	14.26816 44	0	12.04271 13	0
0.7	0	34.66776 93	0	27.11263 90	0	21.82041 41	0	17.97213 82	0	15.08974 65	0
0.8	0	43.43881 71	0	33.83196 98	0	27.11263 87	0	22.23406 87	0	18.58559 04	0
0.9	0	53.56115 08	0	41.56528 08	0	33.18693 57	0	27.11263 87	0	22.57655 99	0
1.0	0	65.18157 85	0	50.42073 33	0	40.12521 12	0	32.67125 22	0	27.11263 89	0

$$10^{-t} {}_1F_1[a; b; x]$$

$$x=3.4$$

b=		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	0	-33.00000	00	0	-16.00000	00	0	-10.33333	33	0	-7.50000	00	0	-5.80000	00
-0.9	0	-40.19903	18	0	-18.97207	29	0	-11.97375	59	0	-8.51881	86	0	-6.47333	05
-0.8	0	-46.41447	08	0	-21.48548	96	0	-13.32787	57	0	-9.33586	77	0	-6.99450	39
-0.7	0	-51.24815	32	0	-23.37072	13	0	-14.29842	98	0	-9.88778	95	0	-7.31913	55
-0.6	0	-54.24282	73	0	-24.43387	45	0	-14.77461	02	0	-10.10266	80	0	-7.39702	47
-0.5	0	-54.87640	50	0	-24.45436	99	0	-14.63079	82	0	-9.89925	13	0	-7.17163	56
-0.4	0	-52.55579	31	0	-23.18245	68	0	-13.72521	54	0	-9.18611	31	0	-6.57954	13
-0.3	0	-46.61025	52	0	-20.33654	47	0	-11.89847	27	0	-7.86076	12	0	-5.54982	95
-0.2	0	-36.28438	36	0	-15.60035	17	0	-8.97201	93	0	-5.80868	22	0	-4.00347	04
-0.1	0	-20.73036	93	0	-8.61984	96	0	-4.74651	29	0	-2.90232	24	0	-1.85264	00
0.0	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00
0.1	0	29.96409	99	0	13.69873	18	0	8.51391	67	0	6.05525	33	0	4.66206	91
0.2	1	6.73377	48	0	29.96410	01	0	18.06811	68	0	12.43692	89	0	9.25228	51
0.3	1	11.44251	25	0	50.33604	75	0	29.96409	98	0	20.33635	56	0	14.90133	25
0.4	1	17.26693	55	1	7.54109	40	0	44.53426	97	0	29.96409	98	0	21.75278	80
0.5	1	24.36638	01	1	10.58460	75	0	62.14436	40	0	41.55145	63	0	29.96410	00
0.6	1	32.91640	89	1	14.23644	86	1	8.31960	39	0	55.35202	71	0	39.70763	42
0.7	1	43.11009	22	1	18.57600	45	1	10.81296	23	1	7.16434	06	0	51.17177	80
0.8	2	5.51593	70	1	23.69029	03	1	13.74270	32	1	9.07289	37	1	6.45621	18
0.9	2	6.92965	05	1	29.67452	76	1	17.16148	75	1	11.29397	40	1	8.01026	82
1.0	2	8.57756	22	1	36.63275	41	1	21.12677	50	1	13.86364	58	1	9.80372	64

b=		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	0	-4.66666	67	0	-3.85714	29	0	-3.25000	00	0	-2.77777	78	0	-2.40000	00
-0.9	0	-5.12774	17	0	-4.17900	43	0	-3.47624	42	0	-2.93605	97	0	-2.50868	63
-0.8	0	-5.46894	77	0	-4.40348	17	0	-3.62153	03	0	-3.02582	04	0	-2.55853	43
-0.7	0	-5.65766	08	0	-4.50574	84	0	-3.66645	82	0	-3.03157	78	0	-2.53697	29
-0.6	0	-5.65710	47	0	-4.45791	09	0	-3.58929	91	0	-2.93604	20	0	-2.43000	27
-0.5	0	-5.42599	69	0	-4.22874	43	0	-3.36579	83	0	-2.71996	61	0	-2.22208	10
-0.4	0	-4.91814	36	0	-3.78341	27	0	-2.96896	69	0	-2.36198	81	0	-1.89599	84
-0.3	0	-4.08204	21	0	-3.08316	96	0	-2.36885	95	0	-1.83846	10	0	-1.43274	83
-0.2	0	-2.86043	14	0	-2.08503	84	0	-1.53233	73	0	-1.12327	39	0	-0.81138	88
-0.1	0	-1.18982	30	0	-0.74147	38	0	-0.42281	63	0	-0.18766	14	0	-0.00889	42
0.0	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00
0.1	0	3.78651	62	0	3.19717	32	0	2.78040	50	0	2.47440	62	0	2.24296	41
0.2	0	7.25503	31	0	5.91355	84	0	4.96699	12	0	4.27355	41	0	3.75024	92
0.3	0	11.49929	00	0	9.21882	42	0	7.61297	05	0	6.43898	40	0	5.55487	36
0.4	0	16.62209	81	0	13.18925	59	0	10.77651	37	0	9.01603	54	0	7.69282	05
0.5	0	22.73602	19	0	17.90824	26	0	14.52111	10	0	12.05411	94	0	10.20324	62
0.6	0	29.96410	00	0	23.46679	62	0	18.91595	40	0	15.60700	67	0	13.03140	67
0.7	0	38.44060	97	0	29.96410	00	0	24.03634	01	0	19.73313	11	0	16.51536	56
0.8	0	48.31188	30	0	37.50808	93	0	29.96410	08	0	24.49591	31	0	20.41329	11
0.9	0	59.73716	04	0	46.21606	75	0	36.78805	04	0	29.96410	00	0	24.87666	91
1.0	1	7.28895	08	0	56.21535	89	0	44.60447	61	0	36.21212	48	0	29.96410	00

$x = 3.5$

$10^{-t} {}_1F_1[a; b; x]$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	0	-34.00000 00	0	-16.50000 00	0	-10.66666 67	0	-7.75000 00	0	-6.00000 00	0
-0.9	0	-42.09416 08	0	-19.85286 15	0	-12.52516 60	0	-8.91048 18	0	-6.77218 48	0
-0.8	0	-49.16827 94	0	-22.72857 82	0	-14.08505 43	0	-9.86007 06	0	-7.38504 71	0
-0.7	0	-54.77495 93	0	-24.93683 81	0	-15.23724 28	0	-10.52776 43	0	-7.78888 76	0
-0.6	0	-58.39780 50	0	-26.25890 37	0	-15.85685 37	0	-10.83259 59	0	-7.92724 03	0
-0.5	0	-59.44449 41	0	-26.44481 71	0	-15.80169 74	0	-10.68269 32	0	-7.73624 57	0
-0.4	0	-57.23930 36	0	-25.21039 03	0	-14.91064 10	0	-9.97427 01	0	-7.14398 16	0
-0.3	0	-51.01508 79	0	-22.23397 79	0	-13.00185 40	0	-8.59054 64	0	-6.06974 53	0
-0.2	0	-39.90465 62	0	-17.15301 22	0	-9.87093 31	0	-6.40059 13	0	-4.42328 76	0
-0.1	0	-22.93152 52	0	-9.56029 12	0	-5.28888 99	0	-3.25808 57	0	-2.10399 19	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	33.11545 19	0	15.03654 56	0	9.28049 72	0	6.55481 93	0	5.01272 30	0
0.2	1	7.47815 75	0	33.11545 21	0	19.86885 66	0	13.60715 76	0	10.07138 46	0
0.3	1	12.75184 96	0	55.86511 24	0	33.11545 18	0	22.37904 10	0	16.32742 67	0
0.4	1	19.30127 25	1	8.39814 04	0	49.40757 52	0	33.11545 18	0	23.94765 68	0
0.5	1	27.31310 58	1	11.82332 36	1	6.91724 24	0	46.08615 63	0	33.11545 21	0
0.6	1	36.99354 11	1	15.94684 57	1	9.28802 84	0	61.58764 98	0	44.03204 04	0
0.7	1	48.56986 28	1	20.86201 73	1	12.10479 13	1	7.99452 28	0	56.91786 42	0
0.8	2	6.22921 37	1	26.67134 49	1	15.42421 54	1	10.15151 89	1	7.20140 23	0
0.9	2	7.84350 09	1	33.48724 72	1	19.30837 79	1	12.66871 77	1	8.95838 18	0
1.0	2	9.72996 16	1	41.43281 55	1	23.82515 50	1	15.58866 62	1	10.99143 77	0
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-4.83333 33	0	-4.00000 00	0	-3.37500 00	0	-2.88888 89	0	-2.50000 00	0
-0.9	0	-5.36670 12	0	-4.37652 51	0	-3.64362 89	0	-3.08068 51	0	-2.63560 61	0
-0.8	0	-5.77436 52	0	-4.65079 37	0	-3.82715 47	0	-3.20038 33	0	-2.70925 14	0
-0.7	0	-6.01982 28	0	-4.79504 86	0	-3.90390 29	0	-3.23069 74	0	-2.70690 86	0
-0.6	0	-6.06174 65	0	-4.77796 88	0	-3.84949 46	0	-3.15224 43	0	-2.61289 54	0
-0.5	0	-5.85354 66	0	-4.56435 32	0	-3.63661 08	0	-2.94336 39	0	-2.40973 47	0
-0.4	0	-5.34290 35	0	-4.11478 22	0	-3.23474 14	0	-2.57992 98	0	-2.07800 76	0
-0.3	0	-4.47126 80	0	-3.38525 70	0	-2.60991 74	0	-2.03514 47	0	-1.59619 62	0
-0.2	0	-3.17332 75	0	-2.32681 38	0	-1.72442 42	0	-1.27932 37	0	-0.94051 60	0
-0.1	0	-1.37643 46	0	-0.88511 28	0	-0.53649 78	0	-0.27966 44	0	-0.08473 82	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	4.04515 71	0	3.39495 99	0	2.93592 72	0	2.59946 04	0	2.34539 62	0
0.2	0	7.85749 99	0	6.37298 04	0	5.32722 85	0	4.56241 06	0	3.98619 61	0
0.3	0	12.54551 05	0	10.01458 20	0	8.23532 62	0	6.93674 11	0	5.96042 09	0
0.4	0	18.22847 28	0	14.40815 45	0	11.72753 51	0	9.77485 32	0	8.30960 70	0
0.5	0	25.03730 63	0	19.65055 51	0	15.87750 26	0	13.13398 97	0	11.07906 12	0
0.6	0	33.11545 18	0	25.84774 27	0	20.76567 72	0	17.07658 85	0	14.31813 15	0
0.7	0	42.61981 23	0	33.11545 21	0	26.47980 44	0	21.67065 54	0	18.08049 21	0
0.8	0	53.72175 20	0	41.57990 92	0	33.11545 18	0	26.99015 89	0	22.42444 77	0
0.9	1	6.66081 60	0	51.37859 03	0	40.77656 88	0	33.11545 18	0	27.41325 27	0
1.0	1	8.14825 71	0	62.66102 47	0	49.57607 50	0	40.13371 11	0	33.11545 20	0

$$10^{-t} {}_1F_1[a; b; x]$$

$$x = 3.6$$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	0	-35.000000	00	0	-17.000000	00	0	-11.000000	00	0	-8.000000	00	0	-6.200000	00
-0.9	0	-44.06895	90	0	-20.76720	54	0	-13.09564	28	0	-9.31446	11	0	-7.07960	36
-0.8	0	-52.08501	27	0	-24.04010	15	0	-14.88101	48	0	-10.40925	71	0	-7.79291	83
-0.7	0	-58.54652	63	0	-26.60550	74	0	-16.23402	84	0	-11.20409	47	0	-8.28441	68
-0.6	0	-62.87154	36	0	-28.21718	05	0	-17.01424	83	0	-11.61070	90	0	-8.49072	01
-0.5	1	-6.43894	38	0	-28.59245	58	0	-17.06106	20	0	-11.52273	66	0	-8.33984	76
-0.4	0	-62.33192	61	0	-27.40862	36	0	-16.19171	56	0	-10.82350	28	0	-7.75041	29
-0.3	0	-55.82336	25	0	-24.29903	50	0	-14.19920	35	0	-9.38022	10	0	-6.63075	99
-0.2	0	-43.87031	79	0	-18.84891	33	0	-10.84999	66	0	-7.04349	50	0	-4.87803	78
-0.1	0	-25.35035	91	0	-10.59085	41	0	-5.88161	16	0	-3.64582	44	0	-2.37721	14
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	0	36.59823	46	0	16.51113	96	0	10.12324	61	0	7.10261	00	0	5.39625	46
0.2	1	8.30296	81	0	36.59823	41	0	21.85388	65	0	14.89389	53	0	10.96979	96
0.3	1	14.20624	06	0	61.99107	17	0	36.59823	41	0	24.63096	37	0	17.89576	32
0.4	1	21.56631	30	1	9.34999	13	0	54.80657	48	0	36.59823	44	0	26.36753	48
0.5	1	30.60142	62	1	13.20222	86	1	7.69768	23	0	51.10967	29	0	36.59823	42
0.6	1	41.55321	26	1	17.85502	47	1	10.36584	22	1	6.85111	74	0	48.82228	26
0.7	2	5.46886	42	1	23.41781	19	1	13.54565	04	1	8.91831	02	0	63.29706	75
0.8	2	7.03021	02	1	30.01107	91	1	17.30363	36	1	11.35429	98	1	8.03047	23
0.9	2	8.87176	18	1	37.76724	95	1	21.71280	34	1	14.20484	61	1	10.01540	20
1.0	2	11.02912	21	1	46.83162	47	1	26.85316	11	1	17.52002	11	1	12.31823	75

b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	0	-5.000000	00	0	-4.14285	71	0	-3.500000	00	0	-3.000000	00	0	-2.600000	00
-0.9	0	-5.61191	63	0	-4.57878	09	0	-3.81469	71	0	-3.22824	01	0	-2.76489	83
-0.8	0	-6.09240	65	0	-4.90763	71	0	-4.04017	60	0	-3.38081	42	0	-2.86470	89
-0.7	0	-6.40071	91	0	-5.09846	02	0	-4.15227	37	0	-3.43846	52	0	-2.88381	53
-0.6	0	-6.49050	90	0	-5.11615	69	0	-4.12369	72	0	-3.37950	98	0	-2.80468	70
-0.5	0	-6.30931	34	0	-4.92112	63	0	-3.92374	14	0	-3.17962	45	0	-2.60771	45
-0.4	0	-5.79799	16	0	-4.46885	54	0	-3.51798	76	0	-2.81161	79	0	-2.27103	18
-0.3	0	-4.89012	29	0	-3.70948	42	0	-2.86798	21	0	-2.24518	52	0	-1.77032	82
-0.2	0	-3.51136	35	0	-2.58734	22	0	-1.93089	18	0	-1.44664	88	0	-1.07864	73
-0.1	0	-1.57875	72	0	-1.04045	22	0	-0.65913	71	0	-0.37867	94	0	-0.16617	29
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	0	4.32734	64	0	3.61023	01	0	3.10479	40	0	2.73402	94	0	2.45610	72
0.2	0	8.51669	42	0	6.87445	88	0	5.71952	10	0	4.87624	92	0	4.24197	47
0.3	0	13.69332	38	0	10.88555	46	0	8.91493	26	0	7.47905	64	0	6.40129	25
0.4	0	19.99534	00	0	15.74571	80	0	12.76876	11	0	10.60378	60	0	8.98190	76
0.5	0	27.57468	74	0	21.56721	04	0	17.36625	64	0	14.31661	48	0	12.03613	28
0.6	0	36.59823	44	0	28.47312	99	0	22.80075	57	0	18.68989	12	0	15.62107	74
0.7	0	47.24892	48	0	36.59823	43	0	29.17429	13	0	23.80259	09	0	19.79899	50
0.8	0	59.72700	42	0	46.08982	11	0	36.59823	44	0	29.74080	15	0	24.63765	53
0.9	1	7.42513	24	0	57.10865	37	0	45.19397	68	0	36.59823	40	0	30.21073	51
1.0	1	9.10607	24	0	69.82995	27	0	55.09365	96	0	44.47676	96	0	36.59823	44

$x=3.7$ $10^{-t} {}_1F_1[a; b; x]$

b=		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	0	-36.00000 00	0	-17.50000 00	0	-11.33333 33	0	-8.25000 00	0	-6.40000 00	0
-0.9	0	-46.12969 66	0	-21.71766 78	0	-13.68660 02	0	-9.73164 23	0	-7.39618 48	0
-0.8	0	-55.17801 37	0	-25.42550 64	0	-15.71875 75	0	-10.98530 64	0	-8.21938 44	0
-0.7	0	-62.58364 65	0	-28.38520 60	0	-17.29344 99	0	-11.92239 67	0	-8.80768 66	0
-0.6	1	-6.76920 39	0	-30.32010 55	0	-18.25305 81	0	-12.44091 94	0	-9.09009 53	0
-0.5	1	-6.97453 80	0	-30.91117 20	0	-18.41651 19	0	-12.42413 48	0	-8.98563 48	0
-0.4	1	-6.78718 08	0	-29.79265 37	0	-17.57693 47	0	-11.73910 45	0	-8.40238 72	0
-0.3	0	-61.07373 90	0	-26.54739 96	0	-15.49910 79	0	-10.23512 96	0	-7.23645 56	0
-0.2	0	-48.21530 18	0	-20.70181 01	0	-11.91673 83	0	-7.74207 16	0	-5.37085 33	0
-0.1	0	-28.00877 46	0	-11.72040 82	0	-6.52952 39	0	-4.06854 83	0	-2.67431 16	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	40.44730 42	0	18.13659 99	0	11.04984 40	0	7.70338 69	0	5.81584 22	0
0.2	1	9.21684 04	0	40.44730 41	0	24.04219 47	0	16.30890 19	0	11.95537 24	0
0.3	1	15.82156 56	1	6.87779 75	0	40.44730 45	0	27.11369 15	0	19.62071 26	0
0.4	1	24.08779 16	1	10.40701 67	0	60.78765 36	0	40.44730 43	0	29.03562 55	0
0.5	1	34.27013 70	1	14.73694 23	1	8.56424 10	0	56.67441 40	0	40.44730 43	0
0.6	1	46.65118 27	1	19.98341 44	1	11.56521 95	1	7.61977 27	0	54.12828 49	0
0.7	2	6.15438 59	1	26.27450 96	1	15.15241 66	1	9.94610 90	1	7.03784 83	0
0.8	2	7.92940 01	1	33.75155 25	1	19.40374 73	1	12.69528 00	1	8.95273 70	0
0.9	2	10.02830 75	1	42.57020 32	1	24.40490 45	1	15.92091 37	1	11.19362 89	0
1.0	2	12.49311 00	1	52.90161 26	1	30.24997 87	1	19.68181 42	1	13.80009 15	0
b=		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-5.16666 67	0	-4.28571 43	0	-3.62500 00	0	-3.11111 11	0	-2.70000 00	0
-0.9	0	-5.86381 10	0	-4.78608 34	0	-3.98968 48	0	-3.37890 64	0	-2.89670 57	0
-0.8	0	-6.42396 91	0	-5.17467 02	0	-4.26109 08	0	-3.56749 63	0	-3.02520 79	0
-0.7	0	-6.80173 89	0	-5.41700 16	0	-4.41233 75	0	-3.65547 21	0	-3.06815 64	0
-0.6	0	-6.94525 26	0	-5.47383 63	0	-4.41293 14	0	-3.61862 69	0	-3.00599 56	0
-0.5	0	-6.79555 09	0	-5.30071 19	0	-4.22842 92	0	-3.42970 06	0	-2.81676 64	0
-0.4	0	-6.28591 14	0	-4.84746 12	0	-3.82007 90	0	-3.05810 69	0	-2.47589 59	0
-0.3	0	-5.34112 92	0	-4.05769 19	0	-3.14443 48	0	-2.46964 19	0	-1.95597 20	0
-0.2	0	-3.87674 25	0	-2.86822 97	0	-2.15294 39	0	-1.62617 14	0	-1.22650 26	0
-0.1	0	-1.79820 55	0	-1.20852 20	0	-0.79150 57	0	-0.48529 69	0	-0.25365 88	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	4.63531 32	0	3.84460 37	0	3.28821 64	0	2.88173 87	0	2.57581 79	0
0.2	0	9.23811 04	0	7.42198 40	0	6.14684 82	0	5.21734 49	0	4.51935 44	0
0.3	0	14.95278 89	0	11.83904 10	0	9.65723 45	0	8.07008 32	0	6.88071 62	0
0.4	0	21.93892 74	0	17.21371 08	0	13.90896 10	0	11.50951 73	0	9.71491 11	0
0.5	0	30.37252 19	0	23.67585 14	0	19.00051 14	0	15.61200 10	0	13.08222 25	0
0.6	0	40.44730 41	0	31.36818 47	0	25.03996 16	0	20.46119 91	0	17.04861 04	0
0.7	0	52.37614 90	0	40.44730 44	0	32.14571 81	0	26.14864 34	0	21.68613 49	0
0.8	1	6.63925 71	0	51.08474 33	0	40.44730 41	0	32.77432 18	0	27.07340 79	0
0.9	1	8.27523 21	0	63.46811 15	0	50.08619 27	0	40.44730 40	0	33.29607 09	0
1.0	1	10.17350 84	1	7.78023 12	0	61.21669 90	0	49.28640 92	0	40.44730 44	0

$$10^{-t} {}_1F_1[a; b; x]$$

$$x = 3.8$$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	0	-37.00000	00	0	-18.00000	00	0	-11.66666	67	0	-8.50000	00	0	-6.60000	00
-0.9	0	-48.28317	53	0	-22.70702	52	0	-14.29956	75	0	-10.16298	33	0	-7.72257	41
-0.8	0	-58.46179	18	0	-26.89070	91	0	-16.60153	58	0	-11.59025	22	0	-8.66581	44
-0.7	1	-6.69089	74	0	-30.28515	67	0	-18.42057	28	0	-12.68313	33	0	-9.36082	32
-0.6	1	-7.28898	57	0	-32.58010	65	0	-19.58010	01	0	-13.32747	92	0	-9.72822	05
-0.5	1	-7.55496	59	0	-33.41612	93	0	-19.87635	56	0	-13.39206	38	0	-9.67707	88
-0.4	1	-7.39007	44	0	-32.37943	54	0	-19.07557	62	0	-12.72684	84	0	-9.10377	21
-0.3	1	-6.68086	40	0	-28.99625	94	0	-16.91096	34	0	-11.16111	22	0	-7.89074	04
-0.2	0	-52.97692	33	0	-22.72680	41	0	-13.07940	47	0	-8.50144	03	0	-5.90515	65
-0.1	0	-30.93089	95	0	-12.95870	74	0	-7.23794	76	0	-4.52955	75	0	-2.99749	62
0.0	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00
0.1	0	44.70118	48	0	19.92846	92	0	12.06875	21	0	8.36238	69	0	6.27497	62
0.2	1	10.22933	22	0	44.70118	43	0	26.45473	25	0	17.86513	30	0	13.03672	99
0.3	1	17.61543	01	1	7.62967	17	0	44.70118	45	0	29.85102	33	0	21.51810	73
0.4	1	26.89428	42	1	11.58071	31	1	6.74132	23	0	44.70118	47	0	31.97753	36
0.5	1	38.36238	38	1	16.44480	71	1	9.52632	53	0	62.83838	87	0	44.70118	49
0.6	2	5.23495	57	1	22.35695	72	1	12.89967	65	1	8.47305	92	0	60.00530	67
0.7	2	6.92218	85	1	29.46676	56	1	16.94385	52	1	11.08947	18	1	7.82387	02
0.8	2	8.93848	31	1	37.93967	25	1	21.74993	44	1	14.19006	98	1	9.97853	75
0.9	2	11.32865	81	1	47.95828	35	1	27.41829	14	1	17.83757	93	1	12.50673	60
1.0	2	14.14216	33	1	59.72379	00	1	34.05936	55	1	22.10092	00	1	15.45477	42

b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	0	-5.33333	33	0	-4.42857	14	0	-3.75000	00	0	-3.22222	22	0	-2.80000	00
-0.9	0	-6.12284	24	0	-4.99876	80	0	-4.16884	46	0	-3.53287	93	0	-3.03118	20
-0.8	0	-6.77002	08	0	-5.45260	27	0	-4.49043	38	0	-3.76084	16	0	-3.19107	17
-0.7	0	-7.22438	39	0	-5.75177	22	0	-4.68492	21	0	-3.88235	50	0	-3.26043	14
-0.6	0	-7.42799	15	0	-5.85247	95	0	-4.71830	40	0	-3.87044	66	0	-3.21748	73
-0.5	0	-7.31470	49	0	-5.70489	62	0	-4.55201	54	0	-3.69462	17	0	-3.03769	56
-0.4	0	-6.80938	51	0	-5.25258	58	0	-4.14250	52	0	-3.32053	93	0	-2.69349	23
-0.3	0	-5.82703	41	0	-4.43188	25	0	-3.44077	57	0	-2.70966	42	0	-2.15402	46
-0.2	0	-4.27186	83	0	-3.17122	61	0	-2.39189	02	0	-1.81889	40	0	-1.38486	34
-0.1	0	-2.03632	56	0	-1.39044	69	0	-0.93444	42	0	-0.60016	00	0	-0.34769	66
0.0	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00
0.1	0	4.97150	16	0	4.09985	51	0	3.48751	85	0	3.04089	83	0	2.70531	32
0.2	0	10.02778	36	0	8.01993	24	0	6.61247	35	0	5.58818	56	0	4.82026	70
0.3	0	16.33497	17	0	12.88305	86	0	10.46820	46	0	8.71437	10	0	7.40222	21
0.4	0	24.07711	59	0	18.82507	65	0	15.15776	80	0	12.49937	90	0	10.51430	18
0.5	0	33.45768	80	0	25.99591	81	0	20.79472	46	0	17.03114	13	0	14.22584	67
0.6	0	44.70118	43	0	34.56074	74	0	27.50398	06	0	22.40622	90	0	18.61289	61
0.7	0	58.05483	38	0	44.70118	48	0	35.42268	55	0	28.73052	56	0	23.75870	49
0.8	1	7.37904	24	0	56.61660	32	0	44.70118	43	0	36.11994	30	0	29.75428	92
0.9	1	9.22062	46	1	7.05255	22	0	55.50403	73	0	44.70118	43	0	36.69900	87
1.0	1	11.36291	68	1	8.66670	80	1	6.80111	01	0	54.61255	32	0	44.70118	45

$x=3.9$

$$10^{-t} {}_1F_1[a; b; x]$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	0	-38.00000 00	0	-18.50000 00	0	-12.00000 00	0	-8.75000 00	0	-6.80000 00	0
-0.9	0	-50.53678 24	0	-23.73828 79	0	-14.93625 25	0	-10.60951 82	0	-8.05946 76	0
-0.8	0	-61.95213 51	0	-28.44213 30	0	-17.53287 69	0	-12.22629 65	0	-9.13368 86	0
-0.7	1	-7.15471 97	0	-32.31539 44	0	-19.62089 98	0	-13.49063 51	0	-9.94613 00	0
-0.6	1	-7.84983 74	0	-35.01073 13	0	-21.00279 51	0	-14.27500 87	0	-10.40819 43	0
-0.5	1	-8.18431 28	0	-36.12389 15	0	-21.44965 36	0	-14.43215 87	0	-10.41795 29	0
-0.4	1	-8.04645 47	0	-35.18751 83	0	-20.69777 67	0	-13.79303 13	0	-9.85877 99	0
-0.3	1	-7.30746 39	0	-31.66445 32	0	-18.44505 24	0	-12.16454 92	0	-8.59787 79	0
-0.2	0	-58.19621 82	0	-24.94047 70	0	-14.34703 64	0	-9.32720 51	0	-6.48468 81	0
-0.1	0	-34.14331 24	0	-14.31647 99	0	-8.01272 59	0	-5.03247 03	0	-3.34917 81	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	49.40244 89	0	21.90389 74	0	13.18929 23	0	9.08537 11	0	6.77749 06	0
0.2	1	11.35102 21	0	49.40244 86	0	29.11461 83	0	19.57686 29	0	14.22336 24	0
0.3	1	19.60735 24	1	8.46257 74	0	49.40244 89	0	32.86922 13	0	23.60539 22	0
0.4	1	30.01751 94	1	12.88382 58	1	7.47523 65	0	49.40244 90	0	35.22151 47	0
0.5	1	42.92614 44	1	18.34507 81	1	10.59436 22	1	6.96658 03	0	49.40244 91	0
0.6	2	5.87174 96	1	25.00338 53	1	14.38421 88	1	9.42020 85	1	6.65145 10	0
0.7	2	7.78190 41	1	33.03317 14	1	18.94082 89	1	12.36122 32	1	8.69626 18	0
0.8	2	10.07052 57	1	42.62774 55	1	24.37044 94	1	15.85602 38	1	11.11935 21	0
0.9	2	12.79017 21	2	5.40009 20	1	30.79043 80	1	19.97784 09	1	13.96992 35	0
1.0	2	15.99894 90	2	6.73887 46	1	38.33017 82	1	24.80729 92	1	17.30207 13	0
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-5.50000 00	0	-4.57142 86	0	-3.87500 00	0	-3.33333 34	0	-2.90000 00	0
-0.9	0	-6.38950 29	0	-5.21719 55	0	-4.35244 85	0	-3.69036 80	0	-3.16849 12	0
-0.8	0	-7.13160 67	0	-5.74219 94	0	-4.72878 05	0	-3.96129 25	0	-3.36264 72	0
-0.7	0	-7.67027 80	0	-6.10395 92	0	-4.97091 95	0	-4.11979 92	0	-3.46117 65	0
-0.6	0	-7.94090 77	0	-6.25367 98	0	-5.04101 10	0	-4.13588 65	0	-3.43987 92	0
-0.5	0	-7.86943 03	0	-6.13561 56	0	-4.89595 19	0	-3.97550 10	0	-3.27137 10	0
-0.4	0	-7.37137 32	0	-5.68638 55	0	-4.48688 12	0	-3.60015 19	0	-2.92478 58	0
-0.3	0	-6.35083 11	0	-4.83423 41	0	-3.75863 42	0	-2.96649 85	0	-2.36545 72	0
-0.2	0	-4.69936 38	0	-3.49823 84	0	-2.64915 58	0	-2.02590 58	0	-1.55457 75	0
-0.1	0	-2.29480 77	0	-1.58745 43	0	-1.08886 90	0	-0.72396 82	0	-0.44883 05	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	5.33859 23	0	4.37792 73	0	3.70414 81	0	3.21351 18	0	2.84544 98	0
0.2	0	10.89234 35	0	8.67310 49	0	7.11997 09	0	5.99149 23	0	5.14682 30	0
0.3	0	17.85204 73	0	14.02641 58	0	11.35439 50	0	9.41690 39	0	7.96967 17	0
0.4	0	26.42960 70	0	20.59405 65	0	16.52576 67	0	13.58141 51	0	11.38630 77	0
0.5	0	36.85984 67	0	28.54883 25	0	22.76480 33	0	18.58611 53	0	15.47634 88	0
0.6	0	49.40244 86	0	38.08154 10	0	30.21560 80	0	24.54227 64	0	20.32730 23	0
0.7	1	6.43440 26	0	49.40244 89	0	39.03675 54	0	31.57216 66	0	26.03518 55	0
0.8	1	8.20003 36	0	62.74287 50	0	49.40244 86	0	39.80991 80	0	32.70518 86	0
0.9	1	10.27186 47	1	7.83568 70	0	61.50375 28	0	49.40244 86	0	40.45237 85	0
1.0	1	12.68802 63	1	9.65230 18	1	7.55499 11	0	60.51045 03	0	49.40244 90	0

$$10^{-t} {}_1F_1[a; b; x]$$

$$x = 4.0$$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	0	-39.000000	00	0	-19.000000	00	0	-12.333333	33	0	-9.000000	00	0	-7.000000	00
-0.9	0	-52.89854	09	0	-24.81471	94	0	-15.59828	84	0	-11.07236	54	0	-8.40761	69
-0.8	1	-6.56662	16	0	-30.08675	62	0	-18.51660	69	0	-12.89582	41	0	-9.62460	70
-0.7	1	-7.65252	34	0	-34.48684	14	0	-20.90041	10	0	-14.34862	52	0	-10.56610	23
-0.6	1	-8.45540	44	0	-37.62675	33	0	-22.52922	15	0	-15.28853	03	0	-11.13337	91
-0.5	1	-8.86704	80	0	-39.05254	90	0	-23.14628	84	0	-15.55055	54	0	-11.21236	11
-0.4	1	-8.76134	25	0	-38.23720	48	0	-22.45461	21	0	-14.94452	35	0	-10.67199	94
-0.3	1	-7.99228	74	0	-34.57263	42	0	-20.11262	98	0	-13.25241	43	0	-9.36252	11
-0.2	1	-6.39183	18	0	-27.36103	63	0	-15.72954	47	0	-10.22550	10	0	-7.11353	67
-0.1	0	-37.67529	28	0	-15.80552	56	0	-8.86027	55	0	-5.58125	38	0	-3.73199	87
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	0	54.59815	00	0	24.08180	76	0	14.42173	48	0	9.87867	70	0	7.32759	68
0.2	1	12.59362	09	0	54.59815	00	0	32.04736	48	0	21.45981	83	0	15.52571	11
0.3	1	21.81897	16	1	9.38520	07	0	54.59815	00	0	36.19726	52	0	25.90178	86
0.4	1	33.49272	51	1	14.33048	29	1	8.28815	42	0	54.59815	00	0	38.79874	87
0.5	1	48.01476	72	1	20.45913	07	1	11.77991	06	1	7.72277	22	0	54.59815	00
0.6	1	65.83201	79	1	27.95353	07	1	16.03550	40	1	10.47145	33	1	7.37235	87
0.7	2	8.74427	44	1	37.01669	32	1	21.16653	18	1	13.77559	90	1	9.66443	28
0.8	2	11.34011	95	1	47.87409	19	1	27.29674	80	1	17.71243	26	1	12.38792	20
0.9	2	14.43226	11	1	60.77563	14	1	34.56312	15	1	22.36729	90	1	15.60008	55
1.0	2	18.08884	85	2	7.59977	65	1	43.11695	71	1	27.83434	69	1	19.36400	48

b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	0	-5.66666	67	0	-4.71428	57	0	-4.00000	00	0	-3.44444	44	0	-3.00000	00
-0.9	0	-6.66432	27	0	-5.44175	41	0	-4.54078	84	0	-3.85159	75	0	-3.30880	93
-0.8	0	-7.50985	55	0	-6.04428	52	0	-4.97675	07	0	-4.16932	54	0	-3.54030	68
-0.7	0	-8.14117	88	0	-6.47484	53	0	-5.27129	21	0	-4.36854	33	0	-3.67096	91
-0.6	0	-8.48636	63	0	-6.67916	15	0	-5.38234	50	0	-4.41593	73	0	-3.67394	52
-0.5	0	-8.46261	03	0	-6.59496	95	0	-5.26181	06	0	-4.27354	17	0	-3.51873	13
-0.4	0	-7.97509	54	0	-6.15120	28	0	-4.85495	91	0	-3.89828	45	0	-3.17082	00
-0.3	0	-6.91578	17	0	-5.26711	67	0	-4.09978	13	0	-3.24149	77	0	-2.59132	27
-0.2	0	-5.16209	26	0	-3.85134	51	0	-2.92629	19	0	-2.24839	05	0	-1.73656	52
-0.1	0	-2.57549	99	0	-1.80088	43	0	-1.25577	95	0	-0.85748	33	0	-0.55765	20
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	0	5.73952	55	0	4.68094	78	0	3.93968	86	0	3.40078	42	0	2.99716	16
0.2	0	11.83907	31	0	9.38676	77	0	7.67325	58	0	6.43024	17	0	5.50132	78
0.3	0	19.51741	06	0	15.27878	98	0	12.32299	40	0	10.18314	18	0	8.58729	04
0.4	0	29.01811	05	0	22.53632	12	0	18.02458	70	0	14.76445	22	0	12.33775	26
0.5	0	40.61173	02	0	31.35820	10	0	24.92825	19	0	20.29019	73	0	16.84398	37
0.6	0	54.59815	00	0	41.96446	90	0	33.19996	36	0	26.88837	50	0	22.20652	06
0.7	1	7.13090	76	0	54.59815	00	0	43.02276	19	0	34.69993	83	0	28.53591	60
0.8	1	9.11107	21	1	6.95271	64	0	54.59815	00	0	43.87983	96	0	35.95353	64
0.9	1	11.44066	71	1	8.70463	66	1	6.81475	87	0	54.59815	00	0	44.59241	30
1.0	1	14.16409	54	1	10.74797	24	1	8.39140	82	1	6.70412	50	0	54.59815	00

$x=4.1$

$10^{-t} {}_1F_1[a; b; x]$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	0	-40.00000 00	0	-19.50000 00	0	-12.66666 67	0	-9.25000 00	0	-7.20000 00	0
-0.9	0	-55.37715 96	0	-25.93986 00	0	-16.28777 31	0	-11.55273 39	0	-8.76783 32	0
-0.8	1	-6.96227 29	0	-31.83216 28	0	-19.55687 62	0	-13.60141 80	0	-10.14030 10	0
-0.7	1	-8.18724 30	0	-36.81138 77	0	-22.26560 75	0	-15.26114 39	0	-11.22344 42	0
-0.6	1	-9.10966 79	0	-40.44428 46	0	-24.16817 66	0	-16.37350 66	0	-11.90742 56	0
-0.5	1	-9.60806 27	0	-42.22187 13	0	-24.97703 99	0	-16.75393 99	0	-12.06476 66	0
-0.4	1	-9.54024 16	0	-41.55071 28	0	-24.35818 54	0	-16.18882 13	0	-11.54843 03	0
-0.3	1	-8.74094 94	0	-37.74344 46	0	-21.92601 58	0	-14.43233 00	0	-10.18974 80	0
-0.2	1	-7.01928 63	0	-30.00847 70	0	-17.23779 78	0	-11.20304 46	0	-7.79617 27	0
-0.1	0	-41.55909 72	0	-17.43882 39	0	-9.78764 39	0	-6.18025 93	0	-4.14885 07	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	60.34028 73	0	26.48308 28	0	15.77739 57	0	10.74927 68	0	7.92992 10	0
0.2	1	13.97008 95	0	60.34028 71	0	35.28112 22	0	23.53132 92	0	16.95526 37	0
0.3	1	24.27427 35	1	10.40715 59	0	60.34028 79	0	39.86713 52	0	28.42847 72	0
0.4	1	37.35901 07	1	15.93634 58	1	9.18853 83	0	60.34028 75	0	42.74364 36	0
0.5	2	5.36875 63	1	22.81069 73	1	13.09578 24	1	8.56028 00	0	60.34028 74	0
0.6	2	7.37788 67	1	31.24167 34	1	17.87202 27	1	11.63813 52	1	8.17074 58	0
0.7	2	9.82127 57	1	41.46516 80	1	23.64674 41	1	15.34840 00	1	10.73881 21	0
0.8	2	12.76355 83	2	5.37437 32	1	30.56384 52	1	19.78073 88	1	13.79838 74	0
0.9	2	16.27662 67	2	6.83689 63	1	38.78291 05	1	25.03444 79	1	17.41599 90	0
1.0	2	20.44027 95	2	8.56640 75	1	48.48058 21	1	31.21928 55	1	21.66508 36	0

b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-5.83333 33	0	-4.85714 28	0	-4.12500 00	0	-3.55555 56	0	-3.10000 00	0
-0.9	0	-6.94787 39	0	-5.67286 16	0	-4.73417 84	0	-4.01680 86	0	-3.45232 50	0
-0.8	0	-7.90598 60	0	-6.35974 94	0	-5.23501 16	0	-4.38545 21	0	-3.72445 07	0
-0.7	0	-8.63898 77	0	-6.86581 65	0	-5.58707 90	0	-4.62938 35	0	-3.89043 03	0
-0.6	0	-9.06693 17	0	-7.13079 15	0	-5.74370 32	0	-4.71166 85	0	-3.92051 87	0
-0.5	0	-9.09737 61	0	-7.08523 49	0	-5.65129 38	0	-4.59004 54	0	-3.78078 99	0
-0.4	0	-8.62405 55	0	-6.64958 27	0	-5.24863 92	0	-4.21638 86	0	-3.43272 40	0
-0.3	0	-7.52544 02	0	-5.73310 96	0	-4.46614 08	0	-3.53612 97	0	-2.83276 15	0
-0.2	0	-5.66318 00	0	-4.23281 17	0	-3.22498 71	0	-2.48763 52	0	-1.93182 53	0
-0.1	0	-2.88042 26	0	-2.03219 99	0	-1.43626 49	0	-1.00153 48	0	-0.67480 43	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	6.17752 64	0	5.01124 85	0	4.19587 26	0	3.60403 18	0	3.16146 81	0
0.2	0	12.87597 33	0	10.16669 81	0	8.27661 75	0	6.90769 05	0	5.88630 04	0
0.3	0	21.34580 07	0	16.65081 39	0	13.38188 88	0	11.01906 79	0	9.25970 20	0
0.4	0	31.86654 96	0	24.66911 41	0	19.66700 79	0	16.05817 57	0	13.37611 52	0
0.5	0	44.74946 09	0	34.45003 80	0	27.30433 38	0	22.15797 41	0	18.34000 52	0
0.6	0	60.34028 64	0	46.24694 91	0	36.48473 22	0	29.46547 17	0	24.26669 84	0
0.7	1	7.90223 08	0	60.34028 71	0	47.41914 71	0	38.14290 36	0	31.28328 32	0
0.8	1	10.12195 66	1	7.70398 66	0	60.34028 64	0	48.36899 09	0	39.52956 75	0
0.9	1	12.74003 32	1	9.66873 38	1	7.55044 25	0	60.34028 63	0	49.15910 44	0
1.0	1	15.80808 08	1	11.96588 34	1	9.31933 24	1	7.42726 17	0	60.34028 76	0

$$10^{-t} {}_1F_1[a; b; x]$$

$$x = 4.2$$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	0	-41.00000	00	0	-20.00000	00	0	-13.00000	00	0	-9.50000	00	0	-7.40000	00
-0.9	0	-57.98210	54	0	-27.11754	71	0	-17.00675	59	0	-12.05193	13	0	-9.14099	24
-0.8	1	-7.38420	15	0	-33.68659	24	0	-20.65818	96	0	-14.34587	78	0	-10.68264	41
-0.7	1	-8.76207	89	0	-39.30197	93	0	-23.72355	54	0	-16.23257	69	0	-11.92108	73
-0.6	1	-9.81697	80	0	-43.48090	09	0	-25.92924	24	0	-17.53587	62	0	-12.73429	91
-0.5	1	-10.41270	93	0	-45.65345	86	0	-26.95367	09	0	-18.04959	41	0	-12.98002	54
-0.4	1	-10.38918	57	0	-45.15236	37	0	-26.42172	53	0	-17.53410	60	0	-12.49352	15
-0.3	1	-9.55961	62	0	-41.20171	09	0	-23.89869	91	0	-15.71262	89	0	-11.08510	24
-0.2	1	-7.70744	56	0	-32.90475	93	0	-18.88371	45	0	-12.26719	12	0	-8.53748	38
-0.1	0	-45.83026	37	0	-19.23065	28	0	-10.80257	20	0	-6.83425	88	0	-4.60290	23
0.0	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00
0.1	1	6.66863	31	0	29.13076	57	0	17.26874	26	0	11.70484	18	0	8.58954	58
0.2	1	15.49477	50	1	6.66863	31	0	38.84695	70	0	25.81049	02	0	18.52466	00
0.3	1	26.99984	80	1	11.53908	42	1	6.66863	31	0	43.91412	31	0	31.20879	72
0.4	1	41.65979	22	1	17.71877	38	1	10.18575	56	1	6.66863	31	0	47.09417	38
0.5	2	6.00104	71	1	25.42611	74	1	14.55617	83	1	9.48780	87	1	6.66863	31
0.6	2	8.26535	09	1	34.90591	39	1	19.91429	79	1	12.93282	48	1	9.05490	39
0.7	2	11.02625	97	1	46.43184	07	1	26.41012	47	1	17.09716	38	1	11.93095	81
0.8	2	14.35902	85	2	6.03091	39	1	34.21071	35	1	22.08477	34	1	15.36644	25
0.9	2	18.34753	16	2	7.68775	09	1	43.50170	85	1	28.01100	06	1	19.43853	13
1.0	2	23.08505	06	2	9.65142	21	1	54.48899	72	1	35.00359	55	1	24.23258	12

b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	0	-6.00000	00	0	-5.00000	00	0	-4.25000	00	0	-3.66666	67	0	-3.20000	00
-0.9	0	-7.24077	31	0	-5.91096	81	0	-4.93295	61	0	-4.18626	04	0	-3.59924	07
-0.8	0	-8.32131	51	0	-6.68955	19	0	-5.50428	25	0	-4.61022	39	0	-3.91550	91
-0.7	0	-9.16576	33	0	-7.27837	11	0	-5.91940	11	0	-4.90317	75	0	-4.12022	92
-0.6	0	-9.68538	42	0	-7.61059	13	0	-6.12659	68	0	-5.02423	51	0	-4.18049	93
-0.5	0	-9.77712	90	0	-7.60888	21	0	-6.06624	60	0	-4.92642	04	0	-4.05864	22
-0.4	0	-9.32206	61	0	-7.18429	08	0	-5.66998	47	0	-4.55603	72	0	-3.71172	01
-0.3	0	-8.18368	19	0	-6.23502	03	0	-4.85980	47	0	-3.85198	80	0	-3.09101	09
-0.2	0	-6.20603	91	0	-4.64510	87	0	-3.54708	00	0	-2.74503	89	0	-2.14144	32
-0.1	0	-3.21178	56	0	-2.28299	93	0	-1.63151	33	0	-1.15702	58	0	-0.80098	73
0.0	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00
0.1	0	6.65613	38	0	5.37138	24	0	4.47459	49	0	3.82469	27	0	3.33948	05
0.2	0	14.01183	57	0	11.01923	50	0	8.93475	58	0	7.42740	20	0	6.30449	28
0.3	0	23.35343	61	0	18.15417	14	0	14.53973	43	0	11.93123	82	0	9.99196	54
0.4	0	35.00128	77	0	27.01141	07	0	21.46707	37	0	17.47321	50	0	14.50959	11
0.5	0	49.31290	42	0	37.85301	12	0	29.91424	94	0	24.20547	81	0	19.97676	39
0.6	1	6.66863	30	0	50.97027	73	0	40.10042	42	0	32.29662	10	0	26.52558	22
0.7	1	8.75637	69	1	6.66863	31	0	52.26833	87	0	41.93309	58	0	34.30192	51
0.8	1	11.24354	69	1	8.53588	86	1	6.66863	30	0	53.32072	79	0	43.46660	20
0.9	1	14.18439	08	1	10.73832	14	1	8.36505	05	1	6.66863	31	0	54.19658	73
1.0	1	17.63882	62	1	13.31953	16	1	10.34870	50	1	8.22794	39	1	6.66863	31

$x=4.3$

$10^{-t} {}_1F_1[a; b; x]$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	0	-42.00000 00	0	-20.50000 00	0	-13.33333 33	0	-9.75000 00	0	-7.60000 00	0
-0.9	0	-60.72366 43	0	-28.35194 72	0	-17.75751 25	0	-12.57137 15	0	-9.52804 11	0
-0.8	1	-7.83462 30	0	-35.65900 54	0	-21.82543 62	0	-15.13223 73	0	-11.25366 30	0
-0.7	1	-9.38052 19	0	-41.97271 69	0	-25.28194 02	0	-17.26768 76	0	-12.66220 99	0
-0.6	1	-10.58208 80	0	-46.75577 58	0	-27.82285 77	0	-18.78210 06	0	-13.61830 62	0
-0.5	1	-11.28684 71	0	-49.37091 84	0	-29.08901 82	0	-19.44545 42	0	-13.96342 09	0
-0.4	1	-11.31479 11	0	-49.06878 50	0	-28.65969 27	0	-18.98930 80	0	-13.51321 15	0
-0.3	1	-10.45505 96	0	-44.97465 84	0	-26.04544 94	0	-17.10242 19	0	-12.05463 61	0
-0.2	1	-8.46231 56	0	-36.07400 40	0	-20.68036 61	0	-13.42599 51	0	-9.34281 48	0
-0.1	0	-50.52794 65	0	-21.19672 00	0	-11.91356 34	0	-7.54848 68	0	-5.09762 36	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	1	7.36997 94	0	32.05028 45	0	18.90951 46	0	12.75381 21	0	9.31205 55	0
0.2	1	17.18355 62	1	7.36997 94	0	42.77915 18	0	28.31834 34	0	20.24780 85	0
0.3	1	30.02516 77	1	12.79276 31	1	7.36997 93	0	48.37717 49	0	34.26846 94	0
0.4	1	46.44326 27	1	19.69700 81	1	11.29017 28	1	7.36997 94	0	51.89224 54	0
0.5	2	6.70567 89	1	28.33462 98	1	16.17683 76	1	10.51499 27	1	7.36997 94	0
0.6	2	9.25622 11	1	38.98860 63	1	22.18511 03	1	14.36945 43	1	10.03400 99	0
0.7	2	12.37410 99	2	5.19759 81	1	29.48852 36	1	19.04135 40	1	13.25368 06	0
0.8	2	16.14683 26	2	6.76510 93	1	38.28072 92	1	24.65102 06	1	17.10950 34	0
0.9	2	20.67209 30	2	8.64090 79	1	48.77735 79	1	31.33224 74	1	21.69087 16	0
1.0	2	26.05876 15	2	10.86896 28	1	61.21802 05	1	39.23349 96	1	27.09684 67	0

b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-6.16666 67	0	-5.14285 71	0	-4.37500 00	0	-3.77777 78	0	-3.30000 00	0
-0.9	0	-7.54368 57	0	-6.15655 85	0	-5.13748 48	0	-4.36023 13	0	-3.74977 37	0
-0.8	0	-8.75726 57	0	-7.03472 84	0	-5.78533 83	0	-4.84423 34	0	-4.11394 38	0
-0.7	0	-9.72373 39	0	-7.71412 93	0	-6.26946 94	0	-5.19085 08	0	-4.36108 59	0
-0.6	0	-10.34474 03	0	-8.12075 09	0	-6.53266 02	0	-5.35488 49	0	-4.45485 71	0
-0.5	0	-10.50556 53	0	-8.16859 13	0	-6.50866 54	0	-5.28419 03	0	-4.35347 15	0
-0.4	0	-10.07327 68	0	-7.75833 29	0	-6.12123 38	0	-4.91893 54	0	-4.00913 13	0
-0.3	0	-8.89473 06	0	-6.77590 59	0	-5.28304 73	0	-4.19080 30	0	-3.36741 17	0
-0.2	0	-6.79439 67	0	-5.09092 99	0	-3.89457 30	0	-3.02212 36	0	-2.36659 68	0
-0.1	0	-3.57200 60	0	-2.55502 77	0	-1.84282 07	0	-1.32494 05	0	-0.93696 20	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	7.17922 91	0	5.76414 78	0	4.77792 91	0	4.06433 84	0	3.53241 27	0
0.2	0	15.25632 04	0	11.95133 45	0	9.65282 06	0	7.99327 57	0	6.75891 30	0
0.3	0	25.55816 45	0	19.80170 25	0	15.80602 78	0	12.92683 81	0	10.78961 85	0
0.4	0	38.45137 91	0	29.58409 36	0	23.44021 89	0	19.02123 65	0	15.74716 42	0
0.5	0	54.34605 56	0	41.59871 82	0	32.78133 03	0	26.45033 00	0	21.76781 82	0
0.6	1	7.36997 94	0	56.18002 58	0	44.08066 75	0	35.40719 71	0	29.00267 90	0
0.7	1	9.70220 42	1	7.36997 95	0	57.61716 17	0	46.10582 08	0	37.61896 18	0
0.8	1	12.48787 76	1	9.45704 44	1	7.36997 93	0	58.78290 13	0	47.80136 99	0
0.9	1	15.78975 11	1	11.92486 07	1	9.26701 91	1	7.36997 93	0	59.75357 56	0
1.0	1	19.67726 99	1	14.82389 48	1	11.49054 24	1	9.11445 86	1	7.36997 94	0

$$10^{-t} {}_1F_1[a; b; x]$$

$$x = 4.4$$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	0	-43.000000	00	0	-21.000000	00	0	-13.666666	67	0	-10.000000	00	0	-7.800000	00
-0.9	0	-63.613018	84	0	-29.647581	14	0	-18.542510	03	0	-13.112585	58	0	-9.930000	17
-0.8	1	-8.315949	93	0	-37.759141	12	0	-23.063924	42	0	-15.963785	50	0	-11.855555	28
-0.7	1	-10.046379	99	0	-44.838959	90	0	-26.949120	08	0	-18.371647	77	0	-13.450258	89
-0.6	1	-11.410192	22	0	-50.289835	50	0	-29.860395	55	0	-20.119209	93	0	-14.564125	57
-0.5	1	-12.236890	00	0	-53.400056	60	0	-31.397091	14	0	-20.950169	92	0	-15.020703	34
-0.4	1	-12.324311	19	0	-53.329128	80	0	-31.087899	86	0	-20.564176	60	0	-14.613974	48
-0.3	1	-11.434717	74	0	-49.092143	33	0	-28.382440	04	0	-18.611671	12	0	-13.104958	82
-0.2	1	-9.290504	43	0	-39.542706	68	0	-22.642090	02	0	-14.688278	81	0	-10.218012	21
-0.1	0	-55.695288	85	0	-23.354307	74	0	-13.129960	00	0	-8.328685	54	0	-5.636815	53
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	1	8.145086	68	0	35.269701	12	0	20.714850	05	0	13.905474	47	0	10.103586	61
0.2	1	19.054008	85	1	8.145086	68	0	47.115539	93	0	31.078075	57	0	22.140014	46
0.3	1	33.382909	09	1	14.181227	78	1	8.145086	68	0	53.299272	24	0	37.635838	81
0.4	2	5.176291	18	1	21.892375	50	1	12.513263	38	1	8.145086	68	0	57.184106	68
0.5	2	7.490799	93	1	31.568686	69	1	17.975204	42	1	11.652494	49	1	8.145086	69
0.6	2	10.362325	56	1	43.536823	32	1	24.709740	08	1	15.963463	34	1	11.118219	95
0.7	2	13.881415	51	2	5.816354	48	1	32.917337	76	1	21.202570	05	1	14.721175	54
0.8	2	18.149629	94	2	7.585961	16	1	42.822161	16	1	27.508912	28	1	19.046897	77
0.9	2	23.280613	37	2	9.708397	79	2	5.467431	15	1	35.037457	72	1	24.198786	67
1.0	2	29.401248	83	2	12.234831	16	2	6.875224	48	1	43.960497	70	1	30.291646	61
0.6	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.7	0	7.751072	26	0	6.192610	02	0	5.108144	40	0	4.324686	61	0	3.741588	83
0.8	0	16.620042	24	0	12.970629	94	0	10.436455	58	0	8.609579	92	0	7.252849	92
0.9	0	27.979627	70	0	21.607517	74	0	17.191193	38	0	14.013742	26	0	11.658723	31
1.0	0	42.248843	33	0	32.410146	66	0	25.603407	72	0	20.715044	49	0	17.098681	13
0.5	0	59.897470	04	0	45.721980	06	0	35.931257	75	0	28.911899	91	0	23.728050	00
0.6	1	8.145086	68	0	61.926495	53	0	48.462524	45	0	38.825130	08	0	31.719434	43
0.7	1	10.749516	63	1	8.145086	69	0	63.517296	60	0	50.699993	30	0	41.264247	76
0.8	1	13.868285	50	1	10.476994	49	1	8.145086	68	1	6.480832	24	0	52.574355	55
0.9	1	17.573880	04	1	13.241052	22	1	10.265685	51	1	8.145086	68	1	6.588383	35
1.0	1	21.946675	59	1	16.495590	05	1	12.757051	18	1	10.095976	66	1	8.145086	69

$x=4.5$

$$10^{-t} {}_1F_1[a; b; x]$$

b =		0.1		0.2		0.3		0.4		0.5		
a	t		t		t		t		t		t	
-1.0	0	-44.00000	00	0	-21.50000	00	0	-14.00000	00	0	-8.00000	00
-0.9	1	-6.66623	20	0	-31.00935	85	0	-19.36442	31	0	-13.67723	01
-0.8	1	-8.83080	77	0	-39.99759	58	0	-24.37941	81	0	-16.84408	79
-0.7	1	-10.76380	85	0	-47.91743	98	0	-28.73419	09	0	-19.55007	73
-0.6	1	-12.30696	85	0	-54.10591	29	0	-32.05425	16	0	-21.55485	22
-0.5	1	-13.26986	04	0	-57.76908	46	0	-33.89318	58	0	-22.57316	84
-0.4	1	-13.42570	42	0	-57.96532	08	0	-33.72362	99	0	-22.26935	57
-0.3	1	-12.50676	03	0	-53.58691	47	0	-30.92738	65	0	-20.25127	31
-0.2	1	-10.19928	10	0	-43.33997	51	0	-24.78461	44	0	-16.06370	45
-0.1	0	-61.37982	55	0	-25.72242	80	0	-14.46202	46	0	-9.18115	44
0.0	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00
0.1	1	9.00171	31	0	38.81998	25	0	22.70143	33	0	15.17005	01
0.2	1	21.12558	27	1	9.00171	32	0	51.89787	20	0	34.11524	08
0.3	1	37.10925	56	1	15.71890	54	1	9.00171	32	0	58.72785	08
0.4	2	5.76781	32	1	24.32851	06	1	13.86772	64	1	9.00171	32
0.5	2	8.36546	34	1	35.16430	48	1	19.97061	07	1	12.91211	52
0.6	2	11.59682	88	1	48.60287	96	1	27.51624	64	1	17.73196	37
0.7	2	15.56666	22	2	6.50679	51	1	36.73590	31	1	23.60478	37
0.8	2	20.39270	26	2	8.50350	25	1	47.88872	11	1	30.69115	30
0.9	2	26.20695	56	2	10.90364	39	2	6.12643	93	1	39.17032	22
1.0	2	33.15707	83	2	13.76668	24	2	7.71860	57	1	49.24196	15
0.0	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00
0.1	0	8.37633	86	0	6.66012	97	0	5.46772	29	0	4.60761	27
0.2	0	18.11466	80	0	14.08549	69	0	11.29184	72	0	9.28098	39
0.3	0	30.63944	02	0	23.58712	44	0	18.70667	39	0	15.20058	45
0.4	0	46.42896	93	0	35.51486	60	0	27.97528	39	0	22.56869	68
0.5	1	6.60207	38	0	50.26117	91	0	39.39229	75	0	31.61147	46
0.6	1	9.00171	31	1	6.82651	99	0	53.28684	63	0	42.58116	70
0.7	1	11.90916	27	1	9.00171	32	1	7.00257	83	0	55.75850	68
0.8	1	15.39954	83	1	11.60629	89	1	9.00171	31	1	7.14552	88
0.9	1	19.55649	22	1	14.70096	97	1	11.37138	01	1	9.00171	32
1.0	1	24.47289	10	1	18.35305	78	1	14.16175	92	1	11.18264	59

$$10^{-t} {}_1F_1[a; b; x]$$

$$x = 4.6$$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	0	-45.000000	00	0	-22.000000	00	0	-14.333333	33	0	-10.500000	00	0	-8.200000	00
-0.9	1	-6.98847	88	0	-32.44260	58	0	-20.22615	05	0	-14.26709	89	0	-10.78317	04
-0.8	1	-9.38206	07	0	-42.38589	17	0	-25.77817	99	0	-17.77701	47	0	-13.16164	78
-0.7	1	-11.53734	26	0	-51.22639	26	0	-30.64704	54	0	-20.80908	17	0	-15.18241	29
-0.6	1	-13.27862	38	0	-58.22893	81	0	-34.41793	60	0	-23.09735	63	0	-16.66198	23
-0.5	1	-14.39344	83	0	-62.50884	96	0	-36.59400	04	0	-24.32473	07	0	-17.38252	29
-0.4	1	-14.62769	38	0	-63.01232	41	0	-36.58579	36	0	-24.11647	22	0	-17.08759	70
-0.3	1	-13.68016	30	0	-58.49489	55	0	-33.69969	12	0	-22.03314	53	0	-15.47750	36
-0.2	1	-11.19664	38	0	-47.49778	57	0	-27.12519	34	0	-17.56286	07	0	-12.20418	48
-0.1	1	-6.76339	39	0	-28.32200	36	0	-15.92103	54	0	-10.11280	57	0	-6.86566	75
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	1	9.94843	15	0	42.73530	34	0	24.88764	72	0	16.55878	45	0	11.92134	77
0.2	1	23.41980	78	1	9.94843	14	0	57.17222	66	0	37.45799	93	0	26.50066	57
0.3	1	41.24436	15	1	17.42176	20	1	9.94843	15	1	6.47152	60	0	45.42180	64
0.4	2	6.42548	00	1	27.03160	94	1	15.36761	39	1	9.94843	15	1	6.94586	56
0.5	2	9.33973	57	1	39.16145	45	1	22.18448	18	1	14.30691	33	1	9.94843	15
0.6	2	12.97438	04	2	5.42449	11	1	30.63576	21	1	19.69391	70	1	13.64810	57
0.7	2	17.45045	19	2	7.27708	70	1	40.98792	86	1	26.27459	11	1	18.15510	91
0.8	2	22.90426	47	2	9.52890	85	2	5.35401	70	1	34.23407	82	1	23.59282	11
0.9	2	29.48894	67	2	12.24162	12	2	6.86275	99	1	43.77945	16	1	30.09903	89
1.0	2	37.37610	57	2	15.48423	52	2	8.66247	20	1	55.14180	54	1	37.82733	56

b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	0	-6.66666	67	0	-5.57142	86	0	-4.75000	00	0	-4.11111	11	0	-3.60000	00
-0.9	0	-8.51997	22	0	-6.94366	35	0	-5.78963	61	0	-4.91236	20	0	-4.22549	35
-0.8	0	-10.20487	08	0	-8.17413	82	0	-6.70790	44	0	-5.60832	42	0	-4.75866	84
-0.7	0	-11.60997	53	0	-9.17887	24	0	-7.43975	98	0	-6.14752	55	0	-5.15806	74
-0.6	0	-12.60237	02	0	-9.85812	12	0	-7.90829	51	0	-6.46936	86	0	-5.37508	60
-0.5	0	-13.02492	07	0	-10.09444	51	0	-8.02331	37	0	-6.50305	53	0	-5.35314	41
-0.4	0	-12.69332	84	0	-9.75059	48	0	-7.67977	22	0	-6.16641	04	0	-5.02678	57
-0.3	0	-11.39290	92	0	-8.66720	56	0	-6.75608	04	0	-5.36460	39	0	-4.32069	62
-0.2	0	-8.87507	90	0	-6.66028	07	0	-5.11225	18	0	-3.98875	80	0	-3.14863	75
-0.1	0	-4.85351	88	0	-3.51845	25	0	-2.58788	97	0	-1.91443	62	0	-1.41229	05
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	0	9.06015	67	0	7.17038	82	0	5.85938	35	0	4.91516	99	0	4.21458	43
0.2	0	19.75301	85	0	15.30513	19	0	12.22577	51	0	10.01260	34	0	8.37397	94
0.3	0	33.56139	50	0	25.75757	00	0	20.36502	64	0	16.49682	71	0	13.63845	92
0.4	0	51.03065	19	0	38.92609	43	0	30.57634	57	0	24.59762	28	0	20.18777	35
0.5	1	7.27750	00	0	55.25861	64	0	43.19556	33	0	34.57246	42	0	28.22300	29
0.6	1	9.94843	14	1	7.52574	06	0	58.59866	03	0	46.70914	99	0	37.96855	83
0.7	1	13.19314	93	1	9.94843	15	1	7.72055	76	0	61.32864	34	0	49.67433	83
0.8	1	17.09804	26	1	12.85664	12	1	9.94843	14	1	7.87881	29	0	63.61805	01
0.9	1	21.75945	87	1	16.32020	71	1	12.59553	72	1	9.94843	14	1	8.01077	07
1.0	1	27.28462	98	1	20.41675	27	1	15.71965	14	1	12.38569	66	1	9.94843	16

$x=4.7$

$10^{-t} {}_1F_1[a; b; x]$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	0	-46.000000	00	0	-22.500000	00	0	-14.666666	67	0	-10.750000	00	0	-8.400000	00
-0.9	1	-7.329480	03	0	-33.953111	19	0	-21.130830	68	0	-14.884130	39	0	-11.236860	54
-0.8	1	-9.972820	68	0	-44.936560	93	0	-27.267010	24	0	-18.766760	26	0	-13.871210	55
-0.7	1	-12.371930	13	0	-54.785690	15	0	-32.698450	38	0	-22.155290	70	0	-16.134970	90
-0.6	1	-14.331940	36	0	-62.686120	66	0	-36.966170	77	0	-24.755780	61	0	-17.825550	47
-0.5	1	-15.616070	47	1	-6.765300	84	0	-39.517760	94	0	-26.216060	53	0	-18.701290	61
-0.4	1	-15.939850	24	1	-6.850840	37	0	-39.695060	77	0	-26.118220	26	0	-18.476550	13
-0.3	1	-14.964780	58	0	-63.855490	43	0	-36.720610	11	0	-23.970320	54	0	-16.816220	34
-0.2	1	-12.291390	33	0	-52.051270	53	0	-29.682750	88	0	-19.197340	63	0	-13.329810	05
-0.1	1	-7.451530	51	0	-31.176050	56	0	-17.519380	25	0	-11.131220	41	0	-7.564890	06
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	1	10.994710	71	0	47.053370	98	0	27.293750	36	0	18.084050	53	0	12.963130	25
0.2	1	25.960500	97	1	10.994710	71	0	62.989450	74	0	41.137380	89	0	29.008040	96
0.3	1	45.832690	38	1	19.307460	87	1	10.994710	71	1	7.131920	76	0	49.912770	81
0.4	2	7.156600	56	1	30.030700	25	1	17.028470	85	1	10.994710	72	1	7.655980	48
0.5	2	10.424790	95	1	43.604490	31	1	24.640560	11	1	15.851340	20	1	10.994710	72
0.6	2	14.511280	29	2	6.052750	17	1	34.102830	73	1	21.870330	67	1	15.119980	51
0.7	2	19.555730	79	2	8.136310	91	1	45.721970	97	1	29.241500	67	1	20.158290	77
0.8	2	25.715780	93	2	10.674630	21	2	5.984290	95	1	38.178060	04	1	26.251540	93
0.9	2	33.168840	47	2	13.739030	44	2	7.685300	58	1	48.918910	86	1	33.558520	07
1.0	2	42.114080	72	2	17.409510	51	2	9.718560	58	1	61.731210	69	1	42.256500	06

b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	0	-6.833333	33	0	-5.714280	57	0	-4.875000	00	0	-4.222222	22	0	-3.700000	00
-0.9	0	-8.870710	06	0	-7.224830	68	0	-6.021380	79	0	-5.107630	27	0	-4.393000	43
-0.8	0	-10.740000	05	0	-8.592920	40	0	-7.045130	85	0	-5.886180	21	0	-4.991970	04
-0.7	0	-12.318960	20	0	-9.726450	93	0	-7.874980	33	0	-6.501510	56	0	-5.451540	08
-0.6	0	-13.460970	50	0	-10.515510	70	0	-8.426240	15	0	-6.886960	89	0	-5.718280	87
-0.5	0	-13.991910	37	0	-10.830090	13	0	-8.599160	44	0	-6.964320	05	0	-5.729750	99
-0.4	0	-13.706710	46	0	-10.517630	67	0	-8.277120	44	0	-6.642420	62	0	-5.413420	46
-0.3	0	-12.365620	18	0	-9.400370	96	0	-7.324630	56	0	-5.815730	05	0	-4.685530	74
-0.2	0	-9.690070	41	0	-7.272370	71	0	-5.585190	31	0	-4.362640	32	0	-3.449890	35
-0.1	0	-5.358210	65	0	-3.896310	28	0	-2.878910	93	0	-2.143770	20	0	-1.596470	86
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	0	9.808150	80	0	7.727420	23	0	6.286100	11	0	5.249600	07	0	4.481700	55
0.2	0	21.549180	64	0	16.639620	70	0	13.245670	28	0	10.810030	65	0	9.009390	96
0.3	0	36.771670	88	0	28.137590	21	0	22.180030	92	0	17.912840	67	0	14.764310	17
0.4	0	56.096760	06	0	42.674480	00	0	33.429120	40	0	26.818760	52	0	21.950160	38
0.5	1	8.022550	37	0	60.760920	14	0	47.375300	80	0	37.820590	81	0	30.795350	86
0.6	1	10.994710	71	1	8.297070	44	1	6.444760	02	0	51.246330	81	0	41.555340	15
0.7	1	14.614760	29	1	10.994710	71	1	8.512610	63	1	6.746250	21	0	54.515150	31
0.8	1	18.981910	45	1	14.240950	19	1	10.994710	71	1	8.687780	64	1	6.999210	80
0.9	1	24.207040	67	1	18.116040	35	1	13.950800	88	1	10.994710	71	1	8.833910	20
1.0	1	30.413780	99	1	22.709370	10	1	17.447330	41	1	13.717550	53	1	10.994710	71

$$10^{-t} {}_1F_1[a; b; x]$$

$$x=4.8$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	0	-47.00000 00	0	-23.00000 00	0	-15.00000 00	0	-11.00000 00	0	-8.60000 00	0
-0.9	1	-7.69080 02	0	-35.54716 60	0	-22.08189 30	0	-15.53044 02	0	-11.71046 32	0
-0.8	1	-10.60650 54	0	-47.66327 23	0	-28.85330 80	0	-19.81788 59	0	-14.62241 52	0
-0.7	1	-13.27297 83	0	-58.61700 28	0	-34.90013 95	0	-23.59593 63	0	-17.15145 28	0
-0.6	1	-15.47434 92	1	-6.75071 94	0	-39.71503 74	0	-26.54001 22	0	-19.07409 24	0
-0.5	1	-16.94696 35	1	-7.32386 84	0	-42.68440 99	0	-28.25939 91	0	-20.12253 47	0
-0.4	1	-17.37267 98	1	-7.44956 10	0	-43.07407 43	0	-28.28847 78	0	-19.97889 35	0
-0.3	1	-16.37146 27	1	-6.97119 50	0	-40.01343 42	0	-26.07707 85	0	-18.26885 56	0
-0.2	1	-13.49321 30	0	-57.03905 00	0	-32.47808 27	0	-20.97987 16	0	-14.55472 35	0
-0.1	1	-8.20876 71	0	-34.30991 79	0	-19.27068 38	0	-12.24473 34	0	-8.32779 17	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	1	12.15104 16	0	51.81583 60	0	29.94208 23	0	19.75948 54	0	14.10518 53	0
0.2	1	28.77405 69	1	12.15104 16	1	6.94056 87	0	45.18761 74	0	31.76272 84	0
0.3	1	50.92354 12	1	21.39558 03	1	12.15104 17	1	7.86036 64	0	54.85687 91	0
0.4	2	7.96930 56	1	33.35796 02	1	18.86753 22	1	12.15104 16	1	8.43929 97	0
0.5	2	11.63308 25	1	48.54264 10	1	27.36515 79	1	17.56139 41	1	12.15104 18	0
0.6	2	16.22567 43	2	6.75224 78	1	37.95580 92	1	24.28450 68	1	16.74964 26	0
0.7	2	21.90809 18	2	9.09460 22	1	50.99201 59	1	32.53827 70	1	22.38014 44	0
0.8	2	28.86238 61	2	11.95454 40	2	6.68711 60	1	42.56795 03	1	29.20555 05	0
0.9	2	37.29384 48	2	15.41451 52	2	8.60400 38	2	5.46488 73	1	37.40860 79	0
1.0	2	47.43337 70	2	19.56711 79	2	10.89998 18	2	6.90894 78	1	47.19374 28	0

b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-7.00000 00	0	-5.85714 29	0	-5.00000 00	0	-4.33333 34	0	-3.80000 00	0
-0.9	0	-9.23567 09	0	-7.51654 76	0	-6.26116 86	0	-5.30916 04	0	-4.56548 32	0
-0.8	0	-11.30480 77	0	-9.03364 62	0	-7.39904 73	0	-6.17700 38	0	-5.23554 13	0
-0.7	0	-13.07340 48	0	-10.30757 09	0	-8.33563 38	0	-6.87523 07	0	-5.76060 44	0
-0.6	0	-14.37988 98	0	-11.21730 06	0	-8.97778 54	0	-7.33057 91	0	-6.08200 76	0
-0.5	0	-15.03149 84	0	-11.61906 27	0	-9.21529 96	0	-7.45671 29	0	-6.13088 14	0
-0.4	0	-14.80028 57	0	-11.34347 58	0	-8.91881 51	0	-7.15263 99	0	-5.82693 87	0
-0.3	0	-13.41875 28	0	-10.19242 19	0	-7.93750 76	0	-6.30098 08	0	-5.07714 99	0
-0.2	0	-10.57505 00	0	-7.93561 87	0	-6.09657 54	0	-4.76608 02	0	-3.77429 70	0
-0.1	0	-5.90772 79	0	-4.30687 50	0	-3.19449 00	0	-2.39194 52	0	-1.79539 94	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	10.62652 26	0	8.33565 60	0	6.75113 30	0	5.61335 77	0	4.77169 79	0
0.2	0	23.51866 24	0	18.10006 15	0	14.35968 90	0	11.67941 34	0	9.70085 38	0
0.3	0	40.29911 55	0	30.74778 85	0	24.16684 90	0	19.46001 98	0	15.99218 45	0
0.4	0	61.67454 84	0	46.79376 01	0	36.55839 00	0	29.25072 56	0	23.87634 27	0
0.5	1	8.84443 95	1	6.68194 89	0	51.96923 62	0	41.38416 90	0	33.61249 45	0
0.6	1	12.15104 16	1	9.14798 52	1	7.08883 81	0	56.23374 86	0	45.49110 25	0
0.7	1	16.18870 59	1	12.15104 18	1	9.38642 40	1	7.42175 96	0	59.83694 25	0
0.8	1	21.07127 00	1	15.77353 93	1	12.15104 16	1	9.58028 74	1	7.70123 55	0
0.9	1	26.92617 90	1	20.10762 30	1	15.45119 46	1	12.15104 16	1	9.74207 53	0
1.0	1	33.89580 43	1	25.25608 92	1	19.36320 52	1	15.19197 25	1	12.15104 17	0

$x=4.9$

$10^{-t} {}_1F_1[a; b; x]$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	0	-48.000000 00	0	-23.500000 00	0	-15.333333 33	0	-11.250000 00	0	-8.800000 00	0
-0.9	1	-8.07414 08	0	-37.23159 78	0	-23.08302 11	0	-16.20829 69	0	-12.20547 64	0
-0.8	1	-11.28680 10	0	-50.58085 50	0	-30.54510 34	0	-20.93532 85	0	-15.41852 33	0
-0.7	1	-14.24638 36	0	-62.74394 95	0	-37.26486 60	0	-25.13884 36	0	-18.23702 40	0
-0.6	1	-16.71395 63	1	-7.27245 97	0	-42.68203 05	0	-28.46078 43	0	-20.41470 11	0
-0.5	1	-18.39621 82	1	-7.93060 02	0	-46.11567 81	0	-30.46806 68	0	-21.65504 45	0
-0.4	1	-18.93769 59	1	-8.10198 03	0	-46.74756 15	0	-30.64239 20	0	-21.60461 82	0
-0.3	1	-17.91209 73	1	-7.61117 02	0	-43.60367 97	0	-28.36901 32	0	-19.84568 32	0
-0.2	1	-14.81275 92	0	-62.50353 56	0	-35.53396 16	0	-22.92436 53	0	-15.88809 09	0
-0.1	1	-9.04209 98	0	-37.75147 09	0	-21.18990 50	0	-13.46246 86	0	-9.16037 65	0
0.0	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0
0.1	1	13.42897 79	0	57.06860 93	0	32.85724 30	0	21.60006 80	0	15.35734 46	0
0.2	1	31.88963 22	1	13.42897 79	1	7.64828 58	0	49.64638 52	0	34.78941 90	0
0.3	2	5.65715 24	1	23.70773 94	1	13.42897 78	1	8.66388 03	0	60.30020 61	0
0.4	2	8.87258 56	1	37.04903 34	1	20.90382 16	1	13.42897 78	1	9.30338 38	0
0.5	2	12.97839 35	2	5.40305 14	1	30.38742 81	1	19.45476 90	1	13.42897 78	0
0.6	2	18.13773 46	2	7.53095 44	1	42.23721 24	1	26.96222 84	1	18.55393 44	0
0.7	2	24.53600 02	2	10.16318 48	2	5.68579 83	1	36.20123 29	1	24.84436 68	0
0.8	2	32.38321 42	2	13.38409 47	2	7.47069 38	1	47.45357 21	1	32.48732 33	0
0.9	3	4.19166 52	2	17.28884 21	2	9.62991 01	2	6.10362 19	1	41.69290 49	0
1.0	3	5.34036 87	2	21.98450 69	2	12.22132 18	2	7.73048 78	1	52.69656 43	0
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-7.16666 67	0	-6.00000 00	0	-5.12500 00	0	-4.44444 44	0	-3.90000 00	0
-0.9	0	-9.61590 84	0	-7.81955 83	0	-6.50954 52	0	-5.51737 59	0	-4.74326 38	0
-0.8	0	-11.90157 33	0	-9.49795 14	0	-7.77085 30	0	-6.48171 72	0	-5.49009 84	0
-0.7	0	-13.87691 28	0	-10.92480 96	0	-8.82364 11	0	-7.27013 38	0	-6.08638 83	0
-0.6	0	-15.36404 99	0	-11.96702 67	0	-9.56555 86	0	-7.80219 25	0	-6.46778 05	0
-0.5	0	-16.14978 46	0	-12.46575 39	0	-9.87497 03	0	-7.98269 18	0	-6.55840 41	0
-0.4	0	-15.98096 87	0	-12.23308 86	0	-9.60852 17	0	-7.69983 06	0	-6.26946 76	0
-0.3	0	-14.55942 19	0	-11.04844 23	0	-8.59846 85	0	-6.82320 20	0	-5.49772 38	0
-0.2	0	-11.53634 66	0	-8.65455 12	0	-6.64975 08	0	-5.20159 67	0	-4.12379 04	0
-0.1	0	-6.50620 26	0	-4.75311 11	0	-3.53679 14	0	-2.66060 57	0	-2.01031 95	0
0.0	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0
0.1	0	11.52203 48	0	8.99994 03	0	7.25804 55	0	6.00912 26	0	5.08661 63	0
0.2	0	25.67847 47	0	19.69859 92	0	15.57675 87	0	12.62744 73	0	10.45348 08	0
0.3	0	44.17543 51	0	33.61080 28	0	26.34207 51	0	21.15082 12	0	17.33161 87	0
0.4	1	6.78160 98	0	51.32107 39	0	39.99137 83	0	31.91392 97	0	25.98191 91	0
0.5	1	9.75110 94	1	7.34909 79	0	57.01885 67	0	45.29428 36	0	36.69816 20	0
0.6	1	13.42897 78	1	10.08671 16	1	7.79813 10	0	61.71654 14	0	49.81029 65	0
0.7	1	17.93124 88	1	13.42897 79	1	10.35044 61	1	8.16572 04	1	6.56879 91	0
0.8	1	23.38838 53	1	17.47023 69	1	13.42897 78	1	10.56496 79	1	8.47444 77	0
0.9	1	29.94672 18	1	22.31615 69	1	17.11218 68	1	13.42897 78	1	10.74406 96	0
1.0	1	37.77002 83	1	28.08483 78	1	21.48764 79	1	16.82416 41	1	13.42897 80	0

$$10^{-t} {}_1F_1[a; b; x]$$

$$x=5.0$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	0	-49.00000 00	0	-24.00000 00	0	-15.66666 67	0	-11.50000 00	0	-9.00000 00	0
-0.9	1	-8.48135 47	0	-39.01383 37	0	-24.13823 63	0	-16.92017 60	0	-12.72354 27	0
-0.8	1	-12.01775 32	0	-53.70548 51	0	-32.35113 40	0	-22.12445 81	0	-16.26309 06	0
-0.7	1	-15.29859 02	1	-6.71922 90	0	-39.80653 33	0	-26.79254 69	0	-19.39733 06	0
-0.6	1	-18.05964 16	1	-7.83737 80	0	-45.88626 24	0	-30.52981 21	0	-21.85511 00	0
-0.5	1	-19.97490 81	1	-8.58991 92	0	-49.83533 93	0	-32.85662 03	0	-23.30841 85	0
-0.4	1	-20.64754 00	1	-8.81313 79	0	-50.74260 82	0	-33.19652 51	0	-23.36463 15	0
-0.3	1	-19.59977 05	1	-8.31068 13	0	-47.51931 03	0	-30.86321 12	0	-21.55794 55	0
-0.2	1	-16.26175 92	1	-6.84913 56	0	-38.87541 23	0	-25.04609 42	0	-17.33994 57	0
-0.1	1	-9.95925 89	0	-41.53139 85	0	-23.29349 29	0	-14.79445 60	0	-10.06922 83	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	1	14.84131 59	1	6.28624 00	0	36.06636 14	0	23.62230 70	0	16.73042 63	0
0.2	1	35.33953 00	1	14.84131 59	1	8.42893 34	0	54.55524 93	0	38.11533 00	0
0.3	1	62.83717 41	1	26.26789 61	1	14.84131 59	1	9.55023 72	1	6.62935 70	0
0.4	2	9.87643 86	1	41.14342 57	1	23.15842 50	1	14.84131 59	1	10.25659 62	0
0.5	2	14.47607 34	1	60.12871 14	1	33.73967 62	1	21.55105 37	1	14.84131 59	0
0.6	2	20.26991 33	2	8.39773 10	1	46.99424 01	1	29.93208 91	1	20.55151 36	0
0.7	2	27.47119 00	2	11.35457 88	1	63.38647 16	1	40.27068 17	1	27.57724 25	0
0.8	2	36.32194 36	2	14.98048 97	2	8.34418 40	1	52.89027 23	1	36.13292 17	0
0.9	2	47.09611 57	2	19.38518 33	2	10.77533 68	2	6.81553 63	1	46.45984 63	0
1.0	2	60.10295 65	2	24.69234 10	2	13.69886 62	2	8.64757 35	1	58.82891 40	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	12.50214 28	0	9.72559 32	0	7.81074 39	0	6.43982 88	0	5.42870 50	0
0.2	0	28.04734 44	0	21.44859 50	0	16.90668 05	0	13.66149 01	0	11.27290 16	0
0.3	0	48.43556 55	0	36.75153 29	0	28.72396 67	0	22.99893 37	0	18.79306 59	0
0.4	1	7.45788 25	0	56.29730 92	0	43.75803 30	0	34.83080 92	0	28.28401 25	0
0.5	1	10.75134 08	1	8.08378 41	0	62.56987 28	0	49.58514 59	0	40.07844 54	0
0.6	1	14.84131 59	1	11.12234 63	1	8.57928 78	1	6.77444 40	0	54.55080 78	0
0.7	1	19.86039 54	1	14.84131 59	1	11.41402 70	1	8.98511 69	1	7.21214 61	0
0.8	1	25.95794 35	1	19.34856 53	1	14.84131 59	1	11.65137 78	1	9.32612 05	0
0.9	1	33.30180 71	1	24.76514 65	1	18.95092 72	1	14.84131 59	1	11.84061 79	0
1.0	1	42.08017 41	1	31.22659 62	1	23.84324 42	1	18.63096 64	1	14.84131 59	0

$x=5 \cdot 1$

$10^{-t} {}_1F_1[a; b; x]$

b =		0.1	0.2	0.3	0.4	0.5				
t	t	t	t	t	t	t				
-1.0	0	-50.00000 00	0	-24.50000 00	0	-16.00000 00	0	-11.75000 00	0	-9.20000 00
-0.9	1	-8.91445 81	0	-40.90194 63	0	-25.25189 99	0	-17.66875 59	0	-13.26643 40
-0.8	1	-12.80376 65	0	-57.05477 31	0	-34.28089 92	0	-23.39110 27	0	-17.15996 93
-0.7	1	-16.43663 52	1	-7.19901 21	0	-42.54028 13	0	-28.56632 50	0	-20.63849 56
-0.6	1	-19.52111 67	1	-8.44934 74	0	-49.34857 55	0	-32.75985 21	0	-23.40372 94
-0.5	1	-21.69516 01	1	-9.30665 62	0	-53.86936 40	0	-35.44093 58	0	-25.09311 21
-0.4	1	-22.51608 25	1	-9.58855 20	0	-55.08884 92	0	-35.96897 39	0	-25.27083 75
-0.3	1	-21.44885 78	1	-9.07544 18	0	-51.79097 68	0	-33.57836 98	0	-23.41792 64
-0.2	1	-17.85311 92	1	-7.50537 55	0	-42.52989 48	0	-27.36179 21	0	-18.92127 23
-0.1	1	-10.96875 91	0	-45.68347 28	0	-25.59952 73	0	-16.25170 21	0	-11.06156 45
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00
0.1	1	16.40219 06	1	6.92531 65	0	39.59933 40	0	25.84436 89	0	18.23631 92
0.2	1	39.15948 98	1	16.40219 07	1	9.29005 66	0	59.96001 63	0	41.77041 19
0.3	2	6.97875 62	1	29.10255 39	1	16.40219 06	1	10.52801 00	1	7.28929 80
0.4	2	10.99195 20	1	45.68491 21	1	25.65466 75	1	16.40219 07	1	11.30816 34
0.5	2	16.14316 85	2	6.69044 67	1	37.45769 90	1	23.87192 08	1	16.40219 06
0.6	2	22.64718 22	2	9.36242 03	2	5.22792 50	1	33.22576 54	1	22.76302 10
0.7	2	30.74900 70	2	12.68270 18	2	7.06514 57	1	44.79133 96	1	30.60788 70
0.8	2	40.72727 64	2	16.76289 76	2	9.31776 40	2	5.89395 23	1	40.18234 07
0.9	3	5.28979 39	2	21.72937 76	2	12.05399 41	2	7.60890 50	2	5.17632 24
1.0	3	6.76182 95	2	27.72485 32	2	15.35076 54	2	9.67115 11	2	6.56618 99

b =		0.6	0.7	0.8	0.9	1.0				
a	t	t	t	t	t	t				
-1.0	0	-7.50000 00	0	-6.28571 43	0	-5.37500 00	0	-4.66666 67	0	-4.10000 00
-0.9	0	-10.42686 46	0	-8.46283 61	0	-7.03457 65	0	-5.95576 13	0	-5.11619 19
-0.8	0	-13.20106 45	0	-10.50457 76	0	-8.57354 05	0	-7.13690 10	0	-6.03532 48
-0.7	0	-15.64711 78	0	-12.27917 96	0	-9.89024 55	0	-8.12995 42	0	-6.79309 30
-0.6	0	-17.55001 68	0	-13.62606 00	0	-10.86144 16	0	-8.83823 67	0	-7.31227 40
-0.5	0	-18.64963 55	0	-14.35181 25	0	-11.33933 13	0	-9.14629 58	0	-7.50103 09
-0.4	0	-18.63454 20	0	-14.22578 43	0	-11.14832 17	0	-8.91747 14	0	-7.25104 70
-0.3	0	-17.13516 68	0	-12.97520 09	0	-10.08144 57	0	-7.99121 16	0	-6.43547 40
-0.2	0	-13.71626 85	0	-10.27979 58	0	-7.89642 94	0	-6.18012 49	0	-4.90668 24
-0.1	0	-7.86864 11	0	-5.76590 80	0	-4.31137 05	0	-3.26674 73	0	-2.49379 71
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00
0.1	0	13.57502 37	0	10.51844 64	0	8.41350 49	0	6.90868 40	0	5.80041 63
0.2	0	30.64585 08	0	23.36471 23	0	18.36019 79	0	14.78959 41	0	12.16526 56
0.3	0	53.11795 97	0	40.19735 06	0	31.33256 30	0	25.01936 15	0	20.38797 47
0.4	1	8.20260 16	0	61.76747 87	0	47.89127 96	0	38.02599 61	0	30.80140 27
0.5	1	11.85482 11	1	8.89289 22	1	6.86725 99	0	54.29437 05	0	43.78199 61
0.6	1	16.40219 05	1	12.26492 71	1	9.43963 76	1	7.43721 87	0	59.75430 64
0.7	1	21.99606 80	1	16.40219 07	1	12.58748 04	1	9.88764 66	1	7.91958 92
0.8	1	28.80728 90	1	21.42791 04	1	16.40219 06	1	12.85005 82	1	10.26426 40
0.9	1	37.02818 71	1	27.48062 52	1	20.98638 52	1	16.40219 05	1	13.06945 38
1.0	1	46.87478 49	1	34.71573 02	1	26.45501 34	1	20.63100 84	1	16.40219 08

$$10^{-t} {}_1F_1[a; b; x]$$

$$x = 5.2$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	0	-51.00000 00	0	-25.00000 00	0	-16.33333 33	0	-12.00000 00	0	-9.40000 00	0
-0.9	1	-9.37564 67	0	-42.90471 03	0	-26.42874 69	0	-18.45694 13	0	-13.83607 00	0
-0.8	1	-13.64964 44	0	-60.64789 23	0	-36.34473 08	0	-24.74159 33	0	-18.11333 67	0
-0.7	1	-17.66820 56	1	-7.71680 91	0	-45.48260 32	0	-30.47027 02	0	-21.96717 21	0
-0.6	1	-21.10900 62	1	-9.11259 89	0	-53.09171 19	0	-35.16480 56	0	-25.06971 11	0
-0.5	1	-23.57026 47	1	-10.08609 72	0	-58.24613 22	0	-38.23834 03	0	-27.02051 97	0
-0.4	1	-24.55854 45	1	-10.43427 07	0	-59.81871 73	0	-38.97951 86	0	-27.33622 96	0
-0.3	1	-23.47515 77	1	-9.91172 23	0	-56.45226 88	0	-36.53495 38	0	-25.43905 63	0
-0.2	1	-19.60104 41	1	-8.22470 55	0	-46.52754 98	0	-29.88980 25	0	-20.64409 56	0
-0.1	1	-12.07998 45	0	-50.24486 48	0	-28.12788 03	0	-17.84628 98	0	-12.14529 75	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	1	18.12722 41	1	7.63026 65	0	43.48911 55	0	28.28625 10	0	19.88809 47	0
0.2	1	43.38906 32	1	18.12722 41	1	10.23998 38	1	6.59111 76	0	45.78763 78	0
0.3	2	7.74970 12	1	32.24104 27	1	18.12722 41	1	11.60666 18	1	8.01601 78	0
0.4	2	12.23142 78	1	50.72199 60	1	28.41836 11	1	18.12722 41	1	12.46826 90	0
0.5	2	17.99861 21	2	7.44323 74	1	41.58115 77	1	26.44135 49	1	18.12722 42	0
0.6	2	25.29731 73	2	10.43595 52	2	5.81503 26	1	36.87835 17	1	25.21129 73	0
0.7	2	34.40879 87	2	14.16303 11	2	7.87351 03	1	49.81281 15	1	33.96855 78	0
0.8	3	4.56535 06	2	18.75265 09	2	10.40275 24	2	6.56696 04	1	44.67994 64	0
0.9	3	5.93954 65	2	24.35022 12	2	13.48114 69	2	8.49292 97	2	5.76627 83	0
1.0	3	7.60470 65	2	31.12024 02	2	17.19725 24	2	10.81340 74	2	7.32745 89	0
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-7.66666 67	0	-6.42857 14	0	-5.50000 00	0	-4.77777 78	0	-4.20000 00	0
-0.9	0	-10.86014 93	0	-8.80495 15	0	-7.31260 05	0	-6.18696 85	0	-5.31214 03	0
-0.8	0	-13.90936 18	0	-11.05090 79	0	-9.00738 91	0	-7.48961 64	0	-6.32772 48	0
-0.7	0	-16.62266 98	0	-13.02267 47	0	-10.47354 60	0	-8.59842 65	0	-7.17675 14	0
-0.6	0	-18.76398 17	0	-14.54409 41	0	-11.57599 32	0	-9.40753 11	0	-7.77473 59	0
-0.5	0	-20.04630 98	0	-15.40201 30	0	-12.15200 94	0	-9.78994 48	0	-8.02076 39	0
-0.4	0	-20.12463 08	0	-15.34118 43	0	-12.00748 71	0	-9.59475 59	0	-7.79534 33	0
-0.3	0	-18.58798 60	0	-14.05863 38	0	-10.91280 25	0	-8.64402 92	0	-6.95803 99	0
-0.2	0	-14.95075 36	0	-11.19744 38	0	-8.59826 51	0	-6.72940 04	0	-5.34487 92	0
-0.1	0	-8.64302 39	0	-6.33990 78	0	-4.74911 04	0	-3.60833 11	0	-2.76549 43	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	14.74965 64	0	11.38489 49	0	9.07101 30	0	7.41919 79	0	6.20442 92	0
0.2	0	33.49662 35	0	25.46305 46	0	19.94909 61	0	16.02058 05	0	13.13730 02	0
0.3	0	58.26495 27	0	43.97835 51	0	34.18987 16	0	27.22856 45	0	22.12888 74	0
0.4	1	9.02274 39	1	6.77811 44	0	52.42732 56	0	41.52654 67	0	33.55468 85	0
0.5	1	13.07224 39	1	9.78401 24	1	7.53824 40	0	59.46332 19	0	47.84028 18	0
0.6	1	18.12722 41	1	13.52553 23	1	10.38726 12	1	8.16600 69	1	6.54666 13	0
0.7	1	24.36031 15	1	18.12722 41	1	13.88219 10	1	10.88179 66	1	8.69757 43	0
0.8	1	31.96671 95	1	23.72972 55	1	18.12722 41	1	14.17264 56	1	11.29770 98	0
0.9	1	41.16662 74	1	30.49143 91	1	23.23954 85	1	18.12722 41	1	14.41542 44	0
1.0	2	5.22077 74	1	38.59035 61	1	29.35066 95	1	22.84490 38	1	18.12722 42	0

$x=5.3$

$10^{-t} {}_1F_1[a; b; x]$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	0	-52.00000	00	0	-25.50000	00	0	-16.66666	67	0	-12.25000	00	0	-9.60000	00
-0.9	1	-9.86731	18	0	-45.03167	16	0	-27.67391	89	0	-19.28788	26	0	-14.43452	99
-0.8	1	-14.56062	70	1	-6.45057	33	0	-38.55386	84	0	-26.18280	69	0	-19.12772	60
-0.7	1	-19.00170	01	1	-8.27596	41	0	-48.65146	92	0	-32.51536	59	0	-23.39058	93
-0.6	1	-22.83493	62	1	-9.83175	59	0	-57.14048	96	0	-37.75982	24	0	-26.86301	63
-0.5	1	-25.61478	51	1	-10.93402	77	0	-62.99666	15	0	-41.26774	73	0	-29.10306	30
-0.4	1	-26.79163	14	1	-11.35692	27	1	-6.49677	15	0	-42.24978	30	0	-29.57499	41
-0.3	1	-25.69603	53	1	-10.82640	60	0	-61.54001	21	0	-39.75536	87	0	-27.63601	79
-0.2	1	-21.52116	87	1	-9.01331	63	0	-50.90146	51	0	-32.65023	63	0	-22.52158	18
-0.1	1	-13.30327	87	0	-55.25649	12	0	-30.90039	78	0	-19.59148	32	0	-13.32910	28
0.0	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00
0.1	1	20.03368	10	1	8.40790	63	0	47.77203	11	0	30.96996	89	0	21.70012	48
0.2	1	48.07201	77	1	20.03368	09	1	11.28790	68	1	7.24643	87	0	50.20330	81
0.3	2	8.60478	78	1	35.71581	86	1	20.03368	10	1	12.79663	73	1	8.81632	24
0.4	2	13.60851	40	2	5.63084	19	1	31.47806	89	1	20.03368	10	1	13.74815	38
0.5	2	20.06343	78	2	8.27951	85	1	46.15399	65	1	29.28589	52	1	20.03368	09
0.6	2	28.25121	06	2	11.63047	99	2	6.46719	06	1	40.92873	15	1	27.92161	62
0.7	2	38.49439	01	2	15.81278	06	2	8.77286	69	2	5.53901	23	1	37.69499	25
0.8	3	5.11611	68	2	20.97351	07	2	11.61173	54	2	7.31563	46	1	49.67495	06
0.9	3	6.66705	57	2	27.27982	09	2	15.07379	00	2	9.47784	18	2	6.42248	72
1.0	3	8.54980	60	2	34.92113	88	2	19.26087	88	2	12.08790	91	2	8.17548	93
b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	0	-7.83333	33	0	-6.57142	86	0	-5.62500	00	0	-4.88888	89	0	-4.30000	00
-0.9	0	-11.31385	75	0	-9.16207	44	0	-7.60196	69	0	-6.42694	45	0	-5.51499	67
-0.8	0	-14.66079	90	0	-11.62886	56	0	-9.46508	61	0	-7.86072	70	0	-6.63458	35
-0.7	0	-17.66505	89	0	-13.81506	71	0	-11.09363	95	0	-9.09521	88	0	-7.58262	24
-0.6	0	-20.06759	44	0	-15.52758	51	0	-12.33970	18	0	-10.01458	79	0	-8.26675	84
-0.5	0	-21.55201	48	0	-16.53167	73	0	-13.02425	40	0	-10.47926	89	0	-8.57618	58
-0.4	0	-21.73635	79	0	-16.54508	44	0	-12.93287	83	0	-10.32273	40	0	-8.37918	18
-0.3	0	-20.16396	92	0	-15.23153	13	0	-11.81099	81	0	-9.34792	67	0	-7.52038	44
-0.2	0	-16.29340	84	0	-12.19354	35	0	-9.35862	16	0	-7.32333	03	0	-5.81779	70
-0.1	0	-9.48729	41	0	-6.96452	05	0	-5.22454	96	0	-3.97863	90	0	-3.05949	34
0.0	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00
0.1	0	16.03590	04	0	12.33195	28	0	9.78839	94	0	7.97521	02	0	6.64367	13
0.2	0	36.62454	36	0	27.76130	51	0	21.68630	03	0	17.36411	29	0	14.19636	73
0.3	0	63.92315	33	0	48.12764	38	0	37.32006	36	0	29.64459	73	0	24.02954	62
0.4	1	9.92600	08	1	7.43928	74	0	57.40598	73	0	45.36217	90	0	36.56647	01
0.5	1	14.41541	25	1	10.76551	19	1	8.27603	98	1	6.51374	92	0	52.28787	05
0.6	1	20.03368	09	1	14.91639	15	1	11.43106	72	1	8.96744	65	1	7.17381	29
0.7	1	26.97751	96	1	20.03368	09	1	15.31072	62	1	11.97692	32	1	9.55320	03
0.8	1	35.46979	79	1	26.27774	70	1	20.03368	09	1	15.63198	40	1	12.43619	48
0.9	1	45.76234	54	1	33.82954	15	1	25.73364	10	1	20.03368	09	1	15.90060	81
1.0	2	5.81390	10	1	42.89274	98	1	32.56091	57	1	25.29546	06	1	20.03368	11

$$10^{-t} {}_1F_1[a; b; x]$$

$$x=5.4$$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
1.0	0	-53.000000	00	0	-26.000000	00	0	-17.000000	00	0	-12.500000	00	0	-9.800000	00
0.9	1	-10.392057	76	0	-47.293211	11	0	-28.993001	17	0	-20.164997	76	0	-15.064065	58
-0.8	1	-15.542430	06	1	-6.865105	53	0	-40.920540	09	0	-27.722217	72	0	-20.208056	64
-0.7	1	-20.446296	60	1	-8.880126	61	0	-52.066463	34	0	-34.713565	52	0	-24.916606	60
-0.6	1	-24.711630	08	1	-10.611872	21	0	-61.521997	70	0	-40.561416	69	0	-28.794448	94
-0.5	1	-27.844683	38	1	-11.856780	00	1	-6.815448	59	0	-44.549804	49	0	-31.354285	52
-0.4	1	-29.233679	95	1	-12.363774	45	1	-7.057470	06	0	-45.803408	81	0	-32.002620	04
-0.3	1	-28.130577	77	1	-11.827048	86	1	-6.709457	77	0	-43.264145	57	0	-30.024868	80
-0.2	1	-23.630702	26	1	-9.878013	38	0	-55.687964	48	0	-35.665144	47	0	-24.568148	87
-0.1	1	-14.650042	26	0	-60.763393	32	0	-33.941096	68	0	-21.501847	77	0	-14.622494	48
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	1	22.140641	15	1	9.265759	95	0	52.488122	22	0	33.919759	93	0	23.688210	08
0.2	2	5.325678	87	1	22.140641	14	1	12.443971	12	1	7.968100	04	0	55.057388	88
0.3	2	9.553140	06	1	39.562797	70	1	22.140641	14	1	14.109464	45	1	9.697714	48
0.4	2	15.138351	17	2	6.250372	26	1	34.865396	67	1	22.140641	15	1	15.160224	49
0.5	2	22.361008	85	2	9.208470	00	2	5.122490	02	1	32.434908	87	1	22.140641	16
0.6	2	31.543217	77	2	12.959482	26	2	7.191546	64	1	45.419982	29	1	30.921944	41
0.7	3	4.305457	74	2	17.651091	14	2	9.773350	08	2	6.158431	11	1	41.826784	43
0.8	3	5.731774	42	2	23.451927	75	2	12.958707	78	2	8.148396	61	2	5.522193	37
0.9	3	7.481457	76	2	30.553962	26	2	16.850842	25	2	10.575016	67	2	7.152316	69
1.0	3	9.609283	34	2	39.175126	60	2	21.566778	86	2	13.509757	76	2	9.120054	47

b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	0	-8.000000	00	0	-6.714285	57	0	-5.750000	00	0	-5.000000	00	0	-4.400000	00
-0.9	0	-11.789549	95	0	-9.535322	52	0	-7.903503	31	0	-6.676314	44	0	-5.725242	25
-0.8	0	-15.458777	88	0	-12.240891	13	0	-9.948431	19	0	-8.251591	11	0	-6.956944	49
-0.7	0	-18.779714	49	0	-14.660246	65	0	-11.753391	19	0	-9.622490	05	0	-8.012363	39
-0.6	0	-21.468333	84	0	-16.581888	80	0	-13.156508	83	0	-10.662373	31	0	-8.790616	61
-0.5	0	-23.176086	63	0	-17.747478	84	0	-13.960970	00	0	-11.217956	60	0	-9.170122	21
-0.4	0	-23.480389	94	0	-17.845100	01	0	-13.930087	74	0	-11.105605	58	0	-9.005776	65
-0.3	0	-21.874161	12	0	-16.501770	08	0	-12.781810	07	0	-10.107239	95	0	-8.125821	17
-0.2	0	-17.754140	02	0	-13.275154	45	0	-10.182672	20	0	-7.965791	16	0	-6.328397	70
-0.1	0	-10.407982	27	0	-7.644395	51	0	-5.741091	16	0	-4.380219	95	0	-3.377738	87
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	0	17.444583	31	0	13.367312	23	0	10.571284	48	0	8.580922	21	0	7.121341	19
0.2	0	40.056971	10	0	30.278888	53	0	23.585988	88	0	18.830778	85	0	15.350522	25
0.3	1	7.014388	81	0	52.681613	39	0	40.749687	71	0	32.287267	71	0	26.105009	94
0.4	1	10.920851	11	1	8.166275	54	0	62.871054	49	0	49.565535	51	0	39.861542	25
0.5	1	15.897355	53	1	11.846615	52	1	9.087364	46	1	7.136691	14	0	57.162736	64
0.6	1	22.140641	14	1	16.451003	30	1	12.580877	70	1	9.848847	71	1	7.862428	88
0.7	1	29.874683	34	1	22.140641	16	1	16.886957	77	1	13.183342	25	1	10.494284	40
0.8	1	39.353707	76	1	29.098239	98	1	22.140641	14	1	17.242252	22	1	13.690455	50
0.9	1	50.865492	28	1	37.530333	03	1	28.494359	94	1	22.140641	14	1	17.539441	16
1.0	2	6.473496	68	1	47.669798	84	1	36.119763	35	1	28.007915	50	1	22.140641	17

$x=5.5$

$10^{-t} {}_1F_1[a; b; x]$

b=		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	0	-54.00000 00	0	-26.50000 00	0	-17.33333 33	0	-12.75000 00	0	-10.00000 00	0
-0.9	1	-10.95272 17	0	-49.70062 35	0	-30.39206 40	0	-21.09199 49	0	-15.72711 88	0
-0.8	1	-16.60129 30	1	-7.31086 53	0	-43.45805 68	0	-29.36794 56	0	-21.35966 68	0
-0.7	1	-22.01202 25	1	-9.53327 79	0	-55.74893 15	0	-37.07787 94	0	-26.55376 54	0
-0.6	1	-26.75301 55	1	-11.45847 09	1	-6.62658 04	0	-43.58759 31	0	-30.87593 76	0
-0.5	1	-30.27745 73	1	-12.86128 61	1	-7.37577 94	0	-48.10705 63	0	-33.78895 46	0
-0.4	1	-31.90481 73	1	-13.46279 16	1	-7.66822 41	0	-49.66624 38	0	-34.63602 49	0
-0.3	1	-30.79976 69	1	-12.92194 52	1	-7.31602 26	0	-47.08814 67	0	-32.62316 55	0
-0.2	1	-25.94859 02	1	-10.82627 98	0	-60.92693 23	0	-38.95870 65	0	-26.79958 68	0
-0.1	1	-16.13284 32	1	-6.68151 53	0	-37.27638 12	0	-23.59337 51	0	-16.03590 62	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	1	24.46919 32	1	10.21213 06	0	57.68152 85	0	37.16230 58	0	25.86972 86	0
0.2	2	5.89969 66	1	24.46919 32	1	13.71937 55	1	8.76286 64	0	60.39388 37	0
0.3	2	10.60486 98	1	43.82172 10	1	24.46919 31	1	15.55786 91	1	10.66846 66	0
0.4	2	16.83773 79	2	6.93738 90	1	38.61531 91	1	24.46919 32	1	16.71817 79	0
0.5	2	24.91727 33	2	10.24027 52	2	5.68478 16	1	35.92089 18	1	24.46919 32	0
0.6	2	35.21154 16	2	14.43794 35	2	7.99602 87	1	50.39983 19	1	34.24322 55	0
0.7	3	4.81436 75	2	19.69924 77	2	10.88621 80	2	6.84630 74	1	46.40779 48	0
0.8	3	6.41984 42	2	26.21734 83	2	14.45923 00	2	9.07459 67	2	6.13814 49	0
0.9	3	8.39294 73	2	34.21252 89	2	18.83336 39	2	11.79710 18	2	7.96394 85	0
1.0	3	10.79671 86	3	4.39352 86	2	24.14296 10	2	15.09576 10	2	10.17202 02	0
b=		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-8.16666 67	0	-6.85714 29	0	-5.87500 00	0	-5.11111 11	0	-4.50000 00	0
-0.9	0	-12.28891 39	0	-9.92591 51	0	-8.21810 25	0	-6.93575 22	0	-5.94339 54	0
-0.8	0	-16.30698 99	0	-12.88962 70	0	-10.45937 30	0	-8.66367 54	0	-7.29593 52	0
-0.7	0	-19.97253 22	0	-15.56243 06	0	-12.45590 66	0	-10.18257 75	0	-8.46776 73	0
-0.6	0	-22.97435 18	0	-17.71281 87	0	-14.03068 66	0	-11.35410 04	0	-9.34877 07	0
-0.5	0	-24.92869 14	0	-19.05667 39	0	-14.96748 63	0	-12.01000 72	0	-9.80563 42	0
-0.4	0	-25.36835 82	0	-19.24952 67	0	-15.00519 79	0	-11.94793 50	0	-9.67861 59	0
-0.3	0	-23.73062 50	0	-17.87794 58	0	-13.83153 63	0	-10.92668 49	0	-8.77795 42	0
-0.2	0	-19.34378 57	0	-14.44998 85	0	-11.07606 05	0	-8.66100 96	0	-6.87990 21	0
-0.1	0	-11.41224 34	0	-8.38461 74	0	-6.30245 46	0	-4.81585 38	0	-3.72234 97	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	18.98759 56	0	14.49941 19	0	11.42582 77	0	9.24093 05	0	7.64093 83	0
0.2	0	43.82399 24	0	33.03712 63	0	25.66371 59	0	20.43217 87	0	16.60858 26	0
0.3	1	7.69836 41	0	57.68029 25	0	44.50790 32	0	35.17830 55	0	28.37177 84	0
0.4	1	12.01664 27	1	8.96569 44	1	6.88706 89	0	54.17247 43	0	43.46711 33	0
0.5	1	17.53245 36	1	13.03749 31	1	9.97961 54	1	7.82066 21	0	62.50659 83	0
0.6	1	24.46919 31	1	18.14426 78	1	13.84751 97	1	10.81825 86	1	8.61860 66	0
0.7	1	33.08166 86	1	24.46919 32	1	18.62619 84	1	14.51242 88	1	11.52943 47	0
0.8	1	43.65963 98	1	32.22026 49	1	24.46919 31	1	19.01910 13	1	15.07232 78	0
0.9	2	5.65316 90	1	41.63301 91	1	31.55013 54	1	24.46919 31	1	19.34786 18	0
1.0	2	7.20694 63	2	5.29735 02	1	40.06488 87	1	31.01018 91	1	24.46919 33	0

$$10^{-t} {}_1F_1[a; b; x]$$

$$x = 5.6$$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	0	-55.00000	00	0	-27.00000	00	0	-17.66666	67	0	-13.00000	00	0	-10.20000	00
-0.9	1	-11.55239	56	0	-52.26620	04	0	-31.87770	08	0	-22.07289	92	0	-16.42633	50
-0.8	1	-17.74402	18	1	-7.79055	62	0	-46.18090	15	0	-31.12882	17	0	-22.58835	39
-0.7	1	-23.70984	16	1	-10.23976	66	0	-59.72214	36	0	-39.62247	54	0	-28.31135	72
-0.6	1	-28.97433	45	1	-12.37759	06	1	-7.14041	96	0	-46.85798	19	0	-33.12021	96
-0.5	1	-32.93228	64	1	-13.95513	52	1	-7.98459	96	0	-51.96412	02	0	-36.42317	80
-0.4	1	-34.82714	09	1	-14.66270	77	1	-8.33369	13	0	-53.86655	97	0	-37.49368	29
-0.3	1	-33.72666	86	1	-14.12020	30	1	-7.97854	94	0	-51.25678	92	0	-35.45011	71
-0.2	1	-28.49568	59	1	-11.86633	98	1	-6.66621	60	0	-42.55743	77	0	-29.23318	98
-0.1	1	-17.76553	29	1	-7.34663	55	0	-40.93528	36	0	-25.88362	76	0	-17.58078	05
0.0	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00
0.1	1	27.04264	07	1	11.25618	52	0	63.40091	06	0	40.72698	80	0	28.26378	60
0.2	2	6.53518	62	1	27.04264	06	1	15.12648	05	1	9.63819	31	1	6.62612	44
0.3	2	11.77117	66	1	48.53656	80	1	27.04264	09	1	17.15589	69	1	11.73769	68
0.4	2	18.72530	50	2	7.69920	01	1	42.76653	59	1	27.04264	07	1	18.43712	99
0.5	2	27.76105	15	2	11.38623	00	2	6.30824	97	1	39.77980	02	1	27.04264	07
0.6	2	39.29866	31	2	16.08249	90	2	8.88943	16	2	5.59211	50	1	37.91970	16
0.7	3	5.38221	66	2	21.98091	40	2	12.12396	79	2	7.61015	03	2	5.14866	14
0.8	3	7.18870	96	2	29.30255	03	2	16.13060	16	2	10.10462	08	2	6.82206	31
0.9	3	9.41290	02	2	38.29996	55	2	21.04479	37	2	13.15815	67	2	8.86646	64
1.0	3	12.12728	04	3	4.92608	42	2	27.02063	63	2	16.86462	61	2	11.34346	38

b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	0	-8.33333	33	0	-7.00000	00	0	-6.00000	00	0	-5.22222	22	0	-4.60000	00
-0.9	0	-12.81377	80	0	-10.33515	33	0	-8.54672	94	0	-7.20598	46	0	-6.17001	36
-0.8	0	-17.20943	16	0	-13.57793	45	0	-11.00001	50	0	-9.09856	47	0	-7.65276	96
-0.7	0	-21.24991	29	0	-16.52619	42	0	-13.20454	58	0	-10.77800	72	0	-8.95076	83
-0.6	0	-24.59448	54	0	-18.92669	50	0	-14.96687	50	0	-12.09325	17	0	-9.94388	63
-0.5	0	-26.82090	50	0	-20.46716	09	0	-16.04959	27	0	-12.85976	56	0	-10.48604	14
-0.4	0	-27.41295	47	0	-20.76740	25	0	-16.16483	05	0	-12.85468	19	0	-10.40148	66
-0.3	0	-25.74653	89	0	-19.36943	25	0	-14.96703	53	0	-11.81139	86	0	-9.48069	82
-0.2	0	-21.07419	93	0	-15.72647	16	0	-12.04494	65	0	-9.41359	01	0	-7.47582	18
-0.1	0	-12.50791	19	0	-9.19075	16	0	-6.91270	24	0	-5.28857	72	0	-4.09563	69
0.0	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00
0.1	0	20.67799	79	0	15.73750	83	0	12.35877	60	0	9.96026	72	0	8.20628	29
0.2	0	47.95869	58	0	36.05945	88	0	27.93654	62	0	22.18102	59	0	17.98019	82
0.3	1	8.45046	54	0	63.16770	28	0	48.62674	66	0	38.34155	65	0	30.84793	81
0.4	1	13.22368	18	1	9.84483	03	1	7.54578	60	0	59.22239	28	0	47.41303	68
0.5	1	19.33658	21	1	14.34935	83	1	10.96093	84	1	8.57171	45	1	6.83652	92
0.6	1	27.04264	05	1	20.01263	39	1	15.24293	49	1	11.88454	63	1	9.44905	40
0.7	1	36.63152	12	1	27.04264	08	1	20.54535	16	1	15.97672	32	1	12.66813	38
0.8	1	48.43322	44	1	35.67597	79	1	27.04264	05	1	20.97980	93	1	16.59486	55
0.9	2	6.28226	26	1	46.18104	49	1	34.93242	76	1	27.04264	05	1	21.34346	13
1.0	2	8.02244	51	2	5.88615	27	1	44.43802	45	1	34.33317	72	1	27.04264	09

$x=5.7$

$10^{-t} {}_1F_1[a; b; x]$

b =		0.1		0.2		0.3		0.4		0.5		
a	t		t		t		t		t		t	
-1.0	0	-56.000000	00	0	-27.500000	00	0	-18.000000	00	0	-10.400000	00
-0.9	1	-12.194444	83	0	-55.003317	74	0	-33.457081	14	0	-23.112079	94
-0.8	1	-18.978044	72	1	-8.307124	45	0	-49.104844	47	0	-33.014444	53
-0.7	1	-25.551736	66	1	-11.004338	84	1	-6.401147	71	0	-42.362777	85
-0.6	1	-31.392276	67	1	-13.375833	34	1	-7.697242	25	0	-50.393999	32
-0.5	1	-35.830199	97	1	-15.146637	71	1	-8.646378	88	0	-56.147888	39
-0.4	1	-38.024910	04	1	-15.973099	91	1	-9.058974	44	0	-58.435273	39
-0.3	1	-36.936642	20	1	-15.431821	17	1	-8.702360	03	0	-55.802292	22
-0.2	1	-31.294947	75	1	-13.007237	76	1	-7.294173	36	0	-46.490419	98
-0.1	1	-19.563382	21	1	-8.077709	96	0	-44.949727	71	0	-28.391891	11
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	1	29.886740	00	1	12.408041	10	1	6.969991	10	0	44.646154	43
0.2	2	7.238710	03	1	29.886740	00	1	16.678929	99	1	10.602301	10
0.3	2	13.064471	12	2	5.375600	00	1	29.886740	01	1	18.919052	25
0.4	2	20.821721	10	2	8.543903	32	1	47.361868	88	1	29.886740	01
0.5	2	30.924349	95	2	12.658863	34	2	6.999514	47	1	44.051420	05
0.6	3	4.385181	14	2	17.911624	40	2	9.881509	93	2	6.204250	08
0.7	3	6.015736	65	2	24.522400	06	2	13.500480	04	2	8.458287	78
0.8	3	8.047712	20	2	32.744016	66	2	17.992054	41	2	11.249998	88
0.9	3	10.554007	73	3	4.286579	98	2	23.511219	99	2	14.673810	02
1.0	3	13.617910	03	3	5.521785	58	2	30.234578	83	2	18.837170	06

b =		0.6		0.7		0.8		0.9		1.0		
a	t		t		t		t		t		t	
-1.0	0	-8.500000	00	0	-7.142857	71	0	-6.125000	00	0	-5.333333	33
-0.9	0	-13.366121	15	0	-10.764455	56	0	-8.890425	59	0	-7.487799	59
-0.8	0	-18.170441	13	0	-14.308913	31	0	-11.572635	56	0	-9.557971	11
-0.7	0	-22.618810	09	0	-17.556500	04	0	-14.002952	26	0	-11.411514	47
-0.6	0	-26.338368	86	0	-20.230381	12	0	-15.970106	63	0	-12.883602	22
-0.5	0	-28.864793	31	0	-21.987530	07	0	-17.213582	24	0	-13.771945	57
-0.4	0	-29.628024	41	0	-22.408576	69	0	-17.416190	07	0	-13.831238	89
-0.3	0	-27.936300	04	0	-20.986462	21	0	-16.195786	60	0	-12.766970	09
-0.2	0	-22.958351	10	0	-17.113811	12	0	-13.096053	30	0	-10.228553	39
-0.1	0	-13.703573	31	0	-10.068886	62	0	-7.576276	62	0	-5.801703	35
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	0	22.530136	60	0	17.091757	72	0	13.377524	40	0	10.744439	91
0.2	0	52.497470	08	0	39.371622	23	0	30.423203	36	0	24.091252	29
0.3	1	9.277543	37	1	6.919226	64	0	53.141409	94	0	41.803185	51
0.4	1	14.553330	09	1	10.811706	61	1	8.269083	35	1	6.475857	72
0.5	1	21.327264	43	1	15.794574	42	1	12.040304	45	1	9.396507	72
0.6	1	29.886739	99	1	22.074258	80	1	16.780286	68	1	13.057473	36
0.7	1	40.560805	55	1	29.886740	00	1	22.663077	71	1	17.590054	40
0.8	2	5.372508	08	1	39.500956	60	1	29.886739	99	1	23.143448	81
0.9	2	6.980671	11	2	5.122252	25	1	38.676043	30	1	29.886740	00
1.0	2	8.929092	20	2	6.539782	25	1	49.285398	88	1	38.011061	13

$$10^{-t} {}_1F_1[a; b; x]$$

$$x=5.8$$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	0	-57.00000	00	0	-28.00000	00	0	-18.33333	33	0	-13.50000	00	0	-10.60000	00
-0.9	1	-12.88255	26	0	-57.92653	80	0	-35.13799	78	0	-24.21427	98	0	-17.94497	15
-0.8	1	-20.31148	12	1	-8.86378	22	0	-52.24705	74	0	-35.03525	48	0	-25.30267	79
-0.7	1	-27.55080	93	1	-11.83217	55	1	-6.86445	84	0	-45.31559	09	0	-32.22913	58
-0.6	1	-34.02511	44	1	-14.46041	85	1	-8.30089	92	0	-54.21897	86	0	-38.15455	68
-0.5	1	-38.99425	40	1	-16.44489	11	1	-9.36596	44	0	-60.68771	65	0	-42.36216	24
-0.4	1	-41.52476	23	1	-17.40446	71	1	-9.84966	15	0	-63.40620	78	0	-43.96435	53
-0.3	1	-40.45756	83	1	-16.86778	16	1	-9.49329	27	0	-60.75994	74	0	-41.87599	39
-0.2	1	-34.37165	11	1	-14.25891	78	1	-7.98184	70	0	-50.78955	12	0	-34.78447	83
-0.1	1	-21.54322	42	1	-8.88135	40	0	-49.35481	53	0	-31.13934	85	0	-21.11634	00
0.0	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00
0.1	1	33.02995	58	1	13.67886	52	1	7.66376	67	0	48.95542	46	0	33.77566	85
0.2	2	8.01753	05	1	33.02995	58	1	18.39178	44	1	11.66425	29	1	7.98073	62
0.3	2	14.49850	24	2	5.95338	63	1	33.02995	61	1	20.86445	00	1	14.21283	81
0.4	2	23.14991	10	2	9.48046	93	2	5.24487	01	1	33.02995	60	1	22.42651	26
0.5	2	34.44271	00	2	14.07207	21	2	7.76591	06	1	48.77977	57	1	33.02995	60
0.6	3	4.89235	07	2	19.94583	34	2	10.98307	98	2	6.88287	90	1	46.49381	51
0.7	3	6.72241	98	2	27.35295	60	2	15.03116	70	2	9.39995	60	2	6.33587	88
0.8	3	9.00726	10	2	36.58235	08	2	20.06496	62	2	12.52353	31	2	8.42439	95
0.9	3	11.83042	55	3	4.79652	07	2	26.26167	37	2	16.36143	49	2	10.98561	78
1.0	3	15.28752	83	3	6.18800	25	2	33.82352	82	2	21.03656	01	2	14.09998	10

b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	0	-8.66666	67	0	-7.28571	43	0	-6.25000	00	0	-5.44444	44	0	-4.80000	00
-0.9	0	-13.94808	74	0	-11.21535	39	0	-9.25031	77	0	-7.78203	23	0	-6.65109	85
-0.8	0	-19.19472	50	0	-15.08592	11	0	-12.17969	99	0	-10.04374	54	0	-8.42532	14
-0.7	0	-24.08678	11	0	-18.65873	48	0	-14.85507	52	0	-12.08606	14	0	-10.00810	47
-0.6	0	-28.21647	77	0	-21.63133	74	0	-17.04584	37	0	-13.72924	36	0	-11.25677	84
-0.5	0	-31.07350	25	0	-23.62713	51	0	-18.46629	56	0	-14.75166	58	0	-11.99623	98
-0.4	0	-32.02867	45	0	-24.18378	38	0	-18.76712	28	0	-14.88346	93	0	-12.01412	50
-0.3	0	-30.31564	24	0	-22.74020	18	0	-17.52594	00	0	-13.79948	87	0	-11.05542	92
-0.2	0	-25.01043	20	0	-18.62207	00	0	-14.23671	82	0	-11.11137	56	0	-8.81652	89
-0.1	0	-15.00863	28	0	-11.02568	46	0	-8.29803	08	0	-6.35885	09	0	-4.93854	63
0.0	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00
0.1	0	24.55976	79	0	18.57330	13	0	14.49017	48	0	11.59947	40	0	9.49132	52
0.2	0	57.48034	15	0	43.00189	22	0	33.14423	34	0	26.17813	00	0	21.10735	49
0.3	1	10.18714	39	1	7.58072	36	0	58.09055	75	0	45.59190	41	0	36.50962	66
0.4	1	16.01811	81	1	11.87515	76	1	9.06337	04	1	7.08285	69	0	56.46019	21
0.5	1	23.52384	41	1	17.38677	18	1	13.22759	35	1	10.30236	52	1	8.18336	09
0.6	1	33.02995	56	1	24.34918	36	1	18.47409	09	1	14.34779	39	1	11.36301	13
0.7	1	44.90997	74	1	33.02995	59	1	24.99997	35	1	19.36766	89	1	15.29900	74
0.8	2	5.95909	82	1	43.73456	53	1	33.02995	56	1	25.53107	33	1	20.12098	25
0.9	2	7.75598	10	2	5.68107	55	1	42.81949	25	1	33.02995	56	1	25.97590	15
1.0	2	9.93698	85	2	7.26533	10	2	5.46582	14	1	42.08166	08	1	33.02995	62

$x=5.9$

$$10^{-t} {}_1F_1[a; b; x]$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	0	-58.00000 00	0	-28.50000 00	0	-18.66666 67	0	-13.75000 00	0	-10.80000 00	0
-0.9	1	-13.62071 19	0	-61.05171 42	0	-36.92892 55	0	-25.38465 21	0	-18.77087 63	0
-0.8	1	-21.75317 99	1	-9.46403 24	0	-55.62624 01	0	-37.20260 57	0	-26.80257 84	0
-0.7	1	-29.72138 53	1	-12.72893 71	1	-7.36516 58	0	-48.49921 37	0	-34.41226 88	0
-0.6	1	-36.89285 64	1	-15.63924 16	1	-8.95559 49	0	-58.35841 35	0	-40.97649 42	0
-0.5	1	-42.44973 25	1	-17.85986 33	1	-10.14865 85	1	-6.56157 02	0	-45.70700 54	0
-0.4	1	-45.35594 57	1	-18.96832 80	1	-10.71187 35	1	-6.88163 61	0	-47.62351 14	0
-0.3	1	-44.32010 40	1	-18.44014 16	1	-10.35774 87	1	-6.61684 16	0	-45.52305 26	0
-0.2	1	-37.75362 43	1	-15.63231 55	1	-8.73503 63	0	-55.48982 02	0	-37.94565 50	0
-0.1	1	-23.72361 64	1	-9.76485 40	0	-54.18915 09	0	-34.14926 49	0	-23.13589 83	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	1	36.50374 67	1	15.08098 47	1	8.42793 81	0	53.69402 14	0	36.94202 10	0
0.2	2	8.87968 23	1	36.50374 67	1	20.28166 84	1	12.83404 09	1	8.76095 62	0
0.3	2	16.08850 15	2	6.59297 37	1	36.50374 66	1	23.01098 13	1	15.64205 47	0
0.4	2	25.73530 26	2	10.51883 62	2	5.80794 59	1	36.50374 67	1	24.73569 71	0
0.5	2	38.35560 04	2	15.64126 92	2	8.61556 22	2	5.40135 74	1	36.50374 69	0
0.6	3	5.45721 18	2	22.20790 61	2	12.20614 09	2	7.63518 71	2	5.14797 55	0
0.7	3	7.51060 54	2	30.50509 38	2	16.73313 97	2	10.44539 81	2	7.02778 18	0
0.8	3	10.07895 45	2	40.86273 95	2	22.37310 07	2	13.93943 74	2	9.36023 45	0
0.9	3	13.25794 58	3	5.36596 68	2	29.32846 04	2	18.24033 95	2	12.22588 15	0
1.0	3	17.15725 97	3	6.93295 27	2	37.83064 22	2	23.48857 01	2	15.71655 98	0

b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-8.83333 33	0	-7.42857 14	0	-6.37500 00	0	-5.55555 56	0	-4.90000 00	0
-0.9	0	-14.56199 80	0	-11.68950 55	0	-9.62762 11	0	-8.08960 72	0	-6.90691 21	0
-0.8	0	-20.28738 74	0	-15.91259 78	0	-12.82387 58	0	-10.55788 84	0	-8.84398 37	0
-0.7	0	-25.66203 33	0	-19.83874 23	0	-15.76519 41	0	-12.80485 32	0	-10.58714 98	0
-0.6	0	-30.24021 37	0	-23.13767 19	0	-18.20001 82	0	-14.63461 38	0	-11.98106 17	0
-0.5	0	-33.46136 16	0	-25.39615 24	0	-19.81517 00	0	-15.80448 45	0	-12.83416 74	0
-0.4	0	-34.63139 33	0	-26.10472 44	0	-20.22616 72	0	-16.01774 97	0	-12.91320 96	0
-0.3	0	-32.90175 80	0	-24.64284 02	0	-18.96638 47	0	-14.91558 08	0	-11.93708 59	0
-0.2	0	-27.24597 19	0	-20.26224 69	0	-15.47495 42	0	-12.06802 47	0	-9.57001 06	0
-0.1	0	-16.43339 72	0	-12.06843 90	0	-9.08327 30	0	-6.96396 99	0	-5.41391 29	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	26.78420 53	0	20.19436 69	0	15.70560 99	0	12.53197 18	0	10.22058 95	0
0.2	0	62.95133 16	0	46.98133 35	0	36.12218 16	0	28.45839 56	0	22.88712 45	0
0.3	1	11.18757 54	1	8.30712 00	0	63.51667 13	0	49.73922 61	0	39.74070 33	0
0.4	1	17.63185 64	1	13.04491 56	1	9.93569 72	1	7.74846 40	0	61.63684 91	0
0.5	1	25.94767 44	1	19.14098 23	1	14.53368 68	1	11.29734 70	1	8.95593 85	0
0.6	1	36.50374 66	1	26.85953 63	1	20.34035 29	1	15.76735 06	1	12.46357 00	0
0.7	1	49.72379 93	1	36.50374 69	1	27.57877 96	1	21.32638 18	1	16.81535 23	0
0.8	2	6.60931 67	1	48.42035 87	1	36.50374 66	1	28.16592 75	1	22.15793 23	0
0.9	2	8.61660 48	2	6.30047 72	1	47.40538 43	1	36.50374 66	1	28.65785 00	0
1.0	2	11.05734 88	2	8.07066 18	2	6.06131 85	1	46.58681 74	1	36.50374 70	0

$$10^{-t} {}_1F_1[a; b; x]$$

$$x=6.0$$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	0	-59.000000	00	0	-29.000000	00	0	-19.000000	00	0	-14.000000	00	0	-11.000000	00
-0.9	1	-14.41329	18	0	-64.39611	37	0	-38.83908	07	0	-26.62879	24	0	-19.64595	73
-0.8	1	-23.31281	44	1	-10.11169	39	0	-59.26276	18	0	-39.52884	93	0	-28.40818	30
-0.7	1	-32.07913	11	1	-13.70080	47	1	-7.90656	12	0	-51.93358	67	0	-36.76189	41
-0.6	1	-40.01741	56	1	-16.92093	79	1	-9.66592	35	1	-6.28400	93	0	-44.02526	70
-0.5	1	-46.22436	26	1	-19.40246	88	1	-11.00026	13	1	-7.09668	98	0	-49.33187	70
-0.4	1	-49.55058	00	1	-20.67731	30	1	-11.65231	50	1	-7.47062	14	0	-51.59957	29
-0.3	1	-48.55796	01	1	-20.16214	46	1	-11.30275	11	1	-7.20700	55	0	-49.49542	78
-0.2	1	-41.47150	69	1	-17.13945	54	1	-9.56011	20	0	-60.62961	20	0	-41.39634	70
-0.1	1	-26.12501	72	1	-10.73623	10	0	-59.49518	87	0	-37.44719	69	0	-25.34491	61
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	1	40.34287	93	1	16.62800	60	1	9.26969	34	0	58.90513	67	0	40.41841	03
0.2	2	9.83405	66	1	40.34287	93	1	22.36693	23	1	14.12268	17	1	9.61906	66
0.3	2	17.85134	19	1	73.00954	56	1	40.34287	93	1	25.37950	09	1	17.21658	40
0.4	2	28.60609	51	2	11.67001	18	2	6.43121	53	1	40.34287	93	1	27.28376	67
0.5	2	42.70684	72	2	17.38354	67	2	9.55746	91	1	59.80671	14	1	40.34287	93
0.6	2	60.86254	18	2	24.72313	19	2	13.56399	89	2	8.46913	68	1	56.99839	71
0.7	3	8.38957	21	2	34.01495	31	2	18.62539	43	2	11.60597	26	2	7.79473	22
0.8	3	11.27571	43	2	45.63546	05	2	24.94286	94	2	15.51349	06	2	10.39905	61
0.9	3	14.85417	91	2	60.01765	48	2	32.74752	73	2	20.33198	25	2	13.60454	96
1.0	3	19.25068	81	3	7.76580	07	2	42.30398	95	2	26.22188	05	2	17.51597	66
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	0	29.22246	67	0	21.96837	11	0	17.03356	43	0	13.54915	78	0	11.01481	29
0.2	1	6.89588	66	0	51.34407	80	0	39.38179	15	0	30.95039	91	0	24.82910	84
0.3	1	12.28798	91	1	9.10486	02	1	6.94664	31	0	54.27973	75	0	43.27265	54
0.4	1	19.40977	67	1	14.33169	72	1	10.89382	04	1	8.47842	06	1	6.73053	67
0.5	1	28.62232	72	1	21.07377	83	1	15.97056	93	1	12.39031	77	1	9.80333	39
0.6	1	40.34287	93	1	29.62974	09	1	22.39672	16	1	17.32918	94	1	13.67265	16
0.7	1	55.05179	80	1	40.34287	93	1	30.42459	72	1	23.48473	26	1	18.48381	24
0.8	2	7.33002	58	1	53.60652	82	1	40.34287	93	1	31.07367	06	1	24.40260	67
0.9	2	9.57187	14	2	6.98699	64	1	52.48086	10	1	40.34287	93	1	31.61763	54
1.0	2	12.30262	18	2	8.96449	42	2	6.72131	28	1	51.57282	55	1	40.34287	93

$x=6.1$

$$10^{-t} {}_1F_1[a; b; x]$$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	0	-60.000000	00	0	-29.500000	00	0	-19.333333	33	0	-14.250000	00	0	-11.200000	00
-0.9	1	-15.26505	34	1	-6.79785	38	0	-40.87848	98	0	-27.95278	15	0	-20.57418	59
-0.8	1	-25.00094	75	1	-10.81093	77	0	-63.17881	27	0	-42.02742	79	0	-30.12826	02
-0.7	1	-34.64118	16	1	-14.75453	27	1	-8.49223	58	0	-55.64043	78	0	-39.29217	11
-0.6	1	-43.42279	15	1	-18.31495	44	1	-10.43690	43	1	-6.76943	47	0	-47.32062	02
-0.5	1	-50.34855	49	1	-21.08466	46	1	-11.92711	89	1	-7.67796	10	0	-53.26168	64
-0.4	2	-5.41439	41	1	-22.54527	73	1	-12.67833	21	1	-8.11200	63	0	-55.92131	36
-0.3	2	-5.32082	07	1	-22.04833	65	1	-12.33600	39	1	-7.85112	73	0	-53.82323	50
-0.2	1	-45.55903	58	1	-18.79356	26	1	-10.46407	15	1	-6.62510	40	0	-45.16385	54
-0.1	1	-28.76998	15	1	-11.80431	77	1	-6.53196	23	0	-41.06121	87	0	-27.76157	60
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	1	44.58577	68	1	18.33495	02	1	10.19695	80	1	6.46363	36	0	44.23558	85
0.2	2	10.89049	16	1	44.58577	67	1	24.66783	33	1	15.54232	20	1	10.56291	34
0.3	2	19.80571	81	2	8.08462	38	1	44.58577	67	1	27.99302	82	1	18.95128	30
0.4	2	31.79357	09	2	12.94619	45	2	7.12109	71	1	44.58577	69	1	30.09550	10
0.5	3	4.75451	03	2	19.31786	58	2	10.60160	07	2	6.62188	20	1	44.58577	69
0.6	3	6.78669	16	2	27.51959	93	2	15.07140	84	2	9.39354	73	2	6.31065	22
0.7	3	9.36965	12	2	37.92270	86	2	20.72902	96	2	12.89427	74	2	8.64483	35
0.8	3	12.61193	36	3	5.09564	76	2	27.80362	97	2	17.26321	35	2	11.55212	57
0.9	3	16.63876	71	3	6.71154	76	2	36.55886	47	2	22.66021	74	2	15.13694	06
1.0	3	21.59415	36	3	8.69676	83	3	4.72971	19	2	29.26839	10	2	19.51871	19
b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	0	-9.16666	67	0	-7.71428	57	0	-6.62500	00	0	-5.77777	78	0	-5.10000	00
-0.9	0	-15.89592	90	0	-12.71489	08	0	-10.43983	03	0	-8.74879	59	0	-7.45285	48
-0.8	0	-22.70048	40	0	-17.73106	71	0	-14.23535	62	0	-11.68011	19	0	-9.75434	84
-0.7	0	-29.17084	79	0	-22.45800	18	0	-17.77837	59	0	-14.38934	97	0	-11.85924	68
-0.6	0	-34.77530	48	0	-26.50250	53	0	-20.76999	22	0	-16.64420	02	0	-13.58365	37
-0.5	0	-38.83840	49	0	-29.36773	82	0	-22.83445	93	0	-18.15408	94	0	-14.69867	48
-0.4	0	-40.51667	13	0	-30.43598	41	0	-23.50662	00	0	-18.56081	02	0	-14.92329	92
-0.3	0	-38.77122	70	0	-28.94926	27	0	-22.21778	99	0	-17.42801	37	0	-13.91642	77
-0.2	0	-32.33701	93	0	-23.98757	00	0	-18.27993	53	0	-14.22944	07	0	-11.26795	80
-0.1	0	-19.68830	16	0	-14.44450	07	0	-10.86797	12	0	-8.33577	79	0	-6.48884	01
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	0	31.89544	76	0	23.91003	80	0	18.48470	98	0	14.65894	37	0	11.87997	33
0.2	1	7.55562	12	0	56.12762	81	0	42.95022	07	0	33.67425	65	0	26.94849	35
0.3	1	13.49846	29	1	9.98103	36	1	7.59911	29	0	59.25140	10	0	47.13411	81
0.4	1	21.36867	27	1	15.74730	62	1	11.94627	32	1	9.27903	71	1	7.35133	06
0.5	1	31.57382	40	1	23.20343	43	1	17.55144	14	1	13.59103	27	1	10.73289	24
0.6	1	44.58577	67	1	32.68676	01	1	24.66265	81	1	19.04767	91	1	15.00106	80
0.7	2	6.09487	67	1	44.58577	69	1	33.56514	01	1	25.86316	64	1	20.31976	20
0.8	2	8.12883	47	2	5.93463	80	1	44.58577	67	1	34.28262	82	1	26.87629	04
0.9	2	10.63212	45	2	7.74787	54	2	5.80980	78	1	44.58577	67	1	34.88410	39
1.0	2	13.68662	33	2	9.95649	62	2	7.45276	20	2	5.70909	02	1	44.58577	73

$$10^{-t} {}_1F_1[a; b; x]$$

$$x=6.2$$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	0	-61.000000	00	0	-30.000000	00	0	-19.666666	67	0	-14.500000	00	0	-11.400000	00
-0.9	1	-16.181118	88	1	-7.181947	74	0	-43.058060	06	0	-29.363225	53	0	-21.559872	29
-0.8	1	-26.829111	65	1	-11.566305	56	1	-6.739857	71	0	-44.712966	67	0	-31.972343	35
-0.7	1	-37.426279	98	1	-15.897489	92	1	-9.126108	88	0	-59.643446	62	0	-42.018513	39
-0.6	1	-47.135273	34	1	-19.831625	58	1	-11.274021	12	1	-7.295427	78	0	-50.884077	75
-0.5	2	-5.485566	66	1	-22.919549	96	1	-12.936174	48	1	-8.309570	05	0	-57.523627	88
-0.4	2	-5.917476	69	1	-24.587419	95	1	-13.797972	28	1	-8.810638	84	0	-60.620187	78
-0.3	2	-5.831116	12	1	-24.114694	40	1	-13.465959	90	1	-8.554292	27	0	-58.539440	00
-0.2	1	-50.053359	99	1	-20.609181	12	1	-11.454601	14	1	-7.240030	08	0	-54.189702	24
-0.1	1	-31.683375	57	1	-12.979111	45	1	-7.171380	06	0	-45.022172	23	0	-30.405831	15
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	1	49.274904	40	1	20.218396	61	1	11.218470	05	1	7.094000	03	0	48.427382	23
0.2	2	13.806463	39	1	49.274904	40	1	27.206732	22	1	17.106354	47	1	11.601138	81
0.3	2	21.972334	41	2	8.952051	17	1	49.274904	39	1	30.876976	64	1	20.862534	45
0.4	2	35.332411	18	2	14.360890	02	2	7.884692	27	1	49.274904	39	1	33.198251	12
0.5	3	5.309837	76	2	21.465245	55	2	11.759000	00	2	7.331587	76	1	49.274904	38
0.6	3	7.566541	11	2	30.628475	51	2	16.744733	31	2	10.418187	70	2	6.986695	53
0.7	3	10.462328	87	3	4.227300	01	2	23.067453	36	2	14.324278	88	2	9.587062	21
0.8	3	14.103646	68	3	5.688802	21	2	30.988001	16	2	19.208049	95	2	12.831930	05
0.9	3	18.633607	75	3	7.503806	68	2	40.806969	94	2	25.251542	27	2	16.840062	25
1.0	3	24.217037	79	3	9.737247	76	3	5.286965	51	2	32.663599	94	2	21.747523	38

b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	0	-9.333333	33	0	-7.857142	29	0	-6.750000	00	0	-5.888888	89	0	-5.200000	00
-0.9	0	-16.621632	27	0	-13.270168	84	0	-10.877695	52	0	-9.102625	53	0	-7.744673	37
-0.8	0	-24.033460	03	0	-18.731773	37	0	-15.009176	65	0	-12.293064	41	0	-10.249763	33
-0.7	0	-31.124655	52	0	-23.911624	44	0	-18.891931	14	0	-15.262891	12	0	-12.558262	29
-0.6	0	-37.314877	76	0	-28.381012	29	0	-22.200384	42	0	-17.759302	28	0	-14.470238	89
-0.5	0	-41.863176	67	0	-31.595513	37	0	-24.523238	85	0	-19.464560	07	0	-15.735647	78
-0.4	0	-43.840325	51	0	-32.875380	07	0	-25.349190	00	0	-19.985344	45	0	-16.046255	58
-0.3	0	-42.097578	87	0	-31.383440	06	0	-24.050847	77	0	-18.840788	85	0	-15.026567	79
-0.2	0	-35.231493	38	0	-26.100233	37	0	-19.866666	73	0	-15.449057	74	0	-12.223667	73
-0.1	0	-21.544438	69	0	-15.796112	24	0	-11.880709	97	0	-9.112326	63	0	-7.095858	89
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	0	34.826108	81	0	26.035528	80	0	20.070745	52	0	15.869993	34	0	12.822609	91
0.2	1	8.280196	69	0	61.373195	54	0	46.857276	68	0	36.652024	40	0	29.261908	82
0.3	1	14.830097	76	1	10.943440	01	1	8.314713	36	1	6.469589	91	0	51.356485	59
0.4	1	23.527062	25	1	17.304743	39	1	13.102443	36	1	10.157249	90	1	8.031286	68
0.5	1	34.830890	07	1	25.550102	25	1	19.290843	37	1	14.910222	29	1	11.752686	64
0.6	1	49.274904	38	1	36.060361	13	1	27.159623	32	1	20.938648	80	1	16.460715	53
0.7	2	6.747533	35	1	49.274904	42	1	37.030998	85	1	28.484229	96	1	22.340130	08
0.8	2	9.014171	12	2	6.569891	10	1	49.274904	38	1	37.824073	30	1	29.602448	84
0.9	2	11.808833	36	2	8.591136	62	2	6.431474	42	1	49.274904	38	1	38.489099	99
1.0	2	15.224693	32	2	11.057387	72	2	8.263371	13	2	6.319771	13	1	49.274904	46

$x=6.3$

$10^{-t} {}_1F_1[a; b; x]$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	0	-62.000000	00	0	-30.500000	00	0	-20.000000	00	0	-14.750000	00	0	-11.600000	00
-0.9	1	-17.16736	25	1	-7.59412	38	0	-45.38966	34	0	-30.86730	49	0	-22.60769	86
-0.8	1	-28.80992	57	1	-12.38275	96	1	-7.19483	87	0	-47.60138	79	0	-33.95079	55
-0.7	1	-40.45493	08	1	-17.13774	32	1	-9.81245	69	0	-63.96842	39	0	-44.95770	50
-0.6	1	-51.18366	02	1	-21.48226	00	1	-12.18326	80	1	-7.86560	20	0	-54.73910	47
-0.5	2	-5.97822	84	1	-24.92147	56	1	-14.03502	79	1	-8.99609	43	0	-62.14738	60
-0.4	2	-6.46856	22	1	-26.82041	34	1	-15.02005	66	1	-9.57182	33	1	-6.57305	77
-0.3	2	-6.39130	11	1	-26.37876	76	1	-14.70188	98	1	-9.32207	50	0	-63.68012	42
-0.2	2	-5.49953	87	1	-22.60230	87	1	-12.54014	65	1	-7.91281	21	0	-53.77188	09
-0.1	1	-34.89261	61	1	-14.27051	19	1	-7.87342	31	0	-49.36394	21	0	-33.29957	87
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	2	5.44571	91	1	22.29664	64	1	12.34386	55	1	7.78738	28	0	53.03100	51
0.2	2	13.35424	32	2	5.44571	90	1	30.00831	33	1	18.82954	76	1	12.74325	82
0.3	2	24.37412	85	2	9.91216	85	2	5.44571	91	1	34.05939	83	1	22.96840	26
0.4	2	39.26108	64	2	15.92906	35	2	8.72985	99	2	5.44571	91	1	36.62221	32
0.5	3	5.89047	08	2	23.84899	99	2	13.04189	89	2	8.11708	79	2	5.44571	91
0.6	3	8.43471	38	2	34.08437	12	2	18.60211	75	2	11.55387	59	2	7.73491	96
0.7	3	11.68039	09	3	4.71154	46	2	25.66666	14	2	15.91146	55	2	10.63135	96
0.8	3	15.76871	14	3	6.34993	55	2	34.53223	97	2	21.36958	58	2	14.25231	66
0.9	3	20.86311	85	3	8.38800	60	3	4.55413	38	2	28.13540	99	2	18.73279	11
1.0	3	27.15214	87	3	10.89994	26	3	5.90879	91	2	36.44698	38	2	24.22769	96
b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	0	-9.500000	00	0	-8.000000	00	0	-6.875000	00	0	-6.000000	00	0	-5.300000	00
-0.9	0	-17.39068	52	0	-13.85681	34	0	-11.33890	97	0	-9.47423	80	0	-8.05029	72
-0.8	0	-25.45998	87	0	-19.80003	74	0	-15.83318	29	0	-12.94416	04	0	-10.77472	66
-0.7	0	-33.22637	85	0	-25.47186	28	0	-20.08453	95	0	-16.19640	24	0	-13.30364	63
-0.6	0	-40.05672	24	0	-30.40506	03	0	-23.73850	06	0	-18.95598	19	0	-15.41978	43
-0.5	0	-45.13854	53	0	-34.00330	44	0	-26.34503	95	0	-20.87559	85	0	-16.85010	85
-0.4	0	-47.44855	85	0	-35.51888	86	0	-27.34235	36	0	-21.52355	16	0	-17.25666	12
-0.3	0	-45.71704	72	0	-34.02754	50	0	-26.03856	14	0	-20.37012	96	0	-16.22625	01
-0.2	0	-38.38768	44	0	-28.40007	33	0	-21.59110	17	0	-16.77231	68	0	-13.25887	84
-0.1	0	-23.57228	79	0	-17.27044	13	0	-12.98360	89	0	-9.95664	36	0	-7.75479	19
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	0	38.03967	63	0	28.36257	85	0	21.80449	74	0	17.19179	58	0	13.84987	30
0.2	1	9.07606	08	1	6.71260	66	0	51.13568	08	0	39.90788	58	0	31.78756	99
0.3	1	16.29511	89	1	12.00066	06	1	9.09963	98	1	7.06589	58	0	55.97418	99
0.4	1	25.90536	61	1	19.01833	07	1	14.37266	17	1	11.12067	83	1	8.77613	74
0.5	1	38.42523	87	1	28.13600	42	1	21.20479	17	1	16.35969	80	1	12.87158	91
0.6	2	5.44571	90	1	39.78340	40	1	29.91128	31	1	23.01953	14	1	18.06468	41
0.7	2	7.46985	76	2	5.44571	91	1	40.85594	65	1	31.37278	50	1	24.56357	29
0.8	2	9.99536	99	2	7.27293	77	2	5.44571	90	1	41.73252	83	1	32.60696	36
0.9	2	13.11471	50	2	9.52566	27	2	7.11946	92	2	5.44571	90	1	42.46778	05
1.0	2	16.93385	40	2	12.27905	19	2	9.16167	52	2	6.99559	49	2	5.44571	91

$$10^{-t} {}_1F_1[a; b; x]$$

$$x=6.4$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	0	-63.00000 00	0	-31.00000 00	0	-20.33333 33	0	-15.00000 00	0	-11.80000 00	0
-0.9	1	-18.22975 53	1	-8.03681 52	0	-47.88621 74	0	-32.47282 64	0	-23.72274 28	0
-0.8	1	-30.95714 66	1	-13.26571 21	1	-7.68569 80	0	-50.71002 33	0	-36.07488 39	0
-0.7	1	-43.74956 92	1	-18.48409 95	1	-10.55594 80	1	-6.86435 11	0	-48.12801 68	0
-0.6	2	-5.55995 05	1	-23.27923 13	1	-13.17119 56	1	-8.48390 18	0	-58.91128 83	0
-0.5	2	-6.51685 50	1	-27.10616 79	1	-15.23199 42	1	-9.74253 47	1	-6.71653 77	0
-0.4	2	-7.07232 42	1	-29.26255 39	1	-16.35424 79	1	-10.40136 83	1	-7.12900 77	0
-0.3	2	-7.00617 17	1	-28.85983 76	1	-16.05397 17	1	-10.16058 24	1	-6.92847 90	0
-0.2	2	-6.04301 64	1	-24.79054 03	1	-13.72998 41	1	-8.64901 17	0	-58.68113 78	0
-0.1	1	-38.42793 07	1	-15.69112 68	1	-8.64430 32	0	-54.12375 92	0	-36.46684 86	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	2	6.01845 04	1	24.58990 47	1	13.58376 70	1	8.55013 51	0	58.08739 50	0
0.2	2	14.78693 04	2	6.01845 04	1	33.09982 61	1	20.72818 40	1	13.99975 60	0
0.3	2	27.03650 85	2	10.97485 47	2	6.01845 04	1	37.57126 14	1	25.28880 70	0
0.4	3	4.36222 52	2	17.66728 95	2	9.66529 21	2	6.01845 04	1	40.40072 15	0
0.5	3	6.55526 15	2	26.49498 02	2	14.46384 58	2	8.98645 93	2	6.01845 04	0
0.6	3	9.40111 28	2	37.92570 60	2	20.66368 06	2	12.81259 84	2	8.56301 54	0
0.7	3	13.03806 09	3	5.25051 84	2	28.55550 53	2	17.67301 08	2	11.78873 77	0
0.8	3	17.62701 60	3	7.08675 18	2	38.47662 87	2	23.77178 19	2	15.82863 53	0
0.9	3	23.35452 19	3	9.37468 26	3	5.08170 27	2	31.34454 19	2	20.83607 74	0
1.0	3	30.43609 41	3	12.19901 38	3	6.60260 78	2	40.66245 82	2	26.98733 52	0

b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-9.66666 67	0	-8.14285 71	0	-7.00000 00	0	-6.11111 11	0	-5.40000 00	0
-0.9	0	-18.20656 31	0	-14.47729 20	0	-11.82527 34	0	-9.86497 74	0	-8.37074 68	0
-0.8	0	-26.98776 84	0	-20.94131 44	0	-16.71135 24	0	-13.63636 62	0	-11.33149 12	0
-0.7	0	-35.48849 07	0	-27.14754 55	0	-21.36262 76	0	-17.19467 08	0	-14.09903 01	0
-0.6	0	-43.01828 39	0	-32.58698 23	0	-25.39331 16	0	-20.24091 04	0	-16.43734 94	0
-0.5	0	-48.68658 72	0	-36.60670 34	0	-28.31119 04	0	-22.39562 10	0	-18.04843 26	0
-0.4	0	-51.36694 97	0	-38.38455 45	0	-29.49920 75	0	-23.18515 43	0	-18.56187 17	0
-0.3	0	-49.65648 24	0	-36.90050 13	0	-28.19465 13	0	-22.02621 35	0	-17.52316 53	0
-0.2	0	-41.83000 07	0	-30.90427 82	0	-23.46568 84	0	-18.20844 34	0	-14.38055 70	0
-0.1	0	-25.78830 76	0	-18.87895 76	0	-14.18496 85	0	-10.87487 40	0	-8.47027 42	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	41.56387 76	0	30.91066 03	0	23.70003 05	0	18.63474 33	0	14.96959 16	0
0.2	1	9.95030 56	1	7.34360 09	0	55.82135 27	0	43.46836 28	0	34.54543 22	0
0.3	1	17.90699 34	1	13.16213 66	1	9.96070 03	1	7.71908 27	0	61.02498 92	0
0.4	1	28.52610 08	1	20.90384 07	1	15.76829 34	1	12.17770 30	1	9.59217 55	0
0.5	1	42.39187 58	1	30.98564 42	1	23.31092 90	1	17.95245 22	1	14.09935 36	0
0.6	2	6.01845 03	1	43.89216 61	1	32.94373 58	1	25.30953 72	1	19.82737 98	0
0.7	2	8.26927 00	2	6.01845 04	1	45.07726 91	1	34.55625 22	1	27.01063 74	0
0.8	2	11.08276 91	2	8.05099 84	2	6.01845 03	1	46.04610 90	1	35.91837 90	0
0.9	2	14.56386 82	2	10.56129 46	2	7.88085 61	2	6.01845 03	1	46.85896 00	0
1.0	2	18.83299 94	2	13.63466 66	2	10.15712 51	2	7.74349 66	2	6.01845 04	0

$x=6.5$

$10^{-t} {}_1F_1[a; b; x]$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	1	-6.400000	00	0	-31.500000	00	0	-20.666666	67	0	-15.250000	00	0	-12.000000	00
-0.9	1	-19.37511	09	1	-8.51267	22	0	-50.56178	62	0	-34.18827	72	0	-24.91052	46
-0.8	1	-33.28582	80	1	-14.22107	11	1	-8.21556	59	0	-54.05774	52	0	-38.35686	42
-0.7	1	-47.33474	37	1	-19.94618	51	1	-11.36167	80	1	-7.36993	88	0	-51.54935	09
-0.6	2	-6.04173	79	1	-25.23608	19	1	-14.24496	44	1	-9.15463	39	0	-63.42853	51
-0.5	2	-7.10585	06	1	-29.49085	98	1	-16.53617	64	1	-10.55436	07	1	-7.26129	97
-0.4	2	-7.73389	82	1	-31.93391	34	1	-17.81113	90	1	-11.30563	11	1	-7.73397	93
-0.3	2	-7.68119	70	1	-31.57908	53	1	-17.53337	10	1	-11.07651	04	1	-7.53966	86
-0.2	2	-6.64073	02	1	-27.19323	14	1	-15.03430	73	1	-9.45473	58	1	-6.40452	80
-0.1	1	-42.32264	87	1	-17.25367	64	1	-9.49085	68	0	-59.34253	34	0	-39.93401	72
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	2	6.65141	63	1	27.12047	43	1	14.94988	88	1	9.38925	56	0	63.64159	46
0.2	2	16.37267	83	2	6.65141	63	1	36.51135	13	1	22.82022	02	1	15.38217	79
0.3	2	29.98761	73	2	12.15104	23	2	6.65141	64	1	41.44675	37	1	27.84571	23
0.4	3	4.84632	10	2	19.59392	70	2	10.70060	82	2	6.65141	63	1	44.57057	43
0.5	3	7.29420	36	2	29.43185	18	2	16.03984	74	2	9.94863	75	2	6.65141	63
0.6	3	10.47674	31	3	4.21951	26	2	22.95173	16	2	14.20762	81	2	9.47949	08
0.7	3	14.55116	13	3	5.85034	81	2	31.76601	14	2	19.62795	56	2	13.07139	21
0.8	3	19.70070	85	3	7.90782	09	3	4.28659	31	2	26.44123	22	2	17.57790	77
0.9	3	26.13816	29	3	10.47556	96	3	5.66952	81	2	34.91529	75	2	23.17317	50
1.0	4	3.41097	25	3	13.65024	54	3	7.37662	38	3	4.53588	63	2	30.05764	55
b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	0	-9.83333	33	0	-8.28571	43	0	-7.12500	00	0	-6.22222	22	0	-5.50000	00
-0.9	0	-19.07303	74	0	-15.13427	66	0	-12.33873	41	0	-10.27629	57	0	-8.70712	62
-0.8	0	-28.62516	40	0	-22.16152	53	0	-17.64799	56	0	-14.37289	16	0	-11.92249	38
-0.7	0	-37.92455	90	0	-28.94826	45	0	-22.73317	16	0	-18.26288	55	0	-14.94834	85
-0.6	0	-46.21856	70	0	-34.94020	06	0	-27.17456	69	0	-21.62133	35	0	-17.52842	13
-0.5	0	-52.53138	31	0	-39.42269	96	0	-30.43401	94	0	-24.03377	99	0	-19.33754	30
-0.4	0	-55.62343	49	0	-41.49206	76	0	-31.83402	48	0	-24.98073	73	0	-19.96988	63
-0.3	0	-53.94524	72	0	-40.02298	31	0	-30.53409	05	0	-23.82013	24	0	-18.92568	82
-0.2	0	-45.58517	04	0	-33.63164	98	0	-25.50403	05	0	-19.76750	54	0	-15.59629	72
-0.1	0	-28.21031	95	0	-20.63422	17	0	-15.49386	81	0	-11.87373	14	0	-9.24736	51
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	0	45.42917	86	0	33.70114	68	0	25.77276	74	0	20.21022	07	0	16.19032	86
0.2	1	10.91073	51	1	8.03577	24	0	60.95372	76	0	47.36253	87	0	37.55735	55
0.3	1	19.68055	48	1	14.43825	68	1	10.90537	84	1	8.43466	44	1	6.65502	98
0.4	1	31.41409	75	1	22.97864	98	1	17.30184	54	1	13.33753	20	1	10.48631	13
0.5	1	46.76945	06	1	34.12604	63	1	25.62869	07	1	19.70278	49	1	15.44670	12
0.6	2	6.65141	62	1	48.42669	52	1	36.28576	25	1	27.82982	37	1	21.76465	51
0.7	2	9.15398	13	2	6.65141	63	1	49.73612	90	1	38.06487	00	1	29.70396	27
0.8	2	12.28781	89	2	8.91206	11	2	6.65141	63	1	50.80689	29	1	39.56817	34
0.9	2	16.17192	45	2	11.70892	93	2	8.72344	90	2	6.65141	63	2	5.17054	88
1.0	2	20.94309	87	2	15.13883	93	2	11.26018	86	2	8.57114	95	2	6.65141	63

$$10^{-t} {}_1F_1[a; b; x]$$

$$x = 6.6$$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	1	-6.500000	00	0	-32.000000	00	0	-21.000000	00	0	-15.500000	00	0	-12.200000	00
-0.9	1	-20.61078	91	1	-9.02458	43	0	-53.43168	22	0	-36.02288	71	0	-26.17703	66
-0.8	1	-35.81241	60	1	-15.25528	57	1	-8.78785	58	0	-57.66510	63	0	-40.81006	72
-0.7	2	-5.12373	21	1	-21.53452	23	1	-12.23521	05	1	-7.91695	16	0	-55.24338	60
-0.6	2	-6.56751	68	1	-27.36763	46	1	-15.41240	33	1	-9.88250	15	1	-6.83212	81
-0.5	2	-7.75004	71	1	-32.09443	74	1	-17.95753	88	1	-11.43755	23	1	-7.85289	08
-0.4	2	-8.45892	57	1	-34.85651	96	1	-19.40233	95	1	-12.29157	19	1	-8.39246	78
-0.3	2	-8.42234	35	1	-34.55978	43	1	-19.15234	18	1	-12.07719	83	1	-8.20631	68
-0.2	2	-7.29814	37	1	-29.83167	32	1	-16.46431	73	1	-10.33668	63	1	-6.99075	06
-0.1	1	-46.61352	01	1	-18.97246	89	1	-10.42060	35	1	-6.50652	18	0	-43.73003	42
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	2	7.35095	19	1	29.91297	58	1	16.45514	85	1	10.31245	25	1	6.97431	57
0.2	2	18.12780	08	2	7.35095	19	1	40.27609	68	1	25.12545	54	1	16.90324	21
0.3	2	33.25862	73	2	13.45282	58	2	7.35095	19	1	45.72361	56	1	30.66334	10
0.4	3	5.38364	12	2	21.72931	08	2	11.84645	01	2	7.35095	19	1	49.17239	73
0.5	3	8.11550	18	2	32.69139	88	2	17.78652	43	2	11.01350	91	2	7.35095	19
0.6	3	11.67383	38	3	4.69399	69	2	25.49100	50	2	15.75366	60	2	10.49375	87
0.7	3	16.23729	30	3	6.51784	16	2	35.33372	64	2	21.79741	12	2	14.49282	98
0.8	3	22.01445	07	3	8.82267	49	3	4.77498	80	2	29.40745	21	2	19.51899	72
0.9	3	29.24785	36	3	11.70372	66	3	6.32441	87	2	38.88805	50	2	25.76988	12
1.0	4	3.82186	06	3	15.27122	31	3	8.24000	96	3	5.05904	99	2	33.47329	65
b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	0	-10.000000	00	0	-8.42857	14	0	-7.250000	00	0	-6.33333	33	0	-5.600000	00
-0.9	0	-19.99419	81	0	-15.83066	22	0	-12.88139	97	0	-10.70976	35	0	-9.06062	74
-0.8	0	-30.38126	36	0	-23.46709	54	0	-18.64778	50	0	-15.15721	28	0	-12.55037	00
-0.7	0	-40.54934	63	0	-30.88444	60	0	-24.20374	47	0	-19.40667	47	0	-15.85586	34
-0.6	0	-49.67827	55	0	-37.47932	10	0	-29.09286	47	0	-23.10511	83	0	-18.69895	08
-0.5	0	-56.69920	36	0	-42.46980	62	0	-32.72694	62	0	-25.80002	63	0	-20.72495	98
-0.4	0	-60.24852	99	0	-44.86291	11	0	-34.36236	21	0	-26.92182	49	0	-21.48940	45
-0.3	0	-58.61545	72	0	-43.41757	83	0	-33.07321	87	0	-25.76397	83	0	-20.44293	89
-0.2	0	-49.68246	35	0	-36.60275	35	0	-27.72099	18	0	-21.46049	26	0	-16.91437	92
-0.1	0	-30.85792	13	0	-22.54999	11	0	-16.92024	15	0	-12.96055	37	0	-10.09158	82
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	0	49.66906	54	0	36.75750	26	0	28.03962	11	0	21.93069	87	0	17.52145	54
0.2	1	11.96593	59	1	8.79513	30	1	6.65761	03	0	51.62231	27	0	40.84728	91
0.3	1	21.63214	30	1	15.84045	20	1	11.94189	96	1	9.21869	62	1	7.25955	42
0.4	1	34.59673	87	1	25.26189	96	1	18.98708	01	1	14.61028	85	1	11.46613	30
0.5	2	5.16006	29	1	37.58701	15	1	28.17948	73	1	21.62643	22	1	16.92541	86
0.6	2	7.35095	18	2	5.34312	07	1	39.96910	40	1	30.60370	19	1	23.89395	65
0.7	2	10.13307	46	2	7.35095	19	2	5.48779	70	1	41.93198	78	1	32.66848	98
0.8	2	13.62320	04	2	9.86496	32	2	7.35095	18	2	5.60613	35	1	43.59106	38
0.9	2	17.95621	40	2	12.98063	54	2	9.65589	27	2	7.35095	18	2	5.70546	76
1.0	2	23.28742	21	2	16.80776	44	2	12.48245	81	2	9.48704	24	2	7.35095	19

$x=6.7$ $10^{-t} {}_1F_1[a; b; x]$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	1	-6.60000 00	0	-32.50000 00	0	-21.33333 33	0	-15.75000 00	0	-12.40000 00	0
-0.9	1	-21.94482 46	1	-9.57570 14	0	-56.51257 87	0	-37.98669 48	0	-27.52879 13	0
-0.8	1	-38.55488 73	1	-16.37539 60	1	-9.40629 01	0	-61.55449 03	0	-43.44899 48	0
-0.7	2	-5.54867 05	1	-23.26061 47	1	-13.18262 04	1	-8.50903 88	0	-59.23373 83	0
-0.6	2	-7.14143 91	1	-29.69011 45	1	-16.68207 17	1	-10.67264 06	1	-7.36227 29	0
-0.5	2	-8.45474 74	1	-34.93760 09	1	-19.50699 07	1	-12.39864 86	1	-8.49553 39	0
-0.4	2	-9.25360 51	1	-38.05454 58	1	-21.14057 63	1	-13.36681 18	1	-9.10938 99	0
-0.3	2	-9.23617 43	1	-37.82750 49	1	-20.92433 38	1	-13.17069 25	1	-8.93360 93	0
-0.2	2	-8.02127 41	1	-32.72928 87	1	-18.03232 12	1	-11.30221 96	1	-7.63151 83	0
-0.1	2	-5.13410 65	1	-20.86326 31	1	-11.44181 62	1	-7.13412 16	0	-47.88668 15	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	2	8.12405 82	1	32.99458 91	1	18.11378 98	1	11.32821 68	1	7.64466 08	0
0.2	2	20.07034 70	2	8.12405 82	1	44.43072 18	1	27.66572 46	1	18.57695 82	0
0.3	2	36.88406 60	2	14.89358 73	2	8.12405 82	1	50.44351 13	1	33.76840 47	0
0.4	3	5.98000 16	2	24.09596 33	2	13.11459 20	2	8.12405 82	2	5.42510 39	0
0.5	3	9.02826 34	2	36.30886 24	2	19.72228 55	2	12.19201 14	2	8.12405 83	0
0.6	3	13.00597 31	3	5.22127 76	2	28.30892 55	2	17.46699 46	2	11.61623 35	0
0.7	3	18.11603 33	3	7.26056 34	2	39.29810 23	2	24.20478 38	2	16.06801 02	0
0.8	3	24.59570 26	3	9.84191 59	3	5.31837 33	2	32.70319 95	2	21.67284 61	0
0.9	3	32.72133 00	3	13.07371 25	3	7.05395 70	3	4.33077 36	2	28.65486 48	0
1.0	4	4.28136 36	3	17.08157 14	3	9.20297 89	3	5.64178 28	2	37.27285 04	0
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-10.16666 67	0	-8.57142 86	0	-7.37500 00	0	-6.44444 44	0	-5.70000 00	0
-0.9	0	-20.97448 54	0	-16.56958 80	0	-13.45555 33	0	-11.16707 85	0	-9.43253 81	0
-0.8	0	-32.26594 23	0	-24.86499 91	0	-19.71578 69	0	-15.99309 46	0	-13.21797 06	0
-0.7	0	-43.37891 60	0	-32.96742 14	0	-25.78256 92	0	-20.63214 13	0	-16.82619 17	0
-0.6	0	-53.41997 07	0	-40.22024 12	0	-31.15972 75	0	-24.70080 92	0	-19.95539 42	0
-0.5	0	-61.21871 36	0	-45.76820 09	0	-35.20458 01	0	-27.70518 15	0	-22.21885 17	0
-0.4	1	-6.52755 74	0	-48.52052 97	0	-37.10117 69	0	-29.02096 49	0	-23.12988 93	0
-0.3	0	-63.70224 10	0	-47.10896 68	0	-35.82986 99	0	-27.87093 56	0	-22.08484 95	0
-0.2	0	-54.15393 69	0	-39.84009 03	0	-30.13281 60	0	-23.29940 16	0	-18.34383 27	0
-0.1	0	-33.75260 16	0	-24.64133 47	0	-18.47495 70	0	-14.14336 04	0	-11.00897 45	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	54.32034 10	0	40.10549 12	0	30.51914 12	0	23.80984 07	0	18.97322 82	0
0.2	1	13.12535 63	1	9.62829 12	1	7.27360 17	0	56.28267 30	0	44.44147 57	0
0.3	1	23.77975 74	1	17.38130 13	1	13.07930 60	1	10.07782 72	1	7.92105 50	0
0.4	1	38.10422 29	1	27.77467 61	1	20.83914 32	1	16.00710 07	1	12.53997 37	0
0.5	2	5.69325 14	1	41.40140 59	1	30.98690 60	1	23.74071 17	1	18.54846 35	0
0.6	2	8.12405 82	2	5.89545 13	1	44.02876 57	1	33.65685 32	1	26.23448 48	0
0.7	2	11.21659 93	2	8.12405 83	2	6.05529 63	1	46.19438 60	1	35.93169 75	0
0.8	2	15.10295 75	2	10.91948 20	2	8.12405 82	2	6.18607 24	1	48.02534 09	0
0.9	2	19.93594 83	2	14.38977 82	2	10.68775 11	2	8.12405 82	2	6.29587 53	0
1.0	2	25.89179 42	2	18.65939 48	2	13.83677 07	2	10.50056 63	2	8.12405 83	0

$$10^{-t} {}_1F_1[a; b; x]$$

$$x=6.8$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	1	-6.70000 00	0	-33.00000 00	0	-21.66666 67	0	-16.00000 00	0	-12.60000 00	0
-0.9	1	-23.38598 76	1	-10.16945 88	0	-59.82263 59	0	-40.09062 16	0	-28.97286 29	0
-0.8	1	-41.53289 32	1	-17.58908 95	1	-10.07492 82	1	-6.57502 84	0	-46.28942 80	0
-0.7	2	-6.01150 81	1	-25.13703 91	1	-14.21054 25	1	-9.15018 11	0	-63.54613 95	0
-0.6	2	-7.76805 53	1	-32.22128 43	1	-18.06332 96	1	-11.53066 18	1	-7.93691 08	0
-0.5	2	-9.22577 01	1	-38.04304 14	1	-21.19647 63	1	-13.44480 16	1	-9.19384 23	0
-0.4	2	-10.12474 66	1	-41.55452 23	1	-23.03980 08	1	-14.53969 62	1	-9.89012 33	0
-0.3	2	-10.12990 95	1	-41.41034 69	1	-22.86410 91	1	-14.36581 45	1	-9.72722 55	0
-0.2	2	-8.81674 89	1	-35.91184 61	1	-19.75184 43	1	-12.35941 03	1	-8.33202 49	0
-0.1	2	-5.65499 67	1	-22.94341 60	1	-12.56359 73	1	-7.82248 20	0	-52.43884 80	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	2	8.97847 29	1	36.39531 91	1	19.94151 99	1	12.44590 10	1	8.38119 45	0
0.2	2	22.22028 70	2	8.97847 29	1	49.01569 74	1	30.46510 53	1	20.41875 88	0
0.3	2	40.90217 55	2	16.48813 21	2	8.97847 29	2	5.56524 38	1	37.19027 66	0
0.4	3	6.64184 90	2	26.71882 40	2	14.51804 61	2	8.97847 27	2	5.98560 13	0
0.5	3	10.04259 69	2	40.32331 58	2	21.86751 89	2	13.49624 59	2	8.97847 29	0
0.6	3	14.48825 90	3	5.80718 60	2	31.43589 78	2	19.36564 65	2	12.85843 63	0
0.7	3	20.20915 63	3	8.08691 71	3	4.37029 25	2	26.87602 21	2	17.81350 08	0
0.8	3	27.47503 54	3	10.97733 44	3	5.92288 78	2	36.36482 83	2	24.06262 64	0
0.9	4	3.66005 68	3	14.60170 50	3	7.86656 01	3	4.82241 49	2	31.85988 05	0
1.0	4	4.79514 92	3	19.10311 87	3	10.27688 01	3	6.29079 64	3	4.14990 63	0

b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-10.33333 33	0	-8.71428 57	0	-7.50000 00	0	-6.55555 56	0	-5.80000 00	0
-0.9	0	-22.01871 67	0	-17.35445 68	0	-14.06366 65	0	-11.65007 85	0	-9.82424 93	0
-0.8	0	-34.28993 52	0	-26.36280 84	0	-20.85749 57	0	-16.88461 56	0	-13.92838 14	0
-0.7	0	-46.43075 26	0	-35.20951 10	0	-27.47857 46	0	-21.94590 62	0	-17.86433 65	0
-0.6	0	-57.46824 01	0	-43.18026 83	0	-33.38768 64	0	-26.41768 72	0	-21.30475 75	0
-0.5	1	-6.61211 92	0	-49.33987 85	0	-37.88282 91	0	-29.76101 61	0	-23.82809 42	0
-0.4	1	-7.07409 93	0	-52.49051 16	0	-40.06895 88	0	-31.29182 40	0	-24.90163 57	0
-0.3	1	-6.92440 32	0	-51.12411 83	0	-38.82351 32	0	-30.15538 09	0	-23.86223 97	0
-0.2	0	-59.03470 32	0	-43.36827 70	0	-32.75725 65	0	-25.29733 07	0	-19.89450 56	0
-0.1	0	-36.91792 54	0	-26.92475 88	0	-20.16990 87	0	-15.43091 81	0	-12.00610 93	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	0	59.42345 76	0	43.77339 86	0	33.23167 37	0	25.86261 76	0	20.55687 35	0
0.2	1	14.39939 30	1	10.54251 05	1	7.94856 75	0	61.38199 82	0	48.36866 97	0
0.3	1	26.14322 71	1	19.07464 74	1	14.32753 74	1	11.01935 92	1	8.64499 87	0
0.4	1	41.96985 23	1	30.54020 99	1	22.87470 20	1	17.54020 29	1	13.71698 51	0
0.5	2	6.28171 10	1	45.60547 46	1	34.07693 26	1	26.06468 29	1	20.33008 22	0
0.6	2	8.97847 29	2	6.50505 03	1	48.50335 39	1	37.01757 18	1	28.80737 32	0
0.7	2	12.41567 33	2	8.97847 29	2	6.68164 97	1	50.89262 87	1	39.52386 07	0
0.8	2	16.74264 33	2	12.08643 37	2	8.97847 29	2	6.82616 84	2	5.29132 35	0
0.9	2	22.13242 59	2	15.95115 91	2	11.82960 49	2	8.97847 29	2	6.94753 87	0
1.0	2	28.78487 17	2	20.71363 18	2	15.33734 29	2	11.62211 05	2	8.97847 29	0

$x=6.9$

$10^{-t} {}_1F_1[a; b; x]$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	1	-6.800000 00	0	-33.500000 00	0	-22.000000 00	0	-16.250000 00	0	-12.800000 00	0
-0.9	1	-24.94385 36	1	-10.80960 19	0	-63.38163 49	0	-42.34655 00	0	-30.51694 19	0
-0.8	1	-44.76791 85	1	-18.90476 14	1	-10.79819 84	1	-7.02790 55	0	-49.34854 10	0
-0.7	2	-6.51576 85	1	-27.17754 83	1	-15.32622 26	1	-9.84472 16	1	-6.82086 33	1
-0.6	2	-8.45235 34	1	-34.98059 21	1	-19.56641 32	1	-12.46269 32	1	-8.55999 45	1
-0.5	2	-10.06950 16	1	-41.43563 29	1	-23.03907 46	1	-14.58383 39	1	-9.95285 64	1
-0.4	2	-11.07983 40	1	-45.38556 87	1	-25.11530 91	1	-15.81936 47	1	-10.74055 13	1
-0.3	2	-11.11149 20	1	-45.33918 80	1	-24.98787 20	1	-15.67223 75	1	-10.59338 60	1
-0.2	2	-9.69186 78	1	-39.40769 60	1	-21.63775 08	1	-13.51712 36	1	-9.09796 56	1
-0.1	2	-6.22894 96	1	-25.23204 67	1	-13.79596 14	1	-8.57757 02	0	-57.42483 64	0
0.0	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0
0.1	2	9.92274 72	1	40.14829 03	1	21.95565 97	1	13.67580 81	1	9.19051 87	1
0.2	2	24.59971 44	2	9.92274 71	2	5.40757 02	1	33.55015 07	1	22.44564 46	1
0.3	3	4.53553 10	2	18.25284 03	2	9.92274 71	2	6.14011 75	1	40.96169 29	1
0.4	3	7.37633 79	2	29.62553 63	2	16.07126 58	2	9.92274 72	2	6.60419 82	2
0.5	3	11.16972 15	3	4.47780 79	2	24.24480 49	2	14.93960 02	2	9.92274 72	2
0.6	3	16.13746 55	3	6.45819 42	2	34.90563 13	2	21.46959 24	2	14.23311 40	2
0.7	3	22.54087 82	3	9.00623 93	3	4.85967 88	2	29.83989 37	2	19.74764 86	2
0.8	3	30.68648 37	3	12.24204 30	3	6.59535 07	2	40.43268 18	2	26.71405 69	2
0.9	4	4.09323 98	3	16.30573 39	3	8.77159 78	3	5.36926 88	2	35.42020 65	2
1.0	4	5.36954 67	3	21.36021 45	3	11.47435 95	3	7.01356 15	3	4.61994 82	2
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-10.500000 00	0	-8.85714 29	0	-7.62500 00	0	-6.66666 67	0	-5.900000 00	0
-0.9	0	-23.13212 32	0	-18.18895 85	0	-14.70841 72	0	-12.16075 08	0	-10.23726 51	0
-0.8	0	-36.46491 04	0	-27.96874 60	0	-22.07887 08	0	-17.83619 46	0	-14.68494 13	0
-0.7	0	-49.72388 86	0	-37.62411 06	0	-29.30145 93	0	-23.35515 34	0	-18.97572 05	0
-0.6	0	-61.84988 22	0	-46.37824 56	0	-35.79036 93	0	-28.26583 54	0	-22.75464 43	0
-0.5	1	-7.14407 76	0	-53.20881 56	0	-40.77901 74	0	-31.98033 47	0	-25.56233 28	0
-0.4	1	-7.66845 92	0	-56.80078 75	0	-43.28586 67	0	-33.74928 84	0	-26.81584 54	0
-0.3	1	-7.52828 76	0	-55.49250 94	0	-42.07540 24	0	-32.63299 40	0	-25.78689 72	0
-0.2	1	-6.43632 24	0	-47.21425 18	0	-35.61371 96	0	-27.46858 17	0	-21.57714 02	0
-0.1	0	-40.37973 65	0	-29.41834 82	0	-22.01811 23	0	-16.83281 11	0	-13.09018 41	0
0.0	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0
0.1	1	6.50228 83	0	47.79228 63	0	36.19953 49	0	28.10543 32	0	22.28467 99	0
0.2	1	15.79948 71	1	11.54577 63	1	8.68824 02	1	6.69623 90	0	52.66038 61	0
0.3	1	28.74439 63	1	20.93572 37	1	15.69752 13	1	12.05131 11	1	9.43738 26	1
0.4	1	46.23035 46	1	33.58409 34	1	25.11210 01	1	19.22304 56	1	15.00721 87	1
0.5	2	6.93118 25	1	50.23919 29	1	37.47819 75	1	28.61932 28	1	22.28593 81	1
0.6	2	9.92274 71	2	7.17786 72	2	5.34354 47	1	40.71703 11	1	31.63588 19	1
0.7	2	13.74259 63	2	9.92274 72	2	7.37297 28	2	5.60714 55	1	43.47833 82	1
0.8	2	18.55948 21	2	13.37778 48	2	9.92274 71	2	7.53267 31	2	5.83013 25	2
0.9	2	24.56925 64	2	17.68116 94	2	13.09315 88	2	9.92274 71	2	7.66682 43	2
1.0	2	31.99845 30	2	22.99253 63	2	16.99991 79	2	12.86316 81	2	9.92274 72	2

$$10^{-t} {}_1F_1[a; b; x]$$

$$x=7.0$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	1	-6.900000 00	0	-34.000000 00	0	-22.333333 33	0	-16.500000 00	0	-13.000000 00	0
-0.9	1	-26.628887 95	1	-11.50021 64	1	-6.721111 28	0	-44.76741 06	0	-32.16938 73	1
-0.8	1	-48.28345 49	1	-20.33157 98	1	-11.58093 15	1	-7.51697 57	0	-52.64502 73	1
-0.7	2	-7.06530 95	1	-29.39718 20	1	-16.53757 64	1	-10.59739 90	1	-7.32517 81	1
-0.6	2	-9.19980 13	1	-37.98933 30	1	-21.20251 83	1	-13.47543 09	1	-9.23583 79	1
-0.5	2	-10.99295 13	1	-45.14264 66	1	-25.04910 92	1	-15.82430 26	1	-10.77808 39	1
-0.4	2	-12.12709 06	1	-49.57964 91	1	-27.38387 28	1	-17.21582 70	1	-11.66711 03	1
-0.3	2	-12.18966 10	1	-49.64796 29	1	-27.31341 12	1	-17.10056 83	1	-11.53890 44	1
-0.2	2	-10.65467 11	1	-43.24803 19	1	-23.70637 73	1	-14.78509 10	1	-9.93558 67	1
-0.1	2	-6.86139 84	1	-27.75021 45	1	-15.14992 81	1	-9.40594 47	0	-62.88670 32	2
0.0	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0
0.1	2	10.96633 15	1	44.29007 07	1	24.17531 13	1	15.02928 74	1	10.07989 86	1
0.2	2	27.23307 36	2	10.96633 15	2	5.96600 61	1	36.95014 36	1	24.67634 46	1
0.3	2	50.29038 28	2	20.20583 31	2	10.96633 15	1	67.74578 37	1	45.11823 13	1
0.4	2	81.91389 98	2	32.84668 23	2	17.79015 39	2	10.96633 15	2	7.28692 95	2
0.5	3	12.42208 88	2	49.72117 98	2	26.87915 06	2	16.53688 54	2	10.96633 14	2
0.6	3	17.97222 81	2	71.81484 65	2	38.75549 59	2	23.80094 95	2	15.75436 75	2
0.7	3	25.13812 95	3	10.02890 22	2	54.03361 58	2	33.12828 99	2	21.89077 28	2
0.8	3	34.26793 28	3	13.65062 25	2	73.43337 64	2	44.95152 89	2	29.65563 89	2
0.9	4	4.57689 87	3	18.20586 21	3	9.77948 65	2	59.77486 65	2	39.37497 93	2
1.0	4	6.01161 31	3	23.87998 11	3	12.80948 80	2	78.18382 70	2	51.42690 21	2
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-10.66666 67	0	-9.000000 00	0	-7.750000 00	0	-6.77777 78	0	-6.000000 00	0
-0.9	0	-24.32038 42	0	-19.07709 56	0	-15.39270 64	0	-12.70124 65	0	-10.67321 11	0
-0.8	0	-38.80355 50	0	-29.69174 19	0	-23.38637 78	0	-18.85262 08	0	-15.49126 54	0
-0.7	0	-53.27904 29	0	-40.22578 87	0	-31.26176 04	0	-24.86767 88	0	-20.16622 14	0
-0.6	0	-66.59411 44	0	-49.83469 34	0	-38.38260 12	0	-30.25621 07	0	-24.31330 64	0
-0.5	0	-77.21472 76	0	-57.40115 84	0	-43.91201 36	0	-34.37706 90	0	-27.43205 03	0
-0.4	1	-8.31498 75	0	-61.48185 13	0	-46.77388 74	0	-36.40957 52	0	-28.88470 91	0
-0.3	1	-8.18647 83	0	-60.24636 03	0	-45.60874 62	0	-35.32087 58	0	-27.87166 53	0
-0.2	0	-70.18163 63	0	-51.40749 47	0	-38.72341 98	0	-29.82877 38	0	-23.40345 53	0
-0.1	0	-44.16638 13	0	-32.14191 55	0	-24.03381 30	0	-18.35951 80	0	-14.26905 49	0
0.0	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0
0.1	0	71.16749 79	0	52.19626 30	0	39.44720 74	0	30.55626 47	0	24.17009 97	0
0.2	1	17.33822 96	1	12.64686 76	1	9.49891 55	0	73.07004 22	0	57.35116 07	0
0.3	1	31.60733 14	1	22.98129 59	1	17.20127 14	1	13.18249 03	1	10.30478 66	1
0.4	1	50.92623 47	1	36.93452 36	1	27.57152 55	1	21.07041 77	1	16.42171 38	1
0.5	2	7.64800 47	1	55.34664 95	1	41.22224 34	1	31.42771 92	1	24.43325 38	1
0.6	2	10.96633 17	2	7.92047 09	1	58.87200 75	1	44.78957 87	1	34.74561 15	1
0.7	2	15.21097 37	2	10.96633 17	2	8.13601 68	2	6.17802 13	1	47.83188 33	1
0.8	2	20.57254 62	2	14.80677 35	2	10.96633 15	2	8.31248 87	2	6.42409 84	2
0.9	2	27.27261 28	2	19.59796 15	2	14.49136 23	2	10.96633 17	2	8.46076 13	2
1.0	2	35.56782 04	2	25.52056 43	2	18.84192 73	2	14.23645 40	2	10.96633 16	2

$x=7.1$ $10^{-t} {}_1F_1[a; b; x]$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	1	-7.00000 00	0	-34.50000 00	0	-22.66666 67	0	-16.75000 00	0	-13.20000 00	0
-0.9	1	-28.45248 59	1	-12.24576 02	1	-7.13345 97	0	-47.36727 77	0	-33.93928 56	0
-0.8	2	-5.21051 93	1	-21.87956 02	1	-12.42839 81	1	-8.04539 43	0	-56.19923 80	0
-0.7	2	-7.66435 49	1	-31.81238 97	1	-17.85325 16	1	-11.41338 50	1	-7.87089 04	1
-0.6	2	-10.01639 34	1	-41.27082 74	1	-22.98389 27	1	-14.57619 10	1	-9.96914 97	1
-0.5	2	-12.00381 32	1	-49.19398 38	1	-27.24226 69	1	-17.17556 97	1	-11.67554 40	1
-0.4	2	-13.27555 43	2	-5.41718 51	1	-29.86388 27	1	-18.74004 83	1	-12.67684 32	1
-0.3	2	-13.37403 12	2	-5.43739 68	1	-29.86025 62	1	-18.66243 97	1	-12.57124 55	1
-0.2	2	-11.71401 55	1	-47.46717 64	1	-25.97567 96	1	-16.17399 84	1	-10.85173 84	1
-0.1	2	-7.55833 47	1	-30.52111 82	1	-16.63762 29	1	-10.31481 55	1	-6.88706 26	1
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	2	12.11967 08	1	48.86103 25	1	26.62153 97	1	16.51884 24	1	11.05732 82	1
0.2	2	30.14740 76	2	12.11967 07	2	6.58232 26	1	40.69737 81	1	27.13149 17	1
0.3	3	5.57593 45	2	22.36716 10	2	12.11967 07	2	7.47481 55	1	49.69948 43	1
0.4	3	9.09579 52	2	36.41615 42	2	19.69236 32	2	12.11967 07	2	8.04045 73	2
0.5	3	13.81351 78	3	5.52058 63	2	29.79825 15	2	18.30448 72	2	12.11967 07	2
0.6	3	20.01324 90	3	7.98502 81	3	4.30269 20	2	26.38421 02	2	17.43779 91	2
0.7	3	28.03085 74	3	11.16642 67	3	6.00732 49	2	36.77656 34	2	24.26537 38	2
0.8	4	3.82615 56	3	15.21928 60	3	8.17526 34	3	4.99710 46	2	32.91897 23	2
0.9	4	5.11684 59	3	20.32441 95	3	10.90180 77	3	6.65390 07	3	4.37676 24	3
1.0	4	6.72921 91	3	26.69263 83	3	14.29792 93	3	8.71449 27	3	5.72399 73	3
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-10.83333 33	0	-9.14285 71	0	-7.87500 00	0	-6.88888 89	0	-6.10000 00	0
-0.9	0	-25.58966 74	0	-20.02320 86	0	-16.11967 67	0	-13.27389 39	0	-11.13384 51	0
-0.8	0	-41.31966 63	0	-31.54149 53	0	-24.78703 21	0	-19.93908 46	0	-16.35126 81	0
-0.7	0	-57.11877 95	0	-43.03039 17	0	-33.37092 48	0	-26.49194 20	0	-21.44221 35	0
-0.6	1	-7.17327 88	0	-53.57195 77	0	-41.18050 88	0	-32.40072 08	0	-25.98970 52	0
-0.5	1	-8.34837 24	0	-61.94541 52	0	-47.30237 16	0	-36.96637 88	0	-29.44864 29	0
-0.4	1	-9.01843 88	1	-6.65669 94	0	-50.55699 81	0	-39.29035 14	0	-31.12149 57	0
-0.3	1	-8.90400 99	1	-6.54208 48	0	-49.44888 64	0	-38.23768 25	0	-30.13054 10	0
-0.2	1	-7.65361 01	0	-55.98026 68	0	-42.10955 08	0	-32.39496 46	0	-25.38623 69	0
-0.1	0	-48.30894 82	0	-35.11716 97	0	-26.23260 43	0	-20.02249 61	0	-15.55130 11	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	1	7.79110 38	0	57.02278 60	0	43.00154 93	0	33.23481 38	0	26.22786 31	0
0.2	1	19.02947 73	1	13.85543 45	1	10.38750 77	1	7.97556 29	0	62.47885 14	0
0.3	1	34.75854 50	1	25.22981 53	1	18.85199 73	1	14.42257 06	1	11.25443 12	1
0.4	2	5.61021 68	1	40.62256 28	1	30.27520 18	1	23.09857 99	1	17.97260 05	1
0.5	2	8.43917 51	2	6.09764 65	1	45.34382 60	1	34.51528 56	1	26.79096 90	1
0.6	2	12.11967 07	2	8.74011 30	2	6.48648 28	1	49.27306 09	1	38.16474 99	1
0.7	2	16.83586 41	2	12.11967 07	2	8.97823 62	2	6.80733 24	2	5.26249 98	2
0.8	2	22.80296 28	2	16.38804 39	2	12.11967 07	2	9.17323 57	2	7.07888 80	2
0.9	2	30.27150 28	2	21.72163 16	2	16.03854 04	2	12.11967 07	2	9.33711 17	2
1.0	2	39.53212 71	2	28.32481 56	2	20.88268 09	2	15.75603 35	2	12.11967 08	2

$$10^{-t} {}_1F_1[a; b; x]$$

$$x = 7.2$$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	1	-7.100000	00	0	-35.000000	00	0	-23.000000	00	0	-17.000000	00	0	-13.400000	00
-0.9	1	-30.42714	60	1	-13.05109	51	1	-7.57776	34	0	-50.16147	24	0	-35.83651	96
-0.8	2	-5.62612	28	1	-23.55964	39	1	-13.34635	01	1	-8.61660	07	0	-60.03333	06
-0.7	2	-8.31753	02	1	-34.44116	40	1	-19.28269	69	1	-12.29832	35	1	-8.46163	22
-0.6	2	-10.90870	26	1	-44.85061	52	1	-24.92393	42	1	-15.77296	87	1	-10.76507	02
-0.5	2	-13.11053	28	2	-5.36224	31	1	-29.63573	08	1	-18.64787	84	1	-12.65181	54
-0.4	2	-14.53515	69	2	-5.92006	96	1	-32.57550	69	1	-20.40403	95	1	-13.77745	62
-0.3	2	-14.67518	23	2	-5.95581	95	1	-32.64984	80	1	-20.37060	85	1	-13.69858	69
-0.2	2	-12.87965	69	2	-5.21028	95	1	-28.46539	42	1	-17.69557	78	1	-11.85393	47
-0.1	2	-8.32636	57	1	-33.57031	25	1	-18.27238	89	1	-11.31210	67	1	-7.54273	15
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	2	13.39430	76	2	5.39057	44	1	29.31757	69	1	18.15824	74	1	12.13160	75
0.2	2	33.37263	68	2	13.39430	75	2	7.26253	19	1	44.82747	06	1	29.83381	81
0.3	3	6.18197	36	2	24.75900	38	2	13.39430	76	2	8.24766	20	2	5.47490	31
0.4	3	10.09929	62	2	40.37148	02	2	21.79740	89	2	13.39430	76	2	8.87213	78
0.5	3	15.35934	36	3	6.12911	53	2	33.03277	64	2	20.26053	36	2	13.39430	76
0.6	3	22.28352	52	3	8.87766	96	3	4.77658	30	2	29.24649	96	2	19.30066	56
0.7	3	31.25236	05	3	12.43161	16	3	6.67820	88	2	40.82390	39	2	26.89636	96
0.8	4	4.27142	87	3	16.96606	03	3	9.10047	43	3	5.55463	54	2	36.53907	53
0.9	4	5.71955	67	3	22.68625	89	3	12.15143	78	3	7.40610	01	3	4.86463	25
1.0	4	7.53114	09	3	29.83184	37	3	15.95710	92	3	9.71215	60	3	6.37038	14
b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	0	-11.000000	00	0	-9.28571	43	0	-8.000000	00	0	-7.000000	00	0	-6.200000	00
-0.9	0	-26.94667	24	0	-21.03200	89	0	-16.89273	59	0	-13.88121	43	0	-11.62106	81
-0.8	0	-44.02824	94	0	-33.52854	73	0	-26.28844	84	0	-21.10121	47	0	-17.26918	86
-0.7	0	-61.26766	91	0	-46.05515	86	0	-35.64139	50	0	-28.23713	04	0	-22.81060	73
-0.6	1	-7.73006	40	0	-57.61438	41	0	-44.20164	39	0	-34.71230	99	0	-27.79357	00
-0.5	1	-9.02921	76	1	-6.68726	82	0	-50.97248	59	0	-39.76476	48	0	-31.62450	02
-0.4	1	-9.78401	08	1	-7.20925	72	0	-54.66135	80	0	-42.41087	30	0	-33.54064	84
-0.3	1	-9.68639	28	1	-7.10546	01	0	-53.62350	24	0	-41.40376	92	0	-32.57877	85
-0.2	1	-8.34772	03	0	-60.96788	50	0	-45.79747	50	0	-35.18578	81	0	-27.53943	80
-0.1	0	-52.84154	55	0	-38.36790	22	0	-28.63155	59	0	-21.83427	61	0	-16.94629	77
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	1	8.53125	83	0	62.31299	66	0	46.89202	85	0	36.16267	51	0	28.47409	67
0.2	1	20.88848	05	1	15.18208	89	1	11.36161	12	1	8.70747	61	1	6.80849	51
0.3	1	38.22725	02	1	27.70159	16	1	20.66422	18	1	15.78217	73	1	12.29423	89
0.4	2	6.18074	23	1	44.68244	10	1	33.24758	47	1	25.32541	24	1	19.67319	75
0.5	2	9.31241	96	2	6.71822	74	1	49.88123	76	1	37.90999	58	1	29.37990	82
0.6	2	13.39430	75	2	9.64480	06	2	7.14710	39	2	5.42091	77	1	41.92431	41
0.7	2	18.63391	55	2	13.39430	76	2	9.90786	07	2	7.50108	24	2	5.79023	07
0.8	2	25.27411	78	2	18.13779	81	2	13.39430	75	2	10.12332	80	2	7.80075	13
0.9	2	33.59808	66	2	24.07444	15	2	17.75054	01	2	13.39430	75	2	10.30444	32
1.0	3	4.39348	05	2	31.43534	09	2	23.14355	00	2	17.43746	67	2	13.39430	77

$x=7.3$

$$10^{-t} {}_1F_1[a; b; x]$$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	1	-7.200000	00	0	-35.500000	00	0	-23.333333	33	0	-17.250000	00	0	-13.600000	00
-0.9	1	-32.56648	59	1	-13.92152	78	1	-8.05681	34	0	-53.16667	55	0	-37.87183	46
-0.8	2	-6.07822	91	1	-25.38378	37	1	-14.34106	44	1	-9.23434	46	1	-6.41714	39
-0.7	2	-9.02990	11	1	-37.30318	84	1	-20.83623	74	1	-13.25837	57	1	-9.10136	45
-0.6	2	-11.88393	53	1	-48.75667	06	1	-27.03730	14	1	-17.07450	19	1	-11.62921	07
-0.5	2	-14.32238	23	2	-5.84639	43	1	-32.24832	26	1	-20.25243	70	1	-13.71408	89
-0.4	2	-15.91681	57	2	-6.47084	77	1	-35.54086	38	1	-22.22095	93	1	-14.97738	38
-0.3	2	-16.10475	55	2	-6.52456	99	1	-35.70573	01	1	-22.23906	86	1	-14.92988	91
-0.2	2	-14.16234	24	2	-5.71967	47	1	-31.19721	71	1	-19.36271	28	1	-12.95041	79
-0.1	2	-9.17277	81	1	-36.92595	02	1	-20.06891	01	1	-12.40653	08	1	-8.26124	54
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	2	14.80299	92	2	5.94734	11	1	32.28904	33	1	19.96267	77	1	13.31241	98
0.2	2	36.94186	12	2	14.80299	92	2	8.01327	09	1	49.37970	15	1	32.80836	91
0.3	3	6.85352	79	2	27.40590	07	2	14.80299	93	2	9.10066	18	2	6.03149	33
0.4	3	11.21269	35	3	4.47542	20	2	24.12688	02	2	14.80299	93	2	9.79009	20
0.5	3	17.07658	39	3	6.80424	84	2	36.61669	10	2	22.42507	89	2	14.80299	92
0.6	3	24.80860	18	3	9.86922	42	3	5.30231	34	2	32.41785	44	2	21.36206	11
0.7	4	3.48396	63	3	13.83867	19	3	7.42338	16	3	4.53137	52	2	29.81135	34
0.8	4	4.76783	66	3	18.91098	78	3	10.12935	10	3	6.17386	16	2	40.55477	08
0.9	4	6.39224	44	3	25.31903	74	3	13.54269	16	3	8.24251	26	3	5.40645	40
1.0	4	8.42716	46	4	3.33350	94	3	17.80642	23	3	10.82278	33	3	7.08907	58

b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	0	-11.16666	67	0	-9.42857	14	0	-8.12500	00	0	-7.11111	11	0	-6.30000	00
-0.9	0	-28.39867	64	0	-22.10860	80	0	-17.71557	70	0	-14.52593	78	0	-12.13693	71
-0.8	0	-46.94562	87	0	-35.66434	91	0	-27.89889	16	0	-22.34511	27	0	-18.24961	92
-0.7	1	-6.57524	77	0	-49.31884	78	0	-38.08669	23	0	-30.11321	68	0	-24.27890	08
-0.6	1	-8.33355	50	0	-61.98849	38	0	-47.46510	51	0	-37.20505	03	0	-29.73546	68
-0.5	1	-9.76885	80	1	-7.22168	76	0	-54.94675	84	0	-42.79018	77	0	-33.97309	57
-0.4	1	-10.61738	57	1	-7.80982	91	0	-59.11550	33	0	-45.79212	34	0	-36.15789	35
-0.3	1	-10.53965	81	1	-7.71895	99	0	-58.16282	58	0	-44.84134	51	0	-35.23300	70
-0.2	1	-9.10603	69	1	-6.64090	06	0	-49.81492	50	0	-38.22160	01	0	-29.87828	42
-0.1	0	-57.80158	71	0	-41.92018	43	0	-31.24935	57	0	-23.80856	23	0	-18.46428	62
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	1	9.34370	86	1	6.81120	77	0	51.15097	85	0	39.36351	77	0	30.92646	00
0.2	1	22.93202	48	1	16.63849	78	1	12.42956	80	1	9.50884	53	1	7.42149	49
0.3	1	42.04563	16	1	30.41897	76	1	22.65391	50	1	17.27298	30	1	13.43290	45
0.4	2	6.80963	44	1	49.15187	13	1	36.51559	68	1	27.77057	72	1	21.53814	04
0.5	2	10.27626	84	2	7.40232	26	2	5.48766	73	1	41.64264	12	1	32.22297	39
0.6	2	14.80299	91	2	10.64337	34	2	7.87536	50	2	5.96438	77	1	46.05845	51
0.7	2	20.62355	27	2	14.80299	92	2	10.93397	72	2	8.26589	24	2	6.37129	86
0.8	2	28.01191	40	2	20.07395	86	2	14.80299	91	2	11.17205	49	2	8.59658	27
0.9	2	37.28800	62	2	26.68103	66	2	19.64489	23	2	14.80299	91	2	11.37221	74
1.0	3	4.88240	48	2	34.88543	87	2	25.64820	67	2	19.29796	65	2	14.80299	93

$$10^{-t} {}_1F_1[a; b; x]$$

$$x=7.4$$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	1	-7.300000	00	0	-36.000000	00	0	-23.666666	67	0	-17.500000	00	0	-13.800000	00
-0.9	1	-34.88539	39	1	-14.86284	87	1	-8.57365	05	0	-56.40105	05	0	-40.05692	23
-0.8	2	-6.57020	02	1	-27.36504	26	1	-15.41939	29	1	-9.90271	46	1	-6.86398	43
-0.7	2	-9.80701	59	1	-40.41999	79	1	-22.52515	75	1	-14.30026	75	1	-9.79440	65
-0.6	2	-12.94999	43	2	-5.30196	37	1	-29.34003	50	1	-18.49034	18	1	-12.56769	84
-0.5	2	-15.64954	11	2	-6.37579	51	1	-35.10066	34	1	-22.00151	24	1	-14.87022	59
-0.4	2	-17.43252	93	2	-7.07416	27	1	-38.78421	41	1	-24.20522	57	1	-16.28585	70
-0.3	2	-17.67556	03	2	-7.14860	09	1	-39.05375	37	1	-24.28316	86	1	-16.27497	05
-0.2	2	-15.57391	13	2	-6.27944	68	1	-34.19499	74	1	-21.18955	17	1	-14.15023	00
-0.1	2	-10.10560	74	1	-40.61904	55	1	-22.04334	57	1	-13.60766	51	1	-9.04872	03
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	2	16.35984	42	2	6.56183	56	1	35.56419	70	1	21.94885	32	1	14.61042	33
0.2	2	40.89170	14	2	16.35984	42	2	8.84186	74	2	5.43973	93	1	36.08273	85
0.3	3	7.59765	41	2	30.33499	94	2	16.35984	43	2	10.04214	36	2	6.64501	98
0.4	3	12.44796	03	3	4.96104	01	2	26.70466	09	2	16.35984	42	2	10.80328	60
0.5	3	18.98412	20	3	7.55323	91	2	40.58760	76	2	24.82030	90	2	16.35984	43
0.6	3	27.61685	27	3	10.97058	26	3	5.88552	63	2	35.93153	66	2	23.64310	71
0.7	4	3.88339	29	3	15.40339	64	3	8.25101	84	3	5.02942	60	2	33.04088	06
0.8	4	5.32119	25	3	21.07635	02	3	11.27342	30	3	6.86156	92	3	4.50090	89
0.9	4	7.14294	42	3	28.25353	10	3	15.09148	29	3	9.17249	84	3	6.00815	82
1.0	4	9.42820	12	4	3.72441	51	3	19.86744	89	3	12.05905	90	3	7.88811	26
b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	0	-11.33333	33	0	-9.57142	86	0	-8.25000	00	0	-7.22222	22	0	-6.40000	00
-0.9	0	-29.95358	79	0	-23.25855	45	0	-18.59220	46	0	-15.21102	07	0	-12.68367	66
-0.8	0	-50.08956	41	0	-37.96134	87	0	-29.62733	50	0	-23.67739	75	0	-19.29753	50
-0.7	1	-7.06023	61	0	-52.84187	09	0	-40.72151	77	0	-32.13103	15	0	-25.85522	61
-0.6	1	-8.98788	35	1	-6.67231	82	0	-50.99168	25	0	-39.89424	23	0	-31.82687	37
-0.5	1	-10.57258	94	1	-7.80149	99	0	-59.25178	25	0	-46.06220	02	0	-36.50908	24
-0.4	1	-11.52477	64	1	-8.46275	14	0	-63.95057	22	0	-49.45697	59	0	-38.99035	45
-0.3	1	-11.47040	78	1	-8.38719	23	0	-63.09988	29	0	-48.57464	77	0	-38.11135	51
-0.2	1	-9.93463	40	1	-7.23459	51	0	-54.19223	29	0	-41.52463	97	0	-32.41939	53
-0.1	0	-63.23012	10	0	-45.80259	06	0	-34.10646	53	0	-25.96034	38	0	-20.11645	80
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	1	10.23559	63	1	7.44696	59	0	55.81387	68	0	42.86328	99	0	33.60429	38
0.2	1	25.17858	48	1	18.23748	97	1	13.60054	13	1	10.38636	68	1	8.09187	15
0.3	1	46.24915	21	1	33.40657	93	1	24.83863	81	1	18.90781	02	1	14.67997	02
0.4	2	7.50288	65	2	5.40724	08	1	40.10887	38	1	30.45569	62	1	23.58350	76
0.5	2	11.34013	90	2	8.15645	66	2	6.03766	26	1	45.74711	96	1	35.34535	24
0.6	2	16.35984	41	2	11.74559	16	2	8.67821	63	2	6.56277	96	1	50.60476	90
0.7	2	22.82515	62	2	16.35984	44	2	12.06661	79	2	9.10906	62	2	7.01112	26
0.8	2	31.04503	26	2	22.21635	22	2	16.35984	41	2	12.32967	32	2	9.47398	65
0.9	3	4.13807	68	2	29.56870	85	2	21.74099	02	2	16.35984	42	2	12.55088	02
1.0	3	5.42533	19	2	38.71201	81	2	28.42286	08	2	21.35657	44	2	16.35984	43

$x=7.5$ $10^{-t} {}_1F_1[a; b; x]$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	1	-7.40000 00	0	-36.50000 00	0	-24.00000 00	0	-17.75000 00	0	-14.00000 00	0
-0.9	1	-37.40013 66	1	-15.88137 86	1	-9.13159 01	0	-59.88438 09	0	-42.40449 86	0
-0.8	2	-7.10571 40	1	-29.51769 54	1	-16.58881 53	1	-10.62616 85	1	-7.34671 72	1
-0.7	2	-10.65495 23	1	-43.81515 65	1	-24.36179 04	1	-15.43134 29	1	-10.54546 91	1
-0.6	2	-14.11554 69	2	-5.76730 84	1	-31.84968 87	1	-20.03092 98	1	-13.58722 48	1
-0.5	2	-17.10318 59	2	-6.95477 05	1	-38.21534 56	1	-23.90852 97	1	-16.12882 18	1
-0.4	2	-19.09548 76	2	-7.73511 39	1	-42.33216 83	1	-26.37263 68	1	-17.71298 06	1
-0.3	2	-19.40169 36	2	-7.83335 72	1	-42.72230 71	1	-26.51974 69	1	-17.74459 09	1
-0.2	2	-17.12740 62	2	-6.89463 87	1	-37.48495 47	1	-23.19163 14	1	-15.46329 08	1
-0.1	2	-11.13371 62	1	-44.68376 73	1	-24.21348 07	1	-14.92603 98	1	-9.91187 29	1
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	2	18.08042 41	2	7.24005 56	1	39.17420 19	1	24.13519 49	1	16.03734 88	1
0.2	3	4.52626 70	2	18.08042 42	2	9.75641 15	2	5.99283 27	1	39.68733 07	1
0.3	3	8.42216 74	2	33.57633 30	2	18.08042 42	2	11.08130 29	2	7.32132 92	2
0.4	3	13.81836 92	3	5.49909 89	2	29.55717 34	2	18.08042 42	2	11.92161 90	2
0.5	3	21.10291 17	3	8.38412 78	3	4.49871 77	2	27.47076 70	2	18.08042 42	2
0.6	3	30.73979 22	3	12.19382 89	3	6.53247 72	2	39.82437 99	2	26.16717 02	2
0.7	4	4.32809 22	3	17.14332 03	3	9.17018 97	3	5.58188 00	2	36.61878 49	2
0.8	4	5.93796 54	3	23.48691 75	3	12.54549 41	3	7.62528 66	3	4.99497 25	3
0.9	4	7.98060 59	3	31.52398 52	3	16.81550 12	3	10.20645 21	3	6.67632 54	3
1.0	4	10.54641 42	4	4.16055 29	3	22.16419 89	3	13.43508 19	3	8.77641 15	3
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-11.50000 00	0	-9.71428 57	0	-8.37500 00	0	-7.33333 33	0	-6.50000 00	0
-0.9	0	-31.62000 18	0	-24.48787 18	0	-19.52696 14	0	-15.93966 70	0	-13.26369 52	0
-0.8	0	-53.47938 13	0	-40.43307 66	0	-31.48352 31	0	-25.10524 88	0	-20.41832 69	0
-0.7	1	-7.58490 85	0	-56.64644 46	0	-43.56185 39	0	-34.30233 94	0	-27.54840 80	0
-0.6	1	-9.69755 72	1	-7.18499 42	0	-54.80400 63	0	-42.79652 69	0	-34.08026 12	0
-0.5	1	-11.44619 59	1	-8.43074 20	0	-63.91654 34	0	-49.60209 49	0	-39.24839 89	0
-0.4	1	-12.51297 63	1	-9.17276 14	1	-6.92005 42	0	-53.43036 43	0	-42.05668 04	0
-0.3	1	-12.48586 90	1	-9.11519 21	1	-6.84707 55	0	-52.63012 95	0	-41.23358 83	0
-0.2	1	-10.84016 92	1	-7.88250 56	0	-58.96257 36	0	-45.11920 84	0	-35.18091 24	0
-0.1	1	-6.91721 86	0	-50.04644 13	0	-37.22528 90	0	-28.30601 81	0	-21.91504 41	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	1	11.21477 42	1	8.14402 60	0	60.91965 81	0	46.69043 67	0	36.52877 95	0
0.2	1	27.64849 69	1	19.99317 17	1	14.88459 72	1	11.34738 92	1	8.82509 25	1
0.3	1	50.87688 63	1	36.69148 07	1	27.23770 22	1	20.70074 61	1	16.04590 91	1
0.4	2	8.26710 99	2	5.94898 39	1	44.06003 76	1	33.40454 85	1	25.82696 34	1
0.5	2	12.51442 89	2	8.98782 54	2	6.64323 30	1	50.26074 69	1	38.77474 46	1
0.6	2	18.08042 41	2	12.96223 12	2	9.56332 41	2	7.22167 29	2	5.56046 44	2
0.7	2	25.26127 21	2	18.08042 42	2	13.31685 97	2	10.03866 01	2	7.71567 42	2
0.8	2	34.40523 27	2	24.58691 24	2	18.08042 41	2	13.60750 74	2	10.44135 14	2
0.9	3	4.59201 60	2	32.76767 65	2	24.06028 72	2	18.08042 42	2	13.85196 53	2
1.0	3	6.02819 33	3	4.29559 86	2	31.49653 34	2	23.63435 41	2	18.08042 42	2

$$10^{-t} {}_1F_1[a; b; x]$$

$$x = 7.6$$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	1	-7.500000	00	0	-37.000000	00	0	-24.333333	33	0	-18.000000	00	0	-14.200000	00
-0.9	1	-40.12848	98	1	-16.98401	67	1	-9.73424	72	0	-63.63821	51	0	-44.92840	24
-0.8	2	-7.68879	47	1	-31.85734	48	1	-17.85749	83	1	-11.40956	88	1	-7.86846	20
-0.7	2	-11.58036	77	1	-47.51444	90	1	-26.35961	96	1	-16.65962	09	1	-11.35969	15
-0.6	2	-15.39009	84	2	-6.27537	89	1	-34.58547	34	1	-21.70768	10	1	-14.69509	76
-0.5	2	-18.69558	90	2	-7.58806	42	1	-41.61712	43	1	-25.98818	35	1	-17.49927	41
-0.4	2	-20.92018	95	2	-8.45930	03	1	-46.21391	78	1	-28.74050	52	1	-19.26981	60
-0.3	2	-21.29866	88	2	-8.58482	40	1	-46.74256	73	1	-28.96727	64	1	-19.35054	25
-0.2	2	-18.83719	61	2	-7.57078	96	1	-41.09591	55	1	-25.38601	76	1	-16.90048	39
-0.1	2	-12.26687	79	1	-49.15776	02	1	-26.59888	90	1	-16.37323	23	1	-10.85808	10
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	2	19.98195	88	2	7.98862	24	1	43.15343	02	1	26.54199	98	1	17.60610	93
0.2	3	5.00995	86	2	19.98195	88	2	10.76583	50	2	6.60252	03	1	43.65564	59
0.3	3	9.33572	19	2	37.16312	65	2	19.98195	89	2	12.22829	24	2	8.06687	03
0.4	3	15.33863	29	3	6.09524	25	2	32.71364	80	2	19.98195	90	2	13.15602	09
0.5	3	23.45620	21	3	9.30582	67	3	4.98615	22	2	30.40360	53	2	19.98195	88
0.6	4	3.42124	21	3	13.55237	03	3	7.25009	95	3	4.41371	70	2	28.96009	93
0.7	4	4.82315	16	3	19.07791	78	3	10.19095	92	3	6.19465	30	2	40.58253	26
0.8	4	6.62535	35	3	26.17022	44	3	13.95978	36	3	8.47336	68	3	5.54295	38
0.9	4	8.91519	68	4	3.51685	02	3	18.73440	95	3	11.35591	61	3	7.41825	55
1.0	4	11.79536	34	4	4.64710	25	3	24.72338	57	3	14.96652	31	3	9.76387	65
b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	0	-11.66666	67	0	-9.85714	29	0	-8.50000	00	0	-7.44444	45	0	-6.60000	00
-0.9	0	-33.40725	86	0	-25.80310	04	0	-20.52455	80	0	-16.71534	70	0	-13.87959	98
-0.8	0	-57.13611	42	0	-43.09424	17	0	-33.47803	88	0	-26.63645	59	0	-21.61783	93
-0.7	1	-8.15272	68	0	-60.75675	42	0	-46.62508	21	0	-36.63991	93	0	-29.36802	34
-0.6	1	-10.46749	33	1	-7.74030	93	0	-58.92671	91	0	-45.93000	33	0	-36.50917	83
-0.5	1	-12.39599	58	1	-9.11381	95	1	-6.89726	38	0	-53.43305	58	0	-42.20838	61
-0.4	1	-13.58941	50	1	-9.94503	43	1	-7.49024	97	0	-57.73947	47	0	-45.37718	38
-0.3	1	-13.59395	47	1	-9.90846	64	1	-7.43148	66	0	-57.03666	70	0	-44.62126	19
-0.2	1	-11.82993	90	1	-8.58970	58	1	-6.41622	37	0	-49.03186	18	0	-38.18264	17
-0.1	1	-7.56772	01	0	-54.68606	94	0	-40.63036	26	0	-30.86352	37	0	-23.87341	11
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	1	12.28987	71	1	8.90837	71	1	6.65110	49	0	50.87614	72	0	39.72311	94
0.2	1	30.36414	54	1	21.92105	67	1	16.29279	48	1	12.39997	84	1	9.62715	35
0.3	2	5.59718	90	1	40.30349	68	1	29.87234	55	1	22.66726	80	1	17.54221	67
0.4	2	9.10959	60	2	6.54546	19	1	48.40500	13	1	36.64328	58	1	28.28791	77
0.5	2	13.81061	86	2	9.90436	71	2	7.31002	42	2	5.52246	04	1	42.54161	70
0.6	2	19.98195	87	2	14.30519	00	2	10.53914	43	2	7.94721	64	2	6.11036	47
0.7	2	27.95684	11	2	19.98195	89	2	14.69693	28	2	11.06356	06	2	8.49153	46
0.8	2	38.12768	06	2	27.20990	18	2	19.98195	88	2	15.01806	11	2	11.50793	05
0.9	3	5.09547	12	2	36.31140	12	2	26.62651	49	2	19.98195	88	2	15.28820	75
1.0	3	6.69756	89	3	4.76626	80	2	34.90135	78	2	26.15460	60	2	19.98195	90

$x=7.7$

$$10^{-t} {}_1F_1[a; b; x]$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	1	-7.600000 00	0	-37.500000 00	0	-24.666666 67	0	-18.250000 00	0	-14.400000 00	0
-0.9	1	-43.08988 42	1	-18.17829 59	1	-10.38556 47	1	-6.76860 29	0	-47.64369 47	0
-0.8	2	-8.32384 61	1	-34.40104 48	1	-19.23435 86	1	-12.25821 96	1	-8.43261 73	1
-0.7	2	-12.59055 64	2	-5.15460 97	1	-28.53338 42	1	-17.99385 99	1	-12.24268 00	1
-0.6	2	-16.78407 53	2	-6.83020 53	1	-37.56841 67	1	-23.53307 68	1	-15.89929 83	1
-0.5	2	-20.44022 71	2	-8.28087 99	1	-45.33312 62	1	-28.25656 08	1	-18.99186 08	1
-0.4	2	-22.92257 46	2	-9.25287 10	1	-50.46148 81	1	-31.32780 32	1	-20.96847 35	1
-0.3	2	-23.38355 87	2	-9.40958 19	1	-51.14877 53	1	-31.64602 38	1	-21.10575 01	1
-0.2	2	-20.71911 25	2	-8.31399 56	1	-45.05957 37	1	-27.79145 25	1	-18.47375 19	1
-0.1	2	-13.51587 25	2	-5.40825 01	1	-29.22111 46	1	-17.96197 29	1	-11.89544 87	1
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	2	22.08347 99	2	8.81484 65	1	47.53979 35	1	29.19163 17	1	19.33091 75	1
0.2	3	5.54520 32	2	22.08347 98	2	11.87999 94	2	7.27461 51	1	48.02459 78	1
0.3	3	10.34790 10	3	4.11321 36	2	22.08347 98	2	13.49432 08	2	8.88875 55	2
0.4	3	17.02505 91	3	6.75572 10	2	36.20642 13	2	22.08347 98	2	14.51855 85	2
0.5	3	26.06978 87	3	10.32821 48	3	5.52617 15	2	33.64885 92	2	22.08347 99	2
0.6	4	3.80736 09	3	15.06108 25	3	8.04607 80	3	4.89150 71	2	32.05048 93	2
0.7	4	5.37422 63	3	21.22881 54	3	11.32449 19	3	6.87430 18	3	4.49736 08	3
0.8	4	7.39136 56	3	29.15687 71	3	15.53208 13	3	9.41507 63	3	6.15071 20	3
0.9	4	9.95781 74	4	3.92294 73	3	20.87006 31	3	12.63370 80	3	8.24204 72	3
1.0	4	13.19016 26	4	5.18983 22	3	27.57472 78	3	16.67079 96	3	10.86150 51	3
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-11.83333 33	0	-10.00000 00	0	-8.62500 00	0	-7.55555 56	0	-6.70000 00	0
-0.9	0	-35.32551 48	0	-27.21134 28	0	-21.59010 45	0	-17.54182 29	0	-14.53421 18	0
-0.8	0	-61.08265 64	0	-45.96084 03	0	-35.62237 81	0	-28.27947 16	0	-22.90240 64	0
-0.7	1	-8.76746 36	1	-6.51991 29	0	-49.93010 59	0	-39.15765 68	0	-31.32446 72	0
-0.6	1	-11.30305 80	1	-8.34200 49	0	-63.38665 25	0	-49.31436 12	0	-39.12834 96	0
-0.5	1	-13.42889 12	1	-9.85554 01	1	-7.44545 14	0	-57.58033 69	0	-45.40791 11	1
-0.4	1	-14.76221 79	1	-10.78522 45	1	-8.10969 14	0	-62.41395 25	0	-48.97399 28	1
-0.3	1	-14.80332 90	1	-10.77303 88	1	-8.06752 96	0	-61.82578 22	0	-48.29788 35	1
-0.2	1	-12.91194 18	1	-9.36175 15	1	-6.98309 24	0	-53.29162 17	0	-41.44620 71	1
-0.1	1	-8.27993 96	0	-59.75911 16	0	-44.34855 63	0	-33.65248 73	0	-26.00617 04	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	1	13.47040 07	1	9.74659 84	1	7.26349 45	0	55.45461 65	0	43.21272 67	0
0.2	1	33.35017 18	1	24.03820 47	1	17.83728 38	1	13.55298 68	1	10.50462 85	1
0.3	2	6.15816 07	1	44.27544 98	1	32.76592 49	1	24.82438 08	1	19.18151 15	1
0.4	2	10.03838 83	2	7.20223 49	2	5.31833 02	1	40.20067 11	1	30.98769 66	1
0.5	2	15.24138 31	2	10.91483 69	2	8.04425 74	2	6.06839 18	1	46.67947 93	1
0.6	2	22.08347 97	2	15.78760 47	2	11.61500 47	2	8.74618 61	2	6.71519 47	2
0.7	2	30.93945 37	2	22.08347 98	2	16.22034 09	2	12.19357 04	2	9.34595 61	2
0.8	3	4.22513 14	2	30.11216 08	2	22.08347 98	2	16.57514 02	2	12.68393 07	2
0.9	3	5.65382 11	2	40.23693 24	2	29.46592 57	2	22.08347 97	2	16.87366 79	2
1.0	3	7.44075 78	3	5.28823 45	2	38.67291 31	2	28.94310 64	2	22.08348 01	2

$$10^{-t} {}_1F_1[a; b; x]$$

$$x=7.8$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	1	-7.70000 00	0	-38.00000 00	0	-25.00000 00	0	-18.50000 00	0	-14.60000 00	0
-0.9	1	-46.30555 48	1	-19.47244 05	1	-11.08984 28	1	-7.20534 01	0	-50.56676 47	1
-0.8	2	-9.01568 74	1	-37.16744 09	1	-20.72913 49	1	-13.17790 67	1	-9.04288 72	1
-0.7	2	-13.69351 03	2	-5.59409 87	1	-30.89920 06	1	-19.44362 56	1	-13.20055 19	1
-0.6	2	-18.30891 49	2	-7.43620 33	1	-40.82153 72	1	-25.52076 64	1	-17.20854 74	1
-0.5	2	-22.35189 97	2	-9.03892 65	1	-49.39307 98	1	-30.73127 23	1	-20.61782 22	1
-0.4	2	-25.12016 65	2	-10.12257 94	2	-5.51100 15	1	-34.15532 65	1	-22.82221 34	1
-0.3	2	-25.67515 37	2	-10.31486 58	2	-5.59785 39	1	-34.57822 74	1	-23.02438 12	1
-0.2	2	-22.79059 67	2	-9.13096 70	1	-49.41077 73	1	-30.42852 50	1	-20.19619 95	1
-0.1	2	-14.89258 91	2	-5.95036 86	1	-32.10387 09	1	-19.70625 89	1	-13.03287 79	1
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	2	24.40601 99	2	9.72679 93	2	5.23751 06	1	32.10873 29	1	21.22742 02	2
0.2	3	6.13748 60	2	24.40601 97	2	13.10979 26	2	8.01552 81	2	5.28348 61	2
0.3	3	11.46931 75	3	4.55240 24	2	24.40601 97	2	14.89176 41	2	9.79483 07	2
0.4	3	18.89572 46	3	7.48745 52	2	40.07126 48	2	24.40601 98	2	16.02255 56	2
0.5	3	28.97229 18	3	11.46224 14	3	6.12443 10	2	37.23975 65	2	24.40601 97	2
0.6	4	4.23665 25	3	16.73646 84	3	8.92893 06	3	5.42080 90	2	35.46997 28	3
0.7	4	5.98760 24	3	23.62002 85	3	12.58317 41	3	7.62809 34	3	4.98379 47	3
0.8	4	8.24491 23	3	32.48089 43	3	17.27992 05	3	10.46069 47	3	6.82474 02	3
0.9	4	11.12082 86	4	4.37540 60	3	23.24675 26	3	14.05405 93	3	9.15668 39	3
1.0	4	14.74765 77	4	5.79516 47	3	30.75128 44	3	18.56726 85	3	12.08150 84	3

b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-12.00000 00	0	-10.14285 71	0	-8.75000 00	0	-7.66666 67	0	-6.80000 00	0
-0.9	0	-37.38581 12	0	-28.72031 41	0	-22.72914 59	0	-18.42317 16	0	-15.23058 74	0
-0.8	1	-6.53439 34	0	-49.05026 81	0	-37.92903 02	0	-30.04347 08	0	-24.27889 62	0
-0.7	1	-9.43323 11	1	-7.00022 42	0	-53.49748 83	0	-41.87064 11	0	-33.42902 43	0
-0.6	1	-12.21010 71	1	-8.99415 96	1	-6.82130 31	0	-52.97102 35	0	-41.95378 14	0
-0.5	1	-14.55242 34	1	-10.66115 13	1	-8.03997 35	0	-62.07144 59	0	-48.86750 54	0
-0.4	1	-16.04027 29	1	-11.69951 10	1	-8.78279 80	1	-6.74861 26	0	-52.87121 43	0
-0.3	1	-16.12347 93	1	-11.71549 89	1	-8.75991 29	1	-6.70318 91	0	-52.28909 40	0
-0.2	1	-14.09494 57	1	-10.20472 76	1	-7.60120 69	0	-57.93021 23	0	-44.99522 18	0
-0.1	1	-9.05982 83	1	-6.53068 33	0	-48.40930 17	0	-36.69438 54	0	-28.32929 38	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	1	14.76678 75	1	10.66591 83	1	7.93428 19	0	60.46334 19	0	47.02544 39	0
0.2	1	36.63370 26	1	26.36337 80	1	19.53141 38	1	14.81613 21	1	11.46472 69	1
0.3	2	6.77583 18	1	48.64347 14	1	35.94412 90	1	27.19077 03	1	20.97764 53	1
0.4	2	11.06235 82	2	7.92542 98	2	5.84384 65	1	44.10834 09	1	33.94973 61	1
0.5	2	16.82071 73	2	12.02889 29	2	8.85278 88	2	6.66884 86	2	5.12251 92	2
0.6	2	24.40601 97	2	17.42397 94	2	12.80119 43	2	9.62604 94	2	7.38047 79	2
0.7	2	34.23963 11	2	24.40601 97	2	17.90199 34	2	13.43950 28	2	10.28693 17	2
0.8	3	4.68192 49	2	33.32338 00	2	24.40601 97	2	18.29398 79	2	13.98061 22	2
0.9	3	6.27302 65	3	4.45852 92	2	32.60756 01	2	24.40601 96	2	18.62387 20	2
1.0	3	8.26585 68	3	5.86706 66	3	4.28505 93	2	32.02836 67	2	24.40601 99	2

$x=7.9$

$10^{-t} {}_1F_1[a; b; x]$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	1	-7.800000	00	0	-38.500000	00	0	-25.333333	33	0	-18.750000	00	0	-14.800000	00
-0.9	1	-49.79871	88	1	-20.87543	28	1	-11.85177	27	1	-7.67682	03	0	-53.71545	85
-0.8	2	-9.76959	31	1	-40.17691	80	1	-22.35246	33	1	-14.17494	34	1	-9.70330	86
-0.7	2	-14.89798	67	2	-6.07329	28	1	-33.47469	22	1	-21.01936	72	1	-14.23998	28
-0.6	2	-19.97716	33	2	-8.09821	22	1	-44.37003	40	1	-27.68567	66	1	-18.63237	12
-0.5	2	-24.44686	06	2	-9.86846	71	2	-5.38295	68	1	-33.43160	07	1	-22.38945	24
-0.4	2	-27.53223	17	2	-11.07584	27	2	-6.01980	49	1	-37.24586	83	1	-24.84555	52
-0.3	2	-28.19413	52	2	-11.30863	09	2	-6.12731	66	1	-37.78828	95	1	-25.12196	58
-0.2	2	-25.07086	66	2	-10.02908	91	2	-5.41878	45	1	-33.31985	11	1	-22.08220	76
-0.1	2	-16.41014	04	2	-6.54716	70	1	-35.27326	04	1	-21.62148	36	1	-14.28014	86
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	2	26.97282	33	2	10.73339	18	2	5.77054	88	1	35.32045	70	1	23.31284	08
0.2	3	6.79287	51	2	26.97282	31	2	14.46723	52	2	8.83233	14	2	5.81312	60
0.3	3	12.71172	27	3	5.03837	72	2	26.97282	32	2	16.43428	64	2	10.79374	97
0.4	3	20.97066	50	3	8.29810	87	3	4.43477	51	2	26.97282	33	2	17.68272	23
0.5	3	32.19546	30	3	12.72004	27	3	6.78719	28	3	4.12130	53	2	26.97282	34
0.6	4	4.71391	04	3	18.59683	72	3	9.90809	77	3	6.00716	01	2	39.25354	32
0.7	4	6.67026	71	3	26.27822	55	3	13.98074	66	3	8.46408	21	3	5.52264	10
0.8	4	9.19590	66	4	3.61800	85	3	19.22277	03	3	11.62162	56	3	7.57223	49
0.9	4	12.41799	47	4	4.87947	23	3	25.89147	34	3	15.63277	22	3	10.17213	11
1.0	4	16.48662	31	4	6.47025	13	4	3.42898	30	3	20.67744	08	3	13.43744	29
b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	0	-12.16666	67	0	-10.28571	43	0	-8.87500	00	0	-7.77777	78	0	-6.90000	00
-0.9	0	-39.60015	59	0	-30.33839	57	0	-23.94770	07	0	-19.36381	44	0	-15.97203	72
-0.8	1	-6.99470	86	0	-52.38144	69	0	-40.41156	68	0	-31.93841	18	0	-25.75475	82
-0.7	1	-10.15451	24	1	-7.51973	16	0	-57.34960	20	0	-44.79527	18	0	-35.69394	79
-0.6	1	-13.19503	15	1	-9.70122	05	1	-7.34376	85	0	-56.92330	06	0	-45.00287	28
-0.5	1	-15.77483	25	1	-11.53638	12	1	-8.68492	61	1	-6.69363	53	0	-52.60951	72
-0.4	1	-17.43330	27	1	-12.69464	60	1	-9.51439	37	1	-7.29912	57	0	-57.09511	80
-0.3	1	-17.56479	58	1	-12.74305	63	1	-9.51378	23	1	-7.26925	74	0	-56.62286	49
-0.2	1	-15.38856	36	1	-11.12529	82	1	-8.27531	98	0	-62.98231	03	0	-48.85547	47
-0.1	1	-9.91391	77	1	-7.13744	73	0	-52.84483	59	0	-40.01271	95	0	-30.86024	42
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	1	16.19052	17	1	11.67427	90	1	8.66911	67	1	6.59434	45	0	51.19177	40
0.2	1	40.24459	99	1	28.91721	20	1	21.38985	28	1	16.20008	17	1	12.51535	56
0.3	2	7.45596	34	2	5.54473	42	1	39.43521	28	1	29.78696	88	1	22.94582	58
0.4	2	12.19129	19	2	8.72179	84	2	6.42184	13	1	48.40109	32	1	37.19978	91
0.5	2	18.56407	34	2	13.25718	85	2	9.74317	43	2	7.32931	31	2	5.62192	99
0.6	2	26.97282	31	2	19.23032	89	2	14.10906	32	2	10.59503	60	2	8.11229	61
0.7	2	37.89113	59	2	26.97282	32	2	19.75835	28	2	14.81328	60	2	11.32327	05
0.8	3	5.18792	22	2	36.87640	35	2	26.97282	31	2	20.19143	56	2	15.41039	59
0.9	3	6.95969	37	3	4.94019	01	2	36.08354	16	2	26.97282	31	2	20.55596	24
1.0	3	9.18184	73	3	6.50893	51	3	4.74780	13	2	35.44192	70	2	26.97282	35

$$10^{-t} {}_1F_1[a; b; x]$$

$$x=8.0$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	1	-7.900000 00	0	-39.000000 00	0	-25.666666 67	0	-19.000000 00	0	-15.000000 00	0
-0.9	2	-5.35947 57	1	-22.39708 15	1	-12.67647 27	1	-8.18608 13	0	-57.10920 22	2
-0.8	2	-10.59133 72	1	-43.45176 54	1	-24.11596 06	1	-15.25621 83	1	-10.41828 27	1
-0.7	2	-16.21358 23	2	-6.59589 37	1	-36.27913 09	1	-22.73250 13	1	-15.36825 81	1
-0.6	2	-21.80258 57	2	-8.82153 59	1	-48.24149 72	1	-30.04413 43	1	-20.18117 86	1
-0.5	2	-26.74296 06	2	-10.77637 36	2	-5.86783 06	1	-36.37865 94	1	-24.32020 02	2
-0.4	2	-30.17995 35	2	-12.12080 81	2	-6.57678 93	1	-40.62441 44	1	-27.05439 94	2
-0.3	2	-30.96326 70	2	-12.39962 42	2	-6.70780 36	1	-41.30298 81	1	-27.41553 04	2
-0.2	2	-27.58109 68	2	-11.01649 14	2	-5.94329 13	1	-36.49027 49	1	-24.14755 90	2
-0.1	2	-18.08298 90	2	-7.20419 31	1	-38.75801 56	1	-23.72457 39	1	-15.64800 49	2
0.0	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0
0.1	2	29.80957 94	2	11.84446 30	2	6.35818 11	1	38.85672 48	1	25.60614 05	2
0.2	3	7.51808 32	2	29.80957 97	2	15.96560 00	2	9.73282 55	2	6.39631 86	2
0.3	3	14.08812 91	3	5.57611 40	2	29.80958 00	2	18.13697 45	2	11.89505 80	2
0.4	3	23.27208 79	3	9.19616 71	3	4.90796 57	2	29.80957 94	2	19.51530 14	2
0.5	3	35.77452 79	3	14.11506 92	3	7.52139 09	3	4.56094 12	2	29.80958 00	2
0.6	4	5.24445 76	3	20.66249 94	3	10.99404 23	3	6.65669 17	3	4.34399 09	3
0.7	4	7.42998 56	3	29.23301 64	3	15.53245 26	3	9.39119 38	3	6.11953 13	3
0.8	4	10.25537 55	4	4.02964 69	3	21.38224 64	3	12.91051 87	3	8.40117 14	3
0.9	4	13.86464 01	4	5.44098 22	3	28.83422 66	3	17.38739 09	3	11.29944 27	3
1.0	4	18.42797 99	4	7.22305 37	4	3.82312 67	3	23.02522 18	3	14.94436 05	3
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-12.33333 33	0	-10.42857 14	0	-9.000000 00	0	-7.88888 89	0	-7.000000 00	0
-0.9	0	-41.98161 06	0	-32.07469 34	0	-25.25229 98	0	-20.36854 51	0	-16.76214 56	0
-0.8	1	-7.49216 65	0	-55.97496 16	0	-43.08473 76	0	-33.97510 79	0	-27.33807 04	0
-0.7	1	-10.93619 48	1	-8.08183 59	0	-61.51078 98	0	-47.94937 76	0	-38.13254 40	0
-0.6	1	-14.26480 78	1	-10.46803 72	1	-7.90952 94	0	-61.19656 49	0	-48.29454 23	0
-0.5	1	-17.10512 35	1	-12.48748 35	1	-9.38477 69	1	-7.22077 10	0	-56.65827 10	0
-0.4	1	-18.95194 42	1	-13.77800 98	1	-10.30974 59	1	-7.89678 13	0	-61.67433 17	0
-0.3	1	-19.13865 77	1	-13.86359 93	1	-10.33476 30	1	-7.88488 72	0	-61.32971 25	0
-0.2	1	-16.80333 46	1	-12.13076 30	1	-9.01063 22	1	-6.84858 28	0	-53.05513 03	0
-0.1	1	-10.84937 63	1	-7.80116 42	0	-57.69047 39	0	-43.63321 01	0	-33.61811 33	0
0.0	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0
0.1	1	17.75423 43	1	12.78040 68	1	9.47420 08	1	7.19400 21	0	55.74513 69	0
0.2	1	44.21574 05	1	31.72240 34	1	23.42871 84	1	17.71654 59	1	13.66518 60	0
0.3	2	8.20490 47	1	58.73085 70	1	43.27025 39	1	32.63553 97	1	25.10274 71	1
0.4	2	13.43598 39	2	9.59878 19	2	7.05759 09	2	5.31172 06	1	40.76615 80	1
0.5	2	20.48851 22	2	14.61147 57	2	10.72374 06	2	8.05582 19	2	6.17064 03	2
0.6	2	29.80958 02	2	21.22433 63	2	15.55113 13	2	11.66221 59	2	8.91734 60	2
0.7	3	4.19313 16	2	29.80958 03	2	21.80759 61	2	16.32807 90	2	12.46468 05	2
0.8	3	5.74840 88	3	4.08075 64	2	29.80958 01	2	22.28606 79	2	16.98698 37	2
0.9	3	7.72114 37	3	5.47370 48	2	39.92940 50	2	29.80958 02	2	22.68886 73	2
1.0	3	10.19869 14	3	7.22067 88	3	5.26034 64	2	39.21867 54	2	29.80958 01	2

$x = 8 \cdot 1$

$10^{-t} {}_1F_1[a; b; x]$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	1	-8.00000 00	0	-39.50000 00	0	-26.00000 00	0	-19.25000 00	0	-15.20000 00	0
-0.9	2	-5.77214 24	1	-24.04810 20	1	-13.56952 76	1	-8.73643 53	0	-60.76915 24	0
-0.8	2	-11.48723 95	1	-47.01636 10	1	-26.03232 21	1	-16.42924 94	1	-11.19260 70	1
-0.7	2	-17.65081 49	2	-7.16595 35	1	-39.33359 75	1	-24.59550 02	1	-16.59333 04	1
-0.6	2	-23.80028 52	2	-9.61198 79	2	-5.24661 36	1	-32.61399 83	1	-21.86634 54	1
-0.5	2	-29.25980 92	2	-11.77018 68	2	-6.39784 40	1	-39.59557 09	1	-26.42477 90	1
-0.4	2	-33.08662 48	2	-13.26642 52	2	-7.18659 70	1	-44.31835 81	1	-29.46616 13	1
-0.3	2	-34.00760 59	2	-13.59746 42	2	-7.34429 95	1	-45.15171 29	1	-29.92374 23	1
-0.2	2	-30.34461 78	2	-12.10212 25	2	-6.51923 19	1	-39.96709 20	1	-26.40957 63	1
-0.1	2	-19.92708 86	2	-7.92756 06	1	-42.58976 64	1	-26.03414 57	1	-17.14825 11	1
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	2	32.94468 06	2	13.07087 61	2	7.00601 88	1	42.75051 04	1	28.12819 42	1
0.2	3	8.32053 62	2	32.94468 05	2	17.61954 12	2	10.72561 43	2	7.03850 70	2
0.3	3	15.61294 59	3	6.17111 76	2	32.94468 06	2	20.01648 59	2	13.10928 55	2
0.4	3	25.82460 56	3	10.19102 69	3	5.43154 07	2	32.94468 07	2	21.53822 60	2
0.5	4	3.97485 65	3	15.66222 91	3	8.33470 26	3	5.04738 11	2	32.94468 06	3
0.6	4	5.83420 45	3	22.95598 67	3	12.19835 99	3	7.37619 24	3	4.80718 89	3
0.7	4	8.27538 87	3	32.51727 92	3	17.25520 09	3	10.41932 10	3	6.78070 10	3
0.8	4	11.43558 62	4	4.48767 46	3	23.78234 91	3	14.34140 55	3	9.32038 79	3
0.9	4	15.47782 78	4	6.06642 67	3	32.10835 08	3	19.33739 37	3	12.55087 98	3
1.0	4	20.59504 21	4	8.06243 57	4	4.26210 92	3	25.63717 44	3	16.61897 09	3
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-12.50000 00	0	-10.57142 86	0	-9.12500 00	0	-8.00000 00	0	-7.10000 00	0
-0.9	0	-44.54438 21	0	-33.93910 50	0	-26.65003 43	0	-21.44256 12	0	-17.60479 82	0
-0.8	1	-8.02998 59	0	-59.85321 08	0	-45.96457 41	0	-36.16529 96	0	-29.03759 82	0
-0.7	1	-11.78360 82	1	-8.69024 21	1	-6.60075 43	0	-51.35233 91	0	-40.75926 68	0
-0.6	1	-15.42705 22	1	-11.29989 86	1	-8.52236 40	1	-6.58184 29	0	-51.84936 27	0
-0.5	1	-18.55313 83	1	-13.52128 62	1	-10.14439 90	1	-7.79211 01	0	-61.04025 24	0
-0.4	1	-20.60783 57	1	-14.95766 94	1	-11.17460 69	1	-8.54577 55	1	-6.66400 61	1
-0.3	1	-20.85753 02	1	-15.08575 99	1	-11.22900 48	1	-8.55456 05	1	-6.64429 36	1
-0.2	1	-18.35081 59	1	-13.22911 81	1	-9.81283 56	1	-7.44822 13	0	-57.62495 52	0
-0.1	1	-11.87407 32	1	-8.52727 41	0	-62.98490 35	0	-47.58400 88	0	-36.62377 93	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	1	19.47181 68	1	13.99389 06	1	10.35634 31	1	7.85025 33	0	60.72215 84	0
0.2	1	48.58331 97	1	34.80391 55	1	25.66572 35	1	19.37838 23	1	14.92373 16	1
0.3	2	9.02965 20	2	6.45422 41	1	47.48344 04	1	35.76127 86	1	27.46674 39	1
0.4	2	14.80834 22	2	10.56458 05	2	7.75690 46	2	5.82987 84	1	44.67994 93	1
0.5	2	22.61287 16	2	16.10471 78	2	11.80366 49	2	8.85502 17	2	6.77355 69	2
0.6	2	32.94468 04	2	23.42552 44	2	17.14121 10	2	12.83758 56	2	9.80300 20	2
0.7	3	4.64014 85	2	32.94468 08	2	24.06979 41	2	17.99839 74	2	13.72186 38	2
0.8	3	6.36923 72	3	4.51570 46	2	32.94468 05	2	24.59840 42	2	18.72549 36	2
0.9	3	8.56548 92	3	6.06464 10	3	4.41844 59	2	32.94468 02	2	25.04348 88	2
1.0	3	11.32743 84	3	8.00987 55	3	5.82804 11	3	4.33972 03	2	32.94468 09	2

$$10^{-t} {}_1F_1[a; b; x]$$

$$x = 8.2$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	1	- 8.10000 00	0	- 40.00000 00	0	- 26.33333 33	0	- 19.50000 00	0	- 15.40000 00	0
-0.9	2	- 6.22090 69	1	- 25.84019 85	1	- 14.53703 20	1	- 9.33149 17	1	- 6.47183 48	1
-0.8	2	- 12.46422 01	1	- 50.89736 38	1	- 28.11541 24	1	- 17.70224 27	1	- 12.03151 35	1
-0.7	2	- 19.22121 25	2	- 7.78790 94	1	- 42.66114 70	1	- 26.62199 46	1	- 17.92388 07	1
-0.6	2	- 25.98683 28	2	- 10.47594 01	2	- 5.70770 33	1	- 35.41480 61	1	- 23.70030 27	1
-0.5	2	- 32.01894 31	2	- 12.85818 08	2	- 6.97728 89	1	- 43.10765 67	1	- 28.71928 63	1
-0.4	2	- 36.27785 92	2	- 14.52252 51	2	- 7.85432 23	1	- 48.35773 08	1	- 32.09991 45	1
-0.3	2	- 37.35473 21	2	- 14.91272 85	2	- 8.04228 09	1	- 49.36671 81	1	- 32.66706 90	1
-0.2	2	- 33.38713 91	2	- 13.29583 34	2	- 7.15170 23	1	- 43.78028 97	1	- 28.88727 33	1
0.1	2	- 21.96003 36	2	- 8.72400 61	1	- 46.80333 30	1	- 28.57067 32	1	- 18.79385 77	1
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	2	36.40950 28	2	14.42462 46	2	7.72025 16	1	47.03814 84	1	30.90198 36	1
0.2	3	9.20844 91	2	36.40950 30	2	19.44523 85	2	11.82018 94	2	7.74568 96	2
0.3	3	17.30212 78	3	6.82947 75	2	36.40950 30	2	22.09121 26	2	14.44804 72	2
0.4	3	28.65549 71	3	11.29309 17	3	6.01085 71	2	36.40950 30	2	23.77129 70	2
0.5	4	4.41609 25	3	17.37804 34	3	9.23562 91	3	5.58560 08	2	36.40950 31	2
0.6	4	6.48971 36	3	25.50229 29	3	13.53390 17	3	8.17317 87	3	5.31968 51	3
0.7	4	9.21606 74	4	3.61675 18	3	19.16774 74	3	11.55942 66	3	7.51305 37	3
0.8	4	12.75018 22	4	4.99728 08	3	26.44972 47	3	15.92985 01	3	10.33967 93	3
0.9	4	17.27655 16	4	6.76302 54	4	3.57508 92	3	21.50440 57	3	13.94004 21	3
1.0	4	23.01378 56	4	8.99826 40	4	4.75099 01	3	28.54281 39	3	18.47982 45	3
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	- 12.66666 67	0	- 10.71428 57	0	- 9.25000 00	0	- 8.11111 11	0	- 7.20000 00	0
-0.9	0	- 47.30392 87	0	- 35.94238 57	0	- 28.14860 53	0	- 22.59150 41	0	- 18.50420 52	0
-0.8	1	- 8.61167 31	1	- 6.40405 66	0	- 49.06850 65	0	- 38.52174 07	0	- 30.86285 09	0
-0.7	1	- 12.70256 50	1	- 9.34898 62	1	- 7.08686 97	0	- 55.02523 24	0	- 43.58981 63	0
-0.6	1	- 16.69008 00	1	- 12.20257 35	1	- 9.18638 95	1	- 7.08189 59	0	- 55.68970 72	0
-0.5	1	- 20.12963 41	1	- 14.64524 55	1	- 10.96910 88	1	- 8.41153 12	1	- 6.57842 96	1
-0.4	1	- 22.41371 21	1	- 16.24244 36	1	- 12.11525 87	1	- 9.25068 71	1	- 7.20263 15	1
-0.3	1	- 22.73506 56	1	- 16.41898 46	1	- 12.20323 08	1	- 9.28317 36	1	- 7.19988 73	1
-0.2	1	- 20.04367 93	1	- 14.42912 39	1	- 10.68815 96	1	- 8.10167 90	0	- 62.59855 80	0
-0.1	1	- 12.99664 52	1	- 9.32174 27	1	- 6.87705 16	0	- 51.89593 10	0	- 39.90007 45	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	1	21.35854 98	1	15.32526 74	1	11.32301 83	1	8.56852 31	1	6.61629 66	1
0.2	2	5.33871 90	1	38.18920 87	1	28.12033 20	1	21.19970 69	1	16.30142 52	1
0.3	2	9.93791 65	2	7.09345 81	2	5.21123 74	1	39.19143 78	1	30.05794 09	1
0.4	2	16.32150 17	2	11.62823 19	2	8.52617 91	2	6.39921 51	1	48.97534 86	1
0.5	2	24.95795 12	2	17.75121 57	2	12.99306 16	2	9.73423 15	2	7.43607 76	2
0.6	2	36.40950 28	2	25.85545 21	2	18.89453 78	2	14.13216 25	2	10.77738 34	2
0.7	3	5.13473 44	2	36.40950 31	2	26.56710 74	2	19.84025 55	2	15.10661 29	2
0.8	3	7.05688 64	3	4.99693 11	2	36.40950 29	2	27.15110 31	2	20.64259 99	2
0.9	3	9.50172 13	3	6.71916 33	3	4.88921 81	2	36.40950 29	2	27.64290 32	2
1.0	3	12.58034 38	3	8.88492 31	3	6.45680 46	3	4.80201 98	2	36.40950 32	2

$x=8.3$ $10^{-t} {}_1F_1[a; b; x]$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	1	-8.200000 00	0	-40.500000 00	0	-26.666666 67	0	-19.750000 00	0	-15.600000 00	0
-0.9	2	-6.709088 83	1	-27.78616 00	1	-15.58563 78	1	-9.97518 70	1	-6.89818 87	1
-0.8	2	-13.52985 54	2	-5.51239 34	1	-30.38038 50	1	-19.08415 55	1	-12.94070 73	1
-0.7	2	-20.93741 10	2	-8.46661 99	1	-46.28700 37	1	-28.82688 01	1	-19.36938 68	1
-0.6	2	-28.38041 34	2	-11.42037 83	2	-6.21104 16	1	-38.46793 24	1	-25.69663 83	1
-0.5	2	-35.04402 33	2	-14.04943 64	2	-7.61087 04	1	-46.94265 19	1	-31.22133 57	1
-0.4	2	-39.78182 23	2	-15.89990 80	2	-8.58555 64	2	-5.27754 61	1	-34.97655 27	1
-0.3	3	-4.10350 08	2	-16.35704 89	2	-8.80776 44	2	-5.39834 06	1	-35.66795 53	1
-0.2	2	-36.73698 87	2	-14.60846 84	2	-7.84630 66	1	-47.96281 80	1	-31.60152 04	1
-0.1	2	-24.20123 23	2	-9.60095 28	2	-5.14370 49	1	-31.35667 40	1	-20.59907 65	1
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	2	40.23872 37	2	15.91895 14	2	8.50770 67	2	5.17596 82	1	33.95281 21	2
0.2	3	10.19090 93	2	40.23872 39	2	21.46055 66	2	13.02702 04	2	8.52447 72	2
0.3	3	19.17334 00	3	7.55792 97	2	40.23872 40	2	24.38146 19	2	15.92415 58	2
0.4	3	31.79499 47	3	12.51388 10	3	6.65184 08	2	40.23872 38	2	26.23637 82	2
0.5	4	4.90596 95	3	19.28082 26	3	10.23358 27	3	6.18110 43	2	40.23872 38	2
0.6	4	7.21827 12	3	28.32914 18	3	15.01490 94	3	9.05597 30	3	5.88671 92	3
0.7	4	10.26268 00	4	4.02242 58	3	21.29089 66	3	12.82365 94	3	8.32423 31	3
0.8	4	14.21433 90	4	5.56423 20	3	29.41395 44	3	17.69311 54	3	11.46989 92	3
0.9	4	19.28195 62	4	7.53880 76	4	3.98030 14	3	23.91243 23	3	15.48201 31	3
1.0	4	25.71315 13	4	10.04152 16	4	5.29539 63	3	31.77493 52	3	20.54751 34	3
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-12.83333 33	0	-10.85714 29	0	-9.37500 00	0	-8.22222 22	0	-7.30000 00	0
-0.9	0	-50.27706 92	0	-38.09622 93	0	-29.75637 45	0	-23.82149 10	0	-19.46492 91	0
-0.8	1	-9.24104 81	1	-6.85635 56	0	-52.41548 63	0	-41.05828 20	0	-32.82414 79	0
-0.7	1	-13.69940 45	1	-10.06246 67	1	-7.61256 42	0	-58.99097 91	0	-46.64125 12	0
-0.6	1	-18.06297 09	1	-13.18235 52	1	-9.90609 31	1	-7.62308 88	0	-59.83991 53	0
-0.5	1	-21.84637 04	1	-15.86750 41	1	-11.86470 69	1	-9.08326 17	1	-7.09218 05	1
-0.4	1	-24.38350 94	1	-17.64197 39	1	-13.13856 31	1	-10.01651 24	1	-7.78701 76	1
-0.3	1	-24.78622 23	1	-17.87361 20	1	-13.26479 24	1	-10.07607 45	1	-7.80371 79	1
-0.2	1	-21.89582 15	1	-15.74037 82	1	-11.64342 28	1	-8.81391 24	1	-6.80126 61	1
-0.1	1	-14.22657 46	1	-10.19111 30	1	-7.50937 55	0	-56.60270 89	0	-43.47196 77	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	1	23.43124 11	1	16.78611 70	1	12.38243 31	1	9.35476 25	1	7.21115 42	1
0.2	2	5.86712 27	1	41.90849 02	1	30.81393 55	1	23.19601 92	1	17.80971 36	1
0.3	2	10.93819 60	2	7.79663 37	2	5.71984 27	1	42.95596 82	1	32.89844 13	1
0.4	2	17.98995 33	2	12.79969 65	2	9.37245 78	2	7.02482 63	2	5.36899 32	2
0.5	2	27.54671 73	2	19.56674 49	2	14.30307 81	2	10.70151 07	2	8.16414 11	2
0.6	2	40.23872 34	2	28.53792 24	2	20.82792 07	2	15.55809 00	2	11.84943 21	2
0.7	3	5.68194 45	2	40.23872 40	2	29.32400 58	2	21.87131 92	2	16.63192 80	2
0.8	3	7.81853 01	3	5.52935 32	2	40.23872 35	2	29.96918 35	2	22.75670 08	2
0.9	3	10.53980 23	3	7.44409 66	3	5.41006 70	2	40.23872 35	2	30.51259 36	2
1.0	3	13.97100 02	3	9.85512 70	3	7.15319 04	3	5.31348 82	2	40.23872 38	2

$$10^{-t} {}_1F_1[a; b; x]$$

$$x=8.4$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	1	- 8.30000 00	0	- 41.00000 00	0	- 27.00000 00	0	- 20.00000 00	0	- 15.80000 00	0
-0.9	2	- 7.24031 71	1	- 29.89995 92	1	- 16.72260 60	1	- 10.67181 35	1	- 7.35871 09	1
-0.8	2	- 14.69244 15	2	- 5.97279 71	1	- 32.84380 02	1	- 20.58476 69	1	- 13.92641 07	1
-0.7	2	- 22.81326 26	2	- 9.20740 63	1	- 50.23876 23	1	- 31.22643 71	1	- 20.94019 83	1
-0.6	2	- 31.00009 27	2	- 12.45296 11	2	- 6.76059 68	1	- 41.79676 58	1	- 27.87020 49	1
-0.5	2	- 38.36104 25	2	- 15.35391 96	2	- 8.30374 75	1	- 51.13093 65	1	- 33.95020 33	1
-0.4	3	- 4.36294 89	2	- 17.41044 00	2	- 9.38643 66	2	- 5.76076 54	1	- 38.11896 50	1
-0.3	3	- 4.50818 57	2	- 17.94321 89	2	- 9.64736 08	2	- 5.90406 38	1	- 38.95101 41	1
-0.2	2	- 40.42538 43	2	- 16.05196 58	2	- 8.60920 91	2	- 5.25508 81	1	- 34.57522 84	1
-0.1	2	- 26.67209 36	2	- 10.56658 16	2	- 5.65331 16	1	- 34.41691 60	1	- 22.57956 83	1
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	3	4.44706 67	2	17.56847 80	2	9.37591 55	2	5.69592 40	1	37.30853 91	1
0.2	3	11.27796 92	3	4.44706 67	2	23.68522 09	2	14.35765 58	2	9.38215 53	2
0.3	3	21.24614 31	3	8.36392 59	3	4.44706 67	2	26.90965 58	2	17.55174 56	2
0.4	4	3.52766 03	3	13.86614 96	3	7.36104 68	3	4.44706 67	2	28.95760 99	2
0.5	4	5.44982 17	3	21.39085 84	3	11.33898 44	3	6.83997 99	3	4.44706 67	3
0.6	4	8.02796 68	3	31.46728 56	3	16.65716 52	3	10.03378 93	3	6.51408 70	3
0.7	4	11.42707 04	4	4.47324 95	3	23.64772 37	3	14.22548 24	3	9.22270 20	3
0.8	4	15.84493 44	4	6.19493 55	3	32.70787 82	3	19.65034 83	3	12.72307 56	3
0.9	4	21.51757 85	4	8.40270 31	4	4.43104 53	3	26.58812 32	3	17.19352 23	3
1.0	5	2.87253 82	4	11.20443 32	4	5.90158 53	4	3.53699 74	3	22.84489 66	3

b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	- 13.00000 00	0	- 11.00000 00	0	- 9.50000 00	0	- 8.33333 33	0	- 7.40000 00	0
-0.9	0	- 53.48211 04	0	- 40.41334 83	0	- 31.48242 52	0	- 25.13916 27	0	- 20.49191 68	0
-0.8	1	- 9.92227 29	1	- 7.34510 56	0	- 56.02612 15	0	- 43.78997 37	0	- 34.93269 16	0
-0.7	1	- 14.78104 25	1	- 10.83547 63	1	- 8.18125 52	0	- 63.27450 92	0	- 49.93210 93	0
-0.6	1	- 19.55564 08	1	- 14.24610 95	1	- 10.68636 55	1	- 8.20898 60	1	- 6.43264 64	1
-0.5	1	- 23.71620 33	1	- 17.19695 58	1	- 12.83752 23	1	- 9.81191 00	1	- 7.64869 82	1
-0.4	1	- 26.53248 05	1	- 19.16680 22	1	- 14.25201 59	1	- 10.84870 45	1	- 8.42120 75	1
-0.3	1	- 27.02738 70	1	- 19.46095 90	1	- 14.42172 90	1	- 10.93910 66	1	- 8.46011 44	1
-0.2	1	- 23.92248 47	1	- 17.17339 70	1	- 12.68608 98	1	- 9.59034 23	1	- 7.39073 88	1
-0.1	1	- 15.57427 09	1	- 11.14256 16	1	- 8.20055 19	0	- 61.74127 40	0	- 47.36677 41	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	1	25.70837 87	1	18.38916 51	1	13.54359 85	1	10.21549 97	1	7.86160 90	1
0.2	2	6.44837 46	1	45.99498 85	1	33.77004 57	1	25.38433 87	1	19.46115 42	1
0.3	2	12.03985 64	2	8.57018 78	2	6.27871 05	1	47.08779 00	1	36.01251 63	1
0.4	2	19.82968 12	2	14.08995 30	2	10.30349 68	2	7.71231 82	2	5.88650 05	2
0.5	2	30.40452 84	2	21.56870 87	2	15.74600 12	2	11.76573 39	2	8.96428 09	2
0.6	3	4.44706 67	2	31.49921 87	2	22.95990 19	2	17.12875 22	2	13.02899 49	2
0.7	3	6.28737 09	3	4.44706 67	2	32.36750 90	2	24.11107 81	2	18.31213 74	2
0.8	3	8.66211 07	3	6.11841 02	3	4.44706 67	2	33.08026 99	2	25.08809 26	2
0.9	3	11.69077 21	3	8.24699 70	3	5.98631 25	3	4.44706 67	2	33.68069 86	2
1.0	3	15.51448 21	3	10.93079 84	3	7.92445 37	3	5.87934 83	3	4.44706 67	3

$x=8.5$

$10^{-t} {}_1F_1[a; b; x]$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	1	- 8.40000 00	0	- 41.50000 00	0	- 27.33333 33	0	- 20.25000 00	0	- 16.00000 00	0
-0.9	2	- 7.81856 45	1	- 32.19686 50	1	- 17.95586 34	1	- 11.42605 25	1	- 7.85637 99	1
-0.8	2	- 15.96106 34	2	- 6.47443 73	1	- 35.52375 55	1	- 22.21475 47	1	- 14.99541 12	1
-0.7	2	- 24.86395 28	2	- 10.01609 62	2	- 5.45466 16	1	- 33.83846 21	1	- 22.64761 56	1
-0.6	2	- 33.87044 24	2	- 13.58208 53	2	- 7.36071 51	1	- 45.42690 05	1	- 30.23724 15	1
-0.5	3	- 4.19985 56	2	- 16.78256 93	2	- 9.06157 69	2	- 5.57057 90	1	- 36.92698 36	1
-0.4	3	- 4.78549 24	2	- 19.06715 84	2	- 10.26369 88	2	- 6.28939 07	1	- 41.55222 67	1
-0.3	3	- 4.95320 75	2	- 19.68530 84	2	- 10.56833 37	2	- 6.45810 73	1	- 42.54323 80	1
-0.2	3	- 4.44867 27	2	- 17.63946 93	2	- 9.44718 94	2	- 5.75842 62	1	- 37.83355 06	1
-0.1	2	- 29.39623 17	2	- 11.62990 73	2	- 6.21380 00	1	- 37.77864 23	1	- 24.75254 29	1
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	3	4.91476 88	2	19.38934 88	2	10.33318 59	2	6.26854 57	1	40.99983 97	1
0.2	3	12.48074 93	3	4.91476 88	2	26.14101 04	2	15.82483 66	2	10.32675 32	2
0.3	3	23.54219 45	3	9.25570 89	3	4.91476 88	2	29.70055 29	2	19.34640 84	2
0.4	4	3.91374 54	3	15.36401 96	3	8.14572 63	3	4.91476 88	2	31.96164 72	2
0.5	4	6.05356 58	3	23.73063 78	3	12.56337 26	3	7.56896 27	3	4.91476 89	3
0.6	4	8.92778 12	4	3.49508 33	3	18.47815 89	3	11.11682 93	3	7.20819 98	3
0.7	4	12.72239 84	4	4.97421 76	3	26.26382 34	3	15.77981 45	3	10.21782 96	3
0.8	4	17.66073 80	4	6.89651 06	4	3.63679 51	3	21.82278 47	3	14.11253 59	3
0.9	4	24.00961 57	4	9.36464 19	4	4.93240 26	3	29.56106 00	3	19.09312 37	3
1.0	5	3.20863 95	4	12.50060 45	4	6.57651 52	4	3.93684 12	3	25.39734 75	3
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	- 13.16666 67	0	- 11.14285 71	0	- 9.62500 00	0	- 8.44444 44	0	- 7.50000 00	0
-0.9	0	- 56.93897 87	0	- 42.90756 77	0	- 33.33662 44	0	- 26.55172 41	0	- 21.59053 32	0
-0.8	1	- 10.65988 31	1	- 7.87345 02	0	- 59.92282 91	0	- 46.73316 80	0	- 37.20064 05	0
-0.7	1	- 15.95502 34	1	- 11.67323 95	1	- 8.79666 39	1	- 6.79029 44	0	- 53.48253 71	1
-0.6	1	- 21.17891 94	1	- 15.40132 86	1	- 11.53253 82	1	- 8.84346 95	1	- 6.91781 64	1
-0.5	1	- 25.75318 93	1	- 18.64331 57	1	- 13.89446 24	1	- 10.60250 07	1	- 8.25170 82	1
-0.4	1	- 28.87732 00	1	- 20.82845 64	1	- 15.46380 54	1	- 11.75321 47	1	- 9.10961 01	1
-0.3	1	- 29.47651 55	1	- 21.19341 29	1	- 15.68283 28	1	- 11.87865 41	1	- 9.17380 19	1
-0.2	1	- 26.14038 47	1	- 18.73970 35	1	- 13.82433 19	1	- 10.43689 74	1	- 8.03265 82	1
-0.1	1	- 17.05116 27	1	- 12.18396 00	1	- 8.95615 83	1	- 6.73520 61	0	- 51.61437 01	1
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	1	28.21030 12	1	20.14839 86	1	14.81640 94	1	11.15789 63	1	8.57294 44	1
0.2	2	7.08779 42	1	50.48525 88	1	37.01450 31	1	27.78335 56	1	21.26952 48	1
0.3	2	13.25322 01	2	9.42120 85	2	6.89284 80	2	5.16230 89	1	39.42682 22	2
0.4	2	21.85831 96	2	15.51110 02	2	11.32783 65	2	8.46785 91	2	6.45459 66	2
0.5	2	33.55938 50	2	23.77630 63	2	17.33537 32	2	12.93667 45	2	9.84368 58	2
0.6	3	4.91476 88	2	34.76836 22	2	25.31093 83	2	18.85890 21	2	14.32691 59	2
0.7	3	6.95719 90	3	4.91476 89	2	35.72745 07	2	26.58103 37	2	20.16303 49	2
0.8	3	9.59642 12	3	6.77011 95	3	4.91476 88	2	36.51486 82	2	27.65916 68	2
0.9	3	12.96686 36	3	9.13622 98	3	6.62383 93	3	4.91476 89	2	37.17828 88	2
1.0	3	17.22750 79	3	12.12336 24	3	8.77862 68	3	6.50537 78	3	4.91476 89	3

$$10^{-t} {}_1F_1[a; b; x]$$

$$x=8.6$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	1	-8.500000	0	-42.000000	0	-27.666666	0	-20.500000	0	-16.200000	0
-0.9	2	-8.448175	1	-34.693566	1	-19.294064	1	-12.243010	1	-8.394442	1
-0.8	2	-17.345671	2	-7.021132	1	-38.440033	1	-23.985779	1	-16.155113	1
-0.7	2	-27.106132	2	-10.890907	2	-5.924360	1	-36.682409	1	-24.503982	1
-0.6	2	-37.012830	2	-14.816957	2	-8.016157	1	-49.386345	1	-32.815501	1
-0.5	3	-4.598794	2	-18.347392	2	-9.890562	2	-6.070367	1	-40.174766	1
-0.4	3	-5.249559	2	-20.884387	2	-11.224737	2	-6.867764	1	-45.303812	1
-0.3	3	-5.442617	2	-21.598792	2	-11.578664	2	-7.065154	1	-46.474233	1
-0.2	3	-4.895892	2	-19.385449	2	-10.367704	2	-6.310667	1	-41.404101	1
-0.1	2	-32.399695	2	-12.800865	2	-6.830285	1	-41.471820	1	-27.136913	1
0.0	0	1.000000	0	1.000000	0	1.000000	0	1.000000	0	1.000000	0
0.1	3	5.431659	2	21.399389	2	11.388682	2	6.899193	1	45.060490	1
0.2	3	13.811550	3	5.431659	2	28.851973	2	17.442616	2	11.367120	1
0.3	3	26.085473	3	10.242396	3	5.431659	2	32.781489	2	21.325345	1
0.4	4	4.341869	3	17.023126	3	9.013900	3	5.431659	2	35.277922	1
0.5	4	6.723764	3	26.325080	3	13.919522	3	8.375502	3	5.431659	1
0.6	4	9.927684	4	3.881761	3	20.497271	3	12.316387	3	7.976149	1
0.7	4	14.163286	4	5.530874	3	29.167581	3	17.503187	3	11.319989	1
0.8	4	19.682622	4	7.676867	4	4.043464	3	24.233976	3	15.653049	1
0.9	5	2.678722	4	10.435666	4	5.490018	4	3.286407	3	21.201394	1
1.0	5	3.583619	4	13.945177	4	7.327922	4	4.381521	3	28.233029	1
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-13.333333	0	-11.285714	0	-9.750000	0	-8.555555	0	-7.600000	0
-0.9	0	-60.669363	0	-45.593922	0	-35.329693	0	-28.066998	0	-22.766593	0
-0.8	1	-11.458821	1	-8.444812	1	-6.412999	0	-49.905637	0	-39.641197	0
-0.7	1	-17.229579	1	-12.581450	1	-9.462842	1	-7.290579	0	-57.314434	0
-0.6	1	-22.944636	1	-16.656187	1	-12.450423	1	-9.530767	1	-7.442636	1
-0.5	1	-27.972699	1	-20.217197	1	-15.043066	1	-11.460513	1	-8.905269	1
-0.4	1	-31.436301	1	-22.639543	1	-16.782879	1	-12.736539	1	-9.857033	1
-0.3	1	-32.153281	1	-23.084535	1	-17.057719	1	-12.901693	1	-9.949939	1
-0.2	1	-28.567858	1	-20.451926	1	-15.067096	1	-11.360063	1	-8.731816	1
-0.1	1	-18.669798	1	-13.323943	1	-9.782307	1	-7.347934	0	-56.247441	1
0.0	0	1.000000	0	1.000000	0	1.000000	0	1.000000	0	1.000000	0
0.1	1	30.959383	1	22.079190	1	16.211731	1	12.189809	1	9.350951	1
0.2	2	7.791239	2	5.541951	1	40.575711	1	30.413594	1	23.249941	1
0.3	2	14.589664	2	10.357501	2	7.567763	2	5.660164	1	43.170628	1
0.4	2	24.095321	2	17.076475	2	12.454882	2	9.298235	2	7.078272	1
0.5	2	37.042204	2	26.210718	2	19.086120	2	14.225095	2	10.810265	1
0.6	3	5.431659	2	38.377398	2	27.903597	2	20.764801	2	15.755137	1
0.7	3	7.698271	3	5.431659	2	39.436775	2	29.304906	2	22.202029	1
0.8	3	10.631196	3	7.491136	3	5.431659	2	40.306660	2	30.494628	1
0.9	3	14.381631	3	10.121056	3	7.329156	3	5.431659	3	4.103967	1
1.0	3	19.128618	3	13.445478	3	9.724601	3	7.197967	3	5.431659	1

$x=8.7$ $10^{-t} {}_1F_1[a; b; x]$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	1	-8.60000 00	0	-42.50000 00	0	-28.00000 00	0	-20.75000 00	0	-16.40000 00	0
-0.9	2	-9.13390 22	1	-37.40830 44	1	-20.74666 02	1	-13.12825 71	1	-8.97643 44	1
-0.8	2	-18.85716 40	2	-7.61706 03	1	-41.61426 04	1	-25.91057 30	1	-17.41359 59	1
-0.7	2	-29.55805 61	2	-11.86332 78	2	-6.43658 84	1	-39.77954 82	1	-26.52277 93	1
-0.6	2	-40.45453 15	2	-16.16767 55	2	-8.73214 07	2	-5.37057 57	1	-35.62439 52	1
-0.5	3	-5.03636 68	2	-20.06156 69	2	-10.79750 57	2	-6.61645 33	1	-43.71882 42	1
-0.4	3	-5.75927 07	2	-22.87786 92	2	-12.27767 12	2	-7.50064 89	1	-49.40382 95	1
-0.3	3	-5.98087 43	2	-23.70069 45	2	-12.68712 54	2	-7.73034 63	1	-50.77647 35	1
-0.2	3	-5.38837 60	2	-21.30583 88	2	-11.37895 81	2	-6.91661 75	1	-45.31719 92	1
-0.1	2	-35.71121 97	2	-14.09040 48	2	-7.50840 18	1	-45.52941 49	1	-29.75346 76	1
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	3	6.00291 22	2	23.61828 11	2	12.55251 48	2	7.59377 48	1	49.52768 30	1
0.2	3	15.28397 99	3	6.00291 22	2	31.84466 14	2	19.22649 94	2	12.51301 19	1
0.3	3	28.90252 92	3	11.33407 58	3	6.00291 22	2	36.18264 85	2	23.50753 16	1
0.4	4	4.81659 25	3	18.86078 06	3	9.97444 13	3	6.00291 22	2	38.93893 33	2
0.5	4	7.46769 65	3	29.20179 96	3	15.42157 61	3	9.26783 96	3	6.00291 22	2
0.6	4	11.03874 42	4	4.31095 96	3	22.73597 84	3	13.64496 90	3	8.82578 05	1
0.7	4	15.76598 05	4	6.14937 65	3	32.39047 91	3	19.41391 87	3	12.54066 48	1
0.8	4	21.93379 89	4	8.54479 29	4	4.49528 68	3	26.91004 44	3	17.36097 92	1
0.9	5	2.98828 56	4	11.62805 39	4	6.11016 61	4	3.65336 27	3	23.54115 29	1
1.0	5	4.00193 55	4	15.55500 36	4	8.16441 09	4	4.87603 54	3	31.38320 12	1
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-13.50000 00	0	-11.42857 14	0	-9.87500 00	0	-8.66666 67	0	-7.70000 00	0
-0.9	1	-6.46968 84	0	-48.48876 78	0	-37.47328 24	0	-29.69347 65	0	-24.02640 82	0
-0.8	1	-12.32447 75	1	-9.06291 97	1	-6.86741 33	0	-53.32669 77	0	-42.26869 94	0
-0.7	1	-18.61369 28	1	-13.56631 84	1	-10.18420 66	1	-7.83151 63	0	-61.45160 98	0
-0.6	1	-24.86571 46	1	-18.01961 00	1	-13.44636 05	1	-10.27548 69	1	-8.01051 91	1
-0.5	1	-30.39154 23	1	-21.93019 52	1	-16.29156 23	1	-12.39192 22	1	-9.61380 33	1
-0.4	1	-34.22943 03	1	-24.61385 12	1	-18.21901 35	1	-13.80577 25	1	-10.66872 23	1
-0.3	1	-35.07924 61	1	-25.14917 31	1	-18.55690 78	1	-14.01584 96	1	-10.79415 98	1
-0.2	1	-31.22501 88	1	-22.32390 47	1	-16.42417 97	1	-12.36693 55	1	-9.49345 02	1
-0.1	1	-20.44395 58	1	-14.57198 52	1	-10.68569 95	1	-8.01716 12	0	-61.30174 91	0
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	1	33.98023 92	1	24.19843 67	1	17.74149 56	1	13.31986 17	1	10.20198 06	1
0.2	2	8.56516 09	2	6.08420 13	1	44.48489 32	1	33.29759 66	1	25.41899 30	1
0.3	2	16.06172 94	2	11.38766 07	2	8.30951 78	2	6.20671 68	1	47.27609 08	1
0.4	2	26.56214 90	2	18.80077 71	2	13.69499 27	2	10.21091 40	2	7.76301 58	1
0.5	2	40.88712 45	2	28.89531 16	2	21.01469 69	2	15.64284 95	2	11.87272 44	1
0.6	3	6.00291 22	3	4.23617 12	2	30.76277 86	2	22.86437 45	2	17.32681 25	1
0.7	3	8.51815 52	3	6.00291 22	3	4.35318 58	2	32.30886 84	2	24.44831 26	1
0.8	3	11.77721 36	3	8.28882 26	3	6.00291 21	3	4.44928 40	2	33.62173 18	1
0.9	3	15.95009 51	3	11.21172 84	3	8.10946 49	3	6.00291 22	3	4.53027 34	1
1.0	3	21.23837 46	3	14.91117 47	3	10.77222 32	3	7.96418 45	3	6.00291 22	1

$$10^{-t} {}_1F_1[a; b; x]$$

$$x=8.8$$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	1	-8.700000	00	0	-43.000000	00	0	-28.333333	33	0	-21.000000	00	0	-16.600000	00
-0.9	2	-9.88094	67	1	-40.36101	80	1	-22.32397	10	1	-14.08787	02	1	-9.60621	32
-0.8	2	-20.50747	90	2	-8.26679	36	1	-45.07007	83	1	-28.00304	55	1	-18.77967	49
-0.7	2	-32.23974	41	2	-12.91652	18	2	-6.99530	27	1	-43.15313	65	1	-28.71873	49
-0.6	3	-4.42245	08	2	-17.64531	00	2	-9.51437	88	2	-5.84186	89	1	-38.68515	46
-0.5	3	-5.51636	20	2	-21.93956	32	2	-11.78986	92	2	-7.21321	38	1	-47.58682	41
-0.4	3	-6.31915	86	2	-25.06490	01	2	-13.43141	12	2	-8.19326	81	2	-5.38852	66
-0.3	3	-6.57288	97	2	-26.00973	66	2	-13.90335	45	2	-8.45932	83	2	-5.54855	78
-0.2	3	-5.93072	73	2	-23.41817	95	2	-12.48997	28	2	-7.58155	57	1	-49.60613	61
-0.1	2	-39.36250	21	2	-15.51059	27	2	-8.25435	02	1	-49.98769	08	1	-32.62505	08
0.0	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00
0.1	3	6.63424	40	2	26.06775	72	2	13.83583	58	2	8.35880	05	2	5.44423	74
0.2	3	16.91308	97	3	6.63424	40	2	35.14839	47	2	21.19358	94	2	13.77517	68
0.3	3	32.02275	61	3	12.54190	29	3	6.63424	40	2	39.93736	09	2	25.91390	06
0.4	4	5.34296	75	3	20.89614	89	3	11.03716	37	3	6.63424	40	3	4.29805	65
0.5	4	8.29343	51	3	32.39139	58	3	17.08519	30	3	10.25508	98	3	6.63424	40
0.6	4	12.27324	64	4	4.78733	06	3	25.21807	77	3	15.11641	73	3	9.76577	01
0.7	4	17.54853	00	4	6.83655	43	4	3.59674	33	3	21.53230	76	3	13.89257	00
0.8	4	24.44007	48	4	9.51005	09	4	4.99724	89	3	29.87995	46	3	19.25445	84
0.9	5	3.33326	21	4	12.95545	67	4	6.79981	17	4	4.06101	59	3	26.13770	67
1.0	5	4.46854	98	4	17.34883	33	4	9.09554	35	4	5.42593	13	4	3.48825	58

b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	0	-13.66666	67	0	-11.57142	86	0	-10.00000	00	0	-8.77777	78	0	-7.80000	00
-0.9	1	-6.90472	55	0	-51.60989	46	0	-39.78005	50	0	-31.44038	09	0	-25.37682	18
-0.8	1	-13.26272	42	1	-9.73183	19	1	-7.35841	25	0	-57.01734	86	0	-45.09871	69
-0.7	1	-20.11716	78	1	-14.63461	13	1	-10.96556	27	1	-8.41660	01	1	-6.59199	51
-0.6	1	-26.95627	27	1	-19.50133	46	1	-14.52726	00	1	-11.08264	57	1	-8.62517	83
-0.5	1	-33.02810	16	1	-23.79498	08	1	-17.64893	48	1	-13.40324	51	1	-10.38212	96
-0.4	1	-37.27860	79	1	-26.76646	22	1	-19.78289	23	1	-14.96865	65	1	-11.55039	74
-0.3	1	-38.27803	74	1	-27.40358	42	1	-20.19190	46	1	-15.22945	49	1	-11.71261	38
-0.2	1	-34.13393	13	1	-24.37080	79	1	-17.90630	92	1	-13.46527	56	1	-10.32328	50
-0.1	1	-22.38876	57	1	-15.93847	76	1	-11.67367	86	1	-8.74819	57	1	-6.68164	22
0.0	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00
0.1	1	37.29995	11	1	26.52471	15	1	19.41880	76	1	14.55751	16	1	11.13299	07
0.2	2	9.41666	17	2	6.68014	26	1	48.77636	91	1	36.46012	03	1	27.79488	11
0.3	2	17.68323	65	2	12.52115	30	2	9.12477	94	2	6.80677	43	2	5.17785	23
0.4	2	29.28247	77	2	20.70020	90	2	15.05957	31	2	11.21411	12	2	8.51485	78
0.5	3	4.51318	43	2	31.85586	49	2	23.13923	79	2	17.20299	15	2	13.04063	89
0.6	3	6.63424	39	3	4.67603	75	2	33.91594	95	2	25.17737	97	2	19.05642	66
0.7	3	9.42522	31	3	6.63424	40	3	4.80528	66	2	35.62179	18	2	26.92303	73
0.8	3	13.04640	41	3	9.17132	13	3	6.63424	39	3	4.91144	79	2	37.07054	79
0.9	3	17.68889	55	3	12.41959	75	3	8.97272	77	3	6.63424	39	3	5.00093	08
1.0	3	23.57957	65	3	16.53599	12	3	11.93238	85	3	8.81184	89	3	6.63424	40

$x = 8.9$

$10^{-t} {}_1F_1[a; b; x]$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	1	- 8.800000 00	0	- 43.500000 00	0	- 28.666666 67	0	- 21.250000 00	0	- 16.800000 00	0
-0.9	2	- 10.695000 03	1	- 43.573500 50	1	- 24.03726 88	1	- 15.12848 04	1	- 10.28798 13	1
-0.8	2	- 22.30969 44	2	- 8.97533 59	1	- 48.83333 87	1	- 30.27838 96	1	- 20.26297 05	1
-0.7	2	- 35.17314 89	2	- 14.06704 69	2	- 7.60483 48	1	- 46.82860 65	1	- 31.10793 71	1
-0.6	3	- 4.83545 55	2	- 19.26200 48	2	- 10.36913 36	2	- 6.35618 74	1	- 42.02099 51	1
-0.5	3	- 6.04294 21	2	- 23.99726 49	2	- 12.87583 59	2	- 7.86544 54	2	- 5.18090 55	2
-0.4	3	- 6.93420 90	2	- 27.46448 87	2	- 14.69574 28	2	- 8.95135 22	2	- 5.87842 77	2
-0.3	3	- 7.22406 99	2	- 28.54651 64	2	- 15.23794 55	2	- 9.25829 91	2	- 6.06406 25	2
-0.2	3	- 6.52802 07	2	- 25.74178 71	2	- 13.71067 57	2	- 8.31128 57	2	- 5.43074 69	2
-0.1	3	- 4.33885 09	2	- 17.07472 88	2	- 9.07496 05	2	- 5.48865 44	1	- 35.77677 32	1
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	3	7.33197 35	2	28.77181 36	2	15.25094 89	2	9.20144 73	2	5.98496 63	2
0.2	3	18.71553 02	3	7.33197 35	2	38.79554 55	2	23.36275 56	2	15.16546 51	2
0.3	4	3.54786 98	3	13.87821 94	3	7.33197 35	3	4.40824 28	2	28.56754 72	2
0.4	4	5.92659 38	3	23.15044 98	3	12.21292 39	3	7.33197 35	3	4.74424 39	3
0.5	4	9.20993 38	4	3.59277 76	3	18.92770 90	3	11.34733 60	3	7.33197 35	3
0.6	4	13.64482 85	4	5.31603 57	3	27.96993 81	3	16.74605 89	3	10.80571 65	3
0.7	4	19.53098 67	4	7.59999 21	4	3.99371 63	3	23.88084 45	3	15.38978 18	3
0.8	5	2.72301 46	4	10.58348 63	4	5.55488 45	4	3.31758 31	3	21.35357 71	3
0.9	5	3.71767 03	4	14.43305 25	4	7.56669 56	4	4.51385 73	3	29.01911 83	3
1.0	5	4.98899 01	4	19.34753 19	4	10.13195 38	4	6.03737 41	4	3.87696 05	4
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	- 13.83333 33	0	- 11.71428 57	0	- 10.12500 00	0	- 8.88888 89	0	- 7.90000 00	0
-0.9	1	- 7.37484 87	0	- 54.97666 13	0	- 42.26377 57	0	- 33.31772 49	0	- 26.82525 98	0
-0.8	1	- 14.27996 63	1	- 10.45597 11	1	- 7.88913 76	0	- 61.00042 13	0	- 48.14816 40	0
-0.7	1	- 21.75070 51	1	- 15.79371 14	1	- 11.81214 98	1	- 9.04963 47	1	- 7.07476 10	1
-0.6	1	- 29.23173 78	1	- 21.11199 39	1	- 15.70066 05	1	- 11.95771 28	1	- 9.29065 77	1
-0.5	1	- 35.90248 60	1	- 25.82539 95	1	- 19.12499 16	1	- 14.50159 06	1	- 11.21549 94	1
-0.4	1	- 40.60781 66	1	- 29.11387 47	1	- 21.48619 29	1	- 16.23364 77	1	- 12.50830 02	1
-0.3	1	- 41.77555 27	1	- 29.86556 96	1	- 21.97529 84	1	- 16.55161 78	1	- 12.71201 88	1
-0.2	1	- 37.31880 35	1	- 26.60926 26	1	- 19.52523 31	1	- 14.66357 72	1	- 11.22757 90	1
-0.1	1	- 24.52084 31	1	- 17.43482 36	1	- 12.75429 49	1	- 9.54685 36	1	- 7.28342 79	1
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	1	40.94831 46	1	29.07843 18	1	21.25805 97	1	15.91314 02	1	12.15161 32	1
0.2	2	10.35356 47	2	7.33513 18	2	5.34878 70	1	39.92835 76	1	30.39758 13	1
0.3	2	19.46942 07	2	13.76840 25	2	10.02088 59	2	7.46562 06	2	5.67167 17	2
0.4	2	32.28242 86	2	22.79263 07	2	16.56118 68	2	12.31686 84	2	9.34043 05	2
0.5	3	4.98179 83	2	35.12082 00	2	25.47973 52	2	18.91989 82	2	14.32454 80	2
0.6	3	7.33197 35	3	5.16165 31	2	37.39341 60	2	27.72559 77	2	20.95993 47	2
0.7	3	10.42873 45	3	7.33197 35	3	5.30441 56	2	39.27553 27	2	29.64952 35	2
0.8	3	14.45197 67	3	10.14763 99	3	7.33197 35	3	5.42169 25	2	40.87425 44	2
0.9	3	19.61647 10	3	13.75722 86	3	9.92775 42	3	7.33197 35	3	5.52055 89	3
1.0	3	26.17750 82	3	18.33714 84	3	13.21716 22	3	9.74960 83	3	7.33197 36	3

$$10^{-t} {}_1F_1[a; b; x]$$

$$x=9.0$$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	1	-8.900000	00	0	-44.000000	00	0	-29.000000	00	0	-21.500000	00	0	-17.000000	00
-0.9	2	-11.58229	18	1	-47.06960	04	1	-25.89886	67	1	-16.25732	51	1	-11.02632	13
-0.8	2	-24.27813	75	2	-9.74816	45	2	-5.29323	09	1	-32.75320	22	1	-21.87398	31
-0.7	2	-38.38234	83	2	-15.32409	84	2	-8.26992	60	1	-50.83377	07	1	-33.70796	64
-0.6	3	-5.28795	75	2	-21.03107	80	2	-11.30326	62	2	-6.91755	26	1	-45.65731	05
-0.5	3	-6.62068	16	2	-26.25211	13	2	-14.06438	22	2	-8.57840	43	2	-5.64186	81
-0.4	3	-7.60990	61	2	-30.09752	64	2	-16.08140	98	2	-9.78118	65	2	-6.41404	87
-0.3	3	-7.94036	79	2	-31.33369	14	2	-16.70254	11	2	-10.13406	43	2	-6.62844	83
-0.2	3	-7.18584	91	2	-28.29793	02	2	-15.05198	66	2	-9.11218	60	2	-5.94613	42
-0.1	3	-4.78278	15	2	-18.79747	15	2	-9.97775	31	2	-6.02698	66	1	-39.23623	79
0.0	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00
0.1	3	8.10308	38	2	31.75694	62	2	16.81142	73	2	10.12962	48	2	6.57992	17
0.2	3	20.70971	85	3	8.10308	38	3	4.28218	60	2	25.75481	43	2	16.69693	81
0.3	4	3.93063	86	3	15.35667	72	3	8.10308	39	3	4.86584	85	2	31.49394	93
0.4	4	6.57367	59	3	25.64717	57	3	13.51373	00	3	8.10308	39	3	5.23683	11
0.5	4	10.22712	25	4	3.98485	11	3	20.96831	62	3	12.55573	12	3	8.10308	39
0.6	4	15.16862	76	4	5.90279	85	3	31.02077	75	3	18.55086	15	3	11.95623	61
0.7	4	21.73562	62	4	8.44810	69	4	4.43426	09	3	26.48444	96	3	17.04788	07
0.8	5	3.03359	16	4	11.77714	66	4	6.17433	58	4	3.68332	96	3	23.68059	57
0.9	5	4.14598	16	4	16.07771	55	4	8.41941	51	4	5.01687	00	3	32.21650	68
1.0	5	5.56941	19	4	21.57431	43	4	11.28546	27	4	6.71721	11	4	4.30870	75

b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	0	-14.00000	00	0	-11.85714	28	0	-10.25000	00	0	-9.00000	00	0	-8.00000	00
-0.9	1	-7.88310	87	0	-58.61013	50	0	-44.93940	93	0	-35.33638	68	0	-28.37978	25
-0.8	1	-15.38318	74	1	-11.24015	50	1	-8.46300	77	1	-6.53007	42	0	-51.43541	70
-0.7	1	-23.52598	52	1	-17.05166	88	1	-12.72967	57	1	-9.73476	07	1	-7.59652	05
-0.6	1	-31.70896	61	1	-22.86319	49	1	-16.97478	44	1	-12.90664	72	1	-10.01136	01
-0.5	1	-39.03669	06	1	-28.03658	35	1	-20.73044	23	1	-15.69471	42	1	-12.11963	70
-0.4	1	-44.24331	42	1	-31.67413	73	1	-23.34167	79	1	-17.60998	03	1	-13.54924	07
-0.3	1	-45.60017	79	1	-32.55462	46	1	-23.92086	27	1	-17.99229	55	1	-13.79971	14
-0.2	1	-40.80619	43	1	-29.05749	37	1	-21.29381	76	1	-15.97113	38	1	-12.21317	47
-0.1	1	-26.85843	43	1	-19.07353	48	1	-13.93637	41	1	-10.41950	44	1	-7.94021	76
0.0	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00	0	1.00000	00
0.1	1	44.95811	23	1	31.88204	29	1	23.27505	92	1	17.39813	92	1	13.26621	51
0.2	2	11.38448	45	2	8.05506	28	2	5.86608	75	1	43.73217	79	1	33.24901	56
0.3	2	21.43707	55	2	15.14088	94	2	11.00591	15	2	8.18906	58	2	6.21332	81
0.4	2	35.59081	83	2	25.09772	90	2	18.21366	96	2	13.52913	41	2	10.24702	54
0.5	3	5.49915	09	2	38.72155	40	2	28.05822	52	2	20.80940	53	2	15.73604	92
0.6	3	8.10308	39	3	5.69778	23	3	4.12286	14	2	30.53303	84	2	23.05490	87
0.7	3	11.53893	23	3	8.10308	40	3	5.85547	02	3	4.33052	37	2	32.65347	74
0.8	3	16.00855	40	3	11.22774	11	3	8.10308	39	3	5.98502	62	3	4.50694	55
0.9	3	21.75325	09	3	15.23853	20	3	10.98428	79	3	8.10308	39	3	6.09425	84
1.0	3	29.06020	65	3	20.33372	45	3	14.63989	96	3	10.78702	83	3	8.10308	39

$x=9.1$

$10^{-t} {}_1F_1[a; b; x]$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	1	-9.00000 00	0	-44.50000 00	0	-29.33333 33	0	-21.75000 00	0	-17.20000 00	0
-0.9	2	-12.54963 90	1	-50.87536 50	1	-27.92221 57	1	-17.48230 08	1	-11.82623 04	1
-0.8	2	-26.42850 76	2	-10.59127 45	2	-5.73979 03	1	-35.44561 69	1	-23.62417 55	1
-0.7	3	-4.18937 47	2	-16.69775 18	2	-8.99576 71	2	-5.51990 50	1	-36.53803 16	1
-0.6	3	-5.78378 93	2	-22.96713 94	2	-12.32429 66	2	-7.53036 74	1	-49.62187 88	1
-0.5	3	-7.25460 78	2	-28.72325 13	2	-15.36535 53	2	-9.35785 13	2	-6.14520 15	2
-0.4	3	-8.35228 29	2	-32.98697 25	2	-17.60020 94	2	-10.68966 77	2	-6.99973 27	2
-0.3	3	-8.72833 87	2	-34.39618 77	2	-18.30993 88	2	-11.09409 84	2	-7.24641 88	2
-0.2	3	-7.91037 64	2	-31.11002 97	2	-16.52592 02	2	-9.99126 83	2	-6.51118 41	2
-0.1	3	-5.27229 73	2	-20.69497 65	2	-10.97100 93	2	-6.61859 50	1	-43.03378 54	1
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	3	8.95529 27	2	35.05241 22	2	18.53224 52	2	11.15205 23	2	7.23457 33	2
0.2	3	22.91602 76	3	8.95529 26	3	4.72668 10	2	28.39273 00	2	18.38399 20	2
0.3	4	4.35457 09	3	16.99237 61	3	8.95529 27	3	5.37104 04	2	34.72121 66	2
0.4	4	7.29108 92	3	28.41233 43	3	14.95286 54	3	8.95529 27	3	5.78064 92	3
0.5	4	11.35601 37	4	4.41952 32	3	23.22826 17	3	13.89261 18	3	8.95529 26	3
0.6	4	16.86144 52	4	6.55396 63	4	3.44029 70	3	20.54960 96	3	13.22907 21	3
0.7	4	24.18719 38	4	9.39024 08	4	4.92313 89	3	29.37073 37	3	18.88411 76	3
0.8	5	3.37928 54	4	13.10441 47	4	6.86241 97	4	4.08918 51	3	26.26017 77	3
0.9	5	4.62317 09	4	17.90820 48	4	9.36752 00	4	5.57558 44	4	3.57643 79	4
1.0	5	6.21666 94	4	24.05500 99	4	12.56921 08	4	7.47304 73	4	4.78823 93	4

b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-14.16666 67	0	-12.00000 00	0	-10.37500 00	0	-9.11111 11	0	-8.10000 00	0
-0.9	1	-8.43282 88	0	-62.53324 16	0	-47.82322 89	0	-37.50818 50	0	-30.04913 42	0
-0.8	1	-16.58000 33	1	-12.08963 27	1	-9.08374 48	1	-6.99453 14	0	-54.98044 28	0
-0.7	1	-25.45576 02	1	-18.41726 41	1	-13.72436 09	1	-10.47648 62	1	-8.16060 42	1
-0.6	1	-34.40638 24	1	-24.76761 24	1	-18.35860 15	1	-13.93594 50	1	-10.79207 94	1
-0.5	1	-42.45477 96	1	-30.44507 26	1	-22.47698 20	1	-16.99107 71	1	-13.10078 24	1
-0.4	1	-48.21385 63	1	-34.46699 90	1	-25.36329 99	1	-19.10774 00	1	-14.68064 94	1
-0.3	1	-49.78302 84	1	-35.49210 00	1	-26.04366 98	1	-19.56237 41	1	-14.98370 57	1
-0.2	1	-44.62524 59	1	-31.73548 27	1	-23.22615 61	1	-17.39811 60	1	-13.28755 28	1
-0.1	1	-29.42158 02	1	-20.86834 03	1	-15.22959 26	1	-11.37312 60	1	-8.65713 97	1
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	1	49.36541 66	1	34.96021 79	1	25.48717 14	1	19.02501 04	1	14.48597 30	1
0.2	2	12.51890 88	2	8.84642 07	2	6.43409 88	1	47.90439 11	1	36.37324 71	1
0.3	2	23.60471 18	2	16.65125 76	2	12.08874 10	2	8.98349 77	2	6.80750 30	2
0.4	2	39.23944 13	2	27.63720 56	2	20.03226 01	2	14.86185 64	2	11.24266 06	2
0.5	3	6.07031 71	3	4.26926 86	2	30.89900 07	2	22.88895 31	2	17.28790 66	2
0.6	3	8.95529 26	3	6.28968 53	3	4.54584 33	2	33.62616 86	2	25.36070 69	2
0.7	3	12.76714 51	3	8.95529 27	3	6.46385 88	3	4.77496 83	2	35.96324 06	2
0.8	3	17.73232 30	3	12.42264 46	3	8.95529 26	3	6.60697 84	3	4.96965 39	3
0.9	3	24.12186 86	3	16.87890 58	3	12.15310 62	3	8.95529 26	3	6.72766 30	3
1.0	3	32.25875 65	3	22.54685 48	3	16.21538 51	3	11.93468 92	3	8.95529 27	3

$$10^{-t} {}_1F_1[a; b; x]$$

$$x=9.2$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	1	-9.100000 00	0	-45.000000 00	0	-29.666666 67	0	-22.000000 00	0	-17.400000 00	0
-0.9	2	-13.60450 42	2	-5.50193 00	1	-30.12201 08	1	-18.81202 95	1	-12.69315 70	1
-0.8	2	-28.77800 48	2	-11.51122 92	2	-6.22639 30	1	-38.37545 05	1	-25.52605 93	1
-0.7	3	-4.57363 13	2	-18.19904 91	2	-9.78804 07	2	-5.99577 20	1	-39.61912 67	1
-0.6	3	-6.32715 84	2	-25.08621 42	2	-13.44046 50	2	-8.19945 25	2	-5.39450 87	2
-0.5	3	-7.95024 57	2	-31.43170 98	2	-16.78955 78	2	-10.21010 06	2	-6.69488 21	2
-0.4	3	-9.16797 65	2	-36.15806 34	2	-19.26509 62	2	-11.68436 07	2	-7.64024 07	2
-0.3	3	-9.59520 05	2	-37.76142 83	2	-20.07420 43	2	-12.14660 95	2	-7.92313 44	2
-0.2	3	-8.70839 61	2	-34.20387 50	2	-18.14569 48	2	-10.95623 94	2	-7.13073 86	2
-0.1	3	-5.81209 29	2	-22.78505 15	2	-12.06384 88	2	-7.26879 27	1	-47.20276 99	1
0.0	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0
0.1	3	9.89712 90	2	38.69051 65	2	20.42992 22	2	12.27833 95	2	7.95494 48	2
0.2	3	25.35699 15	3	9.89712 90	3	5.21739 79	2	31.30184 06	2	20.24249 68	2
0.3	4	4.82408 29	3	18.80201 91	3	9.89712 90	3	5.92877 32	2	38.28036 15	3
0.4	4	8.08645 12	3	31.47471 80	3	16.54502 22	3	9.89712 90	3	6.38103 29	3
0.5	4	12.60881 95	4	4.90140 65	3	25.73106 82	3	15.37162 35	3	9.89712 90	3
0.6	4	18.74192 88	4	7.27657 78	4	3.81523 85	3	22.76309 79	3	14.63721 39	3
0.7	5	2.69131 75	4	10.43676 23	4	5.46563 07	3	32.57028 94	3	20.91758 87	3
0.8	5	3.76403 93	4	14.58015 51	4	7.62670 16	4	4.53953 01	3	29.11964 84	3
0.9	5	5.15477 37	4	19.94537 40	4	10.42161 66	4	6.19613 77	4	3.97009 98	4
1.0	5	6.93839 47	5	2.68183 54	4	13.99780 49	4	8.31332 97	4	5.32081 88	4
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-14.33333 33	0	-12.14285 71	0	-10.50000 00	0	-9.22222 22	0	-8.20000 00	0
-0.9	1	-9.02763 00	1	-6.67709 39	0	-50.93293 05	0	-39.84595 55	0	-31.84281 39	0
-0.8	1	-17.87872 08	1	-13.01012 39	1	-9.75540 06	1	-7.49635 05	0	-58.80493 85	0
-0.7	1	-27.55395 34	1	-19.90007 61	1	-14.80298 40	1	-11.27972 06	1	-8.77063 56	1
-0.6	1	-37.34412 19	1	-26.83908 58	1	-19.86189 79	1	-15.05268 61	1	-11.63803 71	1
-0.5	1	-46.18307 94	1	-33.06894 61	1	-24.37738 36	1	-18.39991 22	1	-14.16573 82	1
-0.4	2	-5.25509 36	1	-37.51406 78	1	-27.56631 10	1	-20.73794 27	1	-15.91063 63	1
-0.3	2	-5.43582 13	1	-38.70138 00	1	-28.36021 24	1	-21.27375 52	1	-16.27275 60	1
-0.2	1	-48.80793 54	1	-34.66513 43	1	-25.33768 55	1	-18.95565 47	1	-14.45889 46	1
-0.1	1	-32.23229 13	1	-22.83430 66	1	-16.64455 87	1	-12.41536 18	1	-9.43980 78	1
0.0	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0
0.1	2	5.42099 20	1	38.34008 60	1	27.91346 85	1	20.80747 55	1	15.82094 65	1
0.2	2	13.76728 68	2	9.71634 18	2	7.05783 48	2	5.24810 42	1	39.79668 41	1
0.3	2	25.99273 45	2	18.31343 34	2	13.27915 12	2	9.85593 97	2	7.45933 88	2
0.4	3	4.32633 74	2	30.43498 29	2	22.03374 19	2	16.32708 42	2	12.33615 10	2
0.5	3	6.70090 15	3	4.70724 12	2	34.02884 14	2	25.17775 28	2	18.99416 59	2
0.6	3	9.89712 89	3	6.94317 15	3	5.01235 75	2	37.03416 69	2	27.89864 69	2
0.7	3	14.12590 46	3	9.89712 90	3	7.13555 37	3	5.26516 55	2	39.61005 32	2
0.8	3	19.64120 41	3	13.74453 97	3	9.89712 90	3	7.29365 60	3	5.48000 68	3
0.9	3	26.74740 08	3	18.69539 69	3	13.44612 98	3	9.89712 90	3	7.42699 21	3
1.0	4	3.58076 31	3	24.99995 85	3	17.95998 39	3	13.20429 35	3	9.89712 91	3

$x=9.3$

$$10^{-t} {}_1F_1[a; b; x]$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	1	-9.200000 00	0	-45.500000 00	0	-30.000000 00	0	-22.250000 00	0	-17.600000 00	0
-0.9	2	-14.75505 63	2	-5.95325 73	1	-32.51430 96	1	-20.25592 20	1	-13.63304 49	1
-0.8	2	-31.34547 95	2	-12.51521 42	2	-6.75673 74	1	-41.56435 60	1	-27.59329 64	1
-0.7	3	-4.99418 16	2	-19.84009 09	2	-10.65296 94	2	-6.51461 80	1	-42.97419 46	1
-0.6	3	-6.92268 47	2	-27.40588 29	2	-14.66080 34	2	-8.93008 69	2	-5.86601 82	2
-0.5	3	-8.71366 81	2	-34.40057 57	2	-18.34884 11	2	-11.14207 32	2	-7.29526 50	2
-0.4	3	-10.06428 86	2	-39.63853 68	2	-21.09029 69	2	-12.77356 71	2	-8.34079 11	2
-0.3	3	-10.54890 17	3	-4.14595 84	2	-22.01079 93	2	-13.30061 17	2	-8.66425 80	2
-0.2	3	-9.58739 65	2	-37.60786 75	2	-19.92585 34	2	-12.01557 18	2	-7.81011 55	2
-0.1	3	-6.40734 72	2	-25.08732 41	2	-13.26631 46	2	-7.98342 60	2	-5.17798 59	2
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	3	10.93801 92	3	4.27069 30	2	22.52268 39	2	13.51908 00	2	8.74767 04	2
0.2	3	28.05753 79	3	10.93801 91	3	5.75914 94	2	34.51009 83	2	22.28994 55	2
0.3	4	5.34406 37	3	20.80408 24	3	10.93801 92	3	6.54451 79	3	4.22055 98	3
0.4	4	8.96820 28	4	3.48662 02	3	18.30645 21	3	10.93801 92	3	7.04387 31	3
0.5	4	13.99908 27	4	5.43561 19	3	28.50277 52	3	17.00786 00	3	10.93801 91	3
0.6	4	20.83077 45	4	8.07843 82	4	4.23087 64	3	25.21434 67	3	16.19502 87	3
0.7	5	2.99441 02	4	11.59917 83	4	6.06759 14	4	3.61170 11	3	23.16943 46	3
0.8	5	4.19223 66	4	16.22087 84	4	8.47557 72	4	5.03922 15	3	32.28928 23	3
0.9	5	5.74694 80	4	22.21240 30	4	11.59348 59	4	6.88534 10	4	4.40687 88	4
1.0	5	7.74308 55	5	2.98963 14	4	15.58748 14	4	9.24743 93	4	5.91228 60	4
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-14.50000 00	0	-12.28571 43	0	-10.62500 00	0	-9.33333 33	0	-8.30000 00	0
-0.9	1	-9.67145 73	1	-7.13503 95	0	-54.28776 15	0	-42.36365 34	0	-33.77112 77	0
-0.8	1	-19.28840 11	1	-14.00786 24	1	-10.48238 72	1	-8.03872 68	0	-62.93249 12	0
-0.7	1	-29.83576 83	1	-21.51055 48	1	-15.97293 42	1	-12.14980 99	1	-9.43055 73	1
-0.6	1	-40.54419 47	1	-29.09273 01	1	-21.49535 19	1	-16.26458 91	1	-12.55492 04	1
-0.5	1	-50.25039 64	1	-35.92796 85	1	-26.44559 81	1	-19.93129 51	1	-15.32192 15	1
-0.4	2	-5.72890 46	1	-40.83898 92	1	-29.96738 78	1	-22.51262 24	1	-17.24805 27	1
-0.3	2	-5.93631 28	1	-42.20808 02	1	-30.88854 03	1	-23.13945 35	1	-17.67642 74	1
-0.2	2	-5.33893 54	1	-37.87046 52	1	-27.64531 60	1	-20.65593 28	1	-15.73614 60	1
-0.1	1	-35.31474 32	1	-24.98796 72	1	-18.19290 58	1	-13.55458 59	1	-10.29436 62	1
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	2	5.95352 98	1	42.05147 21	1	30.57490 38	1	22.76059 51	1	17.28217 01	1
0.2	2	15.14112 70	2	10.67267 94	2	7.74280 83	2	5.75017 25	1	43.54831 54	1
0.3	2	28.62363 74	2	20.14275 47	2	14.58790 24	2	10.81411 48	2	8.17448 53	2
0.4	3	4.77013 19	2	33.51743 26	2	24.23660 23	2	17.93807 70	2	13.53719 06	2
0.5	3	7.39709 24	3	5.19028 73	2	37.47726 82	2	27.69696 41	2	20.87028 63	2
0.6	3	10.93801 91	3	7.66465 55	3	5.52689 46	2	40.78919 95	2	30.69221 42	2
0.7	3	15.62907 16	3	10.93801 92	3	7.87714 93	3	5.80583 61	3	4.36283 58	3
0.8	3	21.75503 42	3	15.20690 83	3	10.93801 90	3	8.05180 20	3	6.04292 08	3
0.9	3	29.65763 04	3	20.70687 50	3	14.87654 35	3	10.93801 91	3	8.19911 48	3
1.0	4	3.97450 49	3	27.71897 83	3	19.89181 29	3	14.60878 49	3	10.93801 92	3

$$10^{-t} {}_1F_1[a; b; x]$$

$$x=9.4$$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	1	-9.300000	00	0	-46.000000	00	0	-30.333333	33	0	-22.500000	00	0	-17.800000	00
-0.9	2	-16.01023	94	2	-6.44492	79	1	-35.11665	99	1	-21.82425	25	1	-14.65237	55
-0.8	2	-34.15158	92	2	-13.61109	70	2	-7.33486	91	1	-45.03599	97	1	-29.84080	51
-0.7	3	-5.45451	13	2	-21.63413	98	2	-11.59736	69	2	-7.08042	54	1	-46.62831	57
-0.6	3	-7.57544	18	2	-29.94543	31	2	-15.99521	23	2	-9.72805	22	2	-6.38035	37
-0.5	3	-9.55154	99	2	-37.65520	38	2	-20.05620	90	2	-12.16135	61	2	-7.95111	98
-0.4	3	-11.04925	25	3	-4.34588	83	2	-23.09143	60	2	-13.96639	53	2	-9.10710	58
-0.3	3	-11.59819	52	3	-4.55238	49	2	-24.13672	08	2	-14.56600	61	2	-9.47600	36
-0.2	3	-10.55563	12	3	-4.13532	84	2	-21.88239	83	2	-13.17857	95	2	-8.55515	61
-0.1	3	-7.06377	39	2	-27.62342	97	2	-14.58946	76	2	-8.76892	76	2	-5.68053	63
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	3	12.08838	07	3	4.71410	43	2	24.83063	74	2	14.88595	11	2	9.62005	70
0.2	3	31.04523	72	3	12.08838	06	3	6.35724	95	2	38.04834	26	2	24.54562	15
0.3	4	5.91992	57	3	23.01900	12	3	12.08838	07	3	7.22431	60	3	4.65346	75
0.4	4	9.94569	63	4	3.86220	74	3	20.25513	14	3	12.08838	08	3	7.77567	48
0.5	4	15.54182	10	4	6.02780	20	3	31.57220	99	3	18.81801	60	3	12.08838	08
0.6	4	23.15095	13	4	8.96820	27	4	4.69161	41	3	27.92884	09	3	17.91840	81
0.7	5	3.33138	86	4	12.89025	89	4	6.73551	38	4	4.00484	50	3	25.66305	68
0.8	5	4.66874	98	4	18.04492	16	4	9.41836	38	4	5.59364	49	4	3.58026	20
0.9	5	6.40654	43	4	24.73505	86	4	12.89621	11	4	7.65075	20	4	4.89147	94
1.0	5	8.64020	23	5	3.33244	51	4	17.35628	64	4	10.28579	47	4	6.56912	15

b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	0	-14.66666	67	0	-12.42857	14	0	-10.75000	00	0	-9.44444	44	0	-8.40000	00
-0.9	1	-10.36861	07	1	-7.63011	93	0	-57.90866	06	0	-45.07644	06	0	-35.84527	08
-0.8	1	-20.81892	90	1	-15.08964	25	1	-11.26950	92	1	-8.62513	64	1	-6.73887	38
-0.7	1	-32.31780	90	1	-23.26010	14	1	-17.24226	69	1	-13.09257	68	1	-10.14465	99
-0.6	1	-44.03066	26	1	-31.54505	37	1	-23.27061	60	1	-17.58007	00	1	-13.54892	48
-0.5	2	-5.46882	54	1	-39.04374	94	1	-28.69686	48	1	-21.59622	27	1	-16.57742	08
-0.4	2	-6.24659	67	1	-44.46764	13	1	-32.58476	51	1	-24.44492	60	1	-18.70256	01
-0.3	2	-6.48387	75	1	-46.04026	08	1	-33.64841	00	1	-25.17370	10	1	-19.20516	97
-0.2	2	-5.84080	10	1	-41.37781	01	1	-30.16757	40	1	-22.51228	60	1	-17.12909	15
-0.1	1	-38.69549	33	1	-27.34746	82	1	-19.88739	01	1	-14.79997	30	1	-11.22754	13
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	2	6.53896	15	1	46.12717	00	1	33.49449	56	1	24.90090	03	1	18.88174	52
0.2	2	16.65310	48	2	11.72407	55	2	8.49508	10	2	6.30099	45	1	47.65996	44
0.3	2	31.52221	66	2	22.15611	56	2	16.02683	53	2	11.86651	50	2	8.95915	00
0.4	3	5.25959	72	2	36.91362	58	2	26.66120	35	2	19.70942	79	2	14.85643	94
0.5	3	8.16572	35	3	5.72305	64	3	4.12768	27	2	30.46989	56	2	22.93328	21
0.6	3	12.08838	06	3	8.46122	06	3	6.09440	95	3	4.49267	29	2	33.76727	76
0.7	3	17.29197	71	3	12.08838	07	3	8.69592	81	3	6.40218	86	3	4.80561	28
0.8	3	24.09577	30	3	16.82466	25	3	12.08838	07	3	8.88886	29	3	6.66382	12
0.9	4	3.28833	35	3	22.93422	86	3	16.45893	10	3	12.08838	06	3	9.05161	62
1.0	4	4.41133	87	3	30.73265	54	3	22.03092	60	3	16.16248	01	3	12.08838	08

$x=9.5$

$$10^{-t} {}_1F_1[a; b; x]$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	1	-9.40000 00	0	-46.50000 00	0	-30.66666 67	0	-22.75000 00	0	-18.00000 00	0
-0.9	2	-17.37984 50	2	-6.98067 10	1	-37.94823 82	1	-23.52823 98	1	-15.75822 16	1
-0.8	2	-37.21897 52	2	-14.80749 22	2	-7.96521 36	1	-48.81624 93	1	-32.28487 48	1
-0.7	3	-5.95844 43	2	-23.59573 35	2	-12.62869 47	2	-7.69755 14	1	-50.60890 51	1
-0.6	3	-8.29100 07	2	-32.72602 66	2	-17.45454 43	2	-10.59967 99	2	-6.94149 60	2
-0.5	3	-10.47122 80	3	-4.12234 39	2	-21.92592 96	2	-13.27626 71	2	-8.66767 09	2
-0.4	3	-12.13170 68	3	-4.76526 19	2	-25.28567 34	2	-15.27284 07	2	-9.94545 73	2
-0.3	3	-12.75272 05	3	-4.99907 48	2	-26.47065 36	2	-15.95366 68	2	-10.36519 03	2
-0.2	3	-11.62219 76	3	-4.54745 70	2	-24.03293 68	2	-14.45550 19	2	-9.37227 63	2
-0.1	3	-7.78767 64	2	-30.41721 59	2	-16.04548 90	2	-9.63237 64	2	-6.23236 05	2
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	3	13.35972 68	3	5.20363 50	2	27.37596 75	2	16.39182 50	2	10.58015 23	2
0.2	4	3.43505 89	3	13.35972 67	3	7.01756 66	3	4.19505 99	2	27.03078 26	2
0.3	4	6.55766 08	3	25.46938 06	3	13.35972 69	3	7.97483 88	3	5.13092 44	3
0.4	4	11.02929 35	4	4.27813 99	3	22.41094 27	3	13.35972 68	3	8.58362 07	3
0.5	4	17.25368 72	4	6.68425 07	4	3.49712 85	3	20.82055 79	3	13.35972 68	3
0.6	4	25.72794 96	4	9.95546 85	4	5.20232 99	3	30.93479 03	3	19.82492 97	3
0.7	5	3.70601 90	4	14.32417 56	4	7.47659 67	4	4.44062 12	3	28.42436 09	3
0.8	5	5.19899 50	4	20.07265 08	4	10.46540 07	4	6.20877 18	4	3.96968 21	4
0.9	5	7.14118 37	5	2.75419 79	4	14.34432 30	4	8.50075 77	4	5.42911 70	4
1.0	5	9.64027 74	5	3.71423 22	4	19.32427 74	4	11.43996 58	4	7.29851 64	4

b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-14.83333 33	0	-12.57142 86	0	-10.87500 00	0	-9.55555 56	0	-8.50000 00	0
-0.9	1	-11.12377 67	1	-8.16555 44	0	-61.81840 67	0	-48.00080 21	0	-38.07740 15	0
-0.8	1	-22.48109 02	1	-16.26287 11	1	-12.12199 93	1	-9.25936 21	1	-7.22015 51	1
-0.7	1	-35.01820 94	1	-25.16115 86	1	-18.61976 46	1	-14.11436 45	1	-10.91761 25	1
-0.6	1	-47.82983 09	1	-34.21408 95	1	-25.20040 76	1	-19.00830 54	1	-14.62679 96	1
-0.5	2	-5.95311 49	1	-42.43991 63	1	-31.14783 28	1	-23.40669 82	1	-17.94105 68	1
-0.4	2	-6.81230 85	1	-48.42834 68	1	-35.43838 29	1	-26.54921 79	1	-20.28470 57	1
-0.3	2	-7.08301 07	1	-50.22866 06	1	-36.66144 69	1	-27.39206 30	1	-20.87040 28	1
-0.2	2	-6.39061 68	1	-45.21604 54	1	-32.92475 82	1	-24.53931 29	1	-18.64843 30	1
-0.1	1	-42.40371 38	1	-29.93272 73	1	-21.74199 99	1	-16.16157 67	1	-12.24669 73	1
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	2	7.18257 65	1	50.60323 79	1	36.69753 16	1	27.24653 76	1	20.63294 46	1
0.2	2	18.31718 20	2	12.88004 10	2	9.32131 84	2	6.90534 98	2	5.21665 64	2
0.3	2	34.71580 67	2	24.37212 46	2	17.60898 25	2	13.02247 85	2	9.82015 37	2
0.4	3	5.79944 47	3	4.06556 13	2	29.32997 57	2	21.65719 85	2	16.30562 04	2
0.5	3	9.01434 09	3	6.31067 85	3	4.54633 96	2	33.52222 14	2	25.20188 02	2
0.6	3	13.35972 67	3	9.34068 87	3	6.72036 91	3	4.94858 50	2	37.15233 45	2
0.7	3	19.13157 81	3	13.35972 68	3	9.59993 18	3	7.05997 01	3	5.29352 26	3
0.8	3	26.68772 79	3	18.61429 61	3	13.35972 68	3	9.81306 13	3	7.34869 39	3
0.9	4	3.64586 15	3	25.40057 99	3	18.20942 23	3	13.35972 67	3	9.99287 22	3
1.0	4	4.89596 28	4	3.40728 31	3	24.39952 16	3	17.88121 40	3	13.35972 68	3

$$10^{-t} {}_1F_1[a; b; x]$$

$$x=9.6$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	1	-9.500000	00 0	-47.000000	00 0	-31.000000	00 0	-23.000000	00 0	-18.200000	00 0
-0.9	2	-18.87459	60 2	-7.56456	68 1	-41.03000	50 1	-25.38013	50 1	-16.95829	77 1
-0.8	2	-40.57245	27 2	-16.11383	43 2	-8.65261	33 2	-5.29333	81 1	-34.94329	77 1
-0.7	3	-6.51017	60 2	-25.74080	68 2	-13.75512	49 2	-8.37076	39 2	-5.49459	34 2
-0.6	3	-9.07548	00 2	-35.77088	32 2	-19.05069	65 2	-11.55190	53 2	-7.55380	07 2
-0.5	3	-11.48076	79 3	-4.51358	64 2	-23.97365	98 2	-14.49592	49 2	-9.45064	09 2
-0.4	3	-13.32137	64 3	-5.22565	89 2	-27.69185	76 2	-16.70387	16 2	-10.86272	40 2
-0.3	3	-14.02309	31 3	-5.49004	70 2	-29.03313	95 2	-17.47553	93 2	-11.33930	25 2
-0.2	3	-12.79712	43 3	-5.00096	60 2	-26.39684	42 2	-15.85759	65 2	-10.26852	42 2
-0.1	3	-8.58600	92 2	-33.49497	09 2	-17.64779	50 2	-10.58156	14 2	-6.83833	11 2
0.0	0	1.00000	00 0	1.00000	00 0	1.00000	00 0	1.00000	00 0	1.00000	00 0
0.1	3	14.76478	15 3	5.74408	78 2	30.18314	89 2	18.05089	14 2	11.63682	02 2
0.2	4	3.80073	25 3	14.76478	15 3	7.74658	02 3	4.62544	09 2	29.76886	26 2
0.3	4	7.26390	25 3	28.18022	32 3	14.76478	15 3	8.80345	26 3	5.65752	54 3
0.4	4	12.23047	50 4	4.73874	21 3	24.79587	93 3	14.76478	15 3	9.47564	30 3
0.5	4	19.15314	68 4	7.41190	89 4	3.87353	26 3	23.03591	00 3	14.76478	15 3
0.6	5	2.85900	57 4	11.05087	69 4	5.76842	87 4	3.42634	25 3	21.93403	52 3
0.7	5	4.12248	44 4	15.91665	46 4	8.29882	25 4	4.92363	88 3	31.48202	42 3
0.8	5	5.78899	20 4	22.32668	37 4	11.62816	12 4	6.89122	30 4	4.40130	52 3
0.9	5	7.95934	28 5	3.06649	95 4	15.95395	75 4	9.44466	42 4	6.02557	45 3
1.0	5	10.75503	53 5	4.13939	23 4	21.51374	59 4	12.72280	09 4	8.10844	96 3

b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-15.00000	00 0	-12.71428	57 0	-11.00000	00 0	-9.66666	67 0	-8.60000	00 0
-0.9	1	-11.94206	32 1	-8.74485	29 1	-6.60417	87 0	-5.11546	60 0	-4.04807	27 0
-0.8	1	-24.28665	27 1	-17.53562	27 1	-13.04555	73 1	-9.94552	07 1	-7.74012	41 1
-0.7	1	-37.95677	74 1	-27.22730	48 1	-20.11500	29 1	-15.22208	30 1	-11.75449	82 1
-0.6	2	-5.19704	63 1	-37.11953	69 1	-27.29860	74 1	-20.55930	28 1	-15.79589	92 1
-0.5	2	-6.48168	36 1	-46.14230	56 1	-33.81669	16 1	-25.37582	46 1	-19.42245	10 1
-0.4	2	-7.43057	40 2	-5.27521	08 1	-38.55004	92 1	-28.84119	39 1	-22.00600	47 1
-0.3	2	-7.73864	20 2	-5.48069	60 1	-39.95132	17 1	-29.81156	34 1	-22.68460	63 1
-0.2	2	-6.99302	16 1	-49.41683	90 1	-35.93910	84 1	-26.75299	57 1	-20.30587	69 1
-0.1	1	-46.47145	14 1	-32.76560	47 1	-23.77207	73 1	-17.65041	30 1	-13.35989	71 1
0.0	0	1.00000	00 0	1.00000	00 0	1.00000	00 0	1.00000	00 0	1.00000	00 0
0.1	2	7.89019	56 2	5.55193	25 1	40.21179	59 1	29.81742	83 1	22.55032	67 1
0.2	2	20.14873	61 2	14.15104	33 2	10.22885	00 2	7.56848	98 2	5.71064	66 2
0.3	2	38.23454	25 2	26.81127	89 2	19.34868	65 2	14.29227	48 2	10.76499	06 2
0.4	3	6.39487	22 3	4.47787	25 2	32.26762	56 2	23.79906	62 2	17.89762	55 2
0.5	3	9.95127	75 3	6.95881	42 3	5.00765	31 2	36.88222	42 2	27.69669	34 2
0.6	3	14.76478	15 3	10.31169	74 3	7.41080	54 3	5.45096	63 2	40.87877	62 2
0.7	3	21.16663	09 3	14.76478	16 3	10.59804	06 3	7.78552	18 3	5.83118	07 2
0.8	3	29.55780	79 3	20.59405	22 3	14.76478	15 3	10.83347	77 3	8.10414	39 2
0.9	4	4.04212	44 3	28.13152	40 3	20.14585	83 3	14.76478	14 3	11.03213	28 2
1.0	4	5.43358	63 4	3.77747	78 3	27.02217	16 3	19.78250	05 3	14.76478	16 2

$x=9.7$

$10^{-t} {}_1F_1[a; b; x]$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	1	-9.60000 00	0	-47.50000 00	0	-31.33333 33	0	-23.25000 00	0	-18.40000 00	0
-0.9	2	-20.50623 33	2	-8.20107 85	1	-44.38487 15	1	-27.39331 76	1	-18.26102 28	1
-0.8	3	-4.42392 24	2	-17.54045 51	2	-9.40236 68	2	-5.74183 09	1	-37.83550 67	2
-0.7	3	-7.11430 98	2	-28.08682 94	2	-14.98560 81	2	-9.10528 02	2	-5.96721 73	2
-0.6	3	-9.93559 95	2	-39.10548 14	2	-20.79671 26	2	-12.59232 51	2	-8.22203 54	2
-0.5	3	-12.58903 55	3	-4.94260 69	2	-26.21658 10	2	-15.83032 80	2	-10.30629 90	2
-0.4	3	-14.62896 18	3	-5.73113 02	2	-30.33069 09	2	-18.27152 59	2	-11.86644 71	2
-0.3	3	-15.42100 44	3	-6.02972 39	2	-31.84676 31	2	-19.14474 44	2	-12.40655 42	2
-0.2	3	-14.09146 49	3	-5.50003 34	2	-28.99544 10	2	-17.39724 13	2	-11.25164 23	2
-0.1	3	-9.46644 48	2	-36.88567 20	2	-19.41116 23	2	-11.62505 50	2	-7.50380 65	2
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	3	16.31760 70	3	6.34076 60	2	33.27918 63	2	19.87879 34	2	12.79982 41	2
0.2	4	4.20527 59	3	16.31760 70	3	8.55144 58	3	5.10011 90	2	32.78569 53	3
0.3	4	8.04599 35	3	31.17918 43	3	16.31760 71	3	9.71829 22	3	6.23834 09	3
0.4	4	13.56196 03	4	5.24880 12	3	27.43426 69	3	16.31760 71	3	10.46050 06	3
0.5	4	21.26067 43	4	8.21847 77	4	4.29034 41	3	25.48666 51	3	16.31760 71	3
0.6	5	3.17686 64	4	12.26622 73	4	6.39589 82	4	3.79493 14	3	24.26722 86	4
0.7	5	4.58543 03	4	17.68514 64	4	9.21104 14	4	5.45900 41	4	3.48677 90	4
0.8	5	6.44543 17	4	24.83213 79	4	12.91937 58	4	7.64833 91	4	4.87969 19	4
0.9	5	8.87045 07	5	3.41394 82	4	17.74303 41	4	10.49279 74	4	6.68726 33	4
1.0	5	11.99752 70	5	4.61281 92	4	23.94946 53	4	14.14856 71	4	9.00777 43	4
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-15.16666 67	0	-12.85714 29	0	-11.12500 00	0	-9.77777 78	0	-8.70000 00	0
-0.9	1	-12.82903 93	1	-9.37183 63	1	-7.06057 77	0	-54.55748 89	0	-43.06958 21	0
-0.8	1	-26.24846 00	1	-18.91670 16	1	-14.04639 29	1	-10.68809 32	1	-8.30207 68	1
-0.7	1	-41.15515 10	1	-29.47336 00	1	-21.73842 49	1	-16.42326 17	1	-12.66085 01	1
-0.6	2	-5.64840 09	1	-40.28291 88	1	-29.58036 78	1	-22.24397 76	1	-17.06423 89	1
-0.5	2	-7.05866 41	1	-50.17917 42	1	-36.72331 70	1	-27.51790 77	1	-21.03209 81	1
-0.4	2	-8.10636 04	2	-5.74728 57	1	-41.94361 46	1	-31.33800 65	1	-23.87903 06	1
-0.3	2	-8.45617 75	2	-5.98120 61	1	-43.54395 10	1	-32.45082 44	1	-24.66142 00	1
-0.2	2	-7.65310 67	2	-5.40149 19	1	-39.23499 52	1	-29.17083 23	1	-22.11422 98	1
-0.1	1	-50.93391 49	1	-35.87009 83	1	-25.99444 91	1	-19.27855 47	1	-14.57596 90	1
0.0	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0	1.00000 00	0
0.1	2	8.66822 52	2	6.09190 30	1	44.06781 69	1	32.63544 14	1	24.64986 20	1
0.2	2	22.16470 60	2	15.54860 26	2	11.22573 65	2	8.29618 50	2	6.25217 78	2
0.3	3	4.21116 45	2	29.49615 72	2	21.26173 46	2	15.68719 76	2	11.80189 59	2
0.4	3	7.05161 51	3	4.93219 14	2	35.50137 01	2	26.15448 86	2	19.64663 34	2
0.5	3	10.98573 51	3	7.67370 90	3	5.51598 40	2	40.58106 05	2	30.44040 92	2
0.6	3	16.31760 70	3	11.38378 52	3	8.17237 44	3	6.00456 91	3	4.49811 85	3
0.7	3	23.41788 06	3	16.31760 72	3	11.70006 08	3	8.58584 08	3	6.42367 59	3
0.8	3	32.73579 89	3	22.78410 86	3	16.31760 71	3	11.96013 96	3	8.93745 95	3
0.9	4	4.48130 72	3	31.15539 28	3	22.28797 19	3	16.31760 71	3	12.17961 21	3
1.0	4	6.02998 45	4	4.18775 71	3	29.92607 54	3	21.88571 13	3	16.31760 73	3

$$10^{-t} {}_1F_1[a; b; x]$$

$$x=9.8$$

b =		0.1		0.2		0.3		0.4		0.5	
a	t		t		t		t		t		t
-1.0	1	-9.700000 00	0	-48.000000 00	0	-31.666666 67	0	-23.500000 00	0	-18.600000 00	0
-0.9	2	-22.28761 64	2	-8.89509 07	1	-48.03788 54	1	-29.58240 10	1	-19.67558 44	1
-0.8	3	-4.82491 10	2	-19.09867 07	2	-10.22027 22	2	-6.23048 28	1	-40.98273 00	0
-0.7	3	-7.77589 68	2	-30.65295 26	2	-16.32994 89	2	-9.90681 04	2	-6.48234 55	2
-0.6	3	-10.87873 97	3	-4.27577 82	2	-22.70689 39	2	-13.72926 13	2	-8.95141 77	2
-0.5	3	-13.80577 81	3	-5.41309 50	2	-28.67354 92	2	-17.29043 98	2	-11.24151 38	2
-0.4	3	-16.06623 80	3	-6.28612 91	2	-33.22491 52	2	-19.98901 56	2	-12.96489 76	2
-0.3	3	-16.95933 06	3	-6.62297 13	2	-34.93635 43	2	-20.97569 66	2	-13.57596 12	2
-0.2	3	-15.51740 48	3	-6.04926 09	2	-31.85218 96	2	-19.08804 58	2	-12.33013 67	2
-0.1	3	-10.43744 83	2	-40.62126 39	2	-21.35186 53	2	-12.77229 06	2	-8.23467 80	2
0.0	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0
0.1	3	18.03374 48	3	6.99952 61	2	36.69387 37	2	21.89277 53	2	14.07991 84	2
0.2	4	4.65281 62	3	18.03374 47	3	9.44006 45	3	5.62366 44	2	36.10976 10	0
0.3	4	8.91206 30	4	3.44968 53	3	18.03374 48	3	10.72834 01	3	6.87896 51	3
0.4	4	15.03784 15	4	5.81361 65	3	30.35301 15	3	18.03374 50	3	11.54786 54	3
0.5	4	23.59897 19	4	9.11248 87	4	4.75189 17	3	28.19781 03	3	18.03374 48	3
0.6	5	3.52986 04	4	13.61460 21	4	7.09137 22	4	4.20307 25	3	26.84829 49	0
0.7	5	5.10001 58	4	19.64901 47	4	10.22306 61	4	6.05237 40	4	3.86167 96	4
0.8	5	7.17574 99	5	2.76169 07	4	14.35317 10	4	8.48825 89	4	5.40989 43	4
0.9	5	9.88499 47	5	3.80047 53	4	19.73145 08	4	11.65661 49	4	7.42129 15	4
1.0	5	13.38227 93	5	5.13995 54	5	2.66589 66	4	15.73310 69	4	10.00631 32	4
b =		0.6		0.7		0.8		0.9		1.0	
a	t		t		t		t		t		t
-1.0	0	-15.33333 33	0	-13.00000 00	0	-11.25000 00	0	-9.88888 89	0	-8.80000 00	0
-0.9	1	-13.79077 68	1	-10.05066 70	1	-7.55397 35	0	-58.23048 58	0	-45.85956 71	0
-0.8	1	-28.38052 96	1	-20.41570 89	1	-15.13127 10	1	-11.49195 76	1	-8.90959 84	1
-0.7	1	-44.63697 03	1	-31.91550 17	1	-23.50141 98	1	-17.72610 48	1	-13.64269 29	1
-0.6	2	-6.14048 64	1	-43.72775 14	1	-32.06223 05	1	-24.07423 56	1	-18.44055 27	1
-0.5	2	-7.68858 00	2	-5.45814 24	1	-39.88942 80	1	-29.84856 63	1	-22.78144 73	1
-0.4	2	-8.84511 04	2	-6.26277 48	1	-45.64516 81	1	-34.05840 27	1	-25.91751 36	1
-0.3	2	-9.24154 52	2	-6.52843 98	1	-47.46770 72	1	-35.33021 73	1	-26.81575 17	1
-0.2	2	-8.37646 12	2	-5.90483 78	1	-42.83912 56	1	-31.81198 33	1	-24.08750 21	1
-0.1	2	-5.58297 85	1	-39.27255 06	1	-28.42757 15	1	-21.05923 19	1	-15.90458 10	1
0.0	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0	1.000000 00	0
0.1	2	9.52371 56	2	6.68503 02	1	48.29913 65	1	35.72458 90	1	26.94906 86	1
0.2	2	24.38375 03	2	17.08539 83	2	12.32084 18	2	9.09477 60	2	6.84587 30	2
0.3	3	4.63837 40	2	32.45163 07	2	23.36550 20	2	17.21966 75	2	12.93991 99	2
0.4	3	7.77600 13	3	5.43281 26	2	39.06119 06	2	28.74488 30	2	21.56823 82	2
0.5	3	12.12787 48	3	8.46225 38	3	6.07614 05	3	4.46530 51	2	33.45800 19	2
0.6	3	18.03374 47	3	12.56748 46	3	9.01241 98	3	6.61463 26	3	4.94976 60	3
0.7	3	25.90827 26	3	18.03374 48	3	12.91682 16	3	9.46864 77	3	7.07661 87	3
0.8	4	3.62546 71	3	25.20678 29	3	18.03374 47	3	13.20412 01	3	9.85668 22	3
0.9	4	4.96804 54	4	3.45035 47	3	24.65758 83	3	18.03374 47	3	13.44659 05	3
1.0	4	6.69155 96	4	4.64244 97	4	3.31413 39	3	24.21227 11	3	18.03374 49	3

$x=9.9$ $10^{-t} {}_1F_1[a; b; x]$

b =		0.1		0.2		0.3		0.4		0.5		
a	t		t		t		t		t		t	
-1.0	1	-9.800000	00	0	-48.500000	00	0	-32.000000	00	0	-18.800000	00
-0.9	2	-24.23282	87	2	-9.65194	76	2	-5.20164	31	1	-31.96334	80
-0.8	3	-5.26348	03	2	-20.80087	65	2	-11.11267	48	2	-6.76298	97
-0.7	3	-8.50048	02	2	-33.46017	51	2	-17.79888	71	2	-10.78160	42
-0.6	3	-11.91300	71	3	-4.67584	68	2	-24.79692	15	2	-14.97183	12
-0.5	3	-15.14171	21	3	-5.92910	34	2	-31.36525	96	2	-18.88828	25
-0.4	3	-17.64616	11	3	-6.89555	19	2	-36.39951	22	2	-21.87084	21
-0.3	3	-18.65225	35	3	-7.27514	48	2	-38.32921	57	2	-22.98423	12
-0.2	3	-17.08837	64	3	-6.65371	68	2	-34.99290	99	2	-20.94497	46
-0.1	3	-11.50835	93	3	-4.47369	61	2	-23.48783	09	2	-14.03365	00
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	3	19.93037	03	3	7.72683	47	2	40.46008	45	2	24.11184	82
0.2	4	5.14791	85	3	19.93037	03	3	10.42116	14	3	6.20111	94
0.3	4	9.87110	91	4	3.81670	64	3	19.93037	04	3	11.84351	59
0.4	4	16.67373	07	4	6.43905	45	4	3.35818	73	3	19.93037	04
0.5	4	26.19321	10	4	10.10339	39	4	5.26296	75	3	31.19698	41
0.6	5	3.92185	30	4	15.11050	57	4	7.86219	98	4	4.65500	16
0.7	5	5.67197	00	4	21.82974	55	4	11.34577	55	4	6.71001	54
0.8	5	7.98821	06	5	3.07119	64	4	15.94522	01	4	9.42000	53
0.9	5	11.01463	82	5	4.23044	99	4	21.94130	30	4	12.94882	84
1.0	5	14.92546	05	5	5.72685	36	5	2.96728	40	4	17.49401	10
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	2	10.46442	72	2	7.33658	66	2	5.29426	13	1	39.11123	28
0.2	2	26.82642	29	2	18.77538	62	2	13.52391	47	2	9.97123	08
0.3	3	5.10912	05	2	35.70509	73	2	25.67911	43	2	18.90334	47
0.4	3	8.57501	32	3	5.98447	05	3	4.29801	15	2	31.59382	33
0.5	3	13.38891	78	3	9.33205	26	3	6.69342	31	3	4.91360	09
0.6	3	19.93037	01	3	13.87442	74	3	9.93904	52	3	7.28693	31
0.7	3	28.66318	50	3	19.93037	04	3	14.26028	13	3	10.44246	16
0.8	4	4.01509	19	3	27.88675	92	3	19.93037	02	3	14.57764	76
0.9	4	5.50747	49	4	3.82107	00	3	27.27884	72	3	19.93037	02
1.0	4	7.42540	97	4	5.14635	05	4	3.67012	88	3	26.78587	70

$$10^{-t} {}_1F_1[a; b; x]$$

$$x = 10.0$$

b =		0.1		0.2		0.3		0.4		0.5					
a	t		t		t		t		t		t				
-1.0	1	-9.900000	00	0	-49.000000	00	0	-32.333333	33	0	-24.000000	00	0	-19.000000	00
-0.9	2	-26.35729	55	2	-10.47749	78	2	-5.63504	47	1	-34.55359	81	1	-22.88123	87
-0.8	3	-5.74321	44	2	-22.66065	03	2	-12.08651	96	2	-7.34339	26	1	-48.13713	31
-0.7	3	-9.29414	29	2	-36.53152	03	2	-19.40418	87	2	-11.73650	24	2	-7.65615	61
-0.6	3	-13.04730	68	3	-5.11412	17	2	-27.08399	11	2	-16.33002	36	2	-10.61701	33
-0.5	3	-16.60861	88	3	-6.49508	42	2	-34.31442	57	2	-20.63704	04	2	-13.38143	48
-0.4	3	-19.38298	97	3	-7.56478	22	2	-39.88192	77	2	-23.93292	27	2	-15.48313	57
-0.3	3	-20.51539	26	3	-7.99213	74	3	-4.20553	66	2	-25.18774	53	2	-16.26179	40
-0.2	3	-18.81918	76	3	-7.31898	35	2	-38.44601	85	2	-22.98448	30	2	-14.81155	72
-0.1	3	-12.68948	23	3	-4.92715	81	2	-25.83880	49	2	-15.42055	91	2	-9.91916	94
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	3	22.02646	57	3	8.52983	28	3	4.46140	89	3	2.65569	71	3	1.70399	66
0.2	4	5.69563	18	3	22.02646	57	3	11.50437	11	3	6.83804	73	3	4.38084	00
0.3	4	10.93309	33	4	4.22272	41	3	22.02646	59	3	13.07477	29	3	8.36496	74
0.4	4	18.48692	42	4	7.13160	87	4	3.71537	68	3	22.02646	57	3	14.07395	39
0.5	5	2.90712	99	4	11.20166	43	4	5.82887	58	4	3.45147	55	3	22.02646	58
0.6	5	4.35713	28	4	16.77002	03	4	8.71652	19	4	5.15540	76	4	3.28620	65
0.7	5	6.30765	46	4	24.25117	87	4	12.59123	13	4	7.43887	06	4	4.73642	75
0.8	5	8.89199	74	5	3.41517	01	4	17.71291	32	4	10.45358	19	4	6.64873	73
0.9	5	12.27235	25	5	4.70872	70	4	24.39712	35	4	14.38354	24	4	9.13874	31
1.0	5	16.64506	57	5	6.38024	53	5	3.30250	83	4	19.45081	11	4	12.34581	91
b =		0.6		0.7		0.8		0.9		1.0					
a	t		t		t		t		t		t				
-1.0	0	-15.66666	67	0	-13.28571	43	0	-11.50000	00	0	-10.11111	11	0	-9.00000	00
-0.9	1	-15.96561	88	1	-11.58241	73	1	-8.66482	25	1	-6.64811	77	0	-52.11212	86
-0.8	1	-33.21805	89	1	-23.81034	03	1	-17.58330	44	1	-13.30527	65	1	-10.27728	97
-0.7	2	-5.25566	59	1	-37.46030	78	1	-27.49694	89	1	-20.67335	53	1	-15.85967	49
-0.6	2	-7.26224	95	2	-5.15669	48	1	-37.70016	71	1	-28.22463	67	1	-21.55604	49
-0.5	2	-9.12749	57	2	-6.46204	49	1	-47.09726	16	1	-35.14540	41	1	-26.75035	87
-0.4	2	-10.53592	65	2	-7.44065	05	2	-5.40890	80	1	-40.25380	86	1	-30.55221	07
-0.3	2	-11.04241	57	2	-7.78122	74	2	-5.64358	20	1	-41.90064	27	1	-31.72367	53
-0.2	2	-10.03811	86	2	-7.05925	88	1	-51.09200	07	1	-37.85014	29	1	-28.59156	80
-0.1	2	-6.70959	41	1	-47.08983	84	1	-34.00900	96	1	-25.13759	12	1	-18.94278	27
0.0	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00	0	1.000000	00
0.1	2	11.49890	10	2	8.05237	12	2	5.80387	49	2	4.28243	19	2	3.22252	42
0.2	2	29.51536	54	2	20.63392	84	2	14.84567	73	2	10.93320	67	2	8.21055	88
0.3	3	5.62785	57	2	39.28674	01	2	28.22362	39	2	20.75325	44	2	15.56011	84
0.4	3	9.45635	52	3	6.59238	55	3	4.72945	29	2	34.72726	12	2	25.99955	85
0.5	3	14.78125	52	3	10.29149	55	3	7.37367	64	3	5.40715	89	2	40.42755	39
0.6	3	22.02646	56	3	15.31745	81	3	10.96119	21	3	8.02783	99	3	5.99449	61
0.7	3	31.71068	86	3	22.02646	60	3	15.74364	61	3	11.51668	31	3	8.58922	61
0.8	4	4.44649	41	3	30.85133	93	3	22.02646	58	3	16.09422	55	3	11.98926	32
0.9	4	6.10528	42	4	4.23152	76	3	30.17844	67	3	22.02646	60	3	16.39016	84
1.0	4	8.23940	35	4	5.70477	13	4	4.06428	06	3	29.63273	81	3	22.02646	59

APPENDIX III

Table of ${}_1F_1[a; b; 1]$ over the range

$$a = -11.0(0.2)2.0,$$

$$b = -4.0(0.2)1.0,$$

$$x = 1$$

$${}_1F_1[a; b; 1]$$

a	b = 1.0	0.8	0.6	0.4	0.2
2.0	5.43656 37	7.00294 07	9.79886 95	15.80393 31	35.11151 42
1.8	4.79690 84	6.11613 41	8.46366 11	13.49005 61	29.60393 79
1.6	4.20817 97	5.30437 02	7.24875 15	11.39848 65	24.66132 19
1.4	3.66738 17	4.56301 12	6.14631 72	9.51398 64	20.24278 60
1.2	3.17165 71	3.88764 24	5.14892 76	7.82211 71	16.30969 10
1.0	2.71828 18	3.27406 39	4.24952 89	6.30920 50	12.82554 08
0.8	2.30465 98	2.71828 18	3.44142 89	4.96230 95	9.75588 81
0.6	1.92831 84	2.21650 01	2.71828 18	3.76919 11	7.06824 32
0.4	1.58690 33	1.76511 25	2.07407 40	2.71828 18	4.73198 60
0.2	1.27817 41	1.36069 55	1.50311 03	1.79865 55	2.71828 18
0.0	1.00000 00	1.00000 00	1.00000 00	1.00000 00	1.00000 00
-0.2	0.75035 51	0.67994 50	0.55964 47	0.31258 99	-0.44836 46
-0.4	0.52731 44	0.39761 03	0.17722 54	-0.27273 93	-1.65076 70
-0.6	0.32905 01	0.15022 99	-0.15180 98	-0.76461 68	-2.62968 42
-0.8	0.15382 72	-0.06481 47	-0.43175 67	-1.17116 05	-3.40618 57
-1.0	0.00000 00	-0.25000 00	-0.66666 67	-1.50000 00	-4.00000 00
-1.2	-0.13399 16	-0.40766 74	-0.86035 72	-1.75829 98	-4.42957 94
-1.4	-0.24962 56	-0.54002 82	-1.01642 26	-1.95278 04	-4.71216 21
-1.6	-0.34830 11	-0.64916 97	-1.13824 36	-2.08973 92	-4.86383 16
-1.8	-0.43134 21	-0.73706 06	-1.22899 73	-2.17507 07	-4.89957 40
-2.0	-0.50000 00	-0.80555 56	-1.29166 67	-2.21428 57	-4.83333 33
-2.2	-0.55545 73	-0.85640 11	-1.32904 90	-2.21252 97	-4.67806 40
-2.4	-0.59882 99	-0.89123 99	-1.34376 49	-2.17460 10	-4.44578 16
-2.6	-0.63117 07	-0.91161 54	-1.33826 65	-2.10496 75	-4.14761 17
-2.8	-0.65347 16	-0.91897 67	-1.31484 56	-2.00778 33	-3.79383 65
-3.0	-0.66666 67	-0.91468 25	-1.27564 10	-1.88690 48	-3.39393 94
-3.2	-0.67163 45	-0.90000 55	-1.22264 64	-1.74590 52	-2.95664 81
-3.4	-0.66920 07	-0.87613 62	-1.15771 71	-1.58808 98	-2.48997 59
-3.6	-0.66014 02	-0.84418 70	-1.08257 68	-1.41650 95	-2.00126 05
-3.8	-0.64517 98	-0.80519 58	-0.99882 48	-1.23397 46	-1.49720 27
-4.0	-0.62500 00	-0.76012 95	-0.90794 16	-1.04306 72	-0.98390 15
-4.2	-0.60023 75	-0.70988 76	-0.81129 52	-0.84615 41	-0.46688 97
-4.4	-0.57148 70	-0.65530 56	-0.71014 72	-0.64539 82	0.04883 40
-4.6	-0.53930 32	-0.59715 76	-0.60565 79	-0.44276 99	0.55877 23
-4.8	-0.50420 29	-0.53616 03	-0.49889 22	-0.24005 84	1.05890 02

$${}_1F_1[a; b; 1]$$

a	b = 1.0	0.8	0.6	0.4	0.2
-5.0	-0.46666 67	-0.47297 50	-0.39082 44	-0.03888 15	1.54563 49
-5.2	-0.42714 04	-0.40821 11	-0.28234 31	0.15930 43	2.01580 93
-5.4	-0.38603 75	-0.34242 86	-0.17425 62	0.35319 37	2.46664 47
-5.6	-0.34374 02	-0.27614 04	-0.06729 54	0.54162 44	2.89572 63
-5.8	-0.30060 10	-0.20981 55	0.03787 98	0.72356 77	3.30097 88
-6.0	-0.25694 44	-0.14388 06	0.14067 75	0.89812 04	3.68064 40
-6.2	-0.21306 86	-0.07872 29	0.24056 97	1.06449 70	4.03325 84
-6.4	-0.16924 62	-0.01469 24	0.33708 83	1.22202 18	4.35763 27
-6.6	-0.12572 61	0.04789 66	0.42982 16	1.37012 18	4.65283 24
-6.8	-0.08273 44	0.10876 26	0.51841 07	1.50832 00	4.91815 84
-7.0	-0.04047 62	0.16765 58	0.60254 63	1.63622 84	5.15312 96
-7.2	0.00086 39	0.22435 56	0.68196 53	1.75354 21	5.35746 56
-7.4	0.04112 01	0.27866 93	0.75644 79	1.86003 32	5.53107 07
-7.6	0.08014 50	0.33043 00	0.82581 48	1.95554 47	5.67401 89
-7.8	0.11780 75	0.37949 52	0.88992 42	2.03998 60	5.78653 88
-8.0	0.15399 31	0.42574 53	0.94866 93	2.11332 67	5.86900 03
-8.2	0.18860 17	0.46908 18	1.00197 58	2.17559 25	5.92190 15
-8.4	0.22154 76	0.50942 62	1.04979 95	2.22686 03	5.94585 63
-8.6	0.25275 83	0.54671 86	1.09212 37	2.26725 35	5.94158 28
-8.8	0.28217 34	0.58091 61	1.12895 76	2.29693 83	5.90989 23
-9.0	0.30974 43	0.61199 20	1.16033 38	2.31611 95	5.85167 91
-9.2	0.33543 30	0.63993 44	1.18630 66	2.32503 68	5.76791 03
-9.4	0.35921 18	0.66474 50	1.20694 99	2.32396 11	5.65961 71
-9.6	0.38106 20	0.68643 81	1.22235 55	2.31319 14	5.52788 59
-9.8	0.40097 40	0.70503 98	1.23263 17	2.29305 16	5.37384 99
-10.0	0.41894 59	0.72058 68	1.23790 12	2.26388 72	5.19868 23
-10.2	0.43498 35	0.73312 53	1.23830 02	2.22606 29	5.00358 80
-10.4	0.44909 94	0.74271 07	1.23397 63	2.17995 97	4.78979 77
-10.6	0.46131 24	0.74940 63	1.22508 79	2.12597 27	4.55856 17
-10.8	0.47164 73	0.75328 25	1.21180 23	2.06450 83	4.31114 34
-11.0	0.48013 42	0.75441 65	1.19429 49	1.99598 23	4.04881 46

$${}_1F_1[a; b; 1]$$

a	b = 0+, 0-	-0.2	-0.4	-0.6	-0.8
2.0	+ ∞ - ∞	-47.98502 29	-34.12099 60	-35.85523 02	-58.92131 88
1.8	+ -	-41.45379 79	-27.76218 83	-28.89144 62	-47.09708 39
1.6	+ -	-33.56896 17	-22.19930 16	-22.85758 17	-36.95321 83
1.4	+ -	-26.64574 07	-17.36396 22	-17.66883 19	-28.32689 66
1.2	+ -	-20.60446 36	-13.19211 74	-13.24602 68	-21.06668 42
1.0	+ -	-15.37031 97	-9.62382 23	-9.51534 17	-15.03192 60
0.8	+ -	-10.87312 73	-6.60303 55	-6.40801 94	-10.09216 16
0.6	+ -	-7.04711 05	-4.07742 27	-3.86010 42	-6.12656 59
0.4	+ -	-3.83068 48	-1.99816 91	-1.81218 79	-3.02341 40
0.2	+ -	-1.16625 15	-0.31979 84	-0.20916 50	-0.67957 05
0.0	*	1.00000 00	1.00000 00	1.00000 00	1.00000 00
-0.2	- ∞ + ∞	2.71828 18	2.00053 68	1.86249 65	2.10269 86
-0.4	- +	4.03538 97	2.71828 18	2.42188 10	2.70873 81
-0.6	- +	4.99487 78	3.18701 60	2.71828 18	2.89156 84
-0.8	- +	5.63722 58	3.43797 72	2.78859 42	2.71828 18
-1.0	- +	6.00000 00	3.50000 00	2.66666 67	2.25000 00
-1.2	- +	6.11800 71	3.39964 87	2.38347 97	1.54224 28
-1.4	- +	6.02344 15	3.16134 52	1.96731 55	0.64527 94
-1.6	- +	5.74602 72	2.80749 14	1.44392 04	-0.39553 68
-1.8	- +	5.31315 25	2.35858 54	0.83666 01	-1.53945 03
-2.0	- +	4.75000 00	1.83333 33	0.16666 67	-2.75000 00
-2.2	- +	4.07967 03	1.24875 56	-0.54702 00	-3.99473 15
-2.4	- +	3.32330 06	0.62028 88	-1.28731 85	-5.24492 32
-2.6	- +	2.50017 85	-0.03811 75	-2.03897 82	-6.47532 63
-2.8	- +	1.62784 98	-0.71390 79	-2.78845 78	-7.66391 78
-3.0	- +	0.72222 22	-1.39583 33	-3.52380 95	-8.79166 67
-3.2	- +	-0.20233 53	-2.07386 68	-4.23456 96	-9.84231 15
-3.4	- +	-1.13289 88	-2.73912 34	-4.91165 35	-10.80215 02
-3.6	- +	-2.05789 86	-3.38378 37	-5.54725 74	-11.65984 09
-3.8	+ ∞ - ∞	-2.96703 39	-4.00102 18	-6.13476 34	-12.40621 34
-4.0	+ -	-3.85119 05	-4.58493 59	-6.66865 08	-13.03409 09
-4.2	+ -	-4.70236 29	-5.13048 29	-7.14441 15	-13.53812 16
-4.4	+ -	-5.51357 98	-5.63341 60	-7.55846 98	-13.91462 00
-4.6	+ -	-6.27883 36	-6.09022 58	-7.90810 66	-14.16141 66
-4.8	+ -	-6.99301 28	-6.49808 39	-8.19138 77	-14.27771 67

The function is indeterminate at the points marked *.

$${}_1F_1[a; b; 1]$$

a	b = 0+, 0-	-0.2	-0.4	-0.6	-0.8
- 5.0	+ ∞ - ∞	- 7.65183 79	- 6.85478 99	- 8.40709 62	- 14.26396 78
- 5.2	+ -	- 8.25180 10	- 7.15872 09	- 8.55466 84	- 14.12173 37
- 5.4	+ -	- 8.79010 76	- 7.40878 39	- 8.63413 34	- 13.85357 76
- 5.6	+ -	- 9.26462 17	- 7.60437 05	- 8.64605 65	- 13.46295 12
- 5.8	+ -	- 9.67381 40	- 7.74531 44	- 8.59148 50	- 12.95409 15
- 6.0	+ -	- 10.01671 28	- 7.83185 08	- 8.47189 86	- 12.33192 41
- 6.2	+ -	- 10.29285 66	- 7.86457 86	- 8.28916 13	- 11.60197 21
- 6.4	+ -	- 10.50225 04	- 7.84442 45	- 8.04547 72	- 10.77027 17
- 6.6	+ -	- 10.64532 38	- 7.77260 90	- 7.74334 91	- 9.84329 34
- 6.8	+ -	- 10.72289 14	- 7.65061 48	- 7.38553 88	- 8.82786 80
- 7.0	+ -	- 10.73611 56	- 7.48015 69	- 6.97503 12	- 7.73111 91
- 7.2	+ -	- 10.68647 12	- 7.26315 43	- 6.51499 96	- 6.56039 92
- 7.4	+ -	- 10.57571 22	- 7.00170 36	- 6.00877 43	- 5.32323 08
- 7.6	+ -	- 10.40584 10	- 6.69805 49	- 5.45981 28	- 4.02725 29
- 7.8	+ -	- 10.17907 85	- 6.35458 80	- 4.87167 22	- 2.68017 00
- 8.0	+ -	- 9.89783 68	- 5.97379 12	- 4.24798 39	- 1.28970 71
- 8.2	+ -	- 9.56469 31	- 5.55824 11	- 3.59242 94	0.13643 28
- 8.4	+ -	- 9.18236 57	- 5.11058 39	- 2.90871 90	1.59060 76
- 8.6	+ -	- 8.75369 10	- 4.63351 80	- 2.20057 16	3.06527 08
- 8.8	+ -	- 8.28160 23	- 4.12977 76	- 1.47169 56	4.55300 35
- 9.0	+ -	- 7.76911 02	- 3.60211 79	- 0.72577 27	6.04654 33
- 9.2	+ -	- 7.21928 40	- 3.05330 15	0.03355 82	7.53880 97
- 9.4	+ -	- 6.63523 45	- 2.48608 53	0.80271 48	9.02292 80
- 9.6	+ -	- 6.02009 80	- 1.90320 88	1.57818 42	10.49224 93
- 9.8	+ -	- 5.37702 17	- 1.30738 36	2.35653 49	11.94036 89
- 10.0	+ -	- 4.70915 01	- 0.70128 33	3.13442 65	13.36114 21
- 10.2	+ -	- 4.01961 21	- 0.08753 50	3.90861 95	14.74869 75
- 10.4	+ -	- 3.31150 97	0.53128 94	4.67598 32	16.09744 94
- 10.6	+ -	- 2.58790 75	1.15268 00	5.43350 33	17.40210 66
- 10.8	+ -	- 1.85182 26	1.77419 57	6.17828 75	18.65768 14
- 11.0	+ -	- 1.10621 62	2.39346 97	6.90757 18	19.85949 50

$${}_1F_1[a; b; 1]$$

b = -1+, -1-		-1.2	-1.4	-1.6	-1.8
a					
2.0	- ∞ + ∞	55.79611 88	32.24629 84	29.35660 75	42.08513 60
1.8	- +	44.05881 80	25.31804 35	22.89084 80	32.51951 71
1.6	- +	34.18794 98	19.54574 08	17.54993 84	24.67855 07
1.4	- +	25.97911 77	14.79488 76	13.19499 30	18.33674 68
1.2	- +	19.24383 68	10.94204 53	9.69920 95	13.28937 78
1.0	- +	13.80859 97	7.87415 87	6.94708 86	9.35107 00
0.8	- +	9.51398 64	5.48790 90	4.83369 41	6.35447 05
0.6	- +	6.21381 51	3.68909 68	3.26394 98	4.14898 49
0.4	- +	3.77433 38	2.39205 75	2.15197 31	2.59958 21
0.2	- +	2.07345 05	1.51910 43	1.42044 27	1.58566 44
0.0	*	1.00000 00	1.00000 00	1.00000 00	1.00000 00
-0.2	+ ∞ - ∞	0.45304 70	0.77145 51	0.82868 20	0.74771 41
-0.4	+ -	0.34122 30	0.77665 20	0.85138 47	0.74533 72
-0.6	+ -	0.58209 65	0.96479 38	1.01935 57	0.91990 76
-0.8	+ -	1.10157 43	1.29067 69	1.28971 46	1.20812 53
-1.0	*	1.83333 33	1.71428 57	1.62500 00	1.55555 56
-1.2	- ∞ + ∞	2.71828 18	2.20040 99	1.99274 23	1.91588 00
-1.4	- +	3.70404 78	2.71828 18	2.36506 03	2.25019 17
-1.6	- +	4.74449 47	3.24123 38	2.71828 18	2.52633 45
-1.8	- +	5.79926 24	3.74637 49	3.03258 59	2.71828 18
-2.0	- +	6.83333 33	4.21428 57	3.29166 67	2.80555 56
-2.2	- +	7.81662 10	4.62873 04	3.48241 71	2.77268 15
-2.4	- +	8.72358 24	4.97638 56	3.59463 25	2.60868 03
-2.6	- +	9.53285 07	5.24658 48	3.62073 21	2.30659 19
-2.8	- +	10.22688 95	5.43107 88	3.55549 85	1.86303 17
-3.0	- +	10.79166 67	5.52380 95	3.39583 33	1.27777 78
-3.2	- +	11.21634 63	5.52069 87	3.14052 96	0.55338 62
-3.4	- +	11.49299 96	5.41944 90	2.79005 84	-0.30516 59
-3.6	- +	11.61633 28	5.21935 80	2.34637 07	-1.29081 16
-3.8	- +	11.58343 10	4.92114 46	1.81271 24	-2.39470 04
-4.0	- +	11.39351 85	4.52678 57	1.19345 24	-3.60648 15
-4.2	- +	11.04773 35	4.03936 53	0.49392 36	-4.91456 46
-4.4	- +	10.54891 73	3.46293 23	-0.27972 50	-6.30636 05
-4.6	- +	9.90141 73	2.80236 97	-1.12066 51	-7.76850 10
-4.8	- +	9.11090 28	2.06327 18	-2.02151 47	-9.28704 12

The function is indeterminate at the points marked *.

$${}_1F_1[a; b; 1]$$

b = -1+, -1-		-1.2	-1.4	-1.6	-1.8
a					
- 5.0	- ∞ + ∞	8.18419 31	1.25183 15	- 2.97445 44	- 10.84764 31
- 5.2	- +	7.12909 78	0.37473 46	- 3.97133 36	- 12.43574 33
- 5.4	- +	5.95426 74	- 0.56093 63	- 5.00376 87	- 14.03670 49
- 5.6	- +	4.66905 59	- 1.54779 16	- 6.06323 17	- 15.63595 47
- 5.8	- +	3.28339 21	- 2.57821 67	- 7.14113 20	- 17.21910 61
- 6.0	- +	1.80766 15	- 3.64444 70	- 8.22888 95	- 18.77206 97
- 6.2	- +	0.25259 69	- 4.73863 74	- 9.31800 13	- 20.28115 09
- 6.4	- +	- 1.37082 22	- 5.85292 48	- 10.40010 21	- 21.73313 69
- 6.6	- +	- 3.05146 21	- 6.97948 49	- 11.46701 71	- 23.11537 20
- 6.8	- +	- 4.77811 96	- 8.11058 41	- 12.51081 02	- 24.41582 36
- 7.0	- +	- 6.53959 91	- 9.23862 61	- 13.52382 61	- 25.62313 86
- 7.2	+ ∞ - ∞	- 8.32478 35	- 10.35619 36	- 14.49872 71	- 26.72669 10
- 7.4	+ -	- 10.12269 75	- 11.45608 51	- 15.42852 53	- 27.71662 12
- 7.6	+ -	- 11.92256 53	- 12.53134 84	- 16.30661 02	- 28.58386 83
- 7.8	+ -	- 13.71386 25	- 13.57530 90	- 17.12677 20	- 29.32019 47
- 8.0	+ -	- 15.48636 21	- 14.58159 53	- 17.88322 06	- 29.91820 48
- 8.2	+ -	- 17.23017 62	- 15.54416 09	- 18.57060 19	- 30.37135 72
- 8.4	+ -	- 18.93579 10	- 16.45730 20	- 19.18400 93	- 30.67397 17
- 8.6	+ -	- 20.59409 94	- 17.31567 33	- 19.71899 32	- 30.82123 09
- 8.8	+ -	- 22.19642 79	- 18.11430 04	- 20.17156 71	- 30.80917 80
- 9.0	+ -	- 23.73455 95	- 18.84858 91	- 20.53821 05	- 30.63470 88
- 9.2	+ -	- 25.20075 38	- 19.51433 31	- 20.81587 02	- 30.29556 12
- 9.4	+ -	- 26.58776 25	- 20.10771 91	- 21.00195 87	- 29.79030 08
- 9.6	+ -	- 27.88884 19	- 20.62532 91	- 21.09435 05	- 29.11830 27
- 9.8	+ -	- 29.09776 32	- 21.06414 15	- 21.09137 69	- 28.27973 16
- 10.0	+ -	- 30.20881 81	- 21.42153 04	- 20.99181 84	- 27.27551 81
- 10.2	+ -	- 31.21682 39	- 21.69526 28	- 20.79489 63	- 26.10733 36
- 10.4	+ -	- 32.11712 46	- 21.88349 42	- 20.50026 19	- 24.77756 30
- 10.6	+ -	- 32.90559 03	- 21.98476 39	- 20.10798 53	- 23.28927 53
- 10.8	+ -	- 33.57861 46	- 21.99798 69	- 19.61854 26	- 21.64619 33
- 11.0	+ -	- 34.13310 98	- 21.92244 71	- 19.03280 19	- 19.85266 14

$${}_1F_1[a; b; 1]$$

b = -2+, -2-		-2.2	-2.4	-2.6	-2.8
a					
2.0	+ ∞ - ∞	-30.63850 84	-15.71685 71	-12.96296 00	-17.37007 36
1.8	+ -	-23.07450 60	-11.67132 06	-9.34593 72	-12.73537 03
1.6	+ -	-16.95547 16	-8.41912 29	-6.78279 31	-9.06397 58
1.4	+ -	-12.07158 95	-5.83961 74	-4.62195 03	-6.19742 25
1.2	+ -	-8.23469 22	-3.82532 63	-2.94685 07	-3.99652 15
1.0	+ -	-5.27663 62	-2.28089 95	-1.67195 71	-2.33966 79
0.8	+ -	-3.04777 05	-1.12213 58	-0.72235 33	-1.12125 71
0.6	+ -	-1.41549 44	-0.27506 42	-0.03281 68	-0.25020 77
0.4	+ -	-0.26289 96	0.32491 91	0.45304 70	0.35141 57
0.2	+ -	0.51250 63	0.73385 92	0.78361 45	0.74968 49
0.0	*	1.00000 00	1.00000 00	1.00000 00	1.00000 00
-0.2	- ∞ + ∞	1.27676 87	1.16449 29	1.13675 98	1.14819 67
-0.4	- +	1.40896 70	1.26205 94	1.22254 85	1.23157 26
-0.6	- +	1.45270 67	1.32160 97	1.28072 89	1.27983 78
-0.8	- +	1.45498 38	1.36681 93	1.32993 86	1.31599 36
-1.0	*	1.45454 55	1.41666 67	1.38461 54	1.35714 29
-1.2	+ ∞ - ∞	1.48269 92	1.48593 25	1.45548 37	1.41523 75
-1.4	+ -	1.56406 84	1.58566 44	1.55000 42	1.49776 45
-1.6	+ -	1.71729 60	1.72360 71	1.67278 88	1.60837 63
-1.8	+ -	1.95569 94	1.90460 14	1.82598 27	1.74746 69
-2.0	*	2.28787 88	2.13095 24	2.00961 54	1.91269 84
-2.2	- ∞ + ∞	2.71828 18	2.40277 00	2.22192 30	2.09948 03
-2.4	- +	3.24772 65	2.71828 18	2.45964 28	2.30140 44
-2.6	- +	3.87388 45	3.07412 12	2.71828 18	2.51063 86
-2.8	- +	4.59172 78	3.46559 09	2.99236 19	2.71828 18
-3.0	- +	5.39393 94	3.88690 48	3.27564 10	2.91468 25
-3.2	- +	6.27129 14	4.33140 77	3.56131 42	3.08972 37
-3.4	- +	7.21299 11	4.79177 58	3.84219 37	3.23307 60
-3.6	- +	8.20699 84	5.26019 82	4.11087 11	3.33442 14
-3.8	- +	9.24031 39	5.72854 04	4.35986 13	3.38365 03
-4.0	- +	10.29924 24	6.18849 21	4.58173 08	3.37103 17
-4.2	- +	11.36963 06	6.63169 88	4.76921 02	3.28736 16
-4.4	- +	12.43708 19	7.04987 95	4.91529 31	3.12408 82
-4.6	- +	13.48714 97	7.43493 07	5.01332 14	2.87341 73
-4.8	- +	14.50550 98	7.77901 73	5.05705 83	2.52840 01

The function is indeterminate at the points marked *.

$${}_1F_1[a; b; 1]$$

b = -2+, -2-		-2.2	-2.4	-2.6	-2.8
a					
- 5.0	- ∞ + ∞	15.47811 45	8.07465 28	5.04075 09	2.08300 26
- 5.2	- +	16.39132 76	8.31476 77	4.95918 09	1.53216 00
- 5.4	- +	17.23204 43	8.49276 79	4.80770 66	0.87181 68
- 5.6	- +	17.98779 39	8.60258 48	4.58229 63	0.09895 27
- 5.8	- +	18.64682 92	8.63871 39	4.27955 25	- 0.78840 03
- 6.0	- +	19.19820 23	8.59624 92	3.89673 00	- 1.79115 57
- 6.2	- +	19.63182 66	8.47090 71	3.43174 49	- 2.90917 68
- 6.4	- +	19.93852 95	8.25904 44	2.88318 02	- 4.14129 19
- 6.6	- +	20.11009 20	7.95767 17	2.25028 41	- 5.48531 68
- 6.8	- +	20.13928 02	7.56445 70	1.53296 31	- 6.93808 11
- 7.0	- +	20.01986 66	7.07772 97	0.73177 25	- 8.49546 63
- 7.2	- +	19.74664 34	6.49647 49	- 0.15210 16	- 10.15244 50
- 7.4	- +	19.31542 85	5.82032 57	- 1.11685 90	- 11.90312 62
- 7.6	- +	18.72306 37	5.04955 30	- 2.16010 71	- 13.74080 67
- 7.8	- +	17.96740 77	4.18504 70	- 3.27888 70	- 15.65801 88
- 8.0	- +	17.04732 15	3.22830 21	- 4.46969 90	- 17.64658 73
- 8.2	- +	15.96265 10	2.18139 43	- 5.72853 51	- 19.69769 17
- 8.4	- +	14.71420 23	1.04695 68	- 7.05090 72	- 21.80191 92
- 8.6	- +	13.30371 58	- 0.17184 21	- 8.43188 02	- 23.94933 10
- 8.8	- +	11.73383 38	- 1.47133 17	- 9.86610 71	- 26.12951 79
- 9.0	- +	10.00806 60	- 2.84736 26	- 11.34786 08	- 28.33166 26
- 9.2	- +	8.13075 27	- 4.29533 94	- 12.87107 42	- 30.54460 50
- 9.4	- +	6.10702 46	- 5.81025 24	- 14.42937 23	- 32.75690 89
- 9.6	- +	3.94276 15	- 7.38670 60	- 16.01610 84	- 34.95676 52
- 9.8	- +	1.64454 84	- 9.01895 69	- 17.62440 22	- 37.13279 70
- 10.0	- +	- 0.78037 01	- 10.70094 14	- 19.24717 25	- 39.27263 27
- 10.2	- +	- 3.32413 54	- 12.42631 16	- 20.87717 81	- 41.36444 82
- 10.4	- +	- 5.97832 20	- 14.18846 86	- 22.50704 87	- 43.39630 19
- 10.6	- +	- 8.73398 49	- 15.98059 31	- 24.12932 01	- 45.35597 14
- 10.8	- +	- 11.58170 75	- 17.79568 25	- 25.73647 02	- 47.23270 24
- 11.0	- +	- 14.51165 10	- 19.62657 90	- 27.32094 88	- 49.01389 23

$${}_1F_1[a; b; 1]$$

a	b = -3+, -3-		-3·2	-3·4	-3·6	-3·8
	2·0	- ∞ + ∞	12·22348 27	6·29345 78	5·06525 48	6·18677 40
1·8	- +	9·16969 64	4·79276 66	3·88038 66	4·67523 27	
1·6	- +	6·79683 62	3·63833 52	2·97628 85	3·52669 67	
1·4	- +	4·98555 83	2·76769 32	2·30081 07	2·67166 77	
1·2	- +	3·63254 75	2·12714 26	1·80943 22	2·05120 14	
1·0	- +	2·64894 88	1·67085 28	1·46443 25	1·61570 21	
0·8	- +	1·95891 33	1·36002 53	1·23412 95	1·32381 94	
0·6	- +	1·49825 13	1·16212 26	1·09217 93	1·14143 99	
0·4	- +	1·21318 66	1·05015 87	1·01693 56	1·04076 71	
0·2	- +	1·05920 62	1·00204 66	0·99086 26	0·99948 52	
0·0	*	1·00000 00	1·00000 00	1·00000 00	1·00000 00	
-0·2	+ ∞ - ∞	1·00648 50	1·02998 53	1·03347 58	1·02875 18	
-0·4	+ -	1·05590 93	1·08122 13	1·08306 35	1·07559 57	
-0·6	+ -	1·13103 05	1·14572 31	1·14278 19	1·13324 49	
-0·8	+ -	1·21936 44	1·21788 75	1·20853 33	1·19677 07	
-1·0	*	1·31250 00	1·29411 76	1·27777 78	1·26315 79	
-1·2	- ∞ + ∞	1·40547 53	1·37248 32	1·34924 24	1·33090 88	
-1·4	- +	1·49621 15	1·45241 53	1·42266 03	1·39969 38	
-1·6	- +	1·58500 13	1·53443 19	1·49853 99	1·47004 43	
-1·8	- +	1·67404 69	1·61989 32	1·57796 06	1·54308 48	
-2·0	*	1·76704 55	1·71078 43	1·66239 32	1·62030 08	
-2·2	+ ∞ - ∞	1·86881 88	1·80952 22	1·75354 34	1·70333 97	
-2·4	+ -	1·98498 29	1·91878 72	1·85321 71	1·79384 24	
-2·6	+ -	2·12165 63	2·04137 51	1·96320 35	1·89330 12	
-2·8	+ -	2·28520 29	2·18007 01	2·08517 81	2·00294 45	
-3·0	*	2·48200 76	2·33753 50	2·22061 97	2·12364 24	
-3·2	- ∞ + ∞	2·71828 18	2·51621 93	2·37074 43	2·25583 45	
-3·4	- +	2·99989 74	2·71828 18	2·53645 12	2·39947 51	
-3·6	- +	3·33224 52	2·94552 84	2·71828 18	2·55399 56	
-3·8	- +	3·72011 79	3·19936 15	2·91638 97	2·71828 18	
-4·0	- +	4·16761 36	3·48074 23	3·13051 99	2·89066 42	
-4·2	- +	4·67806 04	3·79016 27	3·35999 82	3·06891 97	
-4·4	- +	5·25395 72	4·12762 77	3·60372 72	3·25028 45	
-4·6	- +	5·89693 22	4·49264 55	3·86019 05	3·43147 49	
-4·8	- +	6·60771 60	4·88422 64	4·12746 23	3·60871 61	

The function is indeterminate at the points marked *.

$${}_1F_1[a; b; 1]$$

b = -3+, -3-		-3·2	-3·4	-3·6	-3·8
a					
- 5·0	- ∞ + ∞	7·38612 69	5·30088 70	4·40322 29	3·77777 78
- 5·2	- +	8·23106 99	5·74066 23	4·68477 88	3·93401 48
- 5·4	- +	9·14054 52	6·20112 16	4·96908 64	4·07241 30
- 5·6	- +	10·11166 65	6·67938 99	5·25277 97	4·18763 73
- 5·8	- +	11·14068 79	7·17217 27	5·53220 08	4·27408 46
- 6·0	- +	12·22303 77	7·67578 49	5·80343 15	4·32593 64
- 6·2	- +	13·35335 98	8·18618 22	6·06232 90	4·33721 48
- 6·4	- +	14·52555 91	8·69899 45	6·30456 06	4·30183 85
- 6·6	- +	15·73285 22	9·20956 19	6·52563 98	4·21367 74
- 6·8	- +	16·96782 21	9·71297 10	6·72096 56	4·06661 08
- 7·0	- +	18·22247 59	10·20409 04	6·88585 65	3·85457 96
- 7·2	- +	19·48830 56	10·67762 55	7·01559 13	3·57164 18
- 7·4	- +	20·75634 95	11·12812 52	7·10544 41	3·21202 48
- 7·6	- +	22·01725 60	11·55005 36	7·15071 87	2·77017 27
- 7·8	- +	23·26134 74	11·93781 13	7·14678 90	2·24079 99
- 8·0	- +	24·47868 42	12·28577 03	7·08912 66	1·61893 06
- 8·2	- +	25·65913 17	12·58835 33	6·97334 07	0·89994 54
- 8·4	- +	26·79242 09	12·83998 58	6·79520 56	0·07962 30
- 8·6	- +	27·86821 34	13·03521 52	6·55068 90	- 0·84582 94
- 8·8	- +	28·87616 74	13·16870 76	6·23598 74	- 1·87973 11
- 9·0	- +	29·80597 21	13·23526 35	5·84754 35	- 3·02490 81
- 9·2	- +	30·64746 01	13·22993 98	5·38208 07	- 4·28365 80
- 9·4	- +	31·39060 91	13·14791 42	4·83662 05	- 5·65772 42
- 9·6	- +	32·02562 47	12·98467 20	4·20850 00	- 7·14828 54
- 9·8	- +	32·54299 76	12·73596 31	3·49540 26	- 8·75591 96
- 10·0	- +	32·93349 27	12·39779 48	2·69535 99	- 10·48060 82
- 10·2	- +	33·18832 02	11·96660 47	1·80678 20	- 12·32171 25
- 10·4	- +	33·29905 43	11·43901 64	0·82846 18	- 14·27796 35
- 10·6	- +	33·25773 78	10·81210 96	- 0·24041 90	- 16·34747 37
- 10·8	- +	33·05692 76	10·08332 87	- 1·40026 41	- 18·52770 75
- 11·0	- +	32·68962 71	9·25044 92	- 2·65107 69	- 20·81551 02

SYMBOLIC INDEX OF DEFINITIONS

$(a)_n$	1	$L_\nu^{(\mu)}(x)$	95
$((a)_n)$	1	$\text{li}(x)$	97
$C(x)$	97	$M_{k,m}(x)$	9
$\text{Ci}(x)$	97	$M(a; b; x)$	2
$\mathfrak{C}_n(x)$	91	$\mathfrak{M}_{k,m}(x)$	10
$C(z, k)$	97	$m_k^{(\rho)}(x)$	9
$\delta(\xi, \nu)$	99	$N_{k,m}(x)$	10
$E(a, b, x)$	5	$\omega_{n,m}$	96
$\text{Ei}(x)$	97	$P(x, a)$	97
$E_n(x)$	97	$\text{pe}(r, w)$	100
$\text{Erf}(x)$	99	$Q(x, a)$	97
$\text{Erfc}(c)$	99	$\text{qe}(r, w)$	100
$\text{Erfi}(x)$	99	$\Psi(a)$	9
$\mathfrak{F}[a; b; x]$	2	$\Psi(a; b; x)$	5
${}_1F_1[a; b; x]$	2	${}_1R(\nu, \lambda; x)$	98
${}_A F_B[(a); (b); x]$	1	${}_2R(\nu, \lambda; x)$	98
${}_2F_1[a, b; c; x]$	2	$\rho(\xi, \nu)$	99
${}_2F_0[a, 1+a-b; ; -1/x]$	5	$S_\nu^\mu(x)$	95
${}_4F_3[a, 1+\frac{1}{2}a, b, c; \frac{1}{2}a, 1+a-b, 1+a-c; 1]$	31	$S(a, x)$	97
$F_L(\eta, \rho)$	93	$S(x)$	97
$F_1(a; b; x)$	5	$\text{Si}(x)$	97
$F_2(a; b; x)$	98	$T(m, n, r)$	99
$\Phi(a; b; x)$	2	$U(a; b; x)$	5
$\Phi^*(a; b; x)$	4	$u(a, b, x)$	2
$G(a; b; x)$	5	$U_\nu^{(\mu)}(x)$	95
$G_L(\eta, \rho)$	93	$V_\nu^\mu(x)$	95
$\gamma(a, x)$	96	$v(a, b, x)$	5
$\Gamma(a, x)$	96	$W_{k,m}(x)$	10
$\text{He}_n(x)$	99	$\mathfrak{B}_{k,m}(x)$	9
$I_a(x)$	12	$\mathfrak{U}_k^{(\rho)}(x)$	10
$J_a(x)$	12	$Y_a(x)$	13
$k_\nu(x)$	96		
$L(x)$	100		

GENERAL INDEX

- Addition theorems, for Kummer's functions, 21
for Whittaker's functions, 29
- Airy functions, 77, 81, 86
- Approximations, in terms of Bessel functions, 68
when a and x are real, 70
when b is large for Kummer's functions, 65
when k and x are large for Whittaker's functions, 69, 73
when m is large for Whittaker's functions, 66
- Associated functions, 19
- Asymptotic expansions in x , for Kummer's functions, 58
for Whittaker's functions, 61
when a is large, 68, 79
when k and x are large, 71, 81, 83
- Barnes integrals, contour of integration for, 35, 59
for Kummer's functions, 34, 58
for Whittaker's functions, 51, 61
of Gauss's function, 47
- Bateman's function, 96
- Bessel functions, approximations in terms of, 68
as limiting cases of Kummer's functions, 67
as limiting cases of Whittaker's functions, 69
as special cases of Kummer's functions, 12
as special cases of Whittaker's functions, 13
connection with Kummer's second theorem, 12, 92
definition of, 12
equation for, 91
expansions in series of, 30, 32
in Hankel transform, 49
in Laplace transform, 44
in Olver's theorem, 78
Laplace integrals for, 45
solutions of Bessel equation, 91
zeros of, 106
- Bibliography, 121
- Binomial theorem, 6, 12, 43
- Chappell's function, 97
- Confluent hypergeometric functions, 2
as integrals of general equation of second order, 89
as limiting cases of Gauss's functions, 3, 67
definitions, 1
- Contiguous functions, definition, 19
- Continuation formulae, for Kummer's function $U(a; b; x)$, 20
for Whittaker's functions, 28
in asymptotic expansions, 73
- Convergence, of Gauss's function, 2
of generalized hypergeometric series, 2
of Kummer's function, 2
- Converging factors for Kummer's functions, 61
for Whittaker's functions, 65
- Coulomb functions, 93
equation for, 93
- Cunningham's function, 96
- Cyclic approximations to zeros, 110
- Derivatives of Kummer's functions, 15
of Whittaker's functions, 23
- Error functions, 99
- Euler-Beta function, 34
- Euler's integral, connection with asymptotic expansions, 37
for Kummer's functions, 34
for Whittaker's functions, 50
relaxation of restrictions on, 38, 51
transforms of, 34, 38, 50
- Euler's transform, 6, 43
- Exponential integrals, 97
- Exponential transform, 6
- Four- F -three summation theorem, 31
- Fourier transform, 49
- Fresnel integrals, 97
- Frobenius' method, 3, 7
- Fuchian equation, 2
- Gamma function, 3
integral for, 34
- Graphs of Kummer's functions, 115
- Gauss's differential equation, 2
second solution of, 3
- Gauss's function, 2
Barnes integral for, 47
- Gauss's theorem, 43, 52
- Generalized hypergeometric functions, definition of, 1
differential equation for, 1
Inverse Laplace transform of, 45
Laplace transforms of, 43
Mellin transforms of, 48
- Geometric series, 1
- Hankel transforms, 49
- Harmonic oscillator, 100
- Heatley's function, 99
- Hermite polynomials, 99
- Homogenous differential equation of the second order, 89
- Horn's equation, 94
- Incomplete Gamma function, 96
- Indefinite integrals, for Kummer's function, 42
for Whittaker's function, 50
- Indicial equation, for Kummer's equation, 3, 7
for Whittaker's equation, 10

- Integral addition theorems, 45, 54
 Integral cosine, 97
 Integrals, elementary, 34
 indefinite, 42
 involving pairs of functions, 54, 56
 transforms of, 34
 Integral sine, 97
 Inverse Laplace transforms, 45, 52
- Kamke's equation, 101
 Krupp's functions, 98
 Kummer's equation, definition, 2
 general solutions of, 2
 logarithmic solutions, 7
 relations between solutions, 6, 13
 transforms of, 89
 Wronskians of, 17
 Kummer's first theorem, 6
 Kummer's functions, addition theorem for, 21
 Barnes integral for, 35, 58
 convergence of, 2
 definition, 2
 differentiation of, 15
 elementary integrals for, 34
 indefinite integrals for, 42
 integral addition theorem for, 45
 integrals of pairs of, 54
 Laplace transforms of, 43
 Mellin transforms of, 48
 multiplication theorems for, 22
 notations for, 2
 Pochhammer's integrals for, 38
 poles of, 3
 relations with Whittaker's functions, 13
 Kummer's second theorem, 12
 its connection with Bessel functions, 92
 Kummer's transform, 12
- Laguerre polynomials, 95
 Laplace transforms, definition, 43
 for generalized hypergeometric series, 43
 for Kummer's functions, 43, 46
 for Whittaker's functions, 52
 inverse, 45
 inverse for generalized hypergeometric functions, 45
 Legendre polynomials, 1
 Lebesgue integrals, 43
 Linear relations between asymptotic solutions, 76
 Logarithmic integrals, 97
 Logarithms forms of solutions, 7, 11
- Magnus' functions, 99
 Mapping, 81
 Maxima and minima, 119
 Meijer's integral, 47
 Meixner's functions, 97
- Mellin-Barnes integrals, for Kummer's functions, 35
 for Whittaker's functions, 51
 within asymptotic expansions, 58
 Mellin transforms, 47
 of generalized hypergeometric functions, 48
 of Kummer's functions, 47, 49
 of Whittaker's functions, 61
 Modified forms of Kummer's function, 101
 Multiplication theorems, for Kummer's functions, 22
 for Whittaker's functions, 30
- Neilson's functions, 97
 Nesting processes, 110
 Normalized form of the confluent hypergeometric equation, 9, 118
 Normalizing functions, 96
 Notations for Kummer's functions, 2, 5
 for Whittaker's functions, 9, 10
- Olver's theorems, 76
 Omega function, 96
 Oscillatory regions, 118
- Parabolic cylinder functions, 98
 converging factors for, 63, 64
 Paraboloidal reflectors, 94
 Parameters of hypergeometric functions, 1
 Perron's approximation, 68
 Placzek's functions, 97
 Pochhammer's integrals, contours for, 39
 for Kummer's functions, 38
 for Whittaker's functions, 51
 in the Laplace transform, 45
 Poiseuille functions, 100
 Poisson-Charlier function, 96
 Poles of Kummer's function, 3
 Principal value of $U(a; b; x)$, 5
 of Whittaker's functions, 10
 Probability functions, 96
- Recurrence relations for Kummer's functions, 19
 for Whittaker's functions, 26
- Saddle points, 71, 105
 Schlomilch's functions, 97
 Schrödinger's equation, 100
 Second form of solution $U(a; b; x)$
 addition theorems for, 22
 Barnes's integral for, 37
 continuation formulae for, 20
 definition, 5
 derivatives of, 16
 Laplace transforms of, 46
 Mellin transforms of, 49
 multiplication theorems for, 23
 Pochhammer's integrals for, 41
 Principal branch of, 5, 20
 Recurrence relations for, 20
 Separation of variables, 1

- Sharpe's equation, 94
 Singularities, of generalized hypergeometric equation,
 2
 of Kummer's equation, 2
 of Whittaker's equation, 9
 Solutions of Kummer's equation for b integral, 3, 7
 Sonine-Polya theorem, 119
 Spherical co-ordinates, 1
 Steckloff-Langer method, 71
 Steepest-descent method, 71
 Sturmian chains, 103

 Tabulation of Kummer's function, 113
 Tables, 113
 of Kummer's function, 131, 233
 of zeros, 125
 references, 113
 Taylor's theorem, 21, 22
 Toronto function, 99
 Transformations between Whittaker and Kummer
 functions, 9, 13
 Transformations of Kummer's equation, 89
 Tricomi's approximation to zeros, 106

 Ultra-spherical polynomials, 73

 Vandermonde's theorem, 6, 12
 Variable of hypergeometric functions, 1

 Watson's fourth order equation, 94

 Weber functions, 98
 Well-poised series, 31
 Whittaker's equation, 9
 connections between solutions, 11, 26
 further forms, 12
 logarithmic solutions, 11
 Wronskians of, 26
 Whittaker's functions
 addition theorems for, 29
 Barnes's integrals, 51
 definitions, 9
 elementary integrals for, 50
 integrals of pairs of, 56
 multiplication theorems for, 30
 Pochhammer's integrals for, 51
 principal branch of, 9
 recurrence relations for, 26
 zeros of, 102
 Wronskians of Kummer's equation, 17
 of Whittaker's equation, 26

 Zeros, approximations to, 106
 curves of, 104
 distribution of, 102
 expansions for, 107
 nesting processes for, 110
 of Bessel functions, 107
 of derivatives of Kummer's function, 108
 of Whittaker's functions, 102
 tables of, 125

QA 351 .S56
Slater, Lucy Joan.
Confluent hypergeometric funct 010101 000
0 1163 0100953 0
TRENT UNIVERSITY

QA351 .S56

RECON

Slater, Lucy Joan

Confluent hypergeometric functions.

DATE	ISSUED TO
	006037

006037

QA Slater, Lucy Joan
351 Confluent hypergeometric
S56 functions

Trent
University

